Universitat Politècnica de Catalunya

Thesis proposal

Doctoral Program: Automatic Control, Robotics and Computer Vision (ARV)

Optimization-based Control with Population Dynamics for Large-scale Complex Systems

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1 Introduction

In a control design problem, when the desired performance of the closed-loop system can be expressed as the penalization of a cost function, then an optimization-based controller can be implemented. One of the advantage of this control approach is that constraints of the system can be considered. Among the optimization-based controllers, it is found the Model Predictive Control (MPC). However, this control strategy is presented in an individual item for this proposal, since this technique is wanted to be studied deeper than others. This controller is the most important for the objectives in this thesis proposal since its characteristics make it not only an optimization-based controller but also a predictive control strategy. In the same way, there are optimization-based controller with game theory. This is the other principal topic that conforms the global proposed idea and consequently it is presented separated as well.

Model Predictive Control (MPC) has had a significant impact on industry and it is certainly one of the most implemented controllers in industrial applications [8]. It involves a prediction model and an optimization problem to compute the control actions, becoming an efficient controller that performs a pre-established and desired behavior. Other positive features of this control strategy is that it can deal with multiple decision variables (MIMO controllers), and can consider operational and physical constraints. However, one of the main issues of these controllers in real-time applications is the computational burden to generate the control outputs. This issue depends on how dynamics of the system are, how fast the dynamical behavior of the system is, and how reduced the selected sampling time is. According to the fact that MPC can manage a large number of variables in an easy way, this controller is appropriate to be applied in large-scale complex systems where there are many states, control outputs, and constraints (e.g., energy systems, water systems, transport networks, among others).

In this regard, there are two particular critical points to face when designing an MPC and which have been object of research in the last years. First, since the number of decision variables and constraints in the optimization problem (for an established prediction horizon) is generally large, then the problem is computationally costly to solve. Secondly, there are some implementation issues related to communication availability and reliability. In the traditional MPC approach it is necessary to have the information of all system's states in a centralized scheme in order to compute the control outputs, which implies that it is necessary to send the decisions to all actuators to complete the closed-loop control. In this sense, costs associated to communication channels and operational concerns such as delays, noise or the loss of packets must be also considered. In order to mitigate computational burden, there are some immediate possible solutions such as simplify the corresponding model for prediction, reduce the prediction horizon, increase the sampling time to dispose of more time for computing the solution, or develop more sophisticated solvers in order to reduce computational time. However, this versatility of MPC might not be enough to solve this problem in certain design problems or in some complex systems. Another approach to solve this problem is the Non-centralized Model Predictive Control, which allows to design a predictive control that operates in a decentralized or distributed way, i.e., it is possible to have different decentralized or distributed elements in the system computing different control actions based on local information. Moreover, when having different elements computing control outputs, the computational costs are divided by having different hardware processing the problem. On the other hand, when it is not required to have centralized information, the reduction of communication channels is considerable which implies that the costs are reduced and the reliability improves, having less elements that could suffer a fault.

In contrast, the non-centralizing task becomes challenging in cases where the network contains coupled dynamics or coupled constraints. The former case implies to have a coupled prediction model requiring information about the whole system in a centralized point, and also it implies to have to compute all those variables in the optimization problem. Whereas the latter case implies the consideration of the coupled constraint in the optimization problem, having to compute the involved decision variables in the same solver algorithm. i.e., if there is a resource constraint for all the control outputs, it is necessary to solve the problem in a centralized way or with a centralized coordinator.

On the other hand game theory, originated from economic sciences, has spread throughout other areas including control systems theory. The application of this theory in engineering has emerged since this theory allows to model the interaction among different elements (agents), which make individual local decisions pursuing a global and common objective where no agent can improve its benefits (objective known as *Nash equilibrium*). Furthermore, evolutionary game theory (originated from biology) describes the mentioned model of agents interacting, and also considering a determined population structure, i.e., constraints in the interaction among agents. From this point of view, this theory is suitable to design intelligent systems and controllers for systems where there are local decision makers (local controllers) and achieving a global performance and/or global goal under a specific structure, which is given by the topology of the system (e.g., energy systems, water systems, transportation systems, etc). Game theory has become an important and powerful tool for solving optimization problems (e.g., the Nash equilibrium corresponds to the extreme of a potential function satisfying the Karush-Kuhn-Tucker (KKT) first order condition). Then some kind of optimization problems can be solved by finding a Nash equilibrium for an appropriate designed game.

When designing a controller for a large-scale system, the structure of the system is relevant since it determines the availability of information, the possible coupling in information or in dynamics, and constraints regarding information sharing. The structures can be taken into account with evolutionary game theory and have been studied in [32]. The structures that a system define also play an important role in the evolution of the population. When the interaction among agents is restricted to a structure, graph theory is used to describe the interaction subject to the possible connections among elements.

The motivation in this doctoral thesis proposal is to work on these two different topics (optimization-based control and evolutionary game theory), which are currently object of research in the control systems area, and complement these theories each other to propose a solution for non-centralized control systems design. There are two approaches to work with game theory that will be studied in this thesis. In the first approach, game theory is used as the unique element in the design of controllers. In this sense, game theory works as a tool for an optimization-based control design. Moreover in the second approach, game theory is used to complement an existing control strategy (e.g., coordinating different local elements of the controller, making dynamical tuning in a multi-objective controller, designing observers, etc). In this case, game theory acts as part of the optimization-based controller. This work proposes to study, develop and implement optimization techniques based on evolutionary game theory, since this theory allows to model a set of elements that have behavioral rules and that interact each other to achieve a global and common objective (e.g., the maximization/minimization of a cost function). In this way, this theory could be adapted to solve engineering problems, in particular the large-scale complex systems. Besides, by making an analogy, there might be an appropriate set up for populations to be associated to partitions in a large-scale complex systems, and the fact elements act according to individual rules can be developed to design non-centralized models (models in which these decisions are computed by local controllers). Finally, the global objective is related to a cost function satisfying a resource constraint.

2 Background and Literature Review

The main interest of this work is to consider a centralized control problem as the one shown in Figure 1, and then study different optimization tools to make it non-centralized. The design of non-centralized controllers tries to divide the whole system into various sub-systems. Once this division is achieved, then it is possible to decompose the main problem into sub-problems taking advantage of how the system is, afterwards all these sub-problems are solved independently. Consequently, each sub-problem solution is made by independent computational resources allowing to achieve faster global computing in a specific sampling time.

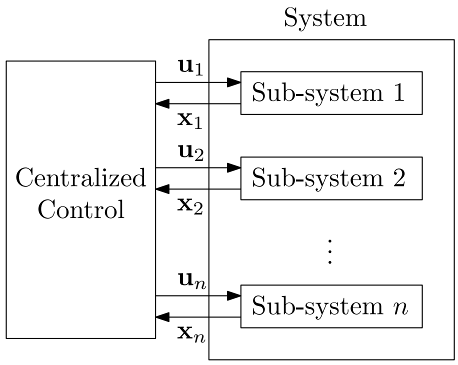


Figure 1: Centralized configuration. The controller disposes of all the states of the system that is composed by various sub-systems.

First, optimization-based control is presented and some optimization techniques to solve the control design are mentioned. Although the MPC is part of the optimization-based controllers, the MPC technique is presented independently since it can be solved with the techniques presented in the first part and because this technique is wanted to be studied deeper than others since it also involves a prediction model. Then, the main structures for non-centralized (implemented mainly in MPC) are presented, and the differences between decentralized and distributed configuration in this topic are pointed out. A distributed scheme by using cooperative game theory is shown. Afterwards, evolutionary game theory is introduced and how this theory can be used in the solution of a constrained optimization problem and to design control systems.

2.1 Optimization-based control

The optimization-based control design, as its name suggests, is applied to systems whose desired performance and behavior can be represented and defined by an optimization problem. Additionally, the optimization-based controllers allow to take into account constraints on the system. Normally, the main objective in a control design is that the difference between the reference and the system outputs tends to zero, however in applications there must be extra constraints besides the final target, these constraints are given by a desired transitory behavior (e.g., smoothness or velocity in response), economic criteria, etc.

Once the control problem is set up through an optimization problem, the method to solve the problem is established. It is necessary to highlight that the nature of the system determines how the optimization-based control could be solved. The method to solve the problem also depends on how large the system is (how many variables are involved in the optimization problem), and how fast dynamical behavior is since it must be taken into account the required computational times. For this specific thesis proposal, the considered systems are the large-scale complex systems, then the optimization-based controller technique is discussed next.

In the simplest case, the large-scale complex system is composed by decoupled dynamical sub-systems and the control objective as the constraints are also decoupled. For this simple case, any optimization strategy may be applied in the solution. In general, the large-scale complex system is composed by several sub-systems with a dynamical coupled behavior. Moreover, the control objective may involve variables from all the sub-systems, and constraints can also be coupled, i.e., decision variables from different sub-systems compose a same constraint. An additional aspect that determines the appropriate method to design an optimization-based control is the topology of the system. The topology of the system helps to determine the information dependency and availability, i.e., the topology of the system also exhibits the information network constraints.

This review focuses on non-centralized proposals. For example, [39] gives wide perspective for coordination among optimization-based controllers that might help in the construction of non-centralized structures. In [6] optimal distributed gradient methods are presented to solve resource allocation problems (a specific well studied control problem over networks). Optimization-based controllers are designed by using different optimization tools, for instance, in [38] a dual decomposition for the design of distributed controllers is shown. In [21] an iterative learning control is discussed based on Pareto Optimization.

Predictive control

MPC has been recognized as a controller with a very good performance since MPC contains an optimization problem that describes the desired operational system behavior. Additionally, the theory of MPC for linear systems is already formally developed and topics such as stability, feasibility and optimality are well studied [22][9][40]. However, there are some current opened research areas for MPC. Among these research areas, this thesis proposal focuses on the issues of MPC applied to large-scale complex systems, non-centralized MPC structures and alternative optimization techniques in the solution of this control design. These three preciously mentioned aspects are tightly related due to the fact that the decentralized or distributed control schemes are commonly applied for large-scale complex systems (e.g., tuning, state estimation), which can be expressed as a composition of several sub-systems. In the same way, the study of optimization techniques allows to propose different schemes and helps in the development of decentralized configurations. Moreover, the increasing number of contributions for non-centralized MPC schemes shows the importance of this topic in the control and automatic systems field [44][25][31].

In general terms, MPC uses a mathematical model of the system to calculate control actions, while optimizing a cost funciton. MPC strategy is composed by three fundamental elements: *i)* a cost function; *ii)* a prediction model; and *iii)* a computational tool to solve the optimization problem subject to certain physical and operational constraints [22] [9]. This control technique requires of a model system, then its performance is highly dependent on how well the model represents the real (commonly non-linear) system dynamics. Secondly, the performance of these controllers depends also on the availability of fast computational resources. In this regard, when designing MPC for large-scale complex systems, it is challenging to take into account a detail model of the dynamics since the number of states and coupling among elements in the system might make the model very complex. As consequence of this, the computing for the MPC becomes more demanding.

Non-centralized MPC Schemes

Regarding non-centralized MPC, there have been proposed some decentralized and distributed structures in the literature. The main difference between the decentralized and distributed schemes, is that for the decentralized structure there are not communication among the local controllers as it is shown in Figure 2, whereas in the distributed structure it is possible to have partial information from other local controllers (but not about the whole system) to establish a coordination as it is shown in Figure 3. Moreover, there are some cases in which it is possible to consider partial groups of communication among different local controllers. In this case, these kind of controllers form a quasi-decentralized structure. The reason is that these control structures can not be considered as a decentralized controller due to the fact information sharing is allowed, neither a distributed control due to the fact that there could be local controllers with absent external information. In this task of distributing or decentralizing, the system to control is considered as a system composed by various sub-system that can be treated by local controllers. In a general way the inconvenient at this stage when dividing the system into several sub-systems, is that the resulting scheme is not a control problem design in which each part is independent, i.e., it is not possible to solve the whole control problem by solving each division of it as an independent control problem. This is because when modeling a system as the composition of various sub-systems, the sub-systems could be coupled/overlapped in different ways.

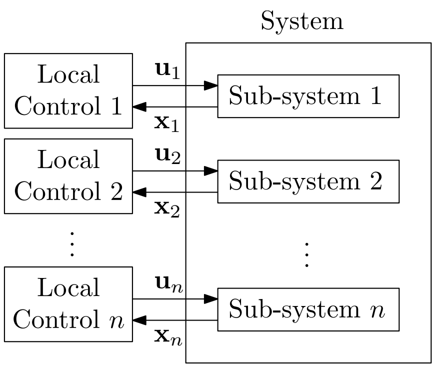


Figure 2: Decentralized configuration. Local controllers dispose of the states of a sub-system but does not have access to information from others.

Another important consideration to take into account is the different types of possible coupling that sub-systems and/or local controllers can have in order to make an MPC non-centralized. This means, that the non-centralizing process could be achieved easily depending on the dynamical structure of the system and/or the cost function established in the MPC. This type of coupling is considered since some proposed solutions only work for specific characteristics and assumptions. Therefore, it is wanted to study these possible situations to establish a clear framework for this research. The possible coupling cases are the following ones:

1. *Dynamical coupling:* when a system is divided into various sub-systems, the resulting dynamical model shows that there is a coupling among states and/or control actions of different sub-systems. In this regard, the subsystems are not dynamically independent.

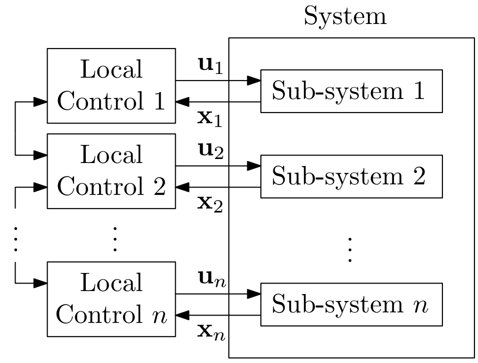


Figure 3: Distributed configuration. Local controllers dispose of the states of a sub-system but does not have access to information from others. Additionally, local controllers share partial information with other local controllers.

2. *Cost function coupling:* when the desired behavior is described by a coupled cost function, i.e., the states of more than a sub-system can not be separated in the cost function and consequently it is not possible to divide the control objective into different local controllers.

3. *Constraints coupling:* when there is a constraint that involves elements from different sub-systems and consequently information from the involved sub-systems has to be available at the same optimization problem.

4. *Combined coupling:* when more than one of the previous cases are presented in the same control problem.

Decentralized MPC

The structure for the decentralized MPC reported in [7] divides the system as various sub-systems. Each sub-system has an MPC that considers an optimization problem in which the variables are given by the actual sub-system parameters and the actual control action, whereas all the other elements in the system (i.e., control actions calculated by other sub-system MPC in the system or coupled states from other subsystem) are considered as external disturbances. Then at this structure, the local controllers do not exchange information each other. This approach allows the computation of the control actions with partial information, but it is not the optimal solution for the whole system (i.e., the solution is different from an MPC in which all the elements in the system are used to compute the control actions). When considering other elements of the system as disturbances, it is not taken into account the effects from other control actions that affect the behavior of the total system, and the actual sub-system behavior. However, this decentralized controller is appropriate in cases where there is not coupled conditions among the sub-systems or when the influence of exogenous inputs is weak. However, the case in which there are a dynamical coupling, in which it is not suitable to consider other elements of the system as external signals has been studied in [14].

Distributed MPC

The distributed control configuration are supposed to have a better performance with respect to the decentralized control configurations. This is because in the distributed schemes, the elements in the system can communicate partially each other, for which a coordination to achieve a better result is possible. In literature, the distributed structures have different classifications depending on how communication among elements is, and how the order of information requirements is (i.e., there are configurations in which the communication is made in both ways since sharing information is required, and others in which it is only required to have a one direction communication). Distributed schemes are also classified depending on the cost function that each element in the system takes into account (i.e., when in a local controller the global cost function is considered and cases where each local controller has a different cost function in comparison with other local controllers cost function). First, the different topologies according to the communication direction is treated, and then the different types of configurations depending on cost functions is discussed.

Two main distributed control configurations according to required information for local controllers coordination are [30]: i) the sequential (serial) configuration and ii) the parallel configuration. Regarding the sequential configuration, a local controller solves its optimization problem and gives information to a second local controller. This second local controller that uses the received information and its inherited information to solve its optimization problem. Then this information is shared to a third local controller and so on. For the parallel configuration, a couple of local controllers required simultaneously information from the other one and they optimize the problem in parallel. These different configurations, in the same way as the different types of coupling, allow to determine an appropriate approach to design the controller and to clarify a framework for a proposed design methodology. For example, since the distributed control configuration proposes that local controllers for sub-systems know information from other local controllers, it is natural that most of the applications in distributed MPC schemes are in parallel configuration, i.e., information is shared and all local MPC controllers optimize at time.

On the other hand, there is a classification for the distributed schemes with respect to the cost function with which the local controllers make the optimization process. In the first case, the cost function that the local controllers use is the same global function, this case is called *cooperative distributed control scheme*; in the second case, the cost function that each local controller uses is a local function and different from others, this case is called *non-cooperative distributed control scheme* [25]. At this point, and owing that this work also proposes the use of game theory, it is significant to highlight that the concept of cooperation from the non-centralized control scheme point of view is different from the game theoretical approach. When a distributed configuration is cooperative or non-cooperative, then it is defined the way in which the cost functions are selected. On the other hand, in the coalitional game theory context the cooperation has a different meaning.

In this proposal, the concepts of cooperation and non-cooperation will be used in the sense of the game theory context. Then, the way in which local controllers consider the cost functions is defined explicitly by saying that the cost function is the global and the same, or that functions are different and independent.

Distributed MPC schemes have been studied for some of the different types of coupling introduced at the beginning of this section. For instance, in [16] a distributed MPC is proposed with coupled states, and in [41] and [17] distributed MPC are proposed with coupling constraints and decoupled dynamics. Finally, in [10] a distributed optimization-based for a hierarchical MPC of large-scale systems with coupled dynamics and also coupled constraints has been proposed.

MPC with Cooperative Games

The use of game theory in the solution of distributed MPC strategies is a relative recent topic [13]. In [25] is shown that the cooperative game theory approach is appropriate to solve distributed MPC configurations where subsystems share the same cost function. In [24] is claim that from the game theory, the useful approach is the cooperative game theory instead of the non-cooperative game theory, since cooperation is needed to obtained a Pareto-optimal solution instead of a Nash equilibrium.

Cooperative game theory [34] allows to model an interaction of players in which they can cooperate each other by groups called coalitions. The general idea is to achieve the maximum utility that the game can produced. For that, and as part of the Shapley axioms [34], each player must receive a benefit according to what it contributes to the game and in this regard the cooperative game theory allows to describe a solution in a fair way where more important players receive more benefits. These concepts have been applied in the design of distributed MPC configurations since various local controllers for sub-systems can cooperate. There are other proposals to work with game theory in the distributed MPC design. For instance, in [23] a reduced version of a classical game called *MIT beer game* is used in order to design a distributed MPC. This approach has been developed to find the optimal decision variables to maximize profit in supply chains.

The main issue to work with cooperative game theory is that in order to compute the coalitions values and the Shapley value (indispensable in the cooperative game theory analysis), it is necessary to make calculus for all the possible coalitions, different also from the order in which this is formed. For this reason, calculus are combinatorial and the use of this theory in large-scale complex system is not suitable. The computational burden to generate the solution in a cooperative game, and the online algorithm to compute coalitions and Shapley value over time varying conditions are still opened to be solved. As it has been shown in [24] and [23], systems have to be of small-scale nature to apply these strategies.

2.2 Evolutionary Game Theory (EGT)

Evolutionary game theory is a branch of game theory that models, in a biological sense, the strategic interaction that agents have in a population. Evolutionary game theory, documented at first by Maynard Smith and George Price [27][45], allows to describe the agents' behavior, agents' decisions and the evolution that a population has.

Suppose that the population is composed by a large and finite number of agents. It is assumed that each agent is programmed to behave in a particular way (this behavior is represented by a strategy). Then, there is a set of n available strategies in the population denoted by $S = \{1, ..., n\}$. The scalar x_i is the proportion of agents in the population that chooses the strategy $i \in S$. Moreover since x_i is a proportion of agents, it holds that $x_i \geq 0$ and $\sum_{i \in S} x_i = 1$, representing all agents in the population. Then, the column vector containing the proportion of agents selecting strategies is known as a state in the population denoted by $x \in \mathbb{R}^n$, and the set of possible population states is given by a simplex denoted by $\Delta = \{x \in \mathbb{R}^n : \sum_{i \in S} x_i = 1, x_i \geq 0\}$. The tangent space associated to the simplex is defined as $\mathrm{T}\Delta = \{z \in \mathbb{R}^n : \sum_{i \in S} z_i = 0\}$. In the agents interaction process each agent tries to improve its benefits or utilities given by a fitness function that depends on the population state, i.e., the function $f_i : \Delta \mapsto \mathbb{R}$ returns the utility that the proportion of x_i receives for selecting the strategy $i \in S$ in the population. Finally, $F : \Delta \mapsto \mathbb{R}^n$ is the column vector of all fitness functions, and $\hat{F}_i(x) = F_i(x) - \sum_{j \in S} x_j F_j(x)$ is the comparison with the average fitness. The equilibrium of the population denoted by $x^* \in \Delta$ is achieved when no agent in the population has an incentive to change strategy, i.e., that it is not possible to obtain a better utility by changing strategy. Then a Nash equilibrium is achieved [29].

In [43], an analysis of the evolution of a population is formally described by using a well-mixed population. A well-mixed population, is a population in which all agents playing different strategies are well-mixed, i.e., if a portion of agents is selected from the population, that proportion contains agents playing all strategies from S with the same probability. Consequently, when an agent is chosen randomly from the population, the probability that the agent is selecting any strategy from S is the same. This is studied as one possible population structure [32] in a graph. The sequential description of the evolution process in a population is as follows:

1. An agent is selected randomly and receives a revision opportunity, i.e., receives the chance to compare to another agent in the population in order to make a decision to improve its benefits.

2. Once an agent has received this opportunity, then selects an opponent. At this point, when it is considered a well-mixed population, the selected opponent could be any agent playing any strategy S with the same probability.

3. The agent who has received the revision opportunity, compares itself with the opponent. In this comparison, both fitness functions and proportion of agents playing both strategies determine the result of the agent's decision.

4. Agent has two possibilities, to select the current strategy or to change strategy to the opponent's one. This is an imitation process.

5. Process is repeated from step 1.

Population dynamics

In [43] a formal deduction of population dynamics from a well-mixed population (Table 1).

Table 1: Fundamental Population Dynamics

Dynamics	**Differential Equation**
Replicator	$\dot{x}_i = x_i \left(F_i(x) - \sum_{j \in S} x_j F_j(x) \right)$
Logit Choice	$\dot{x}_i = \frac{\exp(\eta^{-1}\pi_i)}{\sum_{k \in S} \exp(\eta^{-1}\pi_k)} - x_i, \quad \eta \in [0,1]$
BNN	$\dot{x}_i = [\hat{F}_i(x)]_+ - x_i \sum_{k \in S} [\hat{F}_k(x)]_+$
Smith	$\dot{x}_i = \sum_{j \in S} x_j [F_i(x) - F_j(x)]_+ - x_i \sum_{j \in S} [F_j(x) - F_i(x)]_+$
Projection	$\dot{x}_i = F_i(x) - \frac{1}{n} \sum_{j \in S} F_j(x)$

Among the classical fundamental population dynamics, the most studied ones are the *replicator dynamics* [47], which have been proposed in [46]. The other fundamental dynamics converge to the same equilibrium point and exhibits different properties, and different transitory events which are object of research in this work.

Notice that since all the population dynamics are generated from a well-mixed population, then the differential equations in Table 1 require full information, i.e., each portion of agents requires information about the whole population to evolve. This fact makes the classical population dynamics a centralized model.

Optimization with EGT

The main interest to work with the evolutionary game theory is related to its relationship with the optimization theory, i.e., under some conditions over the functions that conform the game, there are some properties that help in the solution of constrained optimization problems. This conditions are related to a class of games know as full-potential games, in which the incentives that agents have to make decisions can be represented by an unified continuous function. Moreover, another class of games that have been widely used in engineering applications is the stable games. This feature implies, that the decisions that agents make, are made according to an increment in the potential function.

The relationship between stable games [15] *and full potential games* [42] *is that, the corresponding full potential game for a concave potential function is stable.* Moreover, it is known that in a full potential game, the Nash equilibrium points correspond to the extreme points of the potential function satisfying the Karush-Kuhn-Tucker first order condition, what is enough to guarantee optimality if the potential function is concave [43].

Figure 4 illustrates the relationship between extreme points in a potential function with the *Nash equilibrium.* Furthermore, it is shown that an optimization problem may be solved by setting up a game and finding a *Nash equilibrium.* An interesting feature is that the solution is unstable for the convex case, and stable for the concave case.

Control Design by using game theory

The game theory has been used in the modeling of interaction among elements making decisions. A general and complete perspective of the game theory applied to solve engineering problems is presented in [26]. The justification to design learning systems based on this theory is framed on the characteristics that a Nash equilibrium has, i.e., when marginal values are equal then a Nash equilibrium is achieved, and the fact that a Nash equilibrium corresponds to a extreme point of a potential function.

Then, the interest to solve games with limited information has increased in the last years. For instance, in [1] a method to achieve a distributed convergence to Nash equilibria by using only local utility measurements has been proposed. Then, some works to solve constrained optimization problems using game theory were published. Among these work, [20] presents how the design of a game should be in order to solve an optimization problem in a distributed way and in [19] variation of the topology of a system is considered in which an optimization is being

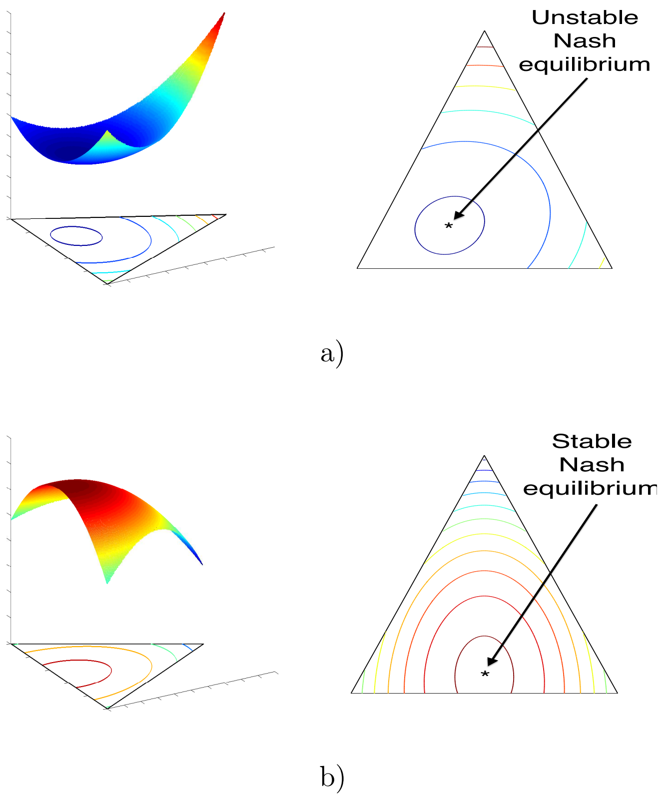


Figure 4: Potential function, and contour over the simplex in the population game. a) Convex function and b) concave function.

executed with game theory. Moreover, evolutionary game theory has been used in the solution of distributed optimization problems [36]. Once games can be designed and implemented to solve optimization problems, the theory can be used in the control design as in [35]. In [4] an agrupation for the whole system is proposed and then a constrained optimization problem is solved by using population dynamics, and [5] presents a respective distributed control design with this method.

Classical population dynamics have been studied in [43] and in [11] it has been shown that these population dynamics have passivity properties [18]. Furthermore, the population dynamics characteristics of passivity has been used to demonstrate how these dynamics can control a dynamical system in [37]. A different approach of only local information dependence in one of the six fundamental population dynamics to design distributed controllers has been proposed in [35].

Proposed approach

According to the state of the art, the problem of decentralizing and distributing optimization-based controllers for large-scale complex systems is relevant within the control systems field. This task is challenging in cases where there are coupling conditions. It has been shown that the distributed optimization tools are significantly important to make an optimization-based controller scheme non-centralized. On the other hand, cooperative game theory has been applied to the design of this kind of systems.

It has been identified that:

1. *It has been presented that the evolutionary game theory allows to solve an optimization problem on-line. This proposal leads to design non-centralized controllers by using games as an optimization tool and to explore the application of this technique in different non-centralized configurations that have not been studied from a game theoretical perspective, e.g., decentralized sequential configuration.*

2. *It has been already proposed to design the utilities of a game to achieve a final result. However, an open problem is to design utilities of a game or fitness functions in a population game in order to obtain a desired transitory performance in a controller.*

3. *Cooperative game theory has been already proposed in the non-centralized control design to obtain a Pareto-optimal solution. However, this solution is not appropriate for large-scale complex systems and it is not an on-line solution. Then, it has been identified that non-cooperative games can be used in the control design in the full potential games framework. The state of the art has shown that this approach solves optimization problems.*

4. *In general, different configurations for non-centralized control assume that the states can be measured. Another issue to research is the estimation of states in a non-centralized manner. Some works have treated this problem by consensus theory [28][12]. This problem can also be set as an optimization problem and consequently some solutions can be proposed by using game theory.*

3 Objectives

3.1 General Objective

Design non-centralized optimization-based controllers for large-scale complex systems using distributed optimization methods, all of them based on evolutionary game theory (EGT).

3.2 Specific Objectives

1. Study the optimization methods proposed with population dynamics.
2. Determine the most suitable approach according to the General Objective of this thesis proposal.
3. Select the most convenient control methodology taking into account the nature of the systems treated in this thesis proposal.
4. Determine how the distributed optimization methods proposed in item 2 can be used in the design of non-centralized controllers for large-scale complex systems.
5. Design EGT-based distributed observers for the implementation of output-feedback non-centralized control topologies for large-scale complex systems.
6. Determine a methodology to design controllers based on the potential function in an EGT.
7. Study and/or propose a dynamical tuning for predictive controllers based on EGT.

4 Methodology

The methodology consists in deeply studying the evolutionary game theory and the population dynamics as a powerful tool to solve constrained optimization problems to propose solution for the non-centralized control scheme design, to complement an existing control for large-scale complex systems, and to design non-centralized estimator. This theory allows to look for the optimal point of a function in a dynamical way, i.e., that the cost function can vary over time. The evolutionary game theory already solves this problem by the classical six fundamental population dynamics in a centralized way. Broadly, it is wanted to study how these population dynamics could be established in a distributed way to solve distributed optimization problems.

Then, evolutionary game theory can be used in the designed of distributed optimization-based controllers and to complement already existing control strategies. Some complementary elements that can be contributed by the game theory according to the proposed approach are mentioned and explained next, [1] and determine a methodology for the research:

i) One of the fundamental elements that composes an MPC is the optimization problem solver, this is the main issue to solve in this work.

ii) Then, once a distributed approach for the distributed population dynamics has been found, it is necessary to study the non-centralized structures, cooperative and non-cooperative problems (from the MPC perspective according to the cost functions), sequence or parallel, and with different types of coupling.

iii) Afterwards, a proposal to combine the non-centralized configurations for optimization-based controllers and evolutionary game theory can create a novel solution for large-scale complex systems. This might imply to choose a certain case(s) with respect to the topology of the system or type of coupling, the identification of this criteria is part of the expected results from this research.

iv) As another complement, this work proposes to design non-centralize observer. This would help in the MPC implementation since this control strategy has a high dependency with the system model.

v) On the other hand, a multi-objective function in MPC is weighted by constant parameters. However, it is wanted to study dynamical strategies to adjust weight parameters to achieve a better performance in an MPC.

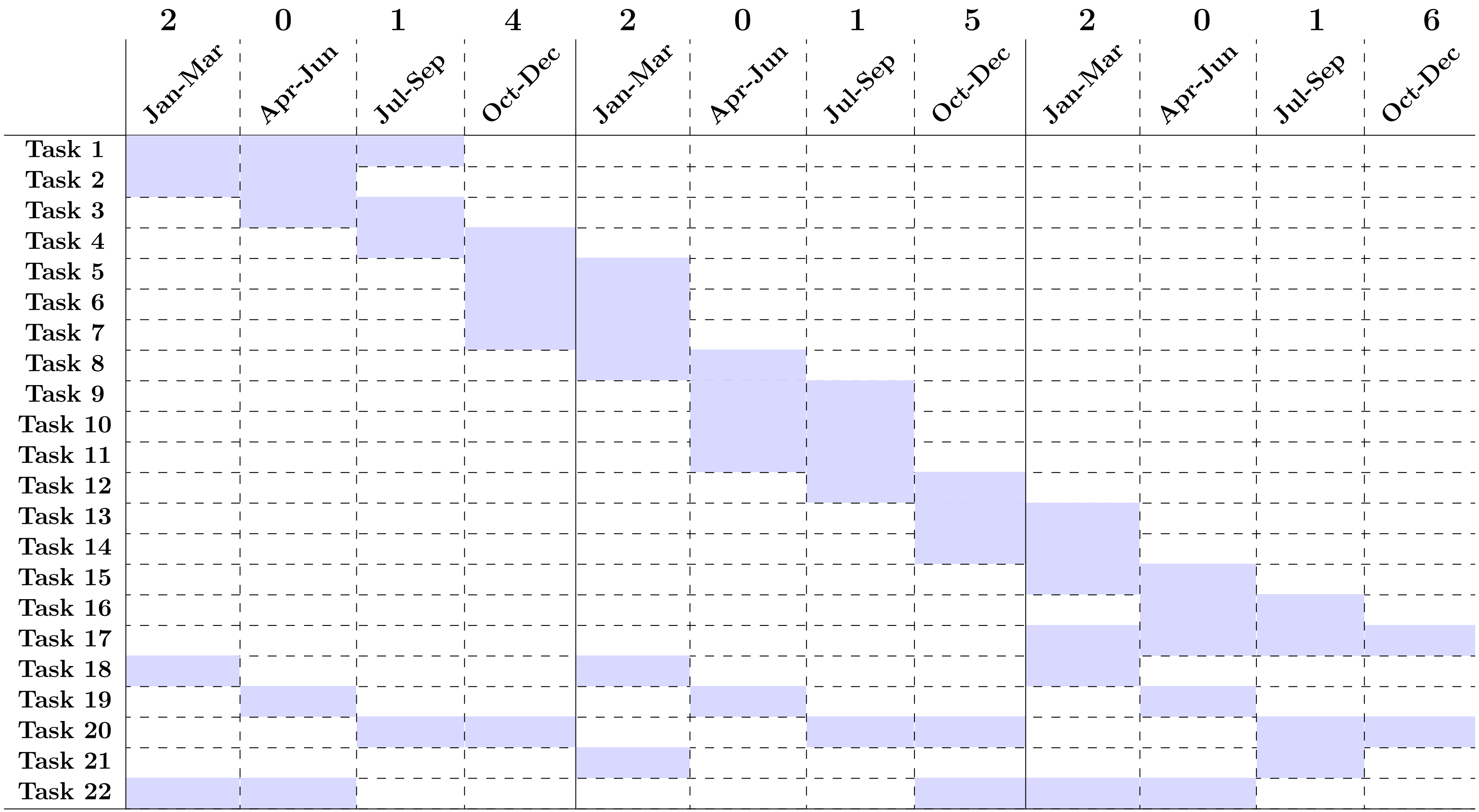
[1] *these are some identified elements so far, which may be designed with game theory and that might help in the non-centralized control performance; however, in the research process more elements with which game theory can contribute may be detected*

5 Working plan

- **Task 1:** Thesis proposal preparation.
- **Task 2:** The study of the main bibliography references related to evolutionary game theory and optimization methods based on games.
- **Task 3:** The proposal of a methodology to solve optimization problem in a distributed way inspired in population dynamics and some proposals already found in literature.
- **Task 4:** The study of the main bibliography references related to the most used non-centralized controllers for large-scale complex systems.
- **Task 5:** A review of how the population dynamics can contribute in the design of non-centralized controllers and make a linear implementation of it.
- **Task 6:** A methodology to design non-centralized controllers with game theory as complement of a existing control strategy.
- **Task 7:** Selection of a case study and simulations of the methodology for the selected system.
- **Task 8:** The study of the main bibliography references related to distributed proposals to design observers.
- **Task 9:** A review of how the techniques of evolutionary game theory can be used in the design of distributed observers.
- **Task 10:** A methodology to design non-centralized estimators using game theory.
- **Task 11:** Simulations that illustrate the performance of the estimator proposed design.
- **Task 12:** The study of the main bibliography references related to the design of utilities and/or fitness functions for evolutionary game theory.
- **Task 13:** The study how the design of fitness functions (or potential functions) can be used in the design of the transitory event of a control system (i.e., settle time, overshoot, etc).
- **Task 14:** Design of controllers based on pole placement for linear plants by using game theory.
- **Task 15:** The study of the main bibliography references of predictive control tuning.
- **Task 16:** The propose of a methodology that allows to make online tuning in a predictive control.
- **Task 17:** The summary of the work and the writing of the final thesis document.
- **Task 18:** The writing of a paper with the last obtained results for the conference on decision and control CDC.
- **Task 19:** The writing of a paper with the last obtained results for the American Control Conference ACC.

- **Task 20:** The writing of a journal paper with topics accepted in conference events.
- **Task 21:** Academic/research stay and cooperation with other national or international research groups.
- **Task 22:** Research stay with the co-advisor.

Table 2: Schedule for the tasks



6 Preliminary results

According to the working plan, a review has been made with respect to optimization problems by using full-potential games and different non-centralized configurations with the corresponding possible coupling cases. Some ideas have been already tested and preliminary results are presented. Results correspond to a parallel decentralized configuration with decoupled dynamics and a coupled constraint, solved with distributed population dynamics.

6.1 Distributed Population Dynamics for Solving Distributed Optimization Problems and Control

Taking as reference the classic six fundamental population dynamics and the mathematical deduction from a well-mixed population, the distributed population dynamics are proposed deduced from a non-well-mixed population. These dynamics are presented in Table 3 and are proposed in [3].

Table 3: Distributed population dynamics generated from a non-well-mixed population

Distributed Dynamics	**Differential Equation**
Distributed Replicator Dynamics (DRD)	$\dot{x}_i = x_i \left(F_i \sum_{j \in \mathcal{N}_i} x_j - \sum_{j \in \mathcal{N}_i} x_j F_j \right)$
Distributed Smith Dynamics (DSD)	$\dot{x}_i = \sum_{j \in \mathcal{N}_i} x_j [F_i - F_j]_+ - x_i \sum_{j \in \mathcal{N}_i} [F_j - F_i]_+$
Distributed Projection Dynamics (DPD)	$\dot{x}_i = \lvert\mathcal{N}_i\rvert F_i - \sum_{j \in \mathcal{N}_i} F_j$
Distributed Logit Dynamics (DLD)	$\dot{x}_i = \sum_{j \in \mathcal{N}_i} x_j e^{\eta^{-1} F_i} - x_i \sum_{j \in \mathcal{N}_i} e^{\eta^{-1} F_j}$

In Table 3, $\mathcal{N}_i$ is the set of neighbors of the i^{th} element in the game and makes possible to require partial information from the whole system, i.e., formally $\mathcal{N}_i = \{j : (i,j) \in \mathcal{E}\}$. The following section presents a control application approach for a non-centralized control configuration by using the distributed population dynamics.

Model-free Control example

The design of a simple MIMO distributed controller for the optimal transportation of drinking water is proposed. This problem is composed by n coupled tanks as shown in Figure 5. The arrows in the graphical representation show how flow directions are. Each tank has an outflow given by an unknown demand considered as a disturbance that is denoted by d_i, and a controlled inflow denoted by u_i from a limited water source, i.e., that control outputs are subject to a constraint given by $\sum_{i=1}^{n} u_i \leq K$, where K is the total available resource. It is assumed that there are local controllers at valves to guarantee the desired inflow.

The dynamics for this system are as follows, $\frac{dh_1}{dt} = u_1 - \sqrt{\rho g h_1} - d_1$, $\frac{dh_i}{dt} = u_i + \sqrt{\rho g h_{i-1}} - \sqrt{\rho g h_i} - d_i$, $i = 2, ..., n-1$, and $\frac{dh_n}{dt} = u_n + \sqrt{\rho g h_{n-1}} - d_n$, where h_i is the water level of the i^{th} tank, ρ is the density of the fluid and g the gravity. The proposed example considers the case of 4 tanks and the control objective is to keep the water level of tanks at a safety value of

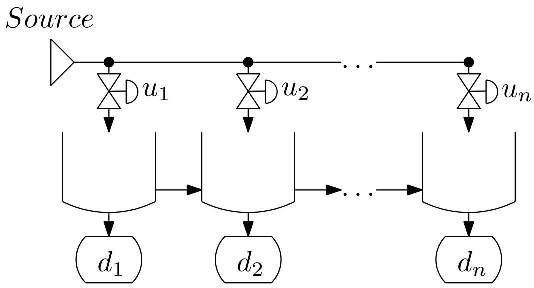


Figure 5: Simple drinking water system with a unique resource and unknown demands

reference in order to satisfy the demand requirement. For this particular example, the reference is set at 0.5 m. The unknown demand profile at each node during two days is shown in Figure 6a) (adapted from [33]).

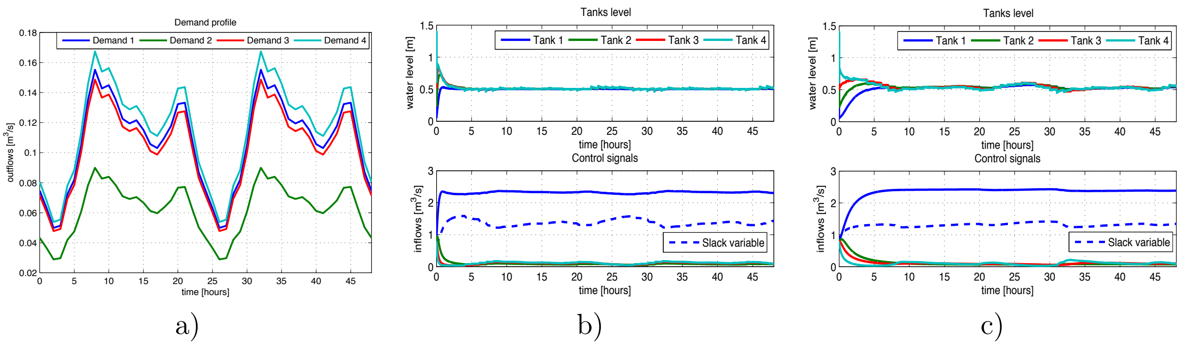


Figure 6: a) Demand profile during 2 days for a 4 tanks case, i.e., d_1, d_2, d_3 and d_4. b) System states evolution for a controller with full information. c) System states evolution for a distributed population dynamics based controller

Figure 6b) shows the control performance considering full information in the classical Smith dynamics, i.e., that at each point of the network, the information related to all the system is available to make decisions. Figure 6c) shows the performance of a distributed controller designed based on DSD. The information graph considered for this example is a path graph, i.e., that the i^{th} tank only has information about the $(i-1)^{\text{th}}$ and the $(i+1)^{\text{th}}$ tanks.

6.2 Non-centralized MPC with Decoupled Dynamics and a Resource Limited Coupled Constraint

The decentralized MPC presented in this section is part of the work in [2]. Consider a large-scale system composed by M controllable sub-systems, and whose communication topology is given by an undirected non-complete connected graph denoted by $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V}$ is the set of vertices that represent the M sub-systems, and $\mathcal{E} = \{(i,j) : i,j \in \mathcal{V}\}$ is the set of links representing the available communication and/or information sharing among sub-systems. Each controllable sub-system has a linear time-invariant discrete-time dynamics

$$x_i(k+1) = A_i x_i(k) + B_i u_i(k), \tag{1}$$

where $k \in \mathbb{Z}$ denotes the discrete time, $i \in \mathcal{V} = \{1, ..., M\}$ is the sub-system index, $x_i(k) \in \mathbb{R}^{n_i}$ denotes the states, $u_i(k) \in \mathbb{R}^{m_i}$ denotes the input of i^{th} sub-system at time k, and the matrices

$A_i \in \mathbb{R}^{n_i \times n_i}$ and $B_i \in \mathbb{R}^{n_i \times m_i}$ have constant elements.

The optimization problem for the MPC can be seen as:

$$\min \ J(k) = \sum_{i=1}^{M} \sum_{j=1}^{H_p} \|x_i(k+j) - r_i(k+j)\|_{Q_i}^2 + \sum_{i=1}^{M} \sum_{l=0}^{H_p-1} \|u_i(k+l)\|_{R_i}^2, \tag{2a}$$

$$\text{subject to } \ x_i(k+1) = A_i x_i(k) + B_i u_i(k) \tag{2b}$$

$$\underline{x}_i \leq x_i(k) \leq \bar{x}_i, \ \ \forall\, i \in \mathcal{V}, \tag{2c}$$

$$\underline{u}_i \leq u_i(k) \leq \bar{u}_i, \ \ \forall\, i \in \mathcal{V}, \tag{2d}$$

$$\sum_{i=1}^{M} u_i(k) \leq K, \tag{2e}$$

where the index $i \in \mathcal{V}$ denotes the sub-system, $Q_i \in \mathbb{R}^{n_i \times n_i}$ is a positive semi-definite weight matrix for states and $R_i \in \mathbb{R}^{m_i \times m_i}$ is a positive definite weight matrix for the control actions. The vector $r_i(k)$ is the reference for the i^{th} sub-system. The vectors $\underline{x}_i$ and $\bar{x}_i$ determine the minimum and maximum possible states of the i^{th} sub-system respectively; and $\underline{u}_i$ and $\bar{u}_i$ determine the minimum and maximum possible control actions respectively. The value of $K \in \mathbb{R}^m$ determines the total available resource as an energy constraint for the whole system.

If the constraint (2e) is omitted, then the optimization problem (2) can be decoupled since sub-systems dynamics are decoupled as well as constraints (2b),(2c), and (2d). Consequently, a local MPC for the i^{th} sub-system can be designed with a cost function given by

$$\min \ J_i(k) = \sum_{j=1}^{H_p} \|x_i(k+j) - r_i(k+j)\|_{Q_i}^2 + \sum_{l=0}^{H_p-1} \|u_i(k+l)\|_{R_i}^2, \tag{3a}$$

$$\text{subject to } \ x_i(k+1) = A_i x_i(k) + B_i u_i(k), \tag{3b}$$

$$\underline{x}_i \leq x_i(k) \leq \bar{x}_i, \tag{3c}$$

$$\underline{u}_i \leq u_i(k) \leq \bar{u}_i. \tag{3d}$$

In order to deal with the constraint (2e), a distributed full potential game with the distributed Smith dynamics is proposed in order to make the traditional MPC a decentralized controller as shown in Figure 7). Since (2e) is not an equality constraint, it is necessary to add a slack variable denoted by p_{M+1} to the game, for which this slack variable is treated as a new node added to the connected graph. Additionally, its fitness function is chosen as $F_{M+1} = 0$. The slack variable allows to use less than the total available resource when it is convenient.

It is proposed a strictly concave full potential function for the distributed population dynamics as follows[2]:

$$f(p) = -\sum_{i=1}^{M} w_i (u_i^* - p_i)^2,$$

where w_i assigns a weight factor for each control action, e.g., if $w_i = e_i, \forall i \in S$ then more priority is assigned to those sub-systems with more error. Consequently, the fitness functions for the

[2] At this applications, the notation is changed. The proportion of agents playing strategy $i \in S$ is denoted by p_i. This change is necessary in order to differentiate population states from the system's states.

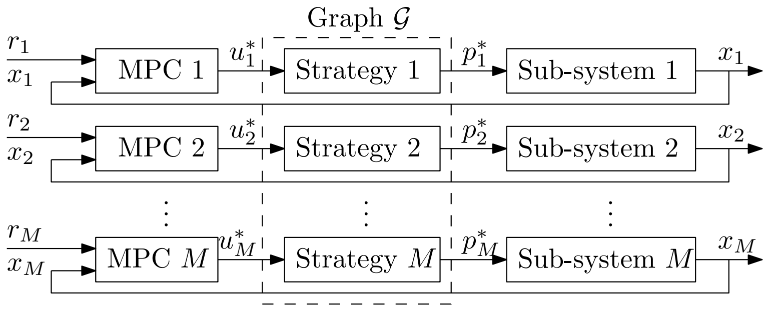


Figure 7: Non-centralized MPC with distributed population dynamics scheme to solve problem in (2). Distributed topology defined by the connected non-complete graph $\mathcal{G}$

game are given by $F(p) = \nabla f(p)$, i.e., $F_i(p_i) = -2w_i(p_i + u_i)$. Note that this methodology does not require full information about all control actions and/or all states of sub-systems since: *i)* the graph $\mathcal{G}$ representing information interaction among sub-systems is a non-complete graph, and *ii)* the proposed fitness functions are decoupled, i.e., F_i depends only on information of the i^{th} sub-system.

Each sub-system has a local MPC in which the optimization problem (3) is solved every $k \in \mathbb{Z}$, then there is a set of M controllers generating an optimal control action $u_i^*(k)$ for all $i \in S$. This optimal control action (with respect to (3)) provides a fitness function $F_i(p_i)$ to the Smith dynamics that calculates in a distributed way the final control action p_i^* satisfying the constraint (2e).

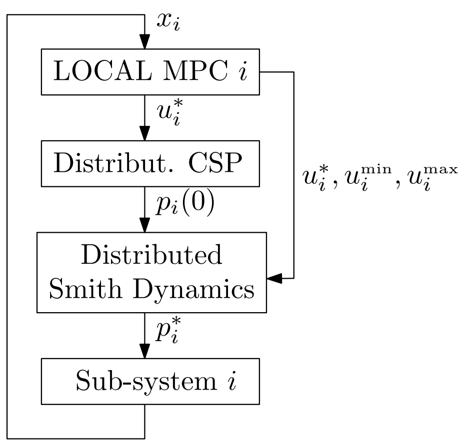


Figure 8: Flow diagram of the proposed methodology.

Figure 8 shows the summary of the proposed non-centralized MPC with distributed Smith dynamics. The distributed Smith dynamics requires u_i^*, for all $i \in S$ to set the fitness functions in the game, and also requires the limits $[u_i^{min}, u_i^{max}]$ (limits between which the problem is feasible) for all $i \in S$ in order to guarantee that the set of final control actions p^* belongs to the feasible set.

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