An Integrated Solution to the Synthesis of Multifinger Grasps

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Objectives

This thesis aims at solving the grasp synthesis problem for multifingered hands. Given a hand and an object to be grasped, the problem entails finding feasible configurations of the hand-object system that simultaneously yield a stable and manipulable grasp. As it will be described in this proposal, this problem remains open in the context of hand manipulation, where the existing approaches either do not integrate all constraints, *i.e.* kinematic, stability and manipulability constraints, in their resolution or use heuristic methods only applicable to simpler grasping devices. The goal is to cover such gap by providing a one-step complete method for the synthesis of stable and manipulable multifinger grasps.

Motivation

When Karel Capek first introduced the term in 1923, *robots* were thought to be human-like machines. Nevertheless, the first industrial robots that appeared two decades after were still far from exhibiting human manipulation abilities. These manipulators allowed more flexibility than *hard automation* machines for large-scale productions, and were able to perform a great variety of tasks, but they still required costly end-effector retoolings for each specific task. This has been the dominant situation in robotic applications since then. In the early eighties, the need of more economical and flexible tools in the industry made robot designers think of versatile grippers, giving rise to the appearance of dextereous end-effectors inspired by the human hand. These robot hands are able to grasp multiple object types and shapes. They also allow their manipulation in a more precise and efficient way, specially on *fine* manipulation tasks where the ranges of valid position and orientation are very small.

A main problem within the context of fine manipulation is grasp synthesis: Given a hand and an object to be grasped, find feasible configurations of the hand-object system that simultaneously yield a stable and manipulable grasp. Enforcing such properties on a grasp is essential, not only to guarantee that the object is firmly held and moveable in any direction, but also to be able to enforce additional properties later on, like dynamic behavior (Shimoga 1996). Although the literature is rich in methods for grasp analysis, the synthesis problem remains largely unsolved so far. Only the following related problems have been treated:

Problem 1 (contact point synthesis): find appropriate contact points on the object, so that any grasp on such points allows to firmly hold the object.

Problem 2 (fingertip force computation): find the fingertip forces that are required to balance a given external force applied on the object.

Problem 3 (inverse kinematics): find a hand configuration that reaches a specified set of contact points.

Problem 4 (dexterous manipulation): determine how the hand should move in order for the object to follow a given trajectory, ensuring along the way that the grasp is stable and manipulable.

Solutions to some of these problems only constitute partial solutions of the grasp synthesis problem as a whole. The solution to Problem 1 only provides a set of contact points on the object, but there is no guarantee that the hand will be able to reach such points, as its structure is always neglected. The solution to Problem 2 allows to compute the fingertip forces required to compensate a given force on the object, but the configuration of the hand is either neglected (assuming that the hand can apply arbitrary forces) or *a priori* known, but it is never synthesized anyway. The solution to Problem 3 gives a way of synthesizing a kinematically feasible grasp, but existing algorithms still do not take stability and manipulability constraints into account in such synthesis. Finally, Problem 4 can be viewed as a generalization of the grasp synthesis problem, as it requires computing stable and manipulable grasps along a trajectory of object poses. However, current solutions to this problem only provide the joint velocities required to move from a feasible configuration to another one infinitesimally close to it, leaving open the question of how to compute an initial stable and manipulable grasp from which such incremental motions would be applied.

Rather than leaving essential constraints out, or introducing simplifying assumptions, this Ph.D. work seeks to obtain a unified solution to the grasp synthesis problem, i.e., one that simultaneously takes all kinematic feasibility, stability and manipulability constraints into account. Applications of the problem are numerous, ranging from autonomous manipulation or assisted teleoperation to dexterous prosthesis control or minimally-invasive surgery, and in general to any setting involving the control of multifingered manipulation devices.

Expected Contributions

This thesis expects to contribute along two lines:

- The derivation of an adequate and generic formulation of the problem, that integrates the kinematic, stability and manipulability constraints.
- The development of a general and complete solution method for multifinger grasp synthesis.

Detailed requirements on each of these two aspects are provided in the following two sections.

On the formulation

Up to the author's knowledge, no work on the literature has provided an integrated formulation of the grasp synthesis problem, and that is why the initial efforts of this work will be headed towards such goal. Based on previous work (Rosales et al. 2008) and an extensive review of the literature, the author conjectures that it is feasible to algebraically formulate the kinematic, manipulatbility and stability constraints, defining an integrated system of polynomial equations, so that the synthesis of the grasp boils down to solving such system.

The requirements for this formulation should be: first, *generality*, *i.e.* it should be applicable to any hand, second, *with low degree*, *i.e.* the equations forming the system should have at most quadratic monomials, for tractability reasons, and finally, *systematically obtainable*, so the user would just need to specify the structure in terms of the bodies and joints connecting them.

On the solution method

Similarly, the solution method must be *general*, in the sense that it should be able to solve any of the formulated equation systems and *complete*, in the sense that it should find at least one solution if it exists, or conclude that the equation system is unsolvable otherwise.

The efficiency, in terms of time complexity, of the solution method will depend mainly on the dimension of the solution space. This dimension depends on two key elements: one, the number of fingers to be used in the grasp, and two, the contact model considered in each fingertip. The first one affects the number of independent loops, hence the number of bodies constraining the object. The second one is related to the mobility of the hand-object system due to the kinematic model representing each contact model (Wen and Wilfinger 1999) plus the fact that the contact point may be known or variable on one or two of the contacting surfaces. Additionaly, the joints may be coupled instead of independently actuated, which can be used as a reasonable tool to reduce the dimensionality of the solution space (Ciocarlie et al. 2007).

This thesis pursues a solution method able to treat a generic system of equations arising from grasp synthesis. This will probably require different techniques depending on the dimensionality of the problem. In principle, the feasibility of solving the equation systems will be explored via:

1. Algebraic-geometry and branch-and-prune methods for lower-dimensional cases.

2. Probabilistic sampling methods for higher-dimensional cases.

These techniques have proved to be efficient in each mentioned range of dimensionality. Techniques of type 1 are complete. Although techniques of type 2 work well in practice, they are not complete. Nevertheless, they are potentially combinable with branch-and-prune methods to make them simultaneously efficient and complete.

State of the Art

Solutions to the grasp synthesis problem

Previous work in grasp synthesis can be grouped according to which of the four problems mentioned before they contribute. The state of the art will be organized following each of these four groups, which can be visualized in the grasp scheme of Fig. 1. This scheme is a representation of the elements that take part in a grasp and how they interact. In there, it can be seen, through the connecting arrows and from top to bottom, that the position, velocity and torque spaces are related to the hand, contact and object domains via three main constraints: the loop closure constraint that ensures the reachability of the contact points by the fingertips, the equilibrium constraint that guarantees the prehensility of the grasp, and the manipulability constraint that ensures the dexterity of the grasp. Additional to the spaces and domains involved in a grasp, there is the contact model, a key ingredient in the formulation. In this work, the contact will be assumed to be one per fingertip, which is known as a *precision grasp* (Schlesinger 1919; Cutkosky 1989) . The three most common models for the contact, as from the point of view of the transmitted forces, are: *frictionless*, *hard* and *soft finger* contact (Kao et al. 2008). In the two latter, the considered friction model is the static Coulomb friction model (Olsson et al. 1998), which is good enough for the applications in robotic grasping.

Most of the previous work correspond to Problem 1, where the objective is the synthesis of the contact points that achieve a stable grasp, without considering the hand that is supposed to execute the grasp. Actually, the earliest studies were in the machine design field, where difference between the joints that needed an external force to be kept in contact and those that needed not were called *force closure* and *form closure*, respectively (Reuleaux 1876). By then, it was concluded that the minimum number of contact points to immobilize a two-dimensional object was four, for a three-dimensional object would be later stated as seven (Somov 1900). About a century later, with the arrival of robot hands, the need to study the mechanic of grasps was inherent, and this concepts were revisited, redefined (Dizioğlu and Lakshiminarayana 1984; Mishra et al. 1987) and proved (Markenscoff et al. 1990), not only for frictionless contacts, but also for contact points using Coulomb friction model, in which these numbers were reduced to three and four, for two- and three-dimensional objects, respectively. Although these results did not determine the contact points, they laid the foundations for finding them. Based on the *wrench* that can be exerted at each of these contact points, qualitative tests were provided to answer whether a set of wrenches constrained an object (Salisbury and Roth 1983), which naturally led to quantitative tests (Trinkle 1992; Ferrari and Canny 1992), defining measures

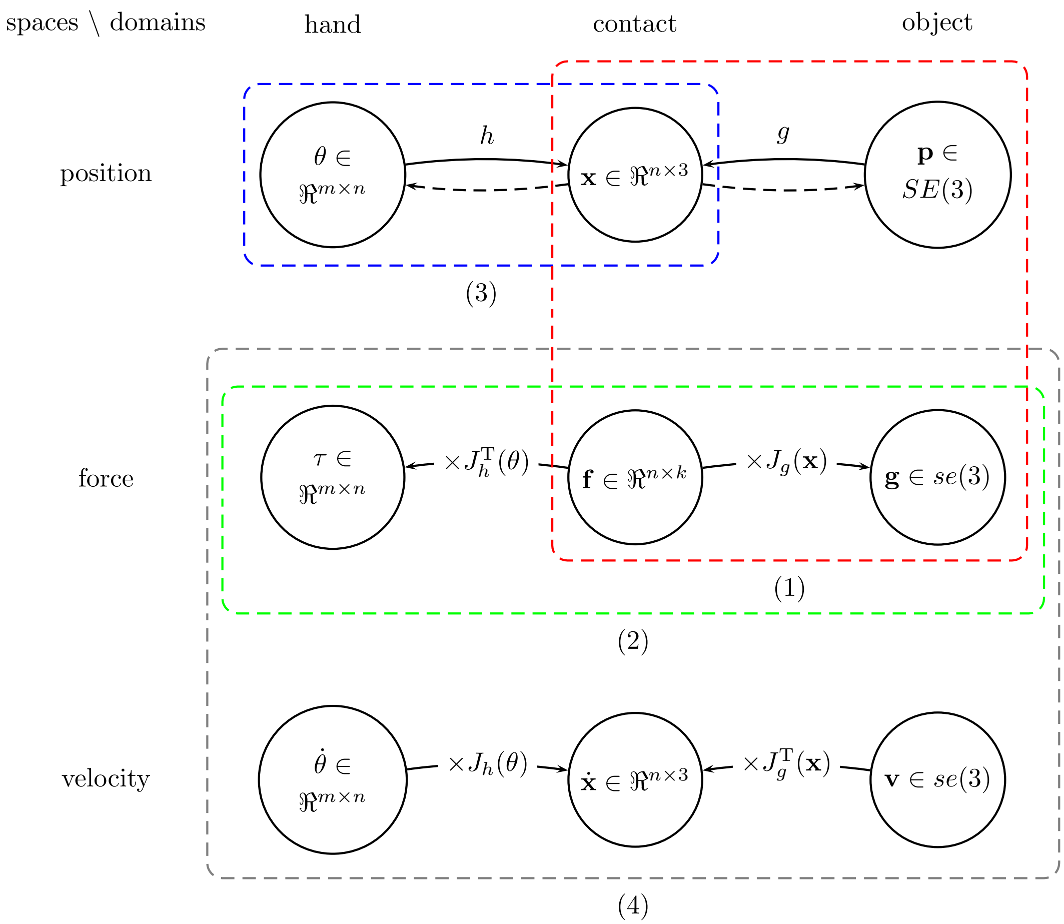

Figure 1: Spaces and domains involved in a grasp. The figure shows the variables and constraints that the grasp synthesis problems studied so far takes into account: contact point synthesis (1), fingertip force computation (2), inverse kinematics (3) and dexterous manipulation (4).

that consider several aspects and characteristics of the grasp (Roa et al. 2008). By using these measures, numerous approaches have been developed for synthesizing the contact points in two-dimensional (Nguyen 1988; Park and Starr 1990; Liu 1998; Cornellà and Suarez 2006) and three-dimensional (Ponce et al. 1997; Borst et al. 1999; Li et al. 2003; Pollard 2004; Roa and Suarez 2007) objects.

In Problem 2, the computation of the fingertip force is usually done by decomposing it into a manipulating force and an internal grasp force (Kumar and Waldron 1989; Yoshikawa and Nagai 1991). The first one produces a resultant force on the object which can balance a given external force or manipulate the object, while the second one guarantees the contact with object surface and allows to firmly hold the object without squeezing it. Both forces must lie within the friction cone; the positivity constraints, *i.e.* the fingertip can only push against the object surface, is implicitly included in the friction constraint. This problem has been tackled by discretizing the non-linear friction cones using half-planes and accounting for the joint torque constraints (Kerr and Roth 1986; Cheng and Orin 1991), so linear programming techniques may be used, or without considering the joint torques using non-linear programming techniques due to the friction cone (Woelfl and Pfeiffer 1994; Buss et al. 1996; Zuo and Qian 2000; Carloni 2006). This problem is also equivalent to the force planning of walking robots (Waldron 1986) and multiple arms manipulating an object (Kwon and Lee 1996).

Problem 3 requires the specification of the grasping device, thus for the sake of this thesis it would be necessary to do so too. A separate section concerning different hand structures is presented below. Solving this problem amounts to determine configurations of any of these hands that make their fingertips to reach a given set of contact points (Borst et al. 2002; Gorce and Rezzoug 2005; Rosell et al. 2005; Rosales et al. 2008) for precision and pinch grasps or that make the whole-hand to contact and grasp the object with multiple parts including the palm (Pollard 1997; Guan and Zhang 2001; Miller et al. 2003) for power grasps.

In the case of Problem 4, the methods proposed need to start from an initial manipulable and stable configuration of the hand-object system (Okamura et al. 2000), that in most of the cases is assumed to be given (Shimoga 1996) or heuristically pre-configured (Bae et al. 2006), in order to determine how the object can moved along a desired trajectory by controlling the hand. Different approaches has been adopted, like optimization-based (Li et al. 1989; Murray et al. 1994; Shimoga 1996; Okamura et al. 2000), control-based (Arimoto 2007) and sampling-based (Saut et al. 2007), where the grasp synthesis is proposed as a module that guarantees the manipulability and stability while moving along the desired trajectory.

Hand models

Many kinematic models of a hand have been proposed for robotic and medical applications, and computer simulations. While the first one includes both the mimicking of the human hand and the meeting of practical requirements, the two latter only seek the mimicking of the human hand in their designs All of them are composed of at least three fingers, otherwise it would resemble a standard gripper, attached to the palm. The problematic of building such hands entails the arrangement of the sensing and actuating hardware in such small rooms, and the unfolding of proper controllers to achieve manipulative dexterity, grasp robustness, and human operability (Bicchi 2000). Robotic hands that meet practical requirements are composed of three fingers with a total of four to nine degrees of freedom (Salisbury and Craig 1982; Okada 1982; Wohlke 1994; Townsend 2000). Although they are not considered to be anthropomorphic, they extend the versatiliy of the standard grippers. More complex structures include four (Jacobsen et al. 1984; Gazeau et al. 2001; Butterfass et al. 2004; Suarez and Grosch 2005) and five (Bekey et al. 1990; Matsuoka 1997; Lovchik and Diftler 1999; Schulz et al. 2001; Kawasaki et al. 2002; Lotti et al. 2005) fingers with different degrees of freedom, about three to four per finger, and three to five for the thumb in most of them. The actuation mechanisms are usually cable- or gear-driven powered by air muscles or electrical motors, ranging from one actuation per each degree of freedom (Grosch and Suarez 2004) to one that actuates all degrees of freedom (Gosselin et al. 2008). In the medical field, prosthetic hands tend to be underactuated for energy and weight constraints as they are supposed to be mounted on a human arm and controlled via myioelectric signals from the user (Kyberd et al. 2001). Virtual hands used in computer simulations have the benefit of exists in a virtual environment, hence the building problems are avoided. Thus the proposed kinematic structures account for over twenty degrees of freedom (Peña et al. 2005; van Nierop et al. 2008), providing very realistic kinematic models of a human hand but unlikely to be built with the current technology. Since it is hard to say the level of anthropomorphism of a hand, there is an index that considers several features, which is useful to compare and design hands when the objective is mimicking the human hand (Biagiotti et al. 2004). It appears that the existence of an opposable thumb is the most important one from the kinematic point of view.

Solution to algebraic equations

An unified solution to the synthesis of multifinger grasps requires to formulate all constraints algebraically, and later solve the resulting system of equations. Therefore, this section will review current methods for the solution of such systems. Broadly speaking, the proposed methods fall into three categories, depending on whether they use algebraic-geometry, continuation, or branch-and-prune techniques.

Algebraic-geometric methods, including those based on resultants and Gröbner bases, use variable elimination to reduce the initial system to a univariate polynomial. The roots of this polynomial, once backsubstituted into other equations, yield all solutions to the original system. These methods have proved quite efficient in fairly non-trivial problems such as the inverse kinematics of general 6R manipulators (Manocha and Canny 1994; Raghavan and Roth 1993), or the forward analysis of general Stewart-Gough platforms (T.-Y. Lee and J.-K. Shim 2001). Recent progress on the theory of sparse resultants, moreover, qualifies them as a very promising set of techniques (Dickenstein and Emiris 2005).

Continuation methods, in contrast, begin with an initial system whose solutions are known, and then transform it gradually to the system whose solutions are sought, while tracking all solution paths along the way. In its original form, this technique was known as the *Bootstrap Method* (Roth and Freudenstein 1963), and subsequent works (Garcia and Li 1980; Garcia and

Zangwill 1981; Morgan and Shapiro 1987; Li et al. 1988), led the procedure into its current highly-developed state (Sommese and Wampler 2005; H.-J. Su et al. 2006). These methods have been responsible for the first solutions to many long-standing problems in Kinematics. For example, using them, it was showed for the first time that the inverse kinematics of the general 6R manipulator has up to sixteen solutions (L. -W. Tsai and Morgan 1985), that the direct kinematics of the general Stewart-Gough platform can have at most forty solutions (Raghavan 1993), and a complete solution to the nine-point path synthesis problems for four-bar linkages (Wampler et al. 1992).

In a different approach, branch-and-prune methods use approximate bounds of the solution set in order to rule out portions of the search space that contain no solution. The initial domain is iteratively reduced as much as possible, and then bisected in two halves. The whole reduction-bisection process is then applied on each half recursively, until a fine-enough approximation of the solution set is derived. The convergence of this scheme is guaranteed by the fact that the bounds get tighter as the intermediate domains get smaller. Two families of branch-and-prune methods can be distinguished, depending on whether they bound the solution set via Taylor expansions, or via polytopes. Within the first family, the interval Newton method is perhaps the most-studied one (Hansen 1992; Rao et al. 1998; Didrit et al. 1998; Castellet and Thomas 1998; J.-P. Merlet 2001a; J.-P. Merlet 2001b). This method is based on the mean value theorem or, what is the same, on a zeroth-order Taylor expansion of the equations with a remainder of order one. The method requires the interval evaluation of the inverse of the Jacobian involved in the Taylor remainder and, therefore, it is only applicable to systems where such Jacobian is non-singular at all points of the input domain. But there are also methods that rely on first-order Taylor approximations of the equations, with bounded second-order reminders using either Linear Programming to determine the area of feasible solutions (Daney et al. 2004) or directly solving a linear system instead, inverting a Jacobian matrix (Gavriliu 2005). The main difference with respect to interval Newton methods is that, here, the matrix to be inverted is real-valued and, therefore, the Jacobian is only required to be non-singular at the linearization point. Results show that first-order methods outperform zeroth-order ones (Gavriliu 2005). Higher-order Taylor approximations are used in the so-called Taylor forms (Berz and Hoffstätter 1998; Neumaier 2002), but they are limited to singe variable, single equation problems. Methods in the second family bound the solutions by deriving, at each iteration, a convex polytope enclosing the solution set. In fact, such polytope is never determined explicitly, as its exact form can be rather complex. Instead, its rectangular hull is readily derived via Linear Programming. Polytope methods have similar convergence properties to Taylor methods (Sherbrooke and Patrikalakis 1993; Kearfott 2006), but present a number of advantages: (1) they avoid the computation of derivatives and Jacobian inverses, (2) they naturally account for inequalities in the problem, and (3) they can directly deal with under- or over-constrained problems. The first methods of this kind appeared in the early nineties, by deriving the polytope from properties of the equations' Bernstein form (Sherbrooke and Patrikalakis 1993). Later on, algorithms were presented where the equations are bounded by polytopes made out of bands. In some cases the bands are aligned according to the linear terms of the equations and the interval evaluation of the non-linear terms defines their width (Yamamura 1998). In other cases, the slopes of trivial functions define the bands' orientation, and the roots of a univariate polynomial yield their widthYamamura (Kolev 1998). More recently, linear relaxation techniques have been proposed (Lebbah et al. 2005), following a research line that can be traced back to the seventies (Adjiman et al. 1998; McCormick 1976). A linear relaxation is a set of linear inequalities that tightly bound a particular type of function. The simplest possible relaxation is the one obtained from a first-order Taylor expansion plus a second-order remainder that, when bounded, defines a polytope similar to those of Yamamura's and Kolev's methods. However, rather than resorting to general Taylor expansions, linear relaxations are defined on an ad-hoc basis for each function, including as many linear constraints as necessary to produce better bounds for the function at hand. Among polytope methods, those based on linear relaxations are usually faster, as they define tighter polytopes with smaller linear programs. Their drawnback is, however, that although linear relaxations are easy to define and yield efficient pruning, they do so only on polynomial equations with simple terms (usually quadratic, or bilinear terms). While in principle any polynomial system could be transformed into such form, the transformation would introduce too extra variables, rendering the solution method inefficient. This is precisely why, being superior in principle, linear relaxation techniques have not been tried in the past on well-established algebraizations of the position analysis problem, but it can be solved with a proper formulation with general and complete solvers (Porta et al. 2006; Porta et al. 2007; Porta et al. 2008).

Project Plan

The following is a description of the tasks and resources required to expand this research on.

Description of the tasks

Task 0. *Literature review.* Research the literature and extract the state of the art on the proposed problem, and particularly an extra-reading will be performed at the begining of Task 7 and 8 since an update on these issues may be required.

Task 1. *Definition of a generic hand model.* According to the review on hand models, specify a generic kinematic model of a hand, which can be particularized to most of the existing robot hands.

Task 2. *Study and formulation of contact models.* Determine the different kinematic models that can be valid for the three most common contact models, namely the frictionless, hard and soft finger contact. Determine how they may affect the dimensionality in correspondence with the degrees of freedom of each kind.

Task 3. *Formulation of the kinematic constraints.* Derive the algebraic equations that characterize the kinematics of the grasp, that is the configuration of the hand-object system.

Task 4. *Formulation of the stability constraints.* Derive the algebraic equations, based on the kinematic formulation, that provide a stable grasp, in other words, add the constraints that make the configurations of the hand-object system counteract any external force while holding the object, accounting for the hand force limits.

Task 5. *Formulation of the manipulability constraints.* Derive systematically the algebraic equations, based on the kinematic formulation, that provide a manipulable grasp, *i.e* add the constraints that make the configurations of the hand-object system to move in arbitrary directions while holding the object.

Task 6. *Identification of problem classes.* Classify the problem according to the dimensionality of the solution space due to the variability of the elements taking part in the grasping task. This may generate different kind of systems or a family of systems represented by a general one, which at this moment is hard to verify.

Task 7. *Development of a solution method for lower-dimensional problems.* The elimination theory based in multivariate resultant (Canny 1988) has proved to be very useful for zero-dimensional cases. Therefore, the derivation and solution of the resultant is to be implemented in this item. When the dimension increases to one or two, it is needed different techniques for instance, a method like the general-purpose CUIK package (Porta et al. 2006; Porta et al. 2007; Porta et al. 2008), from which the one based on linear relaxations has already been used to solve the kinematic formulation of the grasp and documented properly. But, the stability and manipulability constraints may introduce different type of monomials, and their linear relaxations would need to be incorporated in the method during this task.

Task 8. *Development of a solution method for higher-dimensional problems.* For this item, it is expected to visit the Groupe RIS (Ris) in Toulouse, France. There had been conversations with Daniel Sidobre, who is WP2 leader in the Dexmart project (Dexmart 2008), and Juan Cortes, who is experienced in motion planning of closed kinematic chains, both active members of the group. The goal is to develope a probabilisitc method combined with a method designed for lower-dimensional problems to make it complete.

Task 9. *Proposition of a set of objects to test the procedures.* The set of objects to be proposed should be graspable by the hand, but still, some non-graspable objects can be provided to test the completeness of the procedures. These experiments will be performed in a virtual simulated environment or in a real one. Basic objects have been used for initial experiments and results, but this object library may grow when developing the proposed methods if required.

Task 10. *Thesis writing.* The writing of the thesis is scheduled to the end of the work. Nevertheless, it is expected to have contributions along the development of it.

Table 1 shows the time distribution of these tasks in a Gantt chart. T stands for three months, thus T1 represents the first three months of a year.

Resources

This work fits within the research plan of the I+D projects funded by the Spanish Ministry of Education and Science, under contracts DPI2007-63665 and DPI2007-60858, led by the proposed thesis advisors. These projects will provide the necessary support and resources to carry out this work.

Table 2 summarizes additional resources required for this development, showing their description and purpose. All of them are already available at this moment.

time / tasks	2007				2008				2009				2010			
	T1	T2	T3	T4	T1	T2	T3	T4	T1	T2	T3	T4	T1	T2	T3	T4
task 0	•	•	•	•	•											
task 1			•	•												
task 2			•	•												
task 3				•	•											
task 4					•											
task 5																
task 6																
task 7																
task 8																
task 9		•														
task 10																

Table 1: The thesis plan.

Resource	**Status**	**Purpose**
Multi-Processor computer	64-processor grid at IRI	Increased computational capacity to carry out the experiments
Mechanical robot hand	SAHand and MA-I at IOC	Demonstration of the approach using a real robotic hand
Robot arm	Stäubli at IOC	Movement of the 6 d.o.f. of the real robotic hand
Programming software	C++ building environment, Matlab and Maple	Implementation and testing of the algorithms
Access to bibliography	UPC and others	Determination of the state of the art

Table 2: Resources for the development of the thesis.

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