On Model Predictive Control for Economic and Robust Operation of Generalised Flow-based Networks

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To my family...

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Abstract

This thesis is devoted to design Model Predictive Control (MPC) strategies aiming to enhance the management of constrained generalised flow-based networks, with special attention to the economic optimisation and robust performance of such systems. Several control schemes are developed in this thesis to exploit the available economic information of the system operation and the disturbance information obtained from measurements and forecasting models. Dynamic network flows theory is used to develop control-oriented models that serve to design MPC controllers specialised for flow networks with additive disturbances and periodically time-varying dynamics and costs. The control strategies developed in this thesis can be classified in two categories: centralised MPC strategies and non-centralised MPC strategies. Such strategies are assessed through simulations of a real case study: the Barcelona drinking water network (DWN).

Regarding the centralised strategies, different economic MPC formulations are first studied to guarantee recursive feasibility and stability under nominal periodic flow demands and possibly time-varying economic parameters and multi-objective cost functions. Additionally, reliability-based MPC, chance-constrained MPC and tree-based MPC strategies are proposed to address the reliability of both the flow storage and the flow transportation tasks in the network. Such strategies allow to satisfy a customer service level under future flow demand uncertainty and to efficiently distribute overall control effort under the presence of actuators degradation. Moreover, soft-control techniques such as artificial neural networks and fuzzy logic are used to incorporate self-tuning capabilities to an economic certainty-equivalent MPC controller.

Since there are objections to the use of centralised controllers in large-scale networks, two non-centralised strategies are also proposed. First, a multi-layer distributed economic MPC strategy of low computational complexity is designed with a control topology structured in two layers. In a lower layer, a set of local MPC agents are in charge of controlling partitions of the overall network by exchanging limited information on shared resources and solving their local problems in a hierarchical-like fashion. Moreover, to counteract the loss of global economic information due to the decomposition of the overall control task, a coordination layer is designed to influence non-iteratively the decision of local controllers towards the improvement of the overall economic performance. Finally, a cooperative distributed economic MPC formulation based on a periodic terminal cost/region is proposed. Such strategy guarantees convergence to a Nash equilibrium without the need of a coordinator and relies on an iterative and global communication of local controllers, which optimise in parallel their control actions but using a centralised model of the network.

Keywords: MPC, economic optimisation, robust performance, reliability, centralised control, distributed control, non-iterative coordination, cooperative control.

Resumen

Esta tesis se enfoca en el diseño de estrategias de control predictivo basado en modelos (MPC, por sus siglas en inglés) con la meta de mejorar la gestión de sistemas que pueden ser descritos por redes generalizadas de flujo y que están sujetos a restricciones, enfatizando especialmente en la optimización económica y el desempeño robusto de tales sistemas. De esta manera, varios esquemas de control se desarrollan en esta tesis para explotar tanto la información económica disponible de la operación del sistema como la información de perturbaciones obtenida de datos medibles y de modelos de predicción. La teoría de redes dinámicas de flujo es utilizada en esta tesis para desarrollar modelos orientados a control que sirven para diseñar controladores MPC especializados para la gestión de redes de flujo que presentan tanto perturbaciones aditivas como dinámicas y costos periódicamente variables en el tiempo. Las estrategias de control propuestas en esta tesis se pueden clasificar en dos categorías: estrategias de control MPC centralizado y estrategias de control MPC no-centralizado. Dichas estrategias son evaluadas mediante simulaciones de un caso de estudio real: la red de transporte de agua potable de Barcelona en España.

En cuanto a las estrategias de control MPC centralizado, diferentes formulaciones de controladores MPC económicos son primero estudiadas para garantizar factibilidad recursiva y estabilidad del sistema cuya operación responde a demandas nominales de flujo periódico, a parámetros económicos posiblemente variantes en el tiempo y a funciones de costo multi-objetivo. Adicionalmente, estrategias de control MPC basado en fiabilidad, MPC con restricciones probabilísticas y MPC basado en árboles de escenarios son propuestas para garantizar la fiabilidad tanto de tareas de almacenamiento como de transporte de flujo en la red. Tales estrategias permiten satisfacer un nivel de servicio al cliente bajo incertidumbre en la demanda futura, así como distribuir eficientemente el esfuerzo global de control bajo la presencia de degradación en los actuadores del sistema. Por otra parte, técnicas de computación suave como redes neuronales artificiales y lógica difusa se utilizan para incorporar capacidades de auto-sintonía en un controlador MPC económico de certeza-equivalente.

Dado que hay objeciones al uso de control centralizado en redes de gran escala, dos estrategias de control no-centralizado son propuestas en esta tesis. Primero, un controlador MPC económico distribuido de baja complejidad computacional es diseñado con una topología estructurada en dos capas. En una capa inferior, un conjunto de controladores MPC locales se encargan de controlar particiones de la red mediante el intercambio de información limitada de los recursos físicos compartidos y resolviendo sus problemas locales de optimización de forma similar a una secuencia jerárquica de solución. Para contrarrestar la pérdida de información económica global que ocurre tras la descomposición de la tarea de control global, una capa de coordinación es diseñada para influenciar no-iterativamente la decisión de los controles locales con el fin de lograr una mejora global del desempeño económico. La segunda estrategia no-centralizada propuesta en esta tesis es una formulación de control MPC económico distribuido cooperativo basado en una restricción terminal periódica. Tal estrategia garantiza convergencia a un equilibrio de Nash sin la necesidad de una capa de coordinación pero requiere una comunicación iterativa de información global entre todos los controladores locales, los cuales optimizan en paralelo sus acciones de control utilizando un modelo centralizado de la red.

Palabras clave: MPC, optimización económica, desempeño robusto, fiabilidad, control centralizado, control distribuido, coordinación no-iterativa, control cooperativo.

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Nomenclature

Notation

Symbol	**Description**
$\{\cdot, \ldots\}$	set or sequence
$\emptyset$	empty set
$x \in \mathbb{X}$	x is an element of the set $\mathbb{X}$
$\mathbb{R}$	set of real numbers
$\mathbb{R}_+$	set of non-negative real numbers
$\mathbb{R}_{>c}$	$\mathbb{R}_{>c} \coloneqq \{x \in \mathbb{R} \mid x > c\}$ for some $c \in \mathbb{R}_+$
$\mathbb{R}_{\geq c}$	$\mathbb{R}_{\geq c} \coloneqq \{x \in \mathbb{R} \mid x \geq c\}$ for some $c \in \mathbb{R}_+$
$\mathbb{R}^n$	space of n-dimensional (column) vectors with real entries
$\mathbb{R}^{n \times m}$	space of n by m matrices with real entries
$\mathbb{Z}$	set of integers
$\mathbb{Z}_+$	set of non-negative integers including zero
$\mathbb{Z}_{>c}$	$\mathbb{Z}_{>c} \coloneqq \{x \in \mathbb{Z} \mid x > c\}$ for some $c \in \mathbb{Z}_+$
$\mathbb{Z}_{\geq c}$	$\mathbb{Z}_{\geq c} \coloneqq \{x \in \mathbb{Z} \mid x \geq c\}$ for some $c \in \mathbb{Z}_+$
$\mathbb{Z}_{[c_1,c_2)}$	$\mathbb{Z}_{[c_1,c_2)} \coloneqq \{x \in \mathbb{Z} \mid c_1 \leq x < c_2\}$ for some $c_1, c_2 \in \mathbb{Z}_+$
$\mathbb{Z}_{[c_1,c_2]}$	$\mathbb{Z}_{[c_1,c_2]} \coloneqq \{x \in \mathbb{Z} \mid c_1 \leq x \leq c_2\}$ for some $c_1, c_2 \in \mathbb{Z}_+$
$\mathbb{X}(\subset) \subseteq \mathbb{Y}$	set $\mathbb{X}$ is a (strict) subset of $\mathbb{Y}$
$\mathbb{X} \times \mathbb{Y}$	Cartesian product of the sets $\mathbb{X}$ and $\mathbb{Y}$, i.e., $\mathbb{X} \times \mathbb{Y} = \{(x, y) \mid x \in \mathbb{X}, y \in \mathbb{Y}\}$
$\mathbb{X}^N$	N-dimensional Cartesian product $\mathbb{X} \times \mathbb{X} \times \ldots \times \mathbb{X}$, for some $N \in \mathbb{Z}_{\geq 1}$
$\mathrm{Co}\mathbb{X}$	convex hull of the set $\mathbb{X}$
$\mathrm{int}(\mathbb{X})$	interior of the set $\mathbb{X}$
$x^\top$ $(X^\top)$	transpose of a vector $x \in \mathbb{R}^n$ (matrix $X \in \mathbb{R}^{n \times m}$)
X^{-1}	inverse of the matrix $X \in \mathbb{R}^{n \times n}$

$x_{(i)}$ ($X_{(i)}$)	i-th element (row) of the vector $x \in \mathbb{R}^n$ (matrix $X \in \mathbb{R}^{n \times m}$)
X_{ij}	element in the i-th row and j-th column of the matrix $X \in \mathbb{R}^{n \times m}$
$\{x_i\}_{i \in \mathbb{Z}_{[a,b]}}$	ordered sequence of elements $(x_a, \ldots, x_b)$ with $a, b \in \mathbb{Z}_{\geq 1}$ and $a < b$
$\mathrm{diag}(\cdot)$	operator that builds a diagonal matrix with the elements of its argument
$\lvert \cdot \rvert$	(1) $\lvert x \rvert \geq 0$ with $x \in \mathbb{R}$ returns the absolute value of the scalar x (2) $\lvert x \rvert \in (\mathbb{R}^+ \cup \{0\})^n$ returns the component-wise absolute value of the vector $x \in \mathbb{R}^n$ (3) $\lvert \mathcal{A} \rvert$ returns the cardinality of the set $\mathcal{A}$
$\lVert \cdot \rVert$	2-norm (Euclidian norm) of a vector, i.e., $\lVert x \rVert := \sqrt{\sum_{i=1}^{n} x_{(i)}^2}$, where $x \in \mathbb{R}^n$
I_n	identity matrix of dimension $n \times n$, where $n \in \mathbb{Z}_{\geq 1}$
$0_{m \times n}$	zero matrix of dimension $m \times n$, where $m, n \in \mathbb{Z}_{\geq 1}$
$\mathbb{S}^n$	set of all n-dimensional symmetric matrices, i.e., $\mathbb{S}^n := \{X \in \mathbb{R}^{n \times n} \mid X = X^\top\}$
$\mathbb{S}_+^n$ ($\mathbb{S}_-^n$)	set of all n-dimensional positive (negative) semi-definite matrices
$\mathbb{S}_{++}^n$ ($\mathbb{S}_{--}^n$)	set of all n-dimensional positive (negative) definite matrices
$X \succ 0$ ($\prec 0$)	X is a positive (negative) definite matrix, i.e., $X \in \mathbb{S}_{++}^n$
$X \succeq 0$ ($\preceq 0$)	X is a positive (negative) semi-definite matrix, i.e., $X \in \mathbb{S}_+^n$
$\lfloor i \rfloor_j$	$\lfloor i \rfloor_j := \mathrm{mod}(i, j)$ is the modulo operation between integers $i, j \in \mathbb{Z}_+$
$\mathbb{P}[\cdot]$	probability measure
$\mathbb{E}[\cdot]$	expectation with respect to probability measure $\mathbb{P}[\cdot]$
$\mathcal{N}(\bar{x}, \Sigma_x)$	multivariate normal distribution of x with mean $\bar{x}$ and covariance Σ_x
$\Phi(\cdot)$	standard cumulative distribution function
$\Phi^{-1}(\cdot)$	quantile function
σ_x	standard deviation of x
$\frac{\partial f}{\partial x}$	partial derivative of function f with respect to x
$\begin{bmatrix} a & \star \\ b & c \end{bmatrix}$	the symbol "$\star$" denotes the symmetric part of a matrix, i.e., $\begin{bmatrix} a & \star \\ b & c \end{bmatrix} = \begin{bmatrix} a & b^\top \\ b & c \end{bmatrix}$

Acronyms

Acronym	**Description**
MPC	Model Predictive Control
EMPC	Economic Model Predictive Control
CE-MPC	Certainty-Equivalent Model Predictive Control
RB-MPC	Reliability-Based Model Predictive Control
SMPC	Stochastic Model Predictive Control
CC-MPC	Chance-Constrained Model Predictive Control
TB-MPC	Tree-Based Model Predictive Control
LB-MPC	Learning-Based Model Predictive Control
ML-DMPC	Multi-Layer Distributed Model Predictive Control
CDEMPC	Cooperative Distributed Economic Model Predictive Control
FHOP	Finite Horizon Optimisation Problem
LMI	Linear Matrix Inequality
LTI	Linear Time Invariant
RTO	Real-Time Optimiser
QP	Quadratic Programming
LP	Linear Programming
PID	Proportional-Integral-Derivative
DWN	Drinking Water Network

Part I

Preliminaries

Chapter 1

Introduction

1.1 Motivation

The evolution of human civilisations until the current days might be heavily related to the understanding and acceptance of connectivity and networks as tools to enhance the quality of life, considering the importance of both social interactions and physical infrastructure systems. In the daily living people are part of many instances of networks, e.g., communication networks, electrical power networks, public transport networks, road-traffic networks, water networks, financial networks, supply-chains, among others. These networks may be considered as *critical infrastructures* [122], since their proper operation is vital for the normal functioning of modern society. Consequently, maintaining a truly efficient, reliable and sustainable service is a must in network systems.

Physical networks are conceived and designed to supply different specific services. Nevertheless, many of the problems that drive their operation (e.g., minimisation of displacement times, maximisation of plants' throughput, minimisation of energy consumption, maximisation of demand satisfaction, etc.) share a common feature: some commodity (or many at the same time), e.g., information, water, oil, money, people, products, among any other real or abstract entity, need to be transported through the network infrastructure, and this has to be done whilst making best use of available resources and in line with the prevailing regulatory framework. Such similarity in the related transportation problems gave rise to the classic field of *network flows* [2], which forms a large area of optimisation theory (especially in combinatorial and linear programming techniques) and are core problems in operations research, applied mathematics, computer science, and many fields of engineering.

Under the framework of network flows, any network can be generically described with a *graph* consisting of a set of *nodes* (or *vertices*) that represent locations, and a set of *arcs* (or *edges*) that model links between nodes. The network nodes can be classified in *supply* nodes (e.g., production units, oilfields, water reservoirs, factories), *demand* nodes (e.g., houses, refineries, or other consumption points), and intermediate *transshipment* nodes with or without storage capability (e.g., pipe junctions, pumping stations, street crossings, warehouses, water storage deposits, energy storage units). Ordered sequences of arcs (e.g., pipelines, electrical cables, roads, railways, or other channels) form *paths* (or *routes*), which are used to transport the commodity from the supply nodes to the demand nodes. Typically, nodes and arcs of a network are subject to capacity constraints and to some costs associated with their use. The commodity that is being transported over the network, however, determines how the overall system operates.

Network flow problems have been in the focus of research for many years and a mature theory for network analysis and design has been developed with numerous efficient algorithms for solving classical problems such as the minimum cost flow problem, the shortest path problem and the maximum flow problem. A comprehensive discussion of theory, algorithms and applications of network flow problems can be found in [2, 17, 154]. Nevertheless, despite the mature mathematical background supporting the network flows field, there are some aspects that limit the applicability of the available results to real problems. To start, the main drawback of the classical network flow theory is that most developed algorithms rely on the assumptions of static flow conditions and static network structures, but time plays a vital role in many real applications, where capacities, costs, supplies, demands, and therefore flows, evolve over time with possibly different time scales. On other hand, static network flows theory does not consider in general the possibility of storage in transshipment nodes nor the flow of multiple commodities, which are crucial in several real networks.

The aforesaid weaknesses of static network flows have originated the so-called *dynamic network flows* (also called *network flows over time*). In general, dynamic network flows have three main differences with respect to the traditional models; these differences are: (i) flows change over time, (ii) the flow between two nodes takes a finite transit time and, (iii) storage is allowed at the nodes of the network for later transportation. A further extension of the static and dynamic network flows is the so-called *generalised network flows*, which allow to have gains in the arcs of the network. Dynamic networks were introduced in [59, 85]. Since then, several authors have studied different features of flows over time and useful surveys on the topic are available in the literature, see e.g., [9, 80, 95, 97, 168]. A common conclusion reported in the aforementioned surveys is that, when merging from the static models to the dynamic models, several of the arising network flow problems are NP-hard and most of the solution methods are based on approximations of the optimal values and eventually reduce the dynamic problem to a static one to exploit existing algorithms [97, 168].

Recently, some progress on pseudo-polynomial- or polynomial-time algorithms has been achieved for the case of dynamic and generalised network flows, see e.g., [73, 80, 121] and references therein. Nonetheless, there are some practical issues that have not been considered and still hamper the applicability of results to real-size networks. Specifically, besides the general setting of network flows, real networks typically share the following characteristics [122]:

- they span a large geographical area,
- they have a modular structure consisting of multiple interconnected subsystems,
- they have many actuators and sensors,
- they have dynamics evolving with possibly different nature (continuous, discrete, or hybrid),
- there are different actors involved in the network operation with multiple and possibly conflicting objectives.

These features make the management of dynamic networks a complex task and have become an increasingly research subject worldwide due to the lack of strategies to deal with energetic, environmental and economic issues, with special attention to efficient handling of resources and planning against uncertainty of demand and/or supply.

Strategic and tactic decisions in physical networks operation can be addressed by different methods proposed within the supply-chain theory, see e.g., [139, 157], but the modelling framework of systems and control theory has shown to be suitable to handle the problem consisting of time variance, uncertainties, delays, and lack of system information, see e.g., [115, 122, 134, 163, 173]. Most of the approaches developed for dynamic networks management are deterministic and mainly based on efficient linear/quadratic programming techniques to deal with economic optimisation or regulation control. However, due the stochastic nature of demands and the ageing behaviour of the elements in the infrastructure, reliability assessment, uncertainty forecasting, safety mechanisms, robust feasibility, modularity and scalability of the control strategy are still challenging problems in the design of tractable controllers for these large-scale systems, and these latter aspects are the main focus of this research.

In this thesis, a general setting of dynamic flow problems with possibly time-varying capacities, costs, supplies and demands is studied and control algorithms for solving such problems are developed. As discussed in [122], a particularly useful form of control for networks dealing with transportation problems that can cope with constraints and exploit all information available in a systematic way is model predictive control (MPC), see e.g., [109, 149]. Therefore, the contributions in this thesis rely on the use and extensions of recent developments of the MPC framework, specifically, those related to economic MPC and distributed MPC. The work in this thesis is primarily motivated by applications of dynamic flows in drinking water networks control. Hence, the network flows are usually interpreted as water quantities that flow over time and the performance of the strategies is measured with respect to an economic index and to the service level achieved when satisfying time-varying water demands. The global aim of this thesis is then to study the application of the MPC framework for the economic and robust operation of generalised flow-based networks.

1.2 Research Background

This thesis aims to exploit the MPC framework in order to optimise the operations of generalised flow-based networks, seeking to achieve a customer service level and a reliable cost-effective network operation. Therefore, this section presents a short overview regarding the state of the art of the different topics that this thesis synergistically combines to develop MPC controllers as decision-support tools in the management of dynamic network flows. Such topics are: reliability in flow-based networks, decision making under uncertainty, MPC tuning strategies and economic (centralised and non-centralised) MPC schemes for controlling dynamic network flows. For further details on the topics presented in this section, the reader is encouraged to resort to the given bibliography.

1.2.1 Reliability in Flow-based Networks

The behaviour of network flows in a given infrastructure is governed by: (i) the commodity being transported, (ii) the physical laws that describe the flow relationships between the elements conforming the network, (iii) the consumer demand, and (iii) the network topology. Generally, *reliability* can be defined as the probability that units, components, equipments and systems will accomplish their intended function for a specified period of time under some operating conditions and specific environments [67]. Thus, from the perspective of supply chain engineering [69], reliability analysis of a flow-based network is concerned with the α-*service level* (type I), which is an event-oriented performance criterion that measures the probability that all customer demands will be completely served within a given time interval from the stock on hand without delay, under normal and emergency conditions. The required quantities of the transported commodity are defined in terms of the flux to be supplied within given ranges of flow capacity in each element of the network. Traditionally, reliability in flow-based networks has been assumed to be assured by heuristic guidelines and contingency analysis in the design phase (e.g., setting alternative source/demand paths, over-sizing network elements), but the level of reliability is not quantified or measured. Hence, since the reliability of a system involves stochastic events, more emphasis has to be put on its explicit incorporation in the operation phases.

As stated in [136] for the case of a water distribution system (but applicable to other kinds of flow-based networks), reliability assessment of a network can be classified in two main categories: (i) *Topological reliability*, which refers to the probability that a network is connected given the *mechanical reliabilities* of its components, i.e., the probabilities that components will remain operational at any time. For this case, analytical and simulation methods exist to assess topological reliability using graph connectivity and reachability analysis. (ii) *Hydraulic reliability*, which refers directly to the fundamental task of a flow-based network, i.e., the transport of desired quantities of the commodity with a desired quality to the appropriate locations at the appropriate times. Despite this classification, since the network infrastructure is subject to random failures, topological reliability should also be explicitly considered when performing hydraulic reliability assessments in order to guarantee a desired service level. Being the control of water networks the motivational application of this thesis, the reader may refer to [12, 93, 113, 166, 177, 181, 191] as examples of methods for reliability analysis in flow-based networks.

Service reliability and economic optimisation in flow-based networks have been an important research topic in the field of inventory management for planning against uncertainty in demand and/or supply. The main strategy to assure a service level is performing demand forecasting to guarantee a safety stock in storage units (if they exist) as a countermeasure to secure network performance against uncertainty. Obtaining and using advanced demand information enable network operators to be more responsive to customer needs and improve inventory management [138]. Flow demand in a network is a highly variable process due to the range of possible user types and numerous influencing factors categorised as climatic, socio-economic and structural. As a result, it is impossible to forecast demand with certainty. Forecasting methods can be classified in the following categories: econometric, end-use, time-series, regressions, and other non-parametric/soft-computing models, see e.g., [19, 156] for a detailed review on models applied for urban water demand forecasting.

The interaction between forecasting and stock control is well reviewed in [18, 77, 88, 135, 160, 171] and references therein. Most of the results reported in the aforementioned literature of general supply-chains assume that the demand forecast error is stationary and usually normally distributed while replenishment lead time (the time from the moment a supply requirement is placed to the moment it is received) is stationary and usually certain [88], but these assumptions do not generally hold. In practical operation of flow-based networks, the settling of the safety stock is typically determined by experience, estimating risk and assigning a fixed value (i.e., a proportion of the storage capacity) for the entire planning horizon. This approach is too conservative and reduces the manoeuvrability space for economic optimisation because the full excursion in storage nodes is restricted and avoids the use of their plenty capacity to save energy costs in flow-transport actions. Regarding the lead time, it generally fluctuates over time when capacity is limited by a fixed or a time-varying safety constraint. Moreover, models that consider non-stationary flows are unfortunately not very helpful if they just use demand and lead-time information to calculate safety stocks, especially when variations may be caused by the ageing of network supply components.

To the best of this thesis author's knowledge, reliability degradation models for system and components have not been addressed simultaneously with dynamic safety stocks planning in the framework of generalised dynamic network flows optimisation. Reliability in flow-based networks is commonly analysed off-line, i.e., a posteriori of the operation cycle, but without a measure of the capacity degradation that may exist in the arcs of the network (related to actuators). Relevant attempts to compute the required safety stocks considering the network's health were presented in [20, 22] for the control of production-distribution systems with uncertain demands and system failures. In these works, necessary and sufficient conditions to drive and keep the state within the least storage level are obtained, but under the requirement that the controller must be aware of the failure configuration, which is not always possible to identify and isolate. Most of other approaches that present components' health management to assess mechanic reliability in a system are within the framework of fault-tolerant control or in the field of maintenance scheduling, see e.g., [76, 92, 111, 143] and references therein. Commonly, such approaches work in a reactive manner (i.e., executing an action after complete failure of components) or with a monitoring and planning purpose (i.e., programming maintenance periods for repairing or replacing damaged components). Therefore, taking into account that flows through actuators are manipulated and monitored variables, topological and hydraulic reliability in flow-based networks can be assured for a given period of time with optimal control effort allocation policies that explicitly consider the ageing of components and system health, and the MPC framework might be suitable to manage this task online in a proactive manner.

1.2.2 Decision Making under Uncertainty

Decision making under uncertainty is a central issue in almost all disciplines and application areas. The literature in this topic is quite extensive, see e.g., [15, 62, 66, 182] for a survey of mathematical optimisation techniques for decision making under uncertainty.

Especially, in generalised flow-based networks such as water networks, power energy networks, road-traffic networks, supply-chains, among others, uncertainty might be large due to the complexity and size of these systems, and it can be caused by many sources (e.g., exogenous and endogenous demands, noise, equipment degradation, plant model mismatch, other disturbances). Therefore, uncertainty cannot be neglected in the management of dynamic flow-based networks if it is desired to fulfil reliability requirements and quality standards.

In industrial practice, uncertainties are usually compensated by over-design of elements or overestimation of operational parameters by introducing safety factors obtained mostly by experience or application-dependent heuristics, which restrict considerably the economic profit of the network operation. Consequently, several approaches reported in the literature for control applications of flow-based networks, see e.g., [29, 129, 140, 169], addressed the uncertainty by solving, in a receding horizon fashion, a deterministic optimisation problem where uncertain disturbances are replaced by nominal forecasts, which are computed based upon the past and current information available at each decision time instant and assumed as certain. These approaches rely on the so-called *certainty equivalence* property [180], which in the MPC framework leads to a *perturbed nominal deterministic* MPC, also named *certainty-equivalent* MPC (CE-MPC). This strategy is less conservative and is usually complemented with a (de)tuning of the controller, but it can lead to frequent constraint violations due to the ignored effects of future uncertainty.

There is another widely reported class of techniques that face uncertainties explicitly. These strategies use an uncertain process model whose characterisation can be performed under two main paradigms: the deterministic worst-case description, which is exploited in robust optimisation techniques [15], and the stochastic description, which is exploited in stochastic optimisation techniques [94]. The reader is referred to [62] for a detailed overview of the state of the art of these techniques and their application in particular problems related to network flows. A common drawback of most reported approaches is that the practical applicability of uncertain dynamic models turns out to be rather limited to small-size network flow problems, mainly due to the computational burden of the techniques, which generally relies on dynamic programming and two-stage (or multi-stage) decisions with recourse.[1]

[1]A recourse decision means that the decision can be made in the second (or subsequent) stage to

From the perspective of control theory, robustness in dynamic-flow networks has been addressed in [20–24], under a purely deterministic unknown-but-bounded description of the uncertainty (without recourse), studying also special extensions such as periodic network flows, input delays and system failures. The common approach in these works is the characterisation of the maximal robust control invariant (RCI) set, i.e., the set of network states for which there exist network flows that guarantee the demand satisfaction at all time instants. Since in large-scale networks (systems with a large number of states, inputs and disturbances) the computation of the RCI set might be cumbersome, a decentralised design parametrised with respect to arc capacities was proposed in [13]. More recently, thanks to the idea of adjustable solutions of robust optimisation problems proposed in [16], multi-stage decision rules allowing recourse at every step of a planning horizon have been applied to MPC strategies designed for constrained dynamical systems [72] and dynamic network flows, see e.g., [184]. Another approach that has been well exploited in the MPC framework and that is gaining attention under a distributed fashion for dynamic flow-based networks is the so-called tube-based MPC, see, e.g., [43] and references therein. The main problem of the aforementioned robust deterministic approaches is the computational burden and the conservatism of most solutions; if the disturbance bounds used in these methods result to be very wide, a significant deterioration of the performance will take place. It is worth to mention that the assumption of bounded disturbance does not hold in many practical cases, therefore, if the realisation of the disturbances lie outside of the admissible set, no statement about robust stability or feasibility can be made.

A more realistic description of uncertainty is the stochastic paradigm, which leads to less conservative control approaches by including explicit models of uncertainty in the design of control laws and by transforming hard constraints into probabilistic constraints. As reviewed in [31], the stochastic approach is a classic one in the field of optimisation, but due to the advances in technology (which improve computation capacity) and the flexibility of the MPC framework to incorporate models and constraints within an optimal control problem, a renewed attention has been given to stochastic programming [165] as a powerful tool for robust control design, leading to the *Stochastic MPC* and especially

compensate for any bad effects that might be consequence of the first (or previous) stage decision.

the *Chance-Constrained MPC* (CC-MPC). This stochastic control strategy describes robustness in terms of probabilistic (chance) constraints [36], which restrict the probability of violation of any operational requirement or physical constraint to lie below a prescribed value representing the notion of reliability or risk of the system. By setting this value properly, the operator can trade conservatism against performance. Relevant works that exploit the CC-MPC approach can be found in [35, 41, 96, 104, 132, 133, 161, 183], and references therein. Most of the cited publications use the closed-loop prediction scheme to optimise control policies and assume multiplicative and/or additive uncertainties. Feedback control laws are commonly linear or affine with respect to the state, but recently affine disturbance feedback approaches are gaining attention. A review on chance-constrained optimal control of systems related to dynamic flow networks can be read in [137], especially the case of multi-reservoir system optimisation.

1.2.3 MPC Tuning Strategies

An important task in MPC design is the incorporation of preferences and degrees of freedom for the operators. Therefore, as suggested in [103], another approach to aid the decision-making process under uncertainty is to adopt the idea from adaptive control. Some common approaches in this research line are the change of the control law or controller tuning according to the real-time measurement information of the system, see [158, 178], and real-time identification or selection of prediction models, see [49] and references therein. As stated in [112], the limitation of adaptive MPC is that it is challenging to satisfy the stability or even feasibility of the problem, especially when the uncertainty changes frequently.

The tuning task of MPC controllers has been widely investigated and general guidelines are available in the literature, see [64, 162, 164, 178, 189]. Some methods propose heuristics while others are based on stability criteria, closed-loop frequency-domain analysis, optimisation-based algorithms, genetic programming, on-line process identification, among others. The general approach in tuning procedures is to define MPC parameters off-line as constants for all the system operation but this fact could lead to decrease the system performance due to reduction of manoeuvrability. Other methods generate the complete Pareto frontier and select the best solution according to an extra criterion, but these approaches are computationally prohibitive in fast-dynamic or large-scale systems.

In order to face the aforementioned design issues, MPC algorithms have been extended or replaced with soft-computing techniques in different control architectures [176]. Most of these approaches are intended to improve performance by using expert-guidance or iterated experiments in order to simplify models of non-linear systems or to approximate and generalise by learning-based techniques the solution of optimal controllers, see [3, 10, 99, 141, 179, 195]. Intelligent control systems are able to replicate aggressive manoeuvres while performing adaptation, function approximation, knowledge modelling and massive parallel processing. Nevertheless, the main drawback of replacing MPC controllers with predictive soft-controllers is that they may not guarantee safety, stability or robustness due to the lack of feedback correction mechanisms for unmeasured disturbances so the performance is subject to the limited scenarios used in the learning process.

Therefore, implementation of adaptive structures and tractable on-line tuning procedures are still necessary to be integrated with robust MPC techniques to address some uncertainty explicitly in the controller calculation and to assure feasibility, economic efficiency and safety of complex multi-variable systems as generalised flow-based networks.

1.2.4 Economic MPC for the Management of Network Flows

As previously commented, control theory for the management of network flows is an active area of research, see e.g., [134, 157, 163]. Among advanced control techniques, MPC has proven to be one of the most effective and accepted control strategies for large-scale complex systems due to its flexibility to manage constraints and to optimise multi-objective problems as the ones encountered in the management of flow-based networks, see e.g. [124]. The basic idea of MPC is to exploit a model of the network to simulate its future evolution over a prediction horizon and compute an optimal control action (with respect to a predefined cost function) by solving, at each decision time instant, an open-loop optimisation problem in a receding horizon fashion [109].

Within the active research on MPC strategies for economic operation of systems, the predominant approach is to consider a time-invariant model of the system and a hierarchical control structure [175], where standard MPC controllers are designed for tracking economic/operational set-points that are usually computed in an upper layer with a real-time optimiser (RTO) or a steady-state target optimiser (SSTO), which use complex non-linear stationary models and usually larger sampling times than the regulatory MPC layer. Nevertheless, time-varying problems do arise in practice in dynamic networks, either because the plant has time-varying dynamics or because the performance refers to the process economics, which generally encompasses multiple time-dependent objectives, e.g., profitability, reliability, energy consumption, efficiency, etc. Hence, as discussed in [54], model inconsistencies, set-point changes, time-varying parameters, disturbances, and time-scale differences may lead the system to suboptimal economic performance and feasibility loss under the traditional hierarchical control scheme.

In order to tackle some of the main drawbacks of the typical hierarchical scheme and to take more economic profit from the transitory behaviour of the system, some authors have proposed to integrate the economic optimisation within the MPC using either a two-layer approach, see, e.g., [52, 190], or a single-layer approach, i.e., the so-called *economic MPC* [150]. The main challenge of this latter framework is the design of economic controllers with stability guarantees and a priori average performance bounds. Recent studies on control of flow-based networks are focused on the design of MPC controllers that directly optimise a (non-standard) economic cost function, see e.g., [1, 50, 70, 130, 142], not to obtain steady-state set-points but target trajectories for low-level PID controllers. As reviewed in [53], several formulations have been proposed in the literature to design controllers with desired theoretical properties. In former works, e.g., [48] and [7], average performance and Lyapunov-based stability analysis were proposed for schemes using a terminal equality constraint, which have been later relaxed in different ways, e.g., by using a terminal penalty and an ellipsoidal terminal constraint [6], generalised terminal constraints [55, 118], transient average constraints [119], generalised terminal region [120], Lyapunov-based constraints [78], or by removing the terminal constraints [74]. Most of these approaches consider time-invariant systems and time-invariant economic cost functions. Limited extensions are reported in the literature for the time-varying case. In [58], a generalised terminal constraint based MPC is proposed for time-invariant systems with the aim of retaining feasibility under possible changes of the economic cost function (which remains the same along the prediction horizon). In [51], the cost function is considered time-dependent and Lyapunov-based constraints are used to guarantee stability. Other few works are particularly specialised in enforcing periodic operation of the plant by means of MPC schemes relying on periodically time-varying terminal equality constraints, see e.g., [82, 105, 194]. Despite the advances in the economic MPC framework and its practical advantages to be applied in dynamic network flows control, there still are open issues to be addressed [53], such as robustness considerations and non-centralised schemes for large-scale networks.

1.2.5 Non-centralised MPC for Large-scale Networks

It has been already stated that network flow problems are generally associated with large-scale systems or with networks composed by several interacting subsystems, in which achieving high specifications of reliability, efficiency and profitability is a must. Traditional MPC procedures assume that all available information is centralised, i.e., a global dynamical model of the system must be available for control design and all measurements must be collected in one location to estimate all states and to compute all control actions, which give the best possible performance. However, when considering large-scale dynamic flow-based networks, these assumptions usually fail to hold, either because gathering all measurements in one location is not feasible, or because the computational needs of a centralised strategy are too demanding for a real-time implementation. This fact might lead to a lack of scalability. Subsequently, a model change would require the re-tuning of the centralised controller. Thus, the cost of setting up and maintaining the monolithic solution of the control problem is prohibitive. In such a case, a centralised control architecture could not be an adequate choice.

A way of circumventing these issues is to decompose the associated dynamic control problem into a number of smaller problems, and looking into *multi-agent* or *non-centralised* control architectures, such as: *decentralised*, *distributed* or *hierarchical* control, where a set of local controllers (usually denoted as *agents*) are in charge of controlling partitions of the entire system [122]. Those techniques have become one of the hottest topics in control during the early twenty-first century, opening the door to research toward solving new open issues and related problems of the strategy. Many approaches have been developed in this area and relevant surveys are available in the literature, see e.g., [39, 123]. The selection of a specific control architecture for a given network flow problem inherently depends on the application, the process properties (e.g., system type, control objective, coupling sources, randomness) and the technological limitations (e.g., communication and processing constraints). Despite the benefits of non-centralised control architectures, they also have some drawbacks that have to be taken into account, the most important being the loss of performance in comparison with a single centralised controller and the difficulty to guarantee feasibility. The solutions to these issues rely on the degree of interaction between the local subsystems and the coordination/communication mechanisms between their agents. When designing non-centralised controllers for large-scale networks, there is a prior problem to be solved: the *system decomposition* into subsystems, see e.g., [87, 108, 116, 128, 167]. In this thesis, it is assumed that the decomposition of the initial large-scale system into small-scale interacting subsystems is already given.

In [123], a taxonomic discussion of the current state of the art of non-centralised MPC control architectures is presented, with special attention to distributed MPC schemes. In decentralised predictive controllers, local agents usually do not communicate, although in some works information exchange (such as measurements and previous control decisions) is only allowed before and after the decision-making process but without negotiation between agents, i.e., control actions are decided independently [14]. Consequently, the worst overall network performance can be achieved. A full decentralisation of the problem is applicable only for weakly coupled systems, where interactions can be neglected by local agents, otherwise, a loss of feasibility and instability issues may arise. To avoid this latter, some decentralised schemes allow a minimum exchange of information and consider interactions among subsystems as disturbances to be rejected, see e.g., [127, 145, 153]. Nevertheless, it was shown in [148] that modelling the interactions between subsystems and exchanging trajectory information among controllers is insufficient to provide even closed-loop stability due to the inherent competition of local agents. Hence, some level of negotiation or coordination mechanism is necessary to lead local agents to improve overall performance and avoid instability. In this line, there have been developed and published plenty of distributed and hierarchical MPC schemes, lying between the centralised and the fully decentralised extremes, see [123] for details.

Among these latter results, some methods compute their control actions using iterative communication rounds, while others decide them in a non-iterative fashion. In general, non-iterative methods are usually non-cooperative, while the iterative ones are cooperative and usually rely on distributed optimisation techniques. Moreover, the local agents may decide their control actions in parallel or following a hierarchical and sequential updating process. Some approaches are exclusively designed for networks of systems with decoupled dynamics but with coupled costs or coupled constraints, while other approaches are able to cope with coupled dynamics, i.e., state-coupled, input-coupled or both. Few approaches consider the case of having coupled dynamics, coupled costs and coupled constraints, simultaneously. This latter case is the one of interest of this thesis given that in flow-based networks there often appears coupling (equality and/or inequality) constraints and coupled inputs due to subsystem flow interactions and shared resources. In such a case, suitable control strategies arise from cooperative game theory and from decomposition techniques in mathematical programming. A brief discussion of relevant works is presented below. For further details on other distributed MPC techniques, the reader is referred to the aforementioned surveys.

In [68], the distributed optimisation problem is considered as a dynamical game with coupled control sets. The original problem is decomposed into smaller coupled problems in a distributed structure, which is solved iteratively using the theory of potential games. The approach guarantees feasibility of the control algorithm if the starting point is a feasible solution, relying on a candidate control sequence with zero terminal control. Nevertheless, this candidate control sequence might not be feasible in flow problems with input-coupled constraints depending on external signals such as, e.g, time-varying demands. A similar approach is reported in [110], where a stabilising distributed MPC scheme based on agent negotiation is proposed for input-coupled systems with quadratic cost functions, where local agents negotiate asynchronously a cooperative decision at each sampling time, following a proposal-acceptance protocol to improve an initial feasible solution and considering the welfare of the neighbourhood. In such scheme, the controllers are capable to retain feasibility and to guarantee closed-loop stability of the overall system by an optimal design of local feedback control laws and invariant terminal sets. This approach also lacks of mechanisms to cope with coupled equality constraints. In order to cope with the difficulties imposed by the coupling constraints, some distributed schemes based on primal and dual decompositions of the optimisation problem have been proposed. In [124], schemes derived from an overall augmented Lagrange formulation in combination with either a block coordinate descent or the auxiliary problem principle are proposed. The main disadvantage of dual decomposition methods is the fact that primal feasibility is only attained asymptotically. Thus, if early termination of the algorithms is required, no primal feasible solution for the centralised MPC problem can be guaranteed and neither stability. This latter problem was avoided in the cooperative distributed MPC schemes proposed in [57, 170], which rely on subsystems sharing the overall cost function and having knowledge of the centralised model. In these approaches, no coordination layer is employed and terminating the iteration of the distributed controllers prior to convergence retains feasibility and consequently closed-loop stability; besides, in the limit of iterating to convergence, the obtained control action leads to plant-wide Pareto optimality and is equivalent to the centralised solution, even under sparsely input-coupled constraints. This cooperative distributed MPC scheme has been recently extended in the framework of economic MPC [101], but loosing the capability of achieving Pareto optimality in the limit of the iterations due to the appearance of non-sparse coupled constraints in the economic optimisation problem, which leads to converge to non-optimal fixed points. These fixed points have been avoided, within the standard MPC framework, in the scheme proposed in [33] for the control of linear dynamic networks, which relies on a distributed gradient-based algorithm for implementing an interior-point method distributively with a network of agents. Another interesting result, successfully used in the control of energy networks [8, 83], is the distributed optimisation scheme proposed in [126], which is based on the optimality condition decomposition. In this latter approach, a coordinated solution of the global problem is achieved in a decentralised manner; the coordinator does not update information but collects and distributes it, and most important, there is no need to solve sub-problems until optimality in each sampling time (as required in other decomposition approaches).

1.3 Thesis Objectives

This thesis focuses on exploiting the MPC framework to design optimal controllers for networks subject to constraints and to persistent and fluctuating disturbances. Particularly, the management of dynamic network flows within a multi-objective optimisation framework is studied, considering demands as system disturbances. Therefore, the main goal of this thesis is to develop economic MPC flow controllers that use the propagation of uncertainty through the decision-making process and explicitly consider service reliability and actuators degradation to guarantee system availability and demand satisfaction with a given confidence level.

To achieve the main goal of this thesis, some specific objectives have been proposed as follows:

1. To consider the stochastic nature of disturbances and analyse the impact of the open-loop feedforward uncertainty in the MPC strategy.
2. To design robust MPC strategies capable to set optimal safety amounts of commodity storage to face demand uncertainty with an efficient constraint handling.
3. To model degradation of actuators and their reliability as a function of applied control effort.
4. To propose a prognostics and health management method within the MPC formulation in order to efficiently distribute control effort between actuators and guarantee system availability for a given maintenance horizon.
5. To explore the MPC tuning state of the art and propose a practical strategy with computational efficiency for on-line use as a tool for automatic decision making. Attention must be put in memory management and solving time.
6. To design MPC controllers for the finite-time horizon minimum cost dynamic flow problem to achieve an economically optimal operation of a dynamic flow-based network.
7. To design non-centralised economic MPC strategies to control dynamic flow-based networks of large size.
8. To implement the designed controllers and tuning strategies on the Barcelona drinking water network (DWN) as the case study of large-scale complex systems, comparing results with baseline controllers and analysing advantages and disadvantages of the approaches proposed in this thesis.

1.4 Outline of the Thesis

This dissertation is organised in four parts. The first part is dedicated to discuss the state of the art of different topics that are relevant to the control of network flows and introduces fundamental concepts and models for the study of generalised flow-based networks as well as a baseline MPC strategy for the integration of scheduling and control of dynamic network flows. The second part of this dissertation describes different centralised MPC schemes developed here to enhance the economic and robust operation of generalised flow-based networks subject to additive uncertainty and periodic dynamics. The main results in this part are a reliability-based MPC controller, a chance-constrained MPC controller, a scenario tree-based MPC controller, a periodic economic MPC controller, and a learning-based self-tuning MPC controller for the management of dynamic network flows. The third part of the dissertation is devoted to the design of non-centralised economic MPC strategies that cope with some of the difficulties encountered in the centralised control of large-scale networks. Finally, the fourth part of the dissertation summarises the main results and contributions of the thesis and states some avenues for future research.

A detailed summary of the posterior chapters conforming the different parts of this dissertation is given below.

Chapter 2: Generalised Flow-based Networks

This chapter presents mathematical preliminaries about the systems and problems considered in this thesis, in order to support the understanding of the developments that are proposed in this research. Especially, modelling principles and common operational objectives in dynamic-flow control are stated. A baseline centralised MPC strategy for the control of generalised flow-based networks is also introduced. Moreover, a selected case study corresponding to the drinking water network of the city of Barcelona (Spain) is described as an example of the minimum cost dynamic flow problem addressed in this thesis by means of the MPC framework.

Chapter 3: Economic MPC for Periodic Generalised Flow-based Networks

This chapter explores the application of recent results on economic MPC for the periodic operation of generalised flow-based networks. Moreover, an economic MPC formulation with time-varying terminal cost and terminal region is proposed for controlling non-linear periodic systems, relaxing existent results that use more restrictive terminal periodic equality constraints. In addition, some single-layer economic MPC formulations are proposed to cope with the feasibility loss that could happen in standard two-layer control architectures when requiring the pre-calculation of the optimal scheduling trajectory under time-varying cost functions. This chapter is based on the following publications:

- J.M. Grosso, C. Ocampo-Martinez, V. Puig, D. Limón, and M. Pereira. Economic MPC for the management of drinking water networks. In *13th European Control Conference (ECC)*, pages 790-795, Strasbourg, France, June 2014.
- D. Limón, M. Pereira, D. Muñoz de la Peña, T. Alamo and J.M. Grosso. Single-layer economic model predictive control for periodic operation. *Journal of Process Control*, 24(8):1207-1224, 2014.
- J.M. Grosso, M.A. Müller, C. Ocampo-Martinez, V. Puig, and F. Allgöwer. On economic model predictive control for periodic systems. *Automatica*, March 2014 (to be submitted).

Chapter 4: Reliability-based MPC of Generalised Flow-based Networks

This chapter analyses the main ideas on service reliability of networks, dynamic safety stock planning, and degradation of equipment health, in order to develop a reliability-based MPC strategy for the management of generalised flow-based networks. The proposed controller is based on a bi-level optimisation problem with dynamic constraints, with the aim to improve the robust performance of the baseline MPC controller presented in Chapter 2. Two enhancements of the baseline controller are presented and incorporated in the MPC algorithm. The first enhancement considers supply-chain management theory to compute an optimal inventory replenishment policy based on customer service level satisfaction, leading to dynamically allocate safety stocks of water in the network to satisfy periodic demands with stationary stochastic characteristics. The second enhancement computes a smart distribution of the control effort and maximises availability of actuators by estimating their degradation and reliability. This chapter is based on the publication:

- J.M. Grosso, C. Ocampo-Martinez, and V. Puig. Reliability-based economic model predictive control for generalised flow-based networks including health-aware capabilities. *Optimal Control Applications and Methods*, 2015 (submitted).

- J.M. Grosso, C. Ocampo-Martinez, and V. Puig. A service reliability model predictive control with dynamic safety stocks and actuators health monitoring for drinking water networks. In *IEEE 51st Annual Conference on Decision and Control (CDC)*, pages 4568-4573, Maui, Hawaii, USA, December 2012.

Chapter 5: Stochastic MPC for Robustness in Generalised Flow-based Networks

This chapter proposes a robust control strategy for the management of generalised flow-based networks based on finite-horizon stochastic optimisation problems with probabilistic constraints. Particularly, two different stochastic programming approaches are explored: chance-constrained model predictive control (CC-MPC) and tree-based model predictive control (TB-MPC). Under the former approach, flow demands are modelled as stochastic variables with a non-stationary uncertainty description, unbounded support and a known (or approximated) quasi-concave probabilistic distribution. A deterministic equivalent of the stochastic problem is formulated using Boole's inequality to decompose joint chance constraints into a set of individual chance constraints and, by considering a uniform allocation of risk, to bound these latter constraints. In the latter approach, demands are modelled as a disturbance tree. The most probable evolutions of the demand are modelled as branches of the tree. In both cases, an MPC controller is used to optimise the expected value of the system variables taking into account the disturbances. This chapter is based on the following publications:

- J.M. Grosso, C. Ocampo-Martinez, V. Puig, and B. Joseph. Chance-constrained model predictive control for drinking water networks. *Journal of Process Control*, 24(5):504-516, 2014.

- J.M. Grosso, J.M. Maestre, C. Ocampo-Martinez, and V. Puig. On the assessment of tree-based and chance-constrained predictive control approaches applied to drinking water networks. In *19th IFAC World Congress*, pages 6240-6245, Cape Town, South Africa, August 2014.

- A.K. Sampathirao, J.M. Grosso, P. Sopasakis, C. Ocampo-Martinez, A. Bemporad and V. Puig. Water demand forecasting for the optimal operation of large-scale drinking water networks. In *19th IFAC World Congress*, pages 10457-10462, Cape Town, South Africa, August 2014.

Chapter 6: Learning-based Tuning of Supervisory MPC for Generalised Flow-based Networks

In this chapter, a learning-based tuning strategy of MPC controllers for the management of generalised flow-based networks is described. The strategy relies on the use of soft-control techniques to incorporate self-tuning capabilities of controllers. The overall control architecture presents a hierarchical scheme with a learning and planning layer based on artificial neural networks, a supervision and adaptation layer based on a fuzzy inference system and a feedback control layer. The proposed MPC controller leads to improved computational time, especially when the complexity of the problem structure can grow with system dimensionality and non-linear computations. This chapter is based on the following publications:

- J.M. Grosso, C. Ocampo-Martinez, and V. Puig. Learning-based tuning of supervisory model predictive control for drinking water networks. *Engineering Applications of Artificial Intelligence*, 26(7):1741-1750, 2013.
- J.M. Grosso, C. Ocampo-Martinez, and V. Puig. Adaptive multilevel neuro-fuzzy Model Predictive Control for Drinking Water Networks. In *IEEE 20th Mediterranean Conference on Control and Automation (MED)*, Barcelona, Spain, July 2012.

Chapter 7: Multi-layer Non-iterative Distributed Economic MPC

In this chapter, a multi-layer non-iterative distributed economic MPC approach is analysed for its application to large-scale generalised flow-based networks. This approach is based on the periodic nature of the system disturbance and the availability of both static and dynamic models of the network. The topology of the controller is structured in two layers. First, an upper layer is in charge of achieving a set of global objectives and works with a sampling time corresponding to the disturbance period. Second, a lower layer, with a higher frequency of sampling is in charge of computing the references for the system actuators in order to satisfy a set of local objectives. A system partitioning allows to establish a hierarchical flow of information between a set of MPC controllers and restrictive assumptions are required to guarantee recursive feasibility of the control scheme. This chapter is partially based on the following publication:

- C. Ocampo-Martinez, V. Puig, J.M. Grosso, and S. Montes de Oca. Multi-layer decentralized MPC of large-scale networked systems. In *Distributed Model Predictive Control Made Easy*, pages 495-515, Springer Netherlands, 2014.

Chapter 8: Distributed Economic MPC with Global Communication

In this chapter, a cooperative distributed economic MPC approach for the control of dynamic networks formed by input- and cost-coupled subsystems with convex cost functions is explored. The control scheme relies on the availability of global model information of interacting plants and on the broadcasting of predicted trajectories. Conditions to guarantee robust feasibility are introduced by means of robustness constraints based on periodic robust control invariant sets.

Chapter 9: Concluding Remarks

This chapter summarises the contributions made in this thesis and discusses the main concluding ideas and open issues for future research.

Chapter 2

Generalised Flow-based Networks

This chapter presents mathematical preliminaries about the systems and problems considered in this thesis. Especially, modelling principles and common operational objectives in dynamic flows control are stated. A baseline centralised MPC strategy for the control of generalised flow-based networks is also introduced. Moreover, a selected case study corresponding to the drinking water network of the city of Barcelona (Spain) is described as an example of the minimum cost dynamic flow problem.

2.1 Introduction

Several real problems concerning production, transportation and distribution of some commodity(ies) can be addressed by generalised flow-based network models, which are described by a *directed graph* consisting of a set of *nodes* (or *vertices*) that represent locations, and a set of *arcs* (or *edges*) that model links between nodes and flow direction. Basically, such problems consist in determining a strategy to decide the arc flows in order to transport the commodity through the network to satisfy the demand. Typically, nodes and arcs of the graph are subject to capacity constraints and to some costs associated with their use. The commodity that is being transported over the network, however, determines how the overall system functions. The literature on this subject is extensive; the reader is referred to the references discussed in § 1.1.

In particular, dynamic networks (i.e., networks in which flows, storage levels, costs, demands and other network parameters are time-varying quantities) have received great attention and are the main interest of this thesis. A typical problem concerning this kind of systems consists of planning the commodity flows and storage levels at each time in order to minimise transportation and holding costs. If the supply and demand are known in advance for a planning horizon, the dynamic problem can be handled via the time-expanded network method (see, [59]), which requires a duplication of the given network for each time step. This fact makes the time-expanded models non suitable to tackle real-size problems, even more if uncertainty comes to play.

In order to tackle the aforementioned problems, control theory has emerged as a great tool to deal with dynamic networks, see, [20, 22, 23]. In these references, the basic goal was that of keeping the system state (represented by the buffer levels in storage units) inside prescribed box constraints for all possible unknown (but bounded) inputs by using minimum cost flows that are subject to hard bounds. The aforecited works have basically extended, within the control theory framework, the feasibility conditions shown in [59, 63, 85], to networks where all the nodes are dynamic, i.e., they have storage capacity. The main result is that the existence of a feasible control strategy is guaranteed if and only if the storage capacity is large enough to absorb the immediate uncertainty effect and if the controlled admissible flows dominate the demands. Since verifying these conditions could be computationally demanding (or even NP-hard), a decentralised analysis of the feasibility conditions was proposed in [13] for polyhedral constraints following the results in [23]. A common situation found in the state of the art is that all nodes are considered to be either static or dynamic, but there are no conditions for the situation where both kind of nodes are present in the system. In the network flow framework, this latter situation has been analysed transforming dynamic nodes into delayed flows through arcs, but again these models are rather cumbersome.

In this thesis, the *dynamic minimum cost flow* problem is the central object of study, considering both dynamic and static nodes. Throughout the thesis, it is supposed that there is a *single commodity* to be routed through a *multi-terminal* network, that is, a network with multiple source nodes and sink nodes. The capacities, costs and demands are subject to fluctuations over time with constant (zero) transit times. In the following, a detailed description of the problem as well as a mathematical formulation are given.

2.2 Modelling and Problem Statement

2.2.1 Networks as Directed Graphs

A *generalised flow-based network* is here denoted as $\mathcal{N} = (\mathcal{G}, p, \mathcal{S})$, which consists of a *directed graph* $\mathcal{G} = (\mathcal{V}, \mathcal{A})$ formed by a finite set of *nodes* $\mathcal{V} \subseteq \mathbb{Z}_{\geq 1}$, and a finite set of *arcs* $\mathcal{A} \subseteq \mathcal{V} \times \mathcal{V}$, with an arc $a \in \mathcal{A}$ being an ordered link between a pair of nodes (i, j) with $i, j \in \mathcal{V}$, whose order indicates the direction of the flow between the two nodes. The network has a special subset of nodes $\mathcal{S} \subset \mathcal{V}$ called *terminals*. A terminal is either a *source* or a *sink*. The functioning of the network is driven by a vector function p containing the functions that define the dynamic attributes of the graph, i.e., capacities, transit times, gains, supplies, demands. It is assumed that only the attributes conforming p are time varying, while the structure of the network (defined by $\mathcal{G}$ and $\mathcal{S}$) remains unchanged. For an arc $a = (i, j) \in \mathcal{A}$ with $i, j \in \mathcal{V}$, one refers to i as the tail (or start node) and j as the head (or end node). Two arcs $a, a' \in \mathcal{A}$, $a \neq a'$, are called parallel, if they have the same start and end node, i.e., $a = (i, j)$ and $a' = (i, j)$ for some $i, j \in \mathcal{V}$. Moreover, an arc $a = (i, i) \in \mathcal{A}$, with the same start and end node $i \in \mathcal{V}$, is called a loop. For a given node i, the following sets are defined:

$$\delta_i^+ := \{a \in \mathcal{A} \mid a = (j, i) \text{ for some } j \in \mathcal{V}\}, \tag{2.1a}$$

$$\delta_i^- := \{a \in \mathcal{A} \mid a = (i, j) \text{ for some } j \in \mathcal{V}\}, \tag{2.1b}$$

$$\delta_i := \delta_i^+ \cup \delta_i^-, \tag{2.1c}$$

$$\mathcal{N}_i^+ := \{j \in \mathcal{V} \mid a = (j, i) \in \delta_i^+\}, \tag{2.1d}$$

$$\mathcal{N}_i^- := \{j \in \mathcal{V} \mid a = (i, j) \in \delta_i^-\}, \tag{2.1e}$$

$$\mathcal{N}_i := \mathcal{N}_i^+ \cup \mathcal{N}_i^-, \tag{2.1f}$$

where δ_i^+ and δ_i^- are the sets of *incoming* and *outgoing* arcs of node $i \in \mathcal{V}$, respectively. Similarly, $\mathcal{N}_i^+$ is the set of adjacent nodes sending the incoming arcs of node i, and $\mathcal{N}_i^-$ is the set of adjacent nodes receiving the outgoing arcs of node i. Hence, δ_i is the set of all *incident* arcs connected to node i, while $\mathcal{N}_i$ represents its *neighbourhood*, that is, the set of all *adjacent* nodes interacting with node i.

Figure 2.1 shows an example of a network directed graph, with $\mathcal{V} = \{1, 2, 3, 4, 5, 6, 7, 8\}$ and $\mathcal{A} = \{(1, 3), (2, 3), (3, 5), (3, 6), (4, 6), (4, 7), (6, 8)\}$. The terminal nodes in such example form the set of source nodes $\{1, 2\}$ and the set of sinks $\{5, 7, 8\}$. The rest of nodes

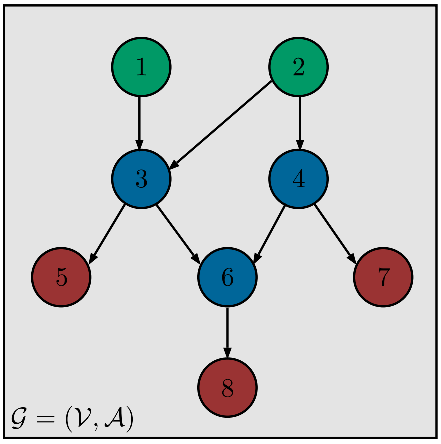


Figure 2.1: Example of a network directed graph. Source nodes (green), sink nodes (red), intermediate nodes (blue)

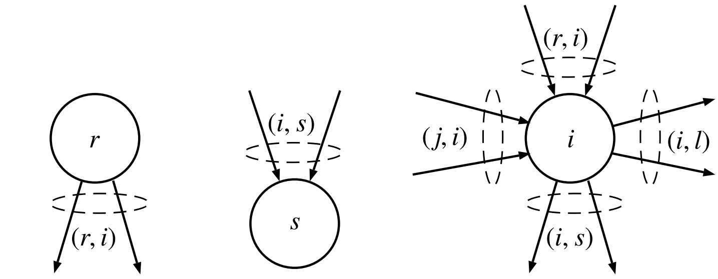


Figure 2.2: Characterisation of network nodes

$\{3, 4, 6\}$ form the set of intermediate nodes. Figure 2.2 shows the structural differences between each sort of node. Note that a source node r has only outgoing arcs (i.e., $\delta_r^- \neq \emptyset$, $\delta_r^+ = \emptyset$), a sink node s has only incoming arcs (i.e., $\delta_s^+ \neq \emptyset$, $\delta_r^- = \emptyset$) and an intermediate node i has both incoming and outgoing arcs (i.e., $\delta_i^- \neq \emptyset$, $\delta_r^+ \neq \emptyset$).

2.2.2 Network Attributes

As mentioned before, the dynamic nature of the network $\mathcal{N}$, with $n_v := |\mathcal{V}|$ nodes, $m_a := |\mathcal{A}|$ arcs and time horizon $T \in \mathbb{Z}_+$, comes from the behaviour of a set of attributes contained in p that define the network functioning. These attributes are described next.

Load and flow rate. In a dynamic model, the commodity being transported could spend a non zero transit time on the arc. Hence, it is important to differentiate two attributes of dynamic flows (also called flows over time): the *load* and the *flow rate*, which are the quantity of commodity on the arc and the amount of flow that enters the arc per unit of time, respectively [97]. The main interest here for dynamic models is then the flow rate, usually denoted just as *flow*. A dynamic flow in network $\mathcal{N}$ is a time-dependent function $u_a : \mathcal{A} \times \mathbb{Z}_{[0,T]} \to \mathbb{R}_+$ that assigns to every arc $a \in \mathcal{A}$, at each time step $k \in \mathbb{Z}_{[0,T]}$, a flow rate value $u_{a,k}$.

Supplies and demands. Each node $i \in \mathcal{V}$, has either a net available supply or net required demand of flow, given by a function $b : \mathcal{V} \times \mathbb{Z}_{[0,T]} \to \mathbb{R}$, which is often called *balance*. Notice that supplies and demands are constrained to be finite. Remind that the network structure contains a special subset $\mathcal{S} \subset \mathcal{V}$ containing the terminal nodes. These nodes are defined from their balance attribute, for all time steps $k \in \mathbb{Z}_{[0,T]}$, as follows: a node $r \in \mathcal{V}$ is denoted as a terminal *source* node if $r \in \mathcal{S}^+ := \{r \in \mathcal{S} \mid b_{r,k} > 0\}$ and their value $b_{r,k}$ is referred to as *supply*, while a node $s \in \mathcal{V}$ is denoted as a terminal *sink* node if $s \in \mathcal{S}^- := \{s \in \mathcal{S} \mid b_{s,k} < 0\}$ and their balance $b_{s,k}$ is referred to as *demand*. It is assumed, without loss of generality, that $\sum_{i \in \mathcal{S}} b_{i,k} = 0$ for all k. Thus, it follows that that $\mathcal{S} = \mathcal{S}^+ \cup \mathcal{S}^-$. The rest of nodes $i \in \mathcal{V} \setminus \mathcal{S}$, are called *intermediate* nodes.

Flow capacity. The physical constraints of the network elements imply a flow capacity given by bounding functions $u_{\max}$ and $u_{\min}$, which represent the maximum and minimum amount of flow on the arc $a \in \mathcal{A}$, respectively. Here, it is assumed, without loss of generality, that $u_{\max} : \mathcal{A} \times \mathbb{Z}_{[0,T]} \to \mathbb{R}_+$ and $u_{\min} : \mathcal{A} \times \mathbb{Z}_{[0,T]} \to \{0\}$. Any non-zero lower bound can be transformed into a zero bound by simple translations of the limits. Hence, a dynamic flow is subject to the following non-negative capacity constraint:

$$0 \le u_{a,k} \le u_{\max}(a,k), \quad \forall a = (i,j) \in \mathcal{A}, \forall i,j \in \mathcal{V}, \forall k \in \mathbb{Z}_{[0,T]}. \tag{2.2}$$

Storage capacity. Each node $i \in \mathcal{V} \setminus \mathcal{S}$ has a storage function $x_i : \mathcal{V} \times \mathbb{Z}_{[0,T]} \to \mathbb{R}_{\ge 0}$, which is induced by the flows from incident arcs $a \in \delta_i$ and a given initial storage value $x_{i,0}$. This function assigns to every node i, at each time step $k \in \mathbb{Z}_{[0,T]}$, a non-negative value $x_{i,k}$, which measures the amount of flow stored at node i from step $k-1$ to k. From this attribute, it is possible to differentiate two other kinds of nodes in a network: static nodes and dynamic nodes. In the static ones, the transshipment of the commodity is immediate since they do not have any storage (also called holdover) capacity, i.e., $x_{i,k} = 0$ for all $k \in \mathbb{Z}_+$. Contrary, dynamic nodes have a finite storage function $x_{i,k} \in \mathbb{R}_{\geq 0}$ for all k. Here, it is assumed that dynamic nodes have finite *storage capacity* given by $x_{\max} : \mathcal{V} \times \mathbb{Z}_{[0,T]} \to \mathbb{R}_+$ and $x_{\min} : \mathcal{V} \times \mathbb{Z}_{[0,T]} \to \{0\}$. Again, any non-zero lower bound can be transformed into a zero bound by simple translations of the limits. Let $\mathcal{Y} \coloneqq \{i \in \mathcal{V} \setminus \mathcal{S} \mid x_{i,k} \geq 0, \ \forall k\}$ denote the set of dynamic nodes. Hence, the storage function satisfies the following capacity constraints:

$$0 \leq x_{i,k} \leq x_{\max}(i,k), \quad \forall i \in \mathcal{Y}, \ \forall k \in \mathbb{Z}_{[0,T]}, \tag{2.3}$$

$$x_{i,k} = 0, \quad \forall i \in \{\mathcal{V} \setminus \mathcal{S}\} \setminus \mathcal{Y}, \ \forall k \in \mathbb{Z}_{[0,T]}. \tag{2.4}$$

Costs. Each arc $a \in \mathcal{A}$ has, without loss of generality, an associated non-negative *flow cost* given by the function $c_a : \mathbb{Z}_{[0,T]} \to \mathbb{R}_+$, which determines the cost per flow unit for transport the commodity through arc a at time $k \in \mathbb{Z}_{[0,T]}$. Similarly, each dynamic node $i \in \mathcal{V} \setminus \mathcal{S}$ has an associated non-negative *storage cost* given by $c_i : \mathbb{Z}_{[0,T]} \to \mathbb{R}_+$, which determines the cost of storing one unit of flow at node i from time step $k-1$ to k.

Transit times. Each arc $a = (i,j) \in \mathcal{A}$, has a transit time $\tau_{a,k} \in \mathbb{Z}_{[0,T]}$ for each time step $k \in \mathbb{Z}_{[0,T]}$, which is the amount of time a unit of flow needs to travel from node i to node j, entering the arc a at time step k, i.e., a flow entering node i at time k will arrive node j at time $k + \tau_{a,k}$.

Gains. Each arc $a \in \mathcal{A}$ has a gain $\gamma_{a,k}$ for each time step $k \in \mathbb{Z}_{[0,T]}$, such that if $u_{a,k}$ units of flow enter an arc a at time k, there will be $\gamma_{a,k} u_{a,k}$ units leaving the arc at time $k + \tau_{a,k}$. Gains determine the amount of flow changes while traversing an arc, and are useful to model network phenomena such as pipe leaks and evaporation in hydraulic networks, taxes or interest in financial networks, energy losses in electrical or thermal networks, among others. A network with $\gamma_{a,k} < 1$ is called *lossy*.

2.2.3 Dynamic Minimum Cost Flow Problem

After defining the attributes of the network elements, it can be stated that a *feasible dynamic flow* is such that satisfies the capacity constraints (2.2) and (2.3), and the following *flow conservation constraints* for all $k \in \mathbb{Z}_{[0,T]}$:

$$x_{i,k+1} = b_{i,k} + x_{i,0} - \sum_{a \in \delta_i^-} \sum_{\theta=0}^{k} u_{a,\theta} + \sum_{a \in \delta_i^+} \sum_{\theta=\tau_a}^{k} \gamma_a u_{a,\theta-\tau_a}, \quad \forall i \in \mathcal{Y}, \tag{2.5a}$$

$$0 = b_{i,k} - \sum_{a \in \delta_i^-} \sum_{\theta=0}^{k} u_{a,\theta} + \sum_{a \in \delta_i^+} \sum_{\theta=\tau_a}^{k} \gamma_a u_{a,\theta-\tau_a}, \quad \forall i \in \{\mathcal{V} \setminus \mathcal{S}\} \setminus \mathcal{Y}. \tag{2.5b}$$

Next, a network flow problem is stated that is of interest for this thesis: the discrete-time dynamic minimum cost flow problem of a nominal capacitated network, usually expressed as a constrained linear programming problem given by

$$\min_{u_{a,[0,T]}} \sum_{t=k}^{T} \left[\sum_{a \in \mathcal{A}} c_{a,k} u_{a,k} + \sum_{i \in \mathcal{Y}} c_{i,k} x_{i,k} \right], \tag{2.6}$$

subject to (2.2), (2.3) and (2.5). Note that due to the possibly time-varying nature of the network attributes, the choice of flows has to be made at successive times, i.e., the operation of the network requires re-scheduling.

In this thesis, the following attribute assumptions are considered for the addressed cost flow problems.

Assumption 2.1 (Zero transit times)**.** *The network operates in a* push-flow *regime with* $\tau_{a,k} = 0$ *for all* $a \in \mathcal{A}$ *and all* $k \in \mathbb{Z}_+$.

Assumption 2.2 (Normal flow conservation)**.** *The flow through each arc does not experience any gain or loss, i.e.,* $\gamma_{a,k} = 1$ *for all* $a \in \mathcal{A}$ *and all* $k \in \mathbb{Z}_+$.

Assumption 2.3 (Constant capacity bounds)**.** *The flow and storage capacities are time invariant.*

Assumption 2.4 (Costs)**.** *The flow cost for each arc and storage cost for each dynamic node are perfectly known for all time instants. Both costs can be time varying.*

Consequently, the flows are computed for the given costs based on measured and forecasted demands and pushed through the arcs as required, maintaining equal inflow rates and outflow rates, and keeping the arcs fully loaded.

It seems contradictory to work with zero transit times in dynamic networks since non-zero times is the main difference from static networks. However, as pointed out in [97], an important additional feature of dynamic networks is their storage capability, which allows to send more flow in a dynamic network with zero transit times than the maximum flow in the corresponding static network, by delaying part of the flow at the source or at intermediate nodes. The storage capability also allows to exploit the forecasting of time-varying demands and costs to achieve an economically efficient operation.

2.3 Integrating Scheduling and Control

Even though the dynamic networks framework considers flows over a time horizon, the dynamic minimum cost flow problem, often used for planning and scheduling of flows under nominal demands, is still a static open-loop optimisation problem based on current available data. Therefore, when deviations from nominal demand patterns occur, reactive scheduling is required. As reviewed in [172], several methods have been proposed in the literature to address such re-scheduling task within both the supply-chain theory and the control theory fields, most of them based on simple re-optimisation that does not necessarily lead to good closed-loop performance. Hence, as discussed in the aforementioned reference, the MPC framework can be successfully used to address some of the scheduling limitations, such as guaranteeing feasibility of the optimisation problem at each time instant and possibly convergence to a desired set-point.

This thesis relies on the MPC framework to integrate, under a single problem, the scheduling of dynamic minimum cost flows with other control objectives, e.g., safety-stocks optimisation, smoothness operation, among others that are not commonly considered in the standard MPC cost function where quadratic costs of deviation from targets are often used. Traditionally, control systems are usually implemented by means of a two-layer architecture, where the scheduling is solved in an upper optimisation layer and the control actions computed in a (follow-up) lower layer usually with feedback PID controllers [174]. Contrary, in this section an MPC controller is designed to compute the control actions by directly optimising a general convex cost function including the process economics, what lies in the so-called economic MPC (see the discussion in § 1.2.4).

2.3.1 Control-oriented Model

In this section, the model of a dynamic network $\mathcal{N}$ is generalised by considering the network as a constrained linear dynamic system, which is represented by a discrete-time state-space model. To do so, consider first the directed graph $\mathcal{G}(\mathcal{V}, \mathcal{A})$ of the network, which has $n_v = |\mathcal{V}|$ nodes and $m_a = |\mathcal{A}|$ arcs, and build the corresponding incidence matrix of the graph.

The incidence matrix, denoted as $B_{\mathcal{N}} \in \mathbb{R}^{n_v \times m_a}$, is defined by its elements

$$[B_{\mathcal{N}}]_{ij} := \begin{cases} 1 & \text{if arc } j \in \delta_i^+ \\ -1 & \text{if arc } j \in \delta_i^- \\ 0 & \text{otherwise} \end{cases}$$

where $i \in \mathcal{V}$ and $j \in \mathcal{A}$ indicate respectively the row and column of the matrix element, and δ_i^+ and δ_i^- are the sets of incoming and outgoing arcs of node i respectively. Moreover, the incidence matrix can be split in two matrices, i.e., $B_{\mathcal{N}} = B_{\mathcal{N}}^u + B_{\mathcal{N}}^d$, whose components are respectively defined as

$$[B_{\mathcal{N}}^u]_{ij} := \begin{cases} 1 & \text{if arc } j \in \delta_i^+ \\ -1 & \text{if arc } j = (i,z) \in \delta_i^- \text{ and node } z \in \mathcal{N}_i^- \setminus \mathcal{S}^- \\ 0 & \text{otherwise} \end{cases}$$

$$[B_{\mathcal{N}}^d]_{ij} := \begin{cases} -1 & \text{if arc } j = (i,z) \in \delta_i^- \text{ and node } z \in \mathcal{S}^- \\ 0 & \text{otherwise} \end{cases}$$

where $\mathcal{N}_i^-$ is the set of adjacent nodes receiving the outgoing arcs of node i, and $\mathcal{S}^-$ is the set of terminal nodes categorised as sinks.

In addition, redefine x (with some abuse of notation) as a state vector $x \in \mathbb{R}^n$ that collects the storage of all dynamic nodes $i \in \mathcal{Y} \subset \mathcal{V}$. Similarly, define the vector $u \in \mathbb{R}^m$ of controlled inputs as the collection of the flow rate through the arcs $j \in \mathcal{A}_u := \{j = (i,z) \in \mathcal{A} \text{ such that } i, z \in \mathcal{V} \setminus \mathcal{S}^-\}$, and the vector $d \in \mathbb{R}^p$ of uncontrolled inputs as the collection of flow rate through the arcs $j \in \mathcal{A}_d := \{j = (i,z) \in \mathcal{A} \text{ such that } i \in \mathcal{V} \setminus \mathcal{S}^- \text{ and } z \in \mathcal{S}^-\}$. The dynamic and static state vectors, and the controlled and uncontrolled input vectors are given respectively by

$$x := T_x^\top x_{\mathcal{N}}, \qquad x_e := T_e^\top x_{\mathcal{N}}, \qquad u := T_u^\top u_{\mathcal{N}}, \qquad d := T_d^\top u_{\mathcal{N}}, \tag{2.7}$$

where $x_{\mathcal{N}} \coloneqq \left(x_{(1)}, \ldots, x_{(n_v)}\right)^\top$ is a vector containing the storage state of all nodes of the network and $u_{\mathcal{N}} \coloneqq \left(u_{(1)}, \ldots, u_{(m_a)}\right)^\top$ is a vector containing the flow rate of all arcs in the network. The static state vector has no storage capacity, i.e., $x_e = 0$ for all time instants. Moreover, $T_x \in \mathbb{R}^{n_v \times n}$ and $T_e \in \mathbb{R}^{n_v \times q}$ are matrices that collect the n columns and the q columns of the identity matrix of order n_v, corresponding to the indices of the dynamic nodes $i \in \mathcal{Y}$ and the static nodes $z \in \{\mathcal{V} \setminus \mathcal{S}\} \setminus \mathcal{Y}$, respectively, with $n + q = n_v$. Similarly, $T_u \in \mathbb{R}^{m_a \times m}$ and $T_d \in \mathbb{R}^{m_a \times p}$ are matrices that collect the m columns and the p columns of the identity matrix of order m_a, corresponding to the indices of the arcs contained in $\mathcal{A}_u$ and $\mathcal{A}_d$, respectively, with $m + p = m_a$. Note that

$$T_x T_x^\top = I_n, \qquad T_e T_e^\top = I_q, \qquad T_u T_u^\top = I_m, \qquad T_d T_d^\top = I_p, \tag{2.8}$$

where $I_{(\cdot)}$ denotes the identity matrix of order $(\cdot)$. Finally, define a matrix $A_{\mathcal{N}} \in \mathbb{R}^{n_v \times n_v}$ to consider the effect of possibly state-couplings between the nodes.

Following the flow conservation constraints (2.5) and Assumptions 2.1, 2.2 and 2.3, a discrete-time model based on a linear difference-algebraic equation can be formulated for the network $\mathcal{N}$ as follows:

$$\begin{cases} x_{k+1} = T_x^\top A_{\mathcal{N}} x_{\mathcal{N},k} + \Delta t\, T_x^\top (B_{\mathcal{N}}^u + B_{\mathcal{N}}^d) u_{\mathcal{N},k}, & \forall k \in \mathbb{Z}_+ \quad (2.9a) \\ 0 = \Delta t\, T_e^\top (B_{\mathcal{N}}^u + B_{\mathcal{N}}^d) u_{\mathcal{N},k}, & \forall k \in \mathbb{Z}_+ \quad (2.9b) \end{cases}$$

where $\Delta t \in \mathbb{Z}_{\geq 1}$ is the sampling time expressed in a time unit compatible with the flow rate unit. This model depends on the overall vectors $x_{\mathcal{N}}$ and $u_{\mathcal{N}}$, but a more appropriate model can be obtained by using (2.7), (2.8) and (2.9), yielding

$$\begin{cases} x_{k+1} = A x_k + B u_k + B_d d_k, & \forall k \in \mathbb{Z}_+ \quad (2.10a) \\ 0 = E_u u_k + E_d d_k, & \forall k \in \mathbb{Z}_+ \quad (2.10b) \end{cases}$$

where $A = T_x^\top A_{\mathcal{N}} T_x \in \mathbb{R}^{n \times n}$, $B = \Delta t\, T_x^\top B_{\mathcal{N}}^u T_u \in \mathbb{R}^{n \times m}$, $B_d = \Delta t\, T_x^\top B_{\mathcal{N}}^d T_d \in \mathbb{R}^{n \times p}$, $E_u = \Delta t\, T_e^\top B_{\mathcal{N}}^u T_u \in \mathbb{R}^{q \times m}$ and $E_d = \Delta t\, T_e^\top B_{\mathcal{N}}^d T_d \in \mathbb{R}^{q \times p}$.

Assumption 2.5. *The pair (A, B) is controllable and (2.10b) is reachable*[1]*, i.e., $q \leq m$ with* $\operatorname{rank}(E_u) = q$.

[1]If $q < m$, then multiple solutions exist, so u_k should be selected by means of an optimisation problem. Equation (2.10b) implies the possible existence of uncontrollable flows d_k at the junction nodes. Therefore, a subset of the control inputs will be restricted by the domain of some flow demands.

Moreover, from (2.2), (2.3) and Assumption 2.3, the storage and flow capacity constraints can be re-written in compact form as follows:

$$x_k \in \mathbb{X} \coloneqq \{x \in \mathbb{R}^n \mid 0 \leq x \leq x_{\max}\}, \quad \forall k \in \mathbb{Z}_+ \tag{2.11a}$$

$$u_k \in \mathbb{U} \coloneqq \{u \in \mathbb{R}^m \mid 0 \leq u \leq u_{\max}\}. \quad \forall k \in \mathbb{Z}_+ \tag{2.11b}$$

Note that requiring zero lower bounds on x and u is not a restrictive assumption. As pointed out in [22], if there is a lower bound $x_{\min} \neq 0$, it suffices to shift the state variables as $\hat{x} = x - x_{\min}$ to obtain $0 \leq \hat{x} \leq x_{\max} - x_{\min}$. Similarly, if some component $u_{(i)}$ of the controlled flows has a lower bound $u_{(i),\min} \neq 0$, then it can be defined $\hat{u}_{(i)} = u_{(i)} - u_{(i),\min}$, where $0 \leq \hat{u}_{(i)} \leq u_{(i),\max} - u_{(i),\min}$. For this latter case, the component $u_{(i),\min}$ has to be considered as a new uncontrolled fixed flow, i.e., $d_{(p+1),k} = u_{(i),\min}$ for all $k \in \mathbb{Z}_+$, with $d_{(p+1),\min} = d_{(p+1),\max}$ since $d_{(p+1),k}$ is a parameter.

A more general constraint set can be used within this modelling framework to easily incorporate any other operational constraints, e.g.,

$$\mathbb{Y}(d) \coloneqq \{(x,u) \in \mathbb{X} \times \mathbb{U} \mid Fx + Gu + Hd \leq g\}, \tag{2.12}$$

where $F \in \mathbb{R}^{r \times n}$, $G \in \mathbb{R}^{r \times m}$, $H \in \mathbb{R}^{r \times p}$ and $g \in \mathbb{R}^r$, with $r \in \mathbb{Z}_+$ being the number of element-wise constraints. Note that the above constraint set is convex and compact. It may depend on the demand d and consequently it could be time varying.

Regarding the operation of the generalised flow-based networks, the following assumptions are considered in this thesis.

Assumption 2.6. *The states are connected only through controlled flows, i.e., there is no direct state couplings, that is $A_{\mathcal{N}} = I_{n_v}$ and $A = I_n$.*

Assumption 2.7. *The states in x and the demands in d are measured at any time instant $k \in \mathbb{Z}_+$.*

Assumption 2.8. *The realisation of demands at any time instant $k \in \mathbb{Z}_+$ can be decomposed as*

$$d_k = \bar{d}_k + e_k, \tag{2.13}$$

where $\bar{d}_k \in \mathbb{R}^p$ is the vector of expected disturbances, and $e_k \in \mathbb{R}^p$ is the vector of forecasting errors with non-stationary uncertainty and a known (or approximated) quasi-concave probability distribution $\mathcal{D}(0, \Sigma(e_{(j),k}))$. The stochastic nature of each j^{th} row of d_k is described by $d_{(j),k} \sim \mathcal{D}_i(\bar{d}_{(j),k}, \Sigma(e_{(j),k}))$, where $\bar{d}_{(j),k}$ denotes its mean, and $\Sigma(e_{(j),k})$ its variance.

Although this chapter considers nominal models, the previous state-space formulation allows to introduce unknown disturbances explicitly in the model and constraints, and is suitable to extend powerful tools of control theory to the design, analysis and optimal control of generalised flow-based networks.

2.3.2 MPC for the Control of Dynamic Network Flows

MPC stands for a family of methods that select control actions based on optimisation problems. It is one of the most successful control technologies applied in a wide variety of application areas. The tractability of an MPC problem, especially when dealing with large-scale systems, is defined by the nature of the elements that are involved in the predictive and optimisation strategy. The use of a *cost function* allows to describe the desired behaviour of the system and is generally defined under two purposes: stability and performance. Such function serves also to specify preferences in a multi-objective optimal control problem and it is application-dependent. There exist within the MPC literature common cost functions that are convex and lead to an easy to solve problem. Common choices are based on linear (i.e., $\|\cdot\|_1$, and $\|\cdot\|_\infty$) and quadratic norm costs (i.e., $\|\cdot\|_2$), which are usually weighted. The explicit handling of *constraints* is the key strength of MPC. It can be found in different applications the following types of constraints: linear (used to upper/lower bound variables), convex quadratic (used to bound a variable to lie within an ellipsoid), probabilistic (used to deal with uncertainty and to reduce conservatism of worst-case approaches), second order cones, switched constraints (used when the inclusion of the constraint depends on meeting a predefined condition), non-linear constraints (comprise any other type of constraint and are difficult to handle when solving the optimisation problem). The most critical element within the MPC framework is the *dynamic model* of the system, since the robustness and performance of the controller depend on the model, which can be deterministic or stochastic, linear or non-linear, continuous or discrete or hybrid.

The MPC strategy can be summarised as follows. At each time step, the controller uses all the available information and a model of the system to solve an open-loop optimisation problem, which gives a sequence of future control actions satisfying system constraints and optimising the desired performance cost; only the first control move is applied. At the next time step, the overall procedure is repeated over a shifted prediction horizon using updated system measurements to compensate for modelling errors and/or disturbances. This scheme is referred also as receding horizon control. Further details on MPC theory, design and applications can be found in [109, 151].

In what follows, an MPC setting is given for the integrated scheduling-control problem in a generalised flow-based network with time-invariant capacity constraints, under the following assumption.

Assumption 2.9. *A priori knowledge of the requested demand is available for a given future time horizon $N \in \mathbb{Z}_+$. The known sequence is denoted by $\mathbf{d}_k := \{d_{k+i}\}_{i \in \mathbb{Z}_{[0,N-1]}}$. Each demand d_{k+i} is assumed* admissible *for all $k, i \in \mathbb{Z}_+$, i.e., there exists a flow satisfying it without violating network's capacity and conservation constraints.*

This setting with perfect knowledge of demand is often called *prescient* MPC and it is based on the following finite horizon optimisation problem (FHOP):

$$\mathcal{P}_N(k, x_k, \mathbf{d}_k): \quad V_N^0(k, x_k) = \min_{\mathbf{u}_k} V_N(k, x_k, \mathbf{u}_k), \tag{2.14a}$$

subject to:

$$x_{k+i+1|k} = Ax_{k+i|k} + Bu_{k+i|k} + B_d d_{k+i}, \qquad \forall i \in \mathbb{Z}_{[0,N-1]} \tag{2.14b}$$

$$E_u u_{k+i|k} + E_d d_{k+i} = 0, \qquad \forall i \in \mathbb{Z}_{[0,N-1]} \tag{2.14c}$$

$$x_{k+i|k} \in \mathbb{X}, \qquad \forall i \in \mathbb{Z}_{[1,N]} \tag{2.14d}$$

$$u_{k+i|k} \in \mathbb{U}, \qquad \forall i \in \mathbb{Z}_{[0,N-1]} \tag{2.14e}$$

$$x_{k|k} = x_k, \tag{2.14f}$$

in which $N \in \mathbb{Z}_+$ is the prediction horizon, $k \in \mathbb{Z}_+$ is the current time step, and the sequence $\mathbf{u}_k = \{u_{k+i|k}\}_{i \in \mathbb{Z}_{[0,N-1]}}$ is the decision variable to be optimised given the current state x_k and the sequence of demands $\mathbf{d}_k = \{d_{k+i}\}_{i \in \mathbb{Z}_{[0,N-1]}}$. The sub-index $k+i|k$ denotes the prediction made at time step k of the associated variable for a future step $k+i$. The cost function $V_N(k, x_k, \mathbf{u}_k)$ is given by

$$V_N(k, x_k, \mathbf{u}_k) := \sum_{i=0}^{N-1} \ell(k+i, x_{k+i|k}, u_{k+i|k}), \tag{2.15}$$

where $\ell : \mathbb{Z}_+ \times \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}_{\geq 0}$ is the stage cost containing the scheduling economic objective function and any other control objective. It is assumed that ℓ is convex and possibly time varying.

The control action to be injected into the system at each time step $k \in \mathbb{Z}_+$ is given by the control law of the receding horizon policy, i.e.,

$$u_k = \kappa_N(k, x_k, \mathbf{d}_k) \coloneqq u^\star_{k|k}, \tag{2.16}$$

hence, the closed-loop evolution of the system is given by

$$x_{k+1} = Ax_k + Bu^\star_{k|k} + B_d d_k, \quad E_u u^\star_{k|k} + B_d d_k = 0. \tag{2.17}$$

The set of control sequences $\mathbf{u}_k$ satisfying the flow conservation constraints and the capacity constraints of $\mathcal{P}_N(k, x_k, \mathbf{d}_k)$ is denoted as

$$\mathcal{U}_{N,k}(x_k, \mathbf{d}_k) \coloneqq \{\mathbf{u}_k \mid (x_k, \mathbf{u}_k) \in \mathcal{F}_{N,k}(\mathbf{d}_k)\}, \tag{2.18}$$

in which, for each $k \in \mathbb{Z}_+$, $\mathcal{F}_{N,k}(\mathbf{d}_k) \subset \mathbb{R}^n \times \mathbb{R}^{Nm}$ is a closed set defined by

$$\begin{aligned}\mathcal{F}_{N,k}(\mathbf{d}_k) \coloneqq \{(x_k, \mathbf{u}_k) \mid \left(x_{k+i|k}, u_{k+i|k}\right) \in \mathbb{X} \times \mathbb{U},\ E_u u_{k+i|k} + E_d d_{k+i} = 0,\\ x_{k+N|k} \in \mathbb{X},\ i \in \mathbb{Z}_{[0,N-1]}\}.\end{aligned} \tag{2.19}$$

The time-varying domain of admissible states is defined as the projection of the feasible set $\mathcal{F}_{N,k}(\mathbf{d}_k)$ onto $\mathbb{R}^n$, i.e.,

$$\mathcal{X}_{N,k}(\mathbf{d}_k) \coloneqq \{x_k \in \mathbb{R}^n \mid \ \exists \mathbf{u}_k \text{ such that } (x_k, \mathbf{u}_k) \in \mathcal{F}_{N,k}(\mathbf{d}_k)\}. \tag{2.20}$$

The overall MPC strategy is summarised in Algorithm 1.

Algorithm 1 MPC Receding Horizon Strategy

1: measure the state x_k and obtain $\mathbf{d}_k = \{d_{k+i}\}_{i \in \mathbb{Z}_{[0,N-1]}}$ at time k
2: compute $\mathbf{u}^*_k(x_k) = \{u^\star_{k+i|k}\}_{i \in \mathbb{Z}_{[0,N-1]}}$ by solving (2.14) with horizon N
3: apply the first element $u^*_{k|k}$ to the system
4: proceed to time step $k+1$
5: go to 1.

In several applications, a priori information of flow demands is not perfectly known. Hence, their characterisation (deterministic or stochastic) should be obtained beforehand. A common approach is to use nominal predictions computed by means of forecasting techniques. In the following section, a description of the case study used throughout the remaining chapters of this thesis is presented along with a baseline MPC controller design based on forecasts of future flow information.

2.4 Case Study

2.4.1 Description

The MPC approaches presented in this thesis will be assessed with a case study of a large-scale real system reported in [129], specifically the Barcelona drinking water network (DWN). The general role of this system is the spatial and temporal re-allocation of water resources from nature to human society, keeping in mind quantitative and qualitative aspects of water availability and human needs. This network is currently managed by AGBAR[1] and it supplies potable water to the Metropolitan Area of Barcelona (Catalunya, Spain). In general, the water network operates as a full-interconnected system driven by endogenous and exogenous flow demands; different hydraulic elements are used to collect, store, distribute and supply drinking water to the associated population. In the Barcelona DWN, the water is taken from both superficial (i.e., rivers) and underground sources (i.e., wells), providing together a flow of around $7\,\mathrm{m}^3/\mathrm{s}$. The main supply comes from rivers Llobregat, Ter, and Besòs, with 52%, 46% and 2% of the total water supply, respectively. These sources are regulated by dams that have an overall capacity of 600 hm^3. The water availability at the sources significantly influences the characteristics and operation of the water supply system. Storage and transmission facilities are used to compensate for the different spatial and temporal distribution of natural water resources and human demands. After collected, water is purified to the level of the drinking water quality standard in four water treatment plants (WTP). The water flow from any of the sources is limited and has an associated price depending on the required treatment and legal extraction canons. Due to the geographical topology of Barcelona and its surroundings, the water supply area is divided in 113 pressure floors and the DWN is structured in two management layers: the *transport network*, which links the water treatment plants with the reservoirs located all over the city, and the *distribution network*, which is sectorised in sub-networks that link reservoirs directly to consumers. This thesis is focused on the transport network. Hence, each sector of the distribution network will be considered as a pooled demand to be served by the transport network. These demands are characterised by patterns of water usage and can be predicted by time-series models, neural networks, among other methods [19, 156].

[1] Aguas de Barcelona, S.A. Company that manages the drinking water transport and distribution in Barcelona (Spain).

Table 2.1: Comparison of considered drinking water network configurations

Model	# dynamic nodes	# static nodes	# source nodes	# demand nodes	# manipulated flows
Full	63	17	10	88	114
Aggregate	17	11	9	25	61
Sector	3	2	2	4	6

Throughout this thesis, three different network examples extracted from the original graph of the Barcelona DWN are used to present the numerical results. The models related to each example are denoted as *full model*, *aggregate model* and *sector model*. The considered configurations differ mainly in the size of the network flow problems and the number of elements that conform the networks (see Table 2.1). The full model represents the original graph of the DWN (see Figure 2.3). The aggregate model is a simplification of the original graph, where groups of elements have been aggregated (not discarded) in single nodes to reduce the size of the original problem (see Figure 2.4). The sector model considers only a sector of the DWN (see Figure 2.5).

2.4.2 System Management Criteria

The general goal in the operation of the Barcelona DWN is to control the hydraulic performance and to minimise the economic expenditures of water provision. Accordingly, the control task for the operation of this system can be formulated as a multi-objective optimisation problem. In this thesis, three operational goals with different nature are considered, i.e., economic, safety, and smoothness objectives, which are stated as follows:

1. To provide a reliable water supply in the most economic way, i.e., minimising water production and transport costs, and (if relevant) costs associated with the storage of water.

2. To guarantee the availability of enough water in each reservoir to satisfy its underlying demand, keeping a safety stock to face uncertainties and avoid stock-outs.

3. To operate the pressurised transport network under smooth control actions.

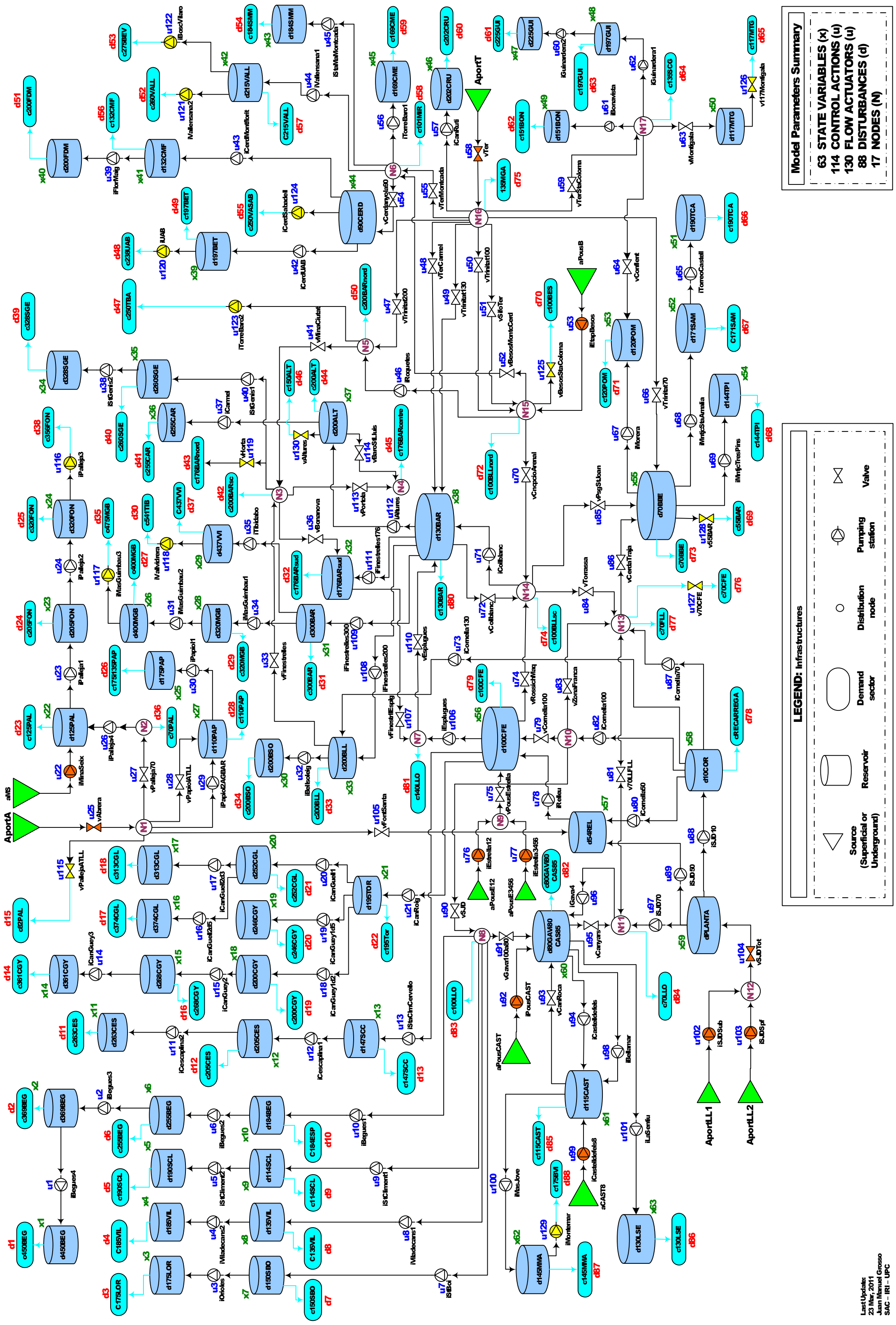


Figure 2.3: Barcelona DWN full diagram

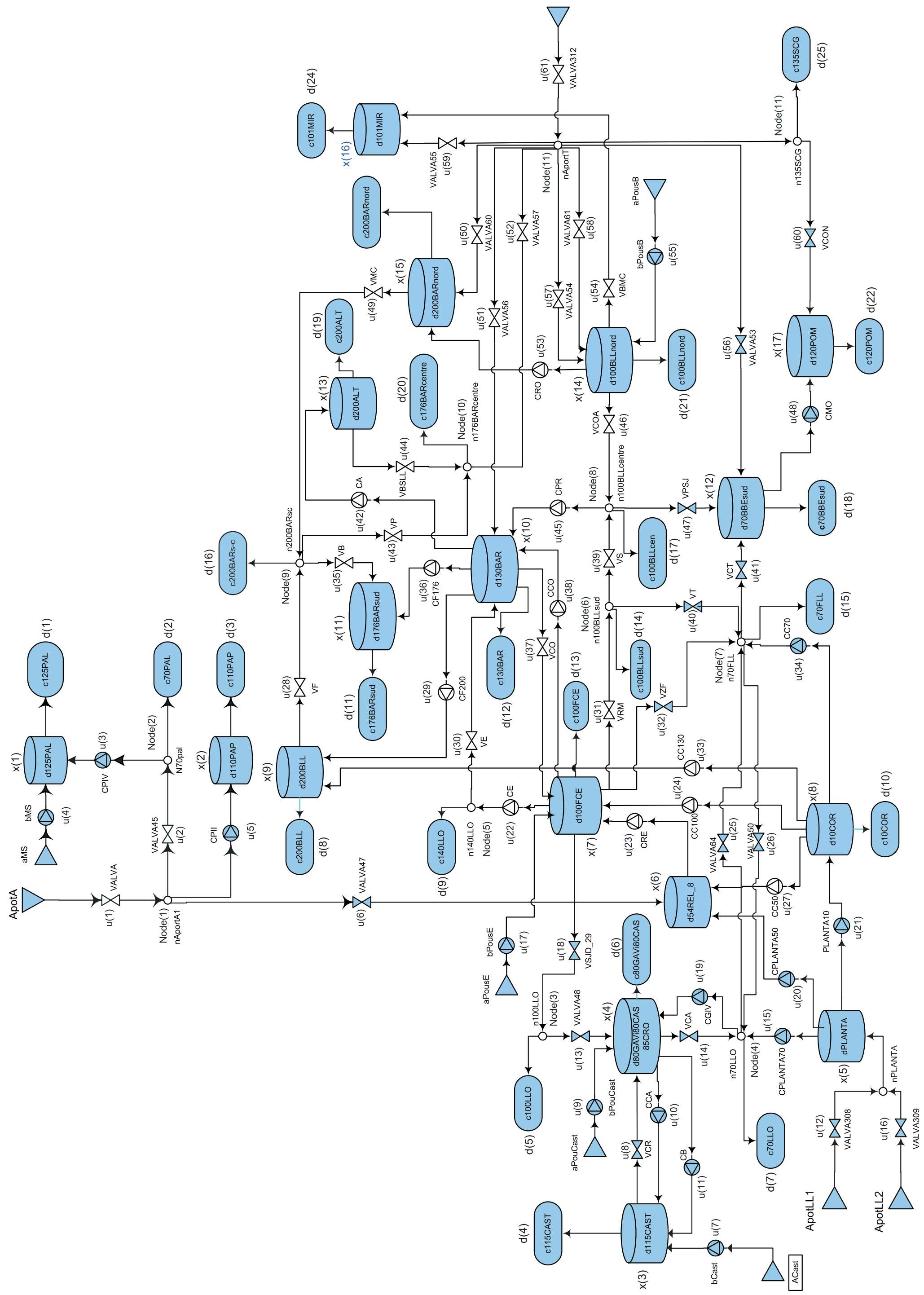


Figure 2.4: Barcelona DWN aggregate diagram

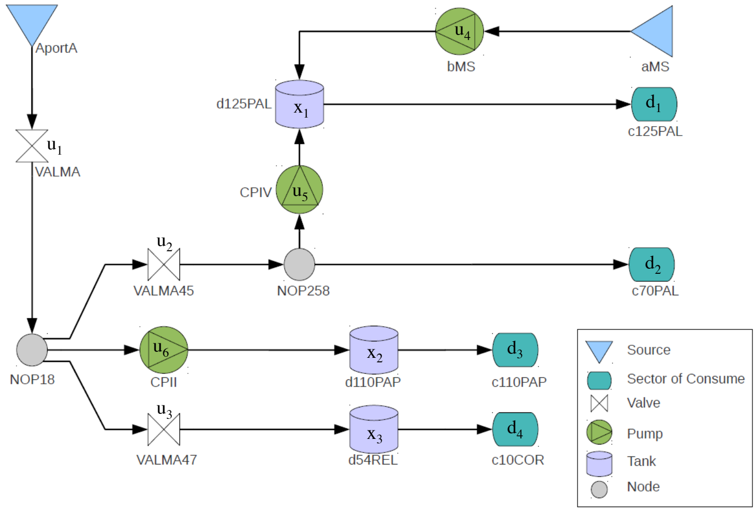


Figure 2.5: Barcelona DWN sector diagram

These objectives can be expressed quantitatively by the following performance indicators[1] for all time steps $k \in \mathbb{Z}_+$:

$$\ell_E(x_k, u_k; c_{u,k}, c_{x,k}) := c_{u,k}^\top W_e \, u_k \Delta t + c_{x,k}^\top W_h x_k, \tag{2.21a}$$

$$\ell_S(x_k; s_k) := \begin{cases} (x_k - s_k)^\top W_s (x_k - s_k) & \text{if } x_k \leq s_k \\ 0 & \text{otherwise,} \end{cases} \tag{2.21b}$$

$$\ell_\Delta(\Delta u_k) := \Delta u_k^\top W_{\Delta u} \, \Delta u_k. \tag{2.21c}$$

The first objective, $\ell_E(x_k, u_k; c_{u,k}, c_{x,k}) \in \mathbb{R}_{\geq 0}$, represents the economic cost of network operation at time step k, which depends on two components. On one hand, on a time-of-use pricing scheme driven by a time-varying price of the flow through arcs $c_{u,k} := (c_1 + c_{2,k}) \in \mathbb{R}_+^m$, which in this application takes into account a fixed water production cost $c_1 \in \mathbb{R}_+^m$ and a water pumping cost $c_{2,k} \in \mathbb{R}_+^m$ that changes according to the electricity tariff (assumed periodically time-varying). On the other hand, on a (possibly) time-varying inventory holding price $c_{x,k} \in \mathbb{R}_+^n$ of storage at dynamic nodes. All prices are given in economic units per cubic meter (e.u./m^3). The second objective, $\ell_S(x_k; s_k) \in \mathbb{R}_{\geq 0}$ for all k, is a performance index that penalises the amount of water volume going below a given safety threshold $s_k \in \mathbb{R}^n$ in m^3, which is desired to be stored in tanks and satisfies the condition $x_{\min} \leq s_k \leq x_{\max}$. Note that this safety objective is a

[1]The performance indicators considered in this thesis may vary or be generalised with the corresponding manipulation to include other control objectives.

piecewise continuous function, but it can be redefined as $\ell_S(\xi_k; x_k, s_k) := \xi_k^\top W_s\, \xi_k$, accompanied with two additional convex constraints, i.e., $x_k \geq s_k - \xi_k$ and $\xi_k \in \mathbb{R}^n_+$, for all k. The last objective, $\ell_\Delta(\Delta u_k) \in \mathbb{R}_{\geq 0}$, represents the penalisation of control signal variations $\Delta u_k := u_k - u_{k-1} \in \mathbb{R}^m$. The inclusion of this latter objective aims to extend actuators life and assure a smooth operation of the dynamic network flows. Furthermore, $W_e \in \mathbb{S}^m_{++}$, $W_x \in \mathbb{S}^n_{++}$, $W_s \in \mathbb{S}^n_{++}$ and $W_{\Delta u} \in \mathbb{S}^m_{++}$ are matrices that weight each decision variable in their corresponding cost function.

To achieve the control task, the above predefined objectives are aggregated in a multi-objective stage cost function, which depends explicitly on time due to the time-varying parameters of the involved individual objectives. The overall stage cost is defined for all $k \in \mathbb{Z}_+$ as

$$\ell(k, x_k, u_k, \xi_k) := \gamma_1 \ell_E(x_k, u_k; c_{u,k}, c_{x,k}) + \gamma_2 \ell_\Delta(\Delta u_k) + \gamma_3 \ell_S(\xi_k; x_k, s_k), \tag{2.22}$$

where $\gamma_1, \gamma_2, \gamma_3 \in \mathbb{R}_+$ are scalarised weights that allow to prioritise the impact of each objective involved in the overall performance of the network.

2.4.3 Baseline MPC Problem Setting

The baseline MPC law used in this thesis for the DWN case study is the one proposed in previous works, see [129, 142], where water demands are considered measured disturbances and their expectations are used as certain in the prediction model. Thus, this baseline approach results in a certainty-equivalent predictive controller (CE-MPC). The problem consists in finding the cheapest possible way of sending a certain amount of water through the network to satisfy a given pattern of water demands, considering available water sources and physical and operative/safety constraints for tanks and actuators. To do so, the CE-MPC law takes into account the cost function (2.22). Ideally, the resultant strategy should fill the tanks during the periods of lower energy cost with water taken from the cheapest sources.

Therefore, for a given $N \in \mathbb{Z}_{\geq 1}$ and given sequences of flow prices, storage prices, safety thresholds and water demands, denoted respectively as $\mathbf{c}_{u,k} = \{c_{u,i}\}_{i \in \mathbb{Z}_{[k,k+N-1]}}$, $\mathbf{c}_{x,k} = \{c_{x,i}\}_{i \in \mathbb{Z}_{[k,k+N-1]}}$, $\mathbf{s}_k = \{s_i\}_{i \in \mathbb{Z}_{[k,k+N-1]}}$ and $\mathbf{d}_k = \{d_i\}_{i \in \mathbb{Z}_{[k,k+N-1]}}$, the baseline MPC controller design is based on the solution of the following finite horizon optimisation problem (FHOP) at each time step k:

$$\mathcal{P}_N(k, x_k, \mathbf{d}_k, \mathbf{s}_k): \quad V_N^0(k, x_k) = \min_{\mathbf{u}_k, \boldsymbol{\xi}_k} V_N(k, x_k, \mathbf{u}_k, \boldsymbol{\xi}_k), \tag{2.23a}$$

subject to:

$$x_{k+i+1|k} = Ax_{k+i|k} + Bu_{k+i|k} + B_d d_{k+i}, \qquad \forall i \in \mathbb{Z}_{[0,N-1]} \tag{2.23b}$$
$$E_u u_{k+i|k} + E_d d_{k+i} = 0, \qquad \forall i \in \mathbb{Z}_{[0,N-1]} \tag{2.23c}$$
$$x_{k+i|k} \in \mathbb{X}, \qquad \forall i \in \mathbb{Z}_{[1,N]} \tag{2.23d}$$
$$u_{k+i|k} \in \mathbb{U}, \qquad \forall i \in \mathbb{Z}_{[0,N-1]} \tag{2.23e}$$
$$\xi_{k+i|k} \geq s_{k+i} - x_{k+i|k}, \qquad \forall i \in \mathbb{Z}_{[0,N]} \tag{2.23f}$$
$$\xi_{k+i|k} \geq 0, \qquad \forall i \in \mathbb{Z}_{[0,N]} \tag{2.23g}$$
$$\Delta u_{k+i|k} = u_{k+i|k} - u_{k+i-1|k}, \qquad \forall i \in \mathbb{Z}_{[0,N-1]} \tag{2.23h}$$
$$x_{k|k} = x_k, \quad u_{k-1|k} = u_{k-1}, \tag{2.23i}$$

in which the cost function $V_N(k, x_k, \mathbf{u}_k, \boldsymbol{\xi}_k)$ is the finite sum of stage costs, i.e.,

$$V_N(k, x_k, \mathbf{u}_k, \boldsymbol{\xi}_k) := \sum_{i=0}^{N-1} \ell(k+i, x_{k+i|k}, u_{k+i|k}, \xi_{k+i|k}), \tag{2.24}$$

with decision variables $\mathbf{u}_k = \{u_{k+i|k}\}_{i \in \mathbb{Z}_{[0,N-1]}}$ and $\boldsymbol{\xi}_k = \{\xi_{k+i|k}\}_{i \in \mathbb{Z}_{[0,N]}}$. The control law is derived from the receding horizon policy, i.e.,

$$u_k = \kappa_N(k, x_k, \mathbf{d}_k, \mathbf{s}_k, \mathbf{c}_{u,k}, \mathbf{c}_{x,k}) := u^{\star}_{k|k}, \tag{2.25}$$

hence, the closed-loop evolution of the system is given by

$$x_{k+1} = Ax_k + Bu^{\star}_{k|k} + B_d d_k, \quad E_u u^{\star}_{k|k} + B_d d_k = 0, \tag{2.26}$$

and the time-varying domain of admissible states is defined as

$$\mathcal{X}_{N,k}(\mathbf{d}_k, \mathbf{s}_k) := \{x_k \in \mathbb{R}^n \mid \exists (\mathbf{u}_k, \boldsymbol{\xi}_k) \text{ such that (2.23) is feasible}\}. \tag{2.27}$$

At the next time step, the optimisation is restarted with new feedback measurements to compensate unmeasured disturbances and model inaccuracies. This scheme is repeated at each future time step for given time-varying parameters. A useful visualization of the MPC approach is presented in Figure 2.6. Notice that a demand forecast signal is

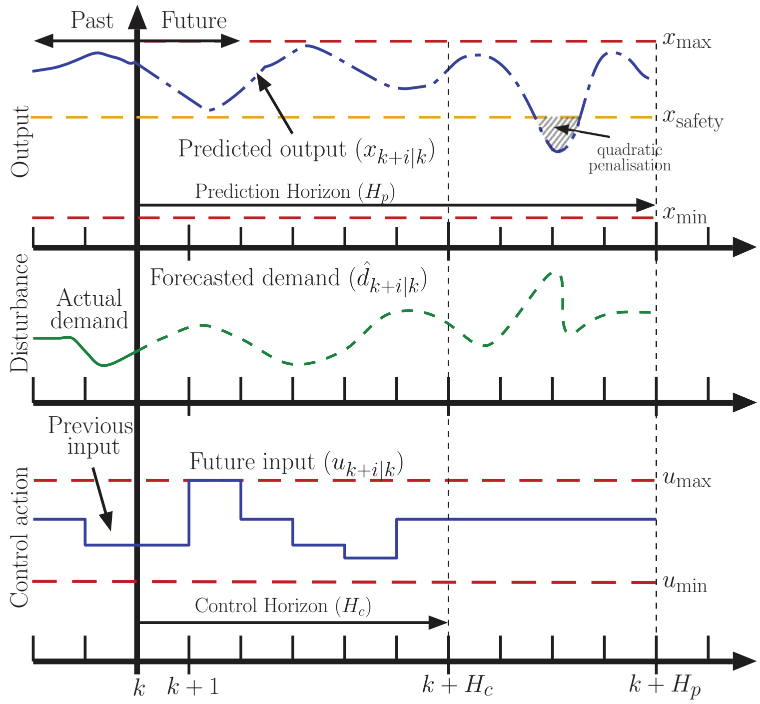


Figure 2.6: CE-MPC receding horizon strategy with hard and soft constraints

used in the receding horizon calculation to anticipate future system behaviour, which plays a significant role in the application of MPC for the management of generalised flow-based networks. This exogenous signal is computed using the method described in Appendix A.1.

In this CE-MPC baseline strategy, the safety volume s_k remains constant for every time step, i.e., $s_k = s_{k+i}$ $\forall k, i \in \mathbb{Z}_+$, and it is defined heuristically as a fraction of the maximum storage capacity, i.e., $s_k = \beta x_{\max}$ with $\beta \in \mathbb{R}_{(0,1)}$. Moreover, it is assumed in the prediction model that perfect demand information is available for the given prediction horizon. This assumption might hold in some applications where the demand follows a predefined contract, but not necessarily in the selected case study, i.e., drinking water networks, where current water demand is measured and future values rely on forecasts that introduce uncertainty in the model. The baseline strategy might work properly due to the inherent robustness of MPC if the realisations of future demands lie near the nominal ones, otherwise, the system will incur a higher risk of constraints violation of the real outputs.

2.4.4 Key Performance Indicators

In order to assess the performance of every control strategy developed throughout this thesis, the baseline framework defines three key performance indicators (KPI) based on the DWN management criteria, which are described below.

Economic KPI. This performance indicator is related with the hourly overall economic cost of meeting water demands. It is defined as follows:

$$\mathrm{KPI}_E := \frac{1}{N_s} \sum_{k=1}^{N_s} (c_1 + c_{2,k})^\top u_k \Delta t, \tag{2.28}$$

where $N_s \in \mathbb{Z}_+$ is the number of hours considered in the experiment, $\Delta t = 3600\mathrm{s}$ is the sampling time in seconds, and c_1 and $c_{2,k}$ are the water production cost and hourly electric cost, respectively, of applying the control law (2.25) at time step k.

Safety KPI. This performance indicator captures the average amount of safety stocks that are used to meet demands. It is defined as follows:

$$\mathrm{KPI}_S := \frac{1}{N_s} \sum_{k=1}^{N_s} \sum_{i=1}^{n} \xi_{(i),k}, \tag{2.29}$$

where $n \in \mathbb{Z}_+$ is the number of storage tanks, $\xi_{(i),k} := \min\{0, x_{(i),k} - x_{s(i),k}\}$ is the level of violation of the soft constraint for the i-th tank at time step k, and N_s the time horizon of the experiment. Ideally, this KPI should be zero, meaning that the operation respects the safety requirements.

Smoothness KPI. This indicator assesses the smooth performance of the control actions. It is defined as follows:

$$\mathrm{KPI}_{\Delta U} := \frac{1}{N_s} \sum_{i=1}^{m} \sum_{k=1}^{N_s} \Delta u_k^\top \Delta u_k, \tag{2.30}$$

where $m \in \mathbb{Z}_+$ is the number of actuators, $\Delta u_k = u_k - u_{k-1}$, is the variation of the control action for the i-th actuator at time step k, and N_s the time horizon of the assessment.

2.5 Numerical Results

This section presents the results of applying the CE-MPC strategy to the full model of the Barcelona DWN (see Figure 2.3). The simulation has been carried out over a time

Table 2.2: Key performance indicators for the CE-MPC strategy

KPI_E (e.u.)	KPI_S (m^3)	$\text{KPI}_{\Delta U}$ (m$^6/s^2$)	CPU Time (s)
2442.97	0.18011	0.84192	202.37

e.u.: economic units

Table 2.3: Water and electric cost for the CE-MPC strategy

	Water Cost (e.u.)	Electric Cost (e.u.)	Total Cost (e.u.)
Day 1	26248.84	29032.71	55281.55
Day 2	31667.56	29895.44	61563.00
Day 3	28788.12	29664.12	58452.24
Day 4	29444.35	29784.28	59228.63

e.u.: economic units

period of four days (96 hours) with a sampling time of one hour. Demand scenarios correspond to real values reported between July 23 and July 27, 2007. Initial conditions, i.e., sources capacity, tank states and safety volumes, are fixed a priori according to real data. The weights of the cost function (2.22) are $\gamma_1 = 100$, $\gamma_2 = 1$ and $\gamma_3 = 10$, while the internal weighting matrices of the individual objectives are $W_e = I_m$, $W_x = 0_n$, $W_s = I_n$ and $W_{\Delta u} = I_m$. The prediction horizon is $N = 24$ hours. The simulation has been carried out using the CPLEX solver of the TOMLAB 7.6 optimisation package, and Matlab R2010b (64 bits), running in a PC Intel Core E8600 at 3.33GHz with 8GB of RAM.

Table 2.2 summarises the baseline control performance according to the KPIs defined in § 2.4.4. Moreover, Table 2.3 shows the economic costs detailed per day. From the numerical results, it is important to notice that the KPI_S is not null, which means that during the experiment some storage tanks violate their safety thresholds. This may imply that some thresholds are too high to retain feasibility for the given demands or that they are not adequate from an economic point of view.

Figure 2.7 shows the excursion of water in a sample of tanks within an important sector of the Barcelona DWN. It can be seen how the CE-MPC controller keeps the volume in tanks within the hard and soft constraints. Notice that the net demand of each tank (exogenous plus endogenous demands) is properly satisfied along the simulation horizon. The selected plots summarise different behaviours that the volume in a tank may present. Tank 59 shows how the relation between storage capacity, safety volumes and demands could lead to a non-smooth excursion of water (increasing pumping costs) if the capacity is low and demand is near to safety volume. Another special case is Tank 53, whose capacity allows to satisfy its underlying demand for several days by filling the tank just once; this behaviour is an ideal one to optimise pumping costs. The other tanks in the Figure 2.7, show the proper replenishment planning that the predictive controller dictates according to the cyclic behaviour of demands. The soft constraints that are shown in Figure 2.7 were set empirically by analysing historic records of demand and supposing an emergency safety factor. Nevertheless, some of the tanks in the complete network, e.g., Tank 50 and Tank 55, have conservative safety volumes when compared with their associated net demand, what leads to higher values in the economic cost function. This fact highlights the need of strategies to set optimal safety volumes.

Figure 2.8 shows the normalised behaviour of a sample of actuators according to a pattern of electric costs. It can be seen how the CE-MPC properly decides to pump water when the electric tariff is cheaper. The actuators of the four top plots of Figure 2.8 have a degree of redundancy with each other because all of them can supply the water required by Tank 38 in the Barcelona DWN. The same situation occurs with the actuators of the four bottom plots, which have redundancy when satisfying the demand plugged to Node 13 in the network (see Figure 2.3). Notice that in both cases, if the control effort is allocated just by economic criteria, some of the actuators will present an accelerated degradation of their health while the other ones are merely used. This fact affects the reliability of actuators and, as a consequence, it could compromise the overall system availability. Therefore, it is necessary to develop additional criteria, e.g., prognostic and health management (PHM) of actuators, to face the uncertainty related with the wear of components.

2.6 Summary

This chapter presented mathematical preliminaries about the systems and problems considered in this thesis. Especially, modelling principles of dynamic network flow problems were introduced and a control-oriented model based on the discrete-time state-space framework was formulated. Furthermore, a baseline centralised MPC strategy for the scheduling-control problem in generalised flow-based networks was introduced. A selected case study corresponding to the drinking water network of the city of Barcelona (Spain) was used as an example of the minimum cost dynamic flow problem addressed in this thesis by means of the MPC framework.

It is important to highlight that in network flow problems working in batch operation with a finite end-point (i.e., no more flow will be demanded after the end of a prefixed time window), the previous MPC settings for re-scheduling can be formulated with a shrinking prediction horizon, i.e., the prediction horizon of $\mathcal{P}_N(k, x_k, \mathbf{d}_k)$ will be time varying with $N_{k+1} = N_k - 1$. A useful result that can be used in networks with periodic re-scheduling or with high computational burden of the receding horizon control is the so-called move blocking strategies [30], which reduce the degrees of freedom by fixing the control actions over several time steps.

Even when the MPC framework is flexible to integrate the scheduling-control problem, both the prescient MPC (with perfect information of demands) and the certainty-equivalent MPC (with estimated information of demands) do not necessarily lead to acceptable closed-loop solutions as has been discussed in [172], mainly because the finite horizon problem does not account for long-term effects of demands. This fact could cause poor performance or loss of feasibility, especially when deciding under uncertainty. Therefore, simple re-scheduling is not enough and appropriate modifications of the FHOP are required to account for this issue. Moreover, in real network flow problems, demands might be unbounded. In such a case, state constraints should be softened to retain feasibility of the optimisation problem.

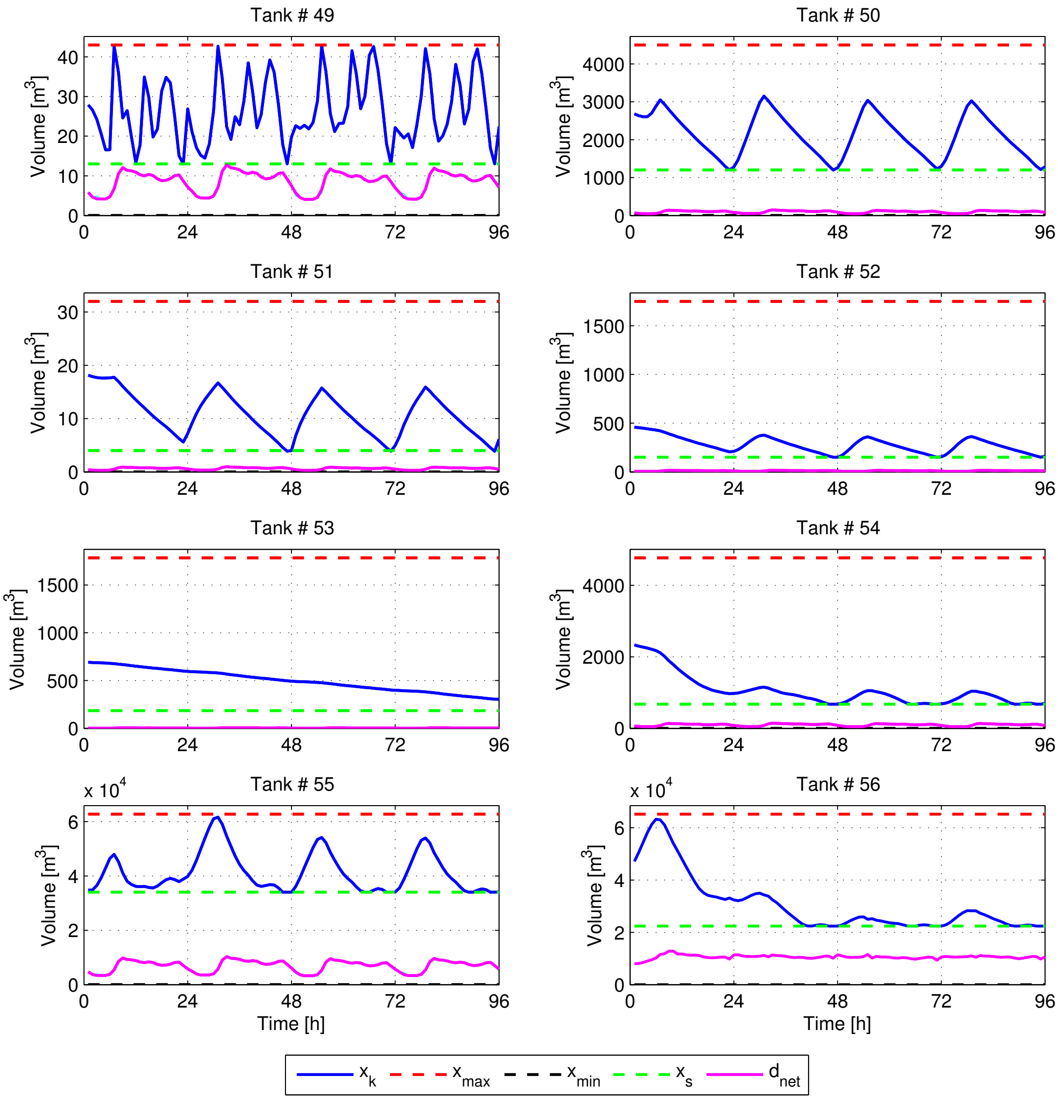


Figure 2.7: Management of water storage with the CE-MPC strategy

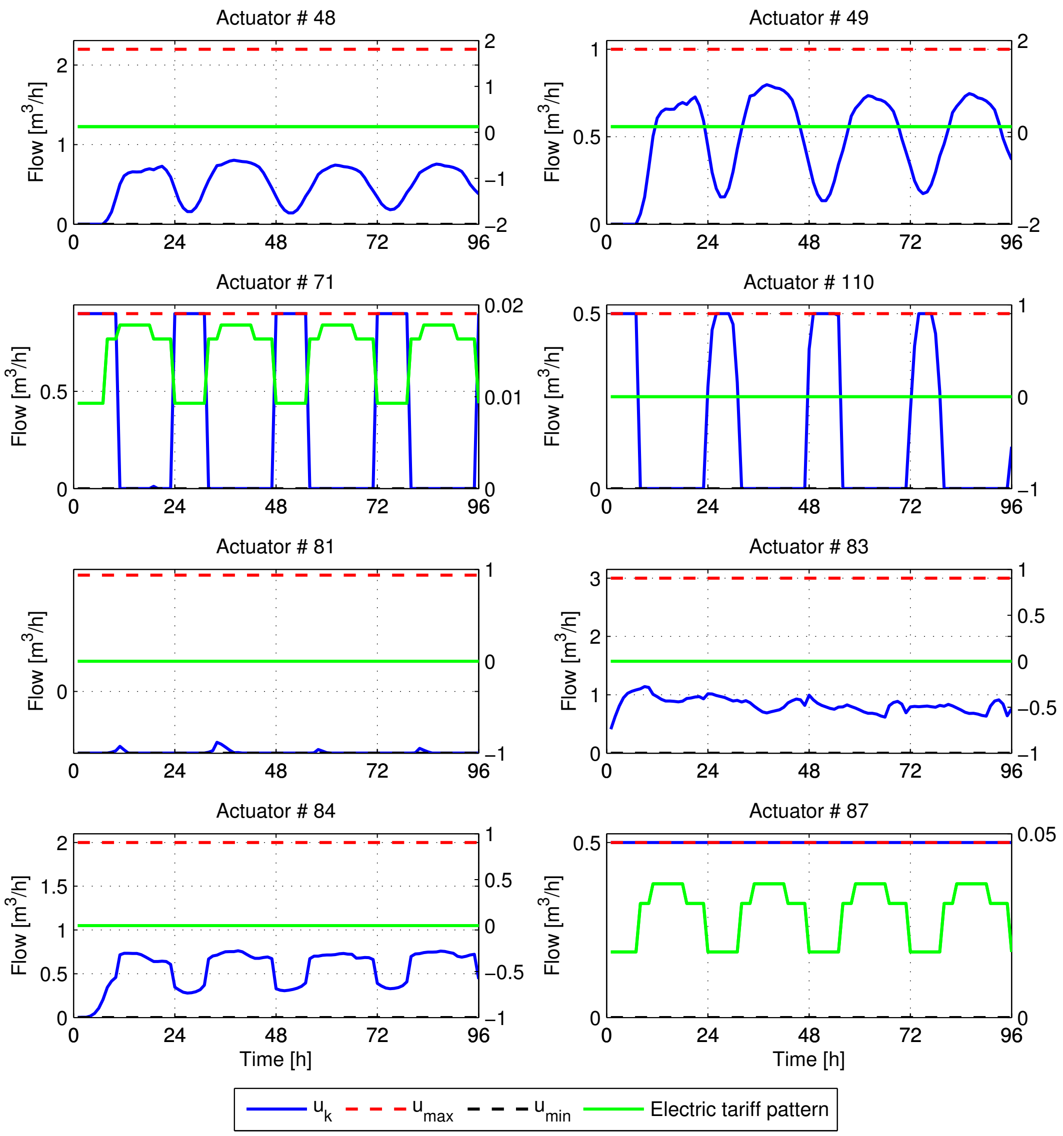


Figure 2.8: Optimal operation of a sample of actuators in the Barcelona DWN

Part II

Centralised MPC Schemes for Economic and Robust Operation

Chapter 3

Economic MPC Strategies for Generalised Flow-based Networks

This chapter addresses the management of generalised flow-based networks regarding a multi-objective cost function by means of economically-oriented MPC strategies. Here, it is shown that the economic MPC framework is flexible to enhance the dynamic minimum cost flow scheduling problem by guaranteeing recursive feasibility, asymptotic economic performance and possibly asymptotic stability, even under changes of the economic cost function. The chapter focuses on the design of economic predictive controllers for networks with periodically-time varying operation.

3.1 Introduction

Recalling the discussion in § 1.2.4, the predominant approach to incorporate economic information in control applications is to consider a hierarchical control structure, where standard MPC controllers are designed for tracking economic operational set-points that are computed usually in an upper layer by means of a real-time optimiser (RTO) or a steady-state target optimiser (SSTO), which usually use complex non-linear stationary models and larger sampling times than the regulatory MPC layer. A more appealing alternative from an economic point of view is to formulate a predictive controller that uses a simplified control-oriented model and integrates the economic and control objectives in a single optimisation problem, as has been introduced in § 2.3.2 for re-scheduling of dynamic network flows. Nevertheless, the main drawback of these aforementioned alternatives is the lack of mechanisms to retain feasibility of the MPC optimisation problem at each time step (even under nominal conditions). This feasibility loss could be caused due to different reasons, e.g., the time-varying nature of disturbances and parameters, the time-scale differences and model mismatch in the hierarchical architecture, the omission of long-term effects of demands within the finite horizon scheduling problem, among others.

In order to tackle the aforementioned feasibility problem without softening the original constraints, additional ingredients are often included in the standard MPC formulation, specifically, terminal constraints and/or terminal penalties [112]. The use of economic cost functions directly in the MPC setting has lead to the so-called economic MPC framework, and several formulations exists [53], where recursive feasibility, closed-loop stability and/or average asymptotic performance can be guaranteed under some form of dissipativity, duality or convexity assumptions.

This chapter is particularly focused on the periodic operation of generalised flow-based networks. Five different MPC formulations are discussed: a hierarchical two-layer approach, an economic MPC with periodic terminal equality constraint, an economic MPC with periodic terminal penalty and terminal region, and two different modifications of the terminal equality constraint based formulation, which are meant to overcome possible feasibility losses in the presence of changing operating patterns. The discussed schemes are tested and compared by means of the Barcelona's drinking water network case study, specifically using the sector model (see Figure 2.5).

3.2 Problem Statement

Consider a generalised flow-based network being described in the form (2.10), satisfying Assumptions 2.6 and 2.7 and with states and controlled flows constrained to lie in the convex and compact sets $\mathbb{X}$ and $\mathbb{U}$ defined in (2.11). The basic problem considered in this section is that of finding conditions under which (2.14) is solvable at time step $k \in \mathbb{Z}_+$, and conditions under which feasibility at k implies feasibility for all subsequent time steps $t > k$, i.e., recursive feasibility. Here, Assumption 2.9 is dropped, and the control synthesis problem is considered under the following interpretation (similar to [146]):

Interpretation 3.1 (Sup-Inf Type Information)**.** *At any time step $k \in \mathbb{Z}_+$ when the decision concerning the controlled flow u_k is taken, both the state x_k and the demand (uncontrolled flow) $d_k \in \mathbb{D}_k := \{d \in \mathbb{R}^p \mid 0 \leq d \leq d_{\max,k},\ d_{\max,k} \in \mathbb{R}^p_+\}$ are known, while future demands d_{k+i} are unknown for all $i \in \mathbb{Z}_+$ and can take arbitrary values $d_{k+i} \in \mathbb{D}_{k+i}$, $i \in \mathbb{Z}_+$. The controller has also knowledge of the sequence of sets $\{\mathbb{D}_{k+i}\}_{i\in\mathbb{Z}_{[1,N-1]}}$ for a given horizon $N \in \mathbb{Z}_+$.*

The above interpretation can be considered as a non-cooperative two-player dynamic game that leads to a *max-min* optimal control problem, whose treatment in the robust MPC literature is quite narrow; see [146] and references therein. In this game, the first player is the controller, which has access to the current state x_k and the current demand d_k when determining the controlled flow u_k at each time step k. Specifically, the controller employs a control law $\kappa_N : \mathbb{Z}_+ \times \mathbb{R}^n \times \mathbb{R}^p \to \mathbb{R}^m$ to set a control action of the form

$$u_k = \kappa_N(k, x_k, d_k). \tag{3.1}$$

The second player, often called the adversary, is the entity (e.g., the customer) which demands a flow of commodity without having information about the state of the system nor the controller. At each time step k, the adversary employs a policy $\delta : \mathbb{Z}_+ \to \mathbb{R}^p$ to determine exogenously the demand, i.e., $d_k = \delta(k) \in \mathbb{D}_k$.

To characterise the set of initial states and the admissible feedback control law (3.1) for which (2.14) is solvable at time step $k = 0$ under Interpretation 3.1, some additional notation is introduced. Let $\phi(i; x_k, \mathbf{u}_k, \mathbf{d}_k)$ denote the solution of the dynamic state update equation (2.10a) at the time step $i \in \mathbb{Z}_+$, starting from the state x_k and reached under the control sequence $\mathbf{u}_k$ and the demand sequence $\mathbf{d}_k$. In addition, define the following set:

$$\mathbb{V}(d) := \{u \in \mathbb{R}^m \mid E_u u + E_d d = 0,\ d \in D\}, \tag{3.2}$$

$$\bar{\mathbb{U}}(d) := \mathbb{U} \cap \mathbb{V}(d), \tag{3.3}$$

where $\mathbb{V}(d)$ is the set of control actions that satisfy the mass balance at static nodes for a given demand $d \in D$. Likewise, some useful definitions are recalled below.

Definition 3.1 (Admissible $\mathbf{u}_k$ and $\mathbf{d}_k$)**.** *Given the state x_k and a target state set $\mathbb{T}$, the control sequence $\mathbf{u}_k = \{u_{k+i|k}\}_{i\in\mathbb{Z}_{[0,N-1]}}$ and the demand sequence $\mathbf{d}_k = \{d_{k+i}\}_{i\in\mathbb{Z}_{[0,N-1]}}$ are admissible if* *(i)* $d_{k+i} \in \mathbb{D}_{k+i}, \quad \forall i \in \mathbb{Z}_{[0,N-1]}$,

(ii) for all $\mathbf{d}_k \in \mathbb{D}_k \times \mathbb{D}_{k+1} \ldots \times \mathbb{D}_{k+N-1}$*, it holds that*

$$(\phi(i; x_k, \mathbf{u}_k, \mathbf{d}_k), u_{k+i|k}(x_k, \mathbf{d}_k)) \in \mathbb{X} \times \bar{\mathbb{U}}(d_{k+i}), \quad \forall i \in \mathbb{Z}_{[0,N-1]}, \text{ and}$$
$$\phi(N; x_k, \mathbf{u}_k, \mathbf{d}_k) \in \mathbb{T}.$$

Definition 3.2 (N-step max-min controllability)**.** *Let* $\mathcal{D}_{N,k}$ *be the set of all admissible demand sequences* $\mathbf{d}_k$ *and* $\mathcal{U}_N(x_k, \mathbf{d}_k)$ *the set of all admissible control sequences* $\mathbf{u}_k$ *at time step* $k \in \mathbb{Z}_+$ *for given time-horizon* $N > k$*. A state* $x_k \in \mathbb{X}$ *is* N-step max-min controllable *to a target set* $\mathbb{T} \subseteq \mathbb{X}$ *if and only if for all admissible demand sequences* $\mathbf{d}_k$ *there exists an admissible control sequence, i.e.,* $\mathcal{U}_N(x_k, \mathbf{d}_k) \neq \emptyset$*. A set* $\mathcal{X}_N$ *is called* N*-step max-min controllable if all* $x_k \in \mathcal{X}_N$ *are* N*-step max-min controllable to the target set.*

An important ingredient in the N-step max-min controllability is the one-step controllable set, which for a generalised flow-based network under Interpretation 3.1 is given by the pre-image mapping

$$\mathcal{B}(X, D) := \{x \in \mathbb{X} \mid \forall d \in D,\ \exists u \in \bar{\mathbb{U}}(d) \text{ such that } Ax + Bu + B_d d \in X\}, \tag{3.4}$$

where X and D are a one-step target set of states and a set of demands, respectively. Therefore, by inspection of Definition 3.2, at $k = 0$, the i-step max-min controllable sets, with $i \in \mathbb{Z}_{[0,N]}$, are given by iteration of the mapping $\mathcal{B}(\cdot, \cdot)$, i.e.,

$$\mathcal{X}_i = \mathcal{B}(\mathcal{X}_{i-1}, \mathbb{D}_{N-i}), \quad i \in \mathbb{Z}_{[1,N]}, \tag{3.5}$$

with the boundary condition $\mathcal{X}_0 := \mathbb{T} \subseteq \mathbb{X}$. Each controllable set has an associated max-min set-valued control map given for all $i \in \mathbb{Z}_{[1,N]}$ by:

$$\mathcal{U}_i(x, d) := \{u \in \mathbb{U} \mid \forall (x, d) \in \mathcal{X}_i \times \mathbb{D}_{N-i},\ E_u u + E_d d = 0\ ,\ Ax + Bu + B_d d \in \mathcal{X}_{i-1}\}. \tag{3.6}$$

Therefore, the set of all admissible control sequences $\mathbf{u} = \{u_i\}_{i \in \mathbb{Z}_{[0,N-1]}}$ is defined as

$$\begin{aligned}\mathcal{U}_N(x, \mathbf{d}) := \{\mathbf{u} \mid u_i \in \mathbb{U},\ \forall i \in \mathbb{Z}_{[0,N-1]} \text{ and } \forall (x, d) \in \mathcal{X}_{N-i} \times \mathbb{D}_i, \\ E_u u_i + E_d d = 0\ , Ax + Bu + B_d d \in \mathcal{X}_{N-i-1}\}.\end{aligned} \tag{3.7}$$

As shown in [146], from continuity of the system dynamics and compactness of $\mathbb{X}$, $\mathbb{U}$ and $\mathbb{D}_i$, if the target set is compact, then the i-step max-min controllable sets and their corresponding set-valued control maps are compact (possibly empty).

3.3 Existence of Admissible Controlled Flows

Showing the resolvability of (2.14) requires first the existence of an admissible control sequence satisfying Definition 3.1. This involves establishing the N-step controllability of the constrained generalised flow-based network following Definition 3.2 with target set $\mathbb{T} = \mathbb{X}$. The following basic result is the starting point for the feasibility analysis.

Proposition 3.1. *The N-horizon scheduling-control problem (2.14) is solvable at given initial time step $k \in \mathbb{Z}_+$ if and only if the N-step max-min controllable set is non-empty, i.e., $\mathcal{X}_N \neq \emptyset$ and the state x_k belongs to any of the one-step controllable sets $\mathcal{X}_j$, $j \in \mathbb{Z}_{[0,N]}$. In such a case, a control sequence exists satisfying $\mathbf{u}_k \in \mathcal{U}_N(x, \mathbf{d})$.*

Proof: The statement follows directly from Definitions 3.1 and 3.2. □

It is important to remark that Proposition 3.1 only ensures feasibility of the MPC optimisation problem (2.14) at time step k, but not at future steps $t > k$. Therefore, it is desired to find conditions under which recursive feasibility of the optimisation problem is guaranteed for any sequence of admissible demand flows. In order to do so, something has to be assumed regarding the limit behaviour of the sequence of demand time-varying sets $\{\mathbb{D}_k\}_{k\in\mathbb{Z}_{[0,\infty)}}$. A suitable assumption is that this sequence of sets is asymptotically convergent either to a single invariant set or to a limit-cycle of sets. For brevity of the exposition, this chapter considers the first case.

Assumption 3.1. *$\mathbb{D}_k \to \bar{\mathbb{D}} := \{d \in \mathbb{R}^p \mid 0 \leq d \leq d_{\max},\ d_{\max} \in \mathbb{R}^p_+\}$ as $k \to \infty$, with $\mathbb{D}_k \subseteq \bar{\mathbb{D}}, \forall k \in \mathbb{Z}_+$.*

Establishing initial conditions that guarantee the existence of admissible control sequences for all time steps requires the following set invariance notions.

Definition 3.3 (Max-min robust control invariance)**.** *A set $\mathcal{C} \subseteq \mathbb{X}$ is a* max-min robust control invariant set *for the system (2.10) with $x \in \mathbb{X}$, $u \in \mathbb{U}$ and $d \in \bar{\mathbb{D}}$ if and only if $\mathcal{C} \subseteq \mathcal{B}(\mathcal{C}, \bar{\mathbb{D}})$, i.e., for all $(x, d) \in \mathcal{C} \times \bar{\mathbb{D}}$ there exists a control $u \in \mathbb{U}$ such that $Ax + Bu + B_d d \in \mathcal{C}$ and $E_u u + E_d d = 0$.*

Definition 3.4 (Maximal max-min robust control invariant set)**.** *A max-min robust control invariant set is called the* maximal max-min robust control invariant set *(here denoted as $\mathcal{C}_\infty$) if and only if it contains all max-min robust control invariant sets.*

Remark 3.1. *The set $\mathcal{C}_\infty$ is unique (possibly empty). Consider the sequence of max-min controllable sets $\{\mathcal{X}_i\}_{i\in\mathbb{Z}_{[0,\infty)}}$ generated by (3.5) with $N \to \infty$. If Assumption 3.1 holds then the sequence $\{\mathcal{X}_i\}_{i\in\mathbb{Z}_{[0,\infty)}}$ converges to the set $\bar{\mathcal{X}} := \bigcap_{i=0}^{\infty} \mathcal{X}_i$. Moreover, from*

Definition 3.3, an i-step controllable set $\mathcal{X}_i$ is max-min robust control invariant if and only if $\mathcal{X}_i \subseteq \mathcal{X}_{i+1} = \mathcal{B}(\mathcal{X}_i, \bar{\mathbb{D}})$. In addition, for the target set $\mathbb{T} = \mathbb{X}$, the maximal max-min robust control invariant set satisfies $\mathcal{C}_\infty \subseteq \bigcap_{i=0}^{\infty} \mathcal{X}_i$ and it is finitely determined, with $\mathcal{C}_\infty = \mathcal{X}_i$, if and only if $\mathcal{X}_i = \mathcal{B}(\mathcal{X}_i, \bar{\mathbb{D}})$ for some $i \in \mathbb{Z}_+$. The limit behaviour of the max-min controllable sets leads to the result in Proposition 3.2. $\Diamond$

Proposition 3.2. *The MPC problem (2.14) is solvable at any time step $k \in \mathbb{Z}_+$ if and only if the maximal max-min controllable set is non-empty, i.e., $\mathcal{C}_\infty \neq \emptyset$, and the state x_k belongs to it. In such a case, a control sequence exists satisfying*

$$\begin{aligned}\mathbf{u}_k \in \mathcal{U}_\infty(x_k, \mathbf{d}_k) = \{\mathbf{u} = \{u_i\}_{i \in \mathbb{Z}_{[0,N-1]}} \in \mathbb{U}^N \mid \forall \mathbf{d}_k = \{d_{k+i}\}_{i \in \mathbb{Z}_{[0,N-1]}} \in \bar{\mathbb{D}}^N, \\ \phi(i; x_k, \mathbf{u}, \mathbf{d}_k) \in \mathcal{C}_\infty \;\; \text{and} \;\; E_u u_i + E_d d_{k+i} = 0 \;, \forall i \in \mathbb{Z}_{[0,N-1]}\}.\end{aligned} \tag{3.8}$$

Proof: The statement follows directly from Definitions 3.3 and 3.4. $\square$

Remark 3.2. *For large-scale systems with more complex constraint sets, the computation of the maximal max-min robust control invariant set $\mathcal{C}_\infty$ might be challenging. Constructing any non-empty robust control invariant set $\Omega \subseteq \mathbb{X}$ could be a simpler approach. From Definition 3.3, for $k = 0$, all $x_0 \in \Omega$ are controllable. Since Ω is control invariant, it follows that $\Omega \subseteq \mathcal{X}_N \subseteq \mathcal{C}_\infty$. Hence, the non-emptiness condition $\mathcal{C}_\infty \neq \emptyset$ holds by construction and a control sequence exists for any time step.* $\Diamond$

Theorem 3.1. *Let Assumptions 2.5, 2.6 and 3.1 hold. Then, under Interpretation 3.1, there exists an admissible max-min feedback control strategy of the form (3.1) such that for all $d_k \in \bar{\mathbb{D}}$ and all $k \in \mathbb{Z}_+$, a generalised flow-based network satisfies flow conservation constraints (2.10) and capacity constraints (2.11), if and only if the following conditions are satisfied:*

$$B_d\bar{\mathbb{D}} \subseteq -B\mathbb{U}, \tag{3.9a}$$

$$E_d\bar{\mathbb{D}} \subseteq -E_u\mathbb{U}. \tag{3.9b}$$

Moreover, the largest admissible initial set $\mathbb{X}_0 \subseteq \mathbb{X}$, for which the MPC algorithm remains feasible for all time and all $(x, d) \in \mathbb{X}_0 \times \bar{\mathbb{D}}$, is non-empty and given by the maximal max-min robust control invariant set, i.e., $\mathbb{X}_0 = \mathcal{C}_\infty \neq \emptyset$, which is explicitly characterised and given by

$$\mathcal{C}_\infty = \mathcal{B}(\mathbb{X}, \bar{\mathbb{D}}) = \big((\mathbb{X} \oplus (-B\mathbb{U})) \ominus B_d\bar{\mathbb{D}}\big) \cap \mathbb{X}. \tag{3.10}$$

Proof: Conditions in (3.9) can be considered as matching conditions for robust control system design, see [187] and references therein. Condition (3.9a) has been previously used under a min-max framework in [22, 23] and proved to be one of the required ingredients to ensure the existence of a min-max winning strategy in a dynamic LTI system. For generalised flow-based networks, it is necessary, in addition, to include condition (3.9b).

Both matching conditions are necessary and sufficient under Interpretation 3.1. The proof follows the line of arguments in [23], and is detailed here for completeness. For necessity, let $d^* \in \bar{\mathbb{D}}$ be a suitable demand flow such that $B_d d^* \notin -B\mathbb{U}$, i.e., there exists a hyperplane in $\mathbb{R}^n$ such that

$$\exists z_1 \in \mathbb{R}^n,\ \epsilon > 0,\ z_1^\top B_d d^* \geq -z_1^\top Bu + \epsilon_1,\ \forall u \in \mathbb{U}. \tag{3.11}$$

Then, for any $x_0 \in \mathbb{X}$ and selecting $d_k = d^*$ for all $k \geq 0$, from (2.10a) it is obtained that $x_k = Ax_0 + \sum_{i=0}^{k-1}(Bu_i + B_d d^*)$ for any feasible sequence $\{u_i \in \mathbb{U}\}_{i=0}^{k-1}$. Multiplying both sides of this latter equality by $z_1^\top$, it follows from (3.11) that

$$z_1^\top x_k = z_1^\top Ax_0 + z_1^\top \sum_{i=0}^{k-1}(Bu_i + B_d d^*) \geq z_1^\top x_0 + \epsilon_1 k,$$

which implies that for k sufficiently large, $x_k \notin \mathbb{X}$. In the same way, assume that there exists a vector $z_2 \neq 0$ in $\mathbb{R}^q$ that strongly separates $E_u\mathbb{U}$ from $-E_d d^*$. Then, there exists a $\epsilon_2 \geq 0$ such that $z_2^\top E_d d^* \geq -z_2^\top E_u u + \epsilon_2$ for every $u \in \mathbb{U}$. This latter implies the violation of the flow conservation in (2.10b). Therefore, both (3.9a) and (3.9b) are necessary conditions to guarantee the possibility for the controller to find admissible flows that counteract the effect of any demand and to keep the states within their feasible domain.

Conditions in (3.9) are also sufficient. First, it follows from (3.9a) and from the Minkowski sum properties, that $(\mathbb{X} \oplus (-B\mathbb{U})) \ominus B_d\bar{\mathbb{D}} \supseteq \mathbb{X}$, which implies that the set $\mathcal{C}_\infty$ defined in (3.10) is not empty. Now, from Definitions 3.3 and 3.4, for each $x_k \in \mathcal{C}_\infty \neq \emptyset$ and all $d_k \in \bar{\mathbb{D}}$, with $k = 0$, there exists a $u_k \in \mathbb{U}$ such that $x_{k+1} = Ax_k + Bu_k + B_d d_k \in \mathcal{C}_\infty$ and $E_u u_k + E_d d_k = 0$. By (3.9a) and (3.9b), for each $d_k \in \bar{\mathbb{D}}$, there exists a $u_k \in \mathbb{U}$ such that $Bu_k = -B_d d_k$ and $E_u u_k = -E_d d_k$. Therefore, $x_{k+1} \in \mathbb{X}$. By repeating the same argument for all subsequent $k \in \mathbb{Z}_{[1,\infty)}$, it can be shown that all the states $x \in \mathcal{C}_\infty$ define admissible initial conditions to keep future state trajectories feasible for all time steps.

Now, to prove that $\mathcal{C}_\infty = \mathcal{B}(\mathbb{X}, \bar{\mathbb{D}})$, recall the recursion (3.5). Assume that the one-step controllable set $\mathcal{X}_1 = \mathcal{B}(\mathcal{X}_0, D) \neq \emptyset$ with $\mathcal{X}_0 = \mathbb{X}$ and $D = \bar{\mathbb{D}}$. From the definition of the pre-image map $\mathcal{B}(\cdot,\cdot)$ in (3.4), it follows that

$$\mathcal{X}_1 = \{x \in \mathbb{X} \mid \forall d \in \bar{\mathbb{D}},\ \exists u \in \mathbb{U},\ \text{such that}\ \ Ax + Bu + B_d d \in \mathbb{X},\ E_u u + E_d d = 0\}. \tag{3.12}$$

In order to find an explicit characterisation of the set $\mathcal{X}_1$, it is needed to eliminate the quantifiers in (3.12) and obtain a single equation. To do so, one can first eliminate the algebraic constraint ($E_u u + E_d d = 0$) by using a suitable parametrisation of u. From Assumption 2.5, the following parametrisation can be performed:

$$\forall w \in \bar{\mathbb{D}},\ \exists v,\ \text{such that}\ \ u = \underbrace{\tilde{P}\begin{bmatrix} -M_1 \\ I_{(m-q)} \end{bmatrix}}_{Q} v + \underbrace{\tilde{P}\begin{bmatrix} -E_d \\ 0_p \end{bmatrix}}_{R} d \in \mathbb{U}, \tag{3.13}$$

where $v \in \mathbb{R}^{m-q}$ is a reordered subset of the control inputs, $\tilde{P} \in \mathbb{R}^{m\times m}$ is a positive permutation matrix, $I_{(m-q)}$ is an $(m-q)$-square identity matrix, 0_p is a p-square null matrix and $M_1 \in \mathbb{R}^{q\times(m-q)}$ is a matrix resultant from the rearrangement of the elements of u in the permutation process. For further details refer to Appendix B. Replacing (3.13) in (3.12) leads to

$$\begin{aligned} \mathcal{B}(\mathbb{X}, \bar{\mathbb{D}}) = \{x \in \mathbb{X} \mid \forall d \in \bar{\mathbb{D}},\ \exists v \in \mathbb{R}^{m-q},\ \text{such that}\ Qv + Rd \in \mathbb{U},\ \text{and} \\ Ax + B(Qv + Rd) + B_d d \in \mathbb{X}\}. \end{aligned} \tag{3.14}$$

Eliminating the quantifiers in (3.14), yields

$$\mathcal{X}_1 = \big((\mathbb{X} \oplus (-B\mathbb{U})) \ominus B_d\bar{\mathbb{D}}\big) \cap \mathbb{X}. \tag{3.15}$$

Similarly, it can be verified that $\mathcal{X}_2 = \mathcal{B}(\mathcal{X}_1, \bar{\mathbb{D}})$ is given explicitly by

$$\begin{aligned} \mathcal{X}_2 &= \big((\mathcal{X}_1 \oplus (-B\mathbb{U})) \ominus B_d\bar{\mathbb{D}}\big) \cap \mathbb{X} \\ &= \Big(\Big(\big(\big((\mathbb{X} \oplus (-B\mathbb{U})) \ominus B_d\bar{\mathbb{D}}\big) \cap \mathbb{X}\big) \oplus (-B\mathbb{U})\Big) \ominus B_d\bar{\mathbb{D}}\Big) \cap \mathbb{X}. \end{aligned} \tag{3.16}$$

From Minkowski sum properties, it can be verified that $(\mathbb{X} \oplus (-B\mathbb{U})) \ominus B_d\bar{\mathbb{D}} \supseteq \mathbb{X}$ if (3.9a) holds. Therefore, it holds that

$$\mathcal{X}_2 = \big((\mathbb{X} \oplus (-B\mathbb{U})) \ominus B_d\bar{\mathbb{D}}\big) \cap \mathbb{X} = \mathcal{X}_1.$$

Hence, from Remark 3.1, the maximal max-min robust control invariant set[1] is finitely determined and given by the fixed point of the pre-image mapping, i.e.,

$$\mathcal{C}_\infty = \big((\mathbb{X} \oplus (-B\mathbb{U})) \ominus B_d\bar{\mathbb{D}}\big) \cap \mathbb{X},$$

which concludes the proof. □

[1] Notice that even when the explicit expression of $\mathcal{C}_\infty$ results to be the same as if considering only (2.10a) for the dynamic nodes, the static balance in (2.10b) requires the extra condition (3.9b) to guarantee $\mathcal{C}_\infty \neq \emptyset$.

3.4 Synthesis of Flow-based Controllers

Initial conditions under which the MPC problem (2.14) ensures feasibility for all time steps has been discussed in the previous section. Nevertheless, in several applications of generalised-flow based networks, properties such as asymptotic average performance or convergence to some economically optimal set-point or periodic trajectory may in addition be required. Therefore, this section presents the design of different economic MPC controllers, which are specialised for the case when the network is operating under periodically time-varying demands and costs. Such periodic operation results to be of interest for the management of critical infrastructures such as water supply networks, energy distribution networks, supply-chains, among others, where the flow costs and demands often follow a periodic pattern.

3.4.1 Economic Scheduling Optimisation for Periodic Operation

A special case of the control synthesis problem is when the demand schedule is a periodic flow that is known a priori. In such a case, the time-varying demand sets $\mathbb{D}_k$ defined in the previous section become periodically time-varying singletons. Other particularity shared in several networks is to operate under periodically time-varying costs, which are usually defined by a time-of-use pricing scheme. Hence, the following assumptions are in order.

Assumption 3.2. *The demand $d_k = \delta(k) \in \mathbb{D}_k$ is perfectly known and follows a nominal T-periodic pattern. Hence, $\mathbb{D}_k = \{\delta(k)\}$ for all $k \in \mathbb{Z}_+$, with the consumer policy δ satisfying $\delta(k) = \delta(k+T)$ and consequently $d_k = d_{k+T}$ for all k. The smallest integer $T \in \mathbb{Z}_{\geq 1}$ fulfilling this latter equality is called the* period *of the uncontrolled flows.*

Assumption 3.3. *The system operates minimising a T-periodic economic stage cost $\ell : \mathbb{Z}_+ \times \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}_+$, which satisfies $\ell(k, x, u) = \ell(k+T, x, u)$, $T \in \mathbb{Z}_{\geq 1}$, for all k. This stage cost is continuous.*

In view of Assumptions 3.2 and 3.3, an economically optimal flow scheduling can be computed by solving the following T-horizon optimisation problem $\mathcal{P}_T(k, \bar{\mathbf{d}}_T, \mathbf{p}_T)$:

$$V_T^0(k, \bar{\mathbf{d}}_T, \mathbf{p}_T) = \min_{\bar{x}_0, \bar{\mathbf{u}}} V_T(k, \bar{x}_0, \bar{\mathbf{u}}) := \sum_{i=0}^{T-1} \ell(k+i, \bar{x}_i, \bar{u}_i) \tag{3.17}$$
$$\text{subject to } (\bar{x}_0, \bar{\mathbf{u}}) \in \mathcal{F}_T(\bar{\mathbf{d}}_T),$$

with the set $\mathcal{F}_T(\bar{\mathbf{d}}_T)$ being compact and defined as

$$\mathcal{F}_T(\bar{\mathbf{d}}_T) := \{(\bar{x}_0, \bar{\mathbf{u}}) \mid \bar{\mathbf{u}} = \{\bar{u}_i\}_{i\in\mathbb{Z}_{[0,N-1]}} \in \mathbb{U}_r^N,\ \phi(i;\bar{x}_0,\bar{\mathbf{u}},\bar{\mathbf{d}}_T) \in \mathbb{X}_r,\ \phi(N;\bar{x}_0,\bar{\mathbf{u}},\bar{\mathbf{d}}_T) = \bar{x}_0, \\ E_u\bar{u}_i + E_d\bar{d}_{\lfloor k+i\rfloor_T} = 0,\ i\in\mathbb{Z}_{[0,N-1]}\}, \tag{3.18}$$

where $\bar{\mathbf{d}}_T := \{\bar{d}_{\lfloor k+i\rfloor_T}\}_{i\in\mathbb{Z}_{[0,T-1]}}$ and $\mathbf{p}_T := \{p_{\lfloor k+i\rfloor_T}\}_{i\in\mathbb{Z}_{[0,T-1]}}$ denote respectively the nominal periodic demand and the vector of economic parameters defining the T-periodic stage cost function $\ell(k+i,\cdot,\cdot)$, $k\in\mathbb{Z}_+$. The sets $\mathbb{X}_r$ and $\mathbb{U}_r$ are closed hyperboxes contained in the relative interior of the original constraint sets in order to avoid that $\mathbb{X}$ and $\mathbb{U}$ are not active at the optimal trajectory. Assume that $\mathcal{F}_T(\bar{\mathbf{d}}_T) \neq \emptyset$, then the minimum of (3.17) exists from compactness of $\mathcal{F}_T(\bar{\mathbf{d}}_T)$ and continuity of $\ell(i,\cdot,\cdot)$ for all i. Moreover, from optimality and periodicity $V_T^0(0,\bar{\mathbf{d}}_T,\mathbf{p}_T) = V_T^0(k,\bar{\mathbf{d}}_T,\mathbf{p}_T)$ for all k. The optimal state and input trajectories can be constructed from the solution of (3.17) and given by $\bar{\mathbf{x}}^\star = \{\bar{x}_i^\star = \phi(i;\bar{x}_0^\star,\bar{\mathbf{u}}^\star,\bar{\mathbf{d}}_T)\}_{i\in\mathbb{Z}_{[0,T-1]}}$ and $\bar{\mathbf{u}}^\star = \{\bar{u}_i^\star\}_{i\in\mathbb{Z}_{[0,T-1]}}$, respectively, where $\phi(i;\bar{x}_0^\star,\bar{\mathbf{u}}^\star,\bar{\mathbf{d}}_T)$, denotes the i-step evolution of the dynamic state vector starting from the initial condition $\bar{x}_0^\star$. In addition, it is possible to build the best T-periodic orbit for system (2.10) as follows:

$$\mathcal{X}_T(\bar{\mathbf{d}}_T,\mathbf{p}_T) := \bigcup_{i=0}^{T-1}\{\bar{x}_i^\star\}. \tag{3.19}$$

In general, $\ell(k+i,\cdot,\cdot)$, $i\in\mathbb{Z}_{[0,T-1]}$, need not be positive definite with respect to any setpoint and there does not necessarily exist a unique optimal feasible solution $(\bar{\mathbf{x}}^\star,\bar{\mathbf{u}}^\star)$ or unique orbit set $\mathcal{X}_T(\bar{\mathbf{d}}_T,\mathbf{p}_T)$. Hence, in the following one of the feasible solutions can be arbitrarily selected or the cost function can be regularised to obtain a unique optimal solution.

3.4.2 Nominal Economic MPC Strategies for Periodic Network Flows

In this section, different economically-oriented MPC strategies are stated for the generalised flow-based networks scheduling-control problem. These schemes are: a two-layer architecture where an economic planner and a tracking MPC are interacting, a standard economic MPC with terminal state constraint, and two one-layer economic MPC strategies that account for changes in the economic criteria.

Hierarchical MPC

This is a two-layer optimal controller, where a separation of objectives, models and/or time-scales may be performed. Below, the optimisation problems involved in this hierarchical approach are stated.

Upper layer Economic MPC. In this layer, a dynamic real-time optimiser (D-RTO) computes the optimal time-varying state and input T-periodic trajectories by solving problem $\mathcal{P}_T(k, \bar{\mathbf{d}}_T, \mathbf{p}_T)$ at the beginning of each operating cycle. Denote with $(\bar{\mathbf{x}}_z^\star, \bar{\mathbf{u}}_z^\star)$ the optimal solution of (3.17) at time step $z \in \mathbb{Z}_+$, then these economically optimal trajectories govern a lower layer MPC described below.

Lower layer Tracking MPC. In this layer, conventional MPC is used to enforce the system to track the pre-computed optimal trajectories. The associated optimisation problem is stated as

$$\min_{\mathbf{u}_k} \sum_{i=0}^{H_p^l-1} \|x_{k+i|k} - \bar{x}^\star_{\lfloor k+i \rfloor_T|z}\|^2_{Q_x} + \|u_{k+i|k} - \bar{u}^\star_{\lfloor k+i \rfloor_T|z}\|^2_{Q_u}, \tag{3.20a}$$

subject to:

$$\begin{aligned}
&x_{k+i+1|k} = Ax_{k+i|k} + Bu_{k+i|k} + B_d\bar{d}_{\lfloor k+i \rfloor_T}, && \forall i \in \mathbb{Z}_{[0,H_p^l-1]} && (3.20b)\\
&E_u u_{k+i|k} + E_d \bar{d}_{\lfloor k+i \rfloor_T} = 0, && \forall i \in \mathbb{Z}_{[0,H_p^l-1]} && (3.20c)\\
&x_{k+i|k} \in \mathbb{X}, && \forall i \in \mathbb{Z}_{[1,H_p^l-1]} && (3.20d)\\
&u_{k+i|k} \in \mathbb{U}, && \forall i \in \mathbb{Z}_{[0,H_p^l-1]} && (3.20e)\\
&x_{k+H_p^l|k} = \bar{x}^\star_{k+H_p^l|z}, && && (3.20f)\\
&x_{k|k} = x_k, && && (3.20g)
\end{aligned}$$

where $H_p^l \in \mathbb{Z}_{\geq 1}$ is the prediction horizon of the lower layer, $x_k \in \mathbb{R}^n$ is the measured initial state at time step $k \in \mathbb{Z}_+$, the weighting matrices satisfy $Q_x \succ 0$, $Q_u \succ 0$ and $\lfloor k+i \rfloor_T$ denotes the modulus operation $\mathrm{mod}(k+i, T)$. Following the receding horizon technique, the control law derived under this hierarchical scheme is given by $\kappa\left(x_k, \bar{\mathbf{d}}_T\right) = u^\star_{k|k}$, i.e., only the first control action of the optimal input sequence obtained in (3.20) is applied to the system. If asymptotic convergence to the upper layer trajectory is desired, the tracking problem can be reformulated in terms of the error $e_{k+i} = x_{k+i|k} - \bar{x}^\star_{\lfloor k+i \rfloor_T|z}$, leading to a set-point (origin) stabilization problem of the error dynamics; see e.g., [56].

Note that the upper layer may have an equal or larger sampling time than the one of the lower layer, i.e., $\Delta t_1 \geq \Delta t_2$. The main drawback of this two-layer MPC approach for the management of generalised flow-based networks is that if the economic parameters of the cost function or the demand patterns change in time with a high rate, then the transitory periods will be so that the interaction between layers could lead to a possible loss of feasibility or to an economic performance degradation.

Standard Economic MPC

The main feature of this approach, in contrast with the hierarchical scheme even when the two-layers may work with the same sampling time, is that the standard economic MPC considers the global economic criteria directly as the stage cost of the MPC controller and avoids penalising the tracking error with respect to the targets [150]. The associated FHOP for the periodic operation of the system is stated as

$$\min_{\mathbf{u}_k} \; V_N(k, x_k, \mathbf{u}_k) = \sum_{i=0}^{N-1} \ell(k+i, x_{k+i|k}, u_{k+i|k}), \tag{3.21a}$$

subject to:

$$x_{k+i+1|k} = Ax_{k+i|k} + Bu_{k+i|k} + B_d \bar{d}_{\lfloor k+i \rfloor_T}, \qquad \forall i \in \mathbb{Z}_{[0,N-1]} \tag{3.21b}$$

$$E_u u_{k+i|k} + E_d \bar{d}_{\lfloor k+i \rfloor_T} = 0, \qquad \forall i \in \mathbb{Z}_{[0,N-1]} \tag{3.21c}$$

$$x_{k+i|k} \in \mathbb{X}, \qquad \forall i \in \mathbb{Z}_{[1,N-1]} \tag{3.21d}$$

$$u_{k+i|k} \in \mathbb{U}, \qquad \forall i \in \mathbb{Z}_{[0,N-1]} \tag{3.21e}$$

$$x_{k+N|k} = \bar{x}^\star_{\lfloor k+N \rfloor_T}, \tag{3.21f}$$

$$x_{k|k} = x_k, \tag{3.21g}$$

where x_k is the measured initial state at time step $k \in \mathbb{Z}_+$, and $\bar{x}^\star_{\lfloor k+N \rfloor_T}$ is the optimal periodic value obtained in (3.17) that corresponds to the time step k. Similarly to conventional tracking MPC, the first optimal control action $u^\star_{k|k}$ obtained from (3.21) is applied to the system. As shown in [7, Theorem 4], standard economic MPC is capable of enhancing the closed-loop economic performance of the system if this is not optimally operated at steady state, achieving an asymptotic average cost that is at least as good as the average cost of the best periodic trajectory obtained in (3.17), i.e.,

$$\limsup_{M \to +\infty} \frac{\sum_{k=0}^{M} \ell(k, x_k, u_k)}{M+1} \leq \frac{\sum_{k=0}^{T-1} \ell(k, \bar{x}^\star_k, \bar{u}^\star_k)}{T}. \tag{3.22}$$

For a general (possibly non-convex) cost functional $\ell(k, x, u)$, the standard economic MPC may not be a stabilising controller. In fact, to guarantee stability and optimal steady-state operation of a system, certain dissipativity conditions must be satisfied (see [117] and references therein). For the particular case of periodic operation, different periodic dissipativity notions were introduced in [75], from where the following definition is extracted (adapted to our setting).

Definition 3.5. *Let Assumptions 3.2 and 3.3 hold. Then, system (2.10) is called* strictly dissipative *with respect to a T-periodic supply rate $s : \mathbb{Z}_{[0,T-1]} \times \mathbb{X} \times \mathbb{U} \to \mathbb{R}$, if there exists a T-periodic storage function $\lambda : \mathbb{Z}_{[0,T-1]} \times \mathbb{X} \to \mathbb{R}$, and a function $\rho \in \mathcal{K}_\infty$, such that the following inequality holds for all $(k, x, u) \in \mathbb{Z}_{[0,T-1]} \times \mathbb{X} \times \bar{\mathbb{U}}(\bar{d}_k)$:*

$$s(k, x, u) + \lambda(k, x) - \lambda(k+1, Ax + Bu + B_d\bar{d}_k) \geq \rho(\|x - \bar{x}_k^\star\|). \tag{3.23}$$

Therefore, in order to enforce convergence to the optimal trajectory $(\bar{\mathbf{x}}^\star, \bar{\mathbf{u}}^\star)$, the following is assumed.

Assumption 3.4. *System (2.10) is strictly dissipative with respect to the supply rate defined as $s(k, x, u) = \ell(k, x, u) - \ell(k, \bar{x}_k^\star, \bar{u}_k^\star)$.*

There exist methods to determine storage functions λ that satisfy Assumption 3.4 for linear systems with convex constraints and convex cost functions, see e.g., [45, 48] for time-invariant systems and [194] for periodically time-varying systems.

Nominal asymptotic stability of the closed-loop system operating with periodic demand and periodic cost is guaranteed by problem (3.21) and can be proved directly from [194, Theorem 1] under certain additional assumptions, i.e., (weak) controllability of system (2.10) and strong duality of problem (3.17), and with minor modifications to include the algebraic equation (2.10b). Adapting [7, Definition 6.1] to the periodic case, it can be said that when the system is forced to *optimally operate* at the best periodic trajectory $(\bar{\mathbf{x}}^\star, \bar{\mathbf{u}}^\star)$, its asymptotic average performance is under-bounded by

$$\limsup_{M \to +\infty} \frac{\sum_{k=0}^{M} \ell(k, x_k, u_k)}{M+1} \geq \frac{\sum_{k=0}^{T-1} \ell(k, \bar{x}_k^\star, \bar{u}_k^\star)}{T}. \tag{3.24}$$

If the above inequality is strict ($>$), then the system is *sub-optimally operated off* its best periodic trajectory.

Even when the economic MPC controller improves the asymptotic average economic performance of the system when not enforcing convergence, its main weakness is a possible loss of feasibility due to changes in the periodic demand pattern or in the economic parameters affecting the cost function, which make the previous target state to be possibly unreachable under the given prediction horizon. Consequently, the standard economic MPC controller still depends on the pre-computed optimal T-periodic trajectory, which should be computed when detecting demand or costs changes before the MPC runs in order to obtain compatible terminal states.

Economic MPC for Changing Operating Patterns

In order to overcome the possible loss of feasibility due to changing operating patterns caused by the parameters of the cost function or by the demands, two approaches following the ideas in [82] and [106] are proposed here to be solved in a one-layer architecture. These schemes integrate in different ways the optimal trajectory problem (3.17) with the standard economic MPC in (3.21).

Option A: Enlargement of the prediction horizon

$$\min_{\mathbf{u}_k} \sum_{i=0}^{N+T-1} \ell(k+i, x_{k+i|k}, u_{k+i|k}), \tag{3.25a}$$

subject to:

$$x_{k+i+1|k} = Ax_{k+i|k} + Bu_{k+i|k} + B_d\bar{d}_{\lfloor k+i \rfloor_T}, \quad \forall i \in \mathbb{Z}_{[0,N+T-1]} \tag{3.25b}$$

$$E_u u_{k+i|k} + E_d\bar{d}_{\lfloor k+i \rfloor_T} = 0, \quad \forall i \in \mathbb{Z}_{[0,N+T-1]} \tag{3.25c}$$

$$x_{k+i|k} \in \mathbb{X}, \quad \forall i \in \mathbb{Z}_{[1,N+T-1]} \tag{3.25d}$$

$$u_{k+i|k} \in \mathbb{U}, \quad \forall i \in \mathbb{Z}_{[0,N+T-1]} \tag{3.25e}$$

$$x_{k+N+T|k} = x_{k+N|k} \tag{3.25f}$$

$$x_{k|k} = x_k, \tag{3.25g}$$

where $T \in \mathbb{Z}_+$ is the period of the demands and $N = cT$ with $c \in \mathbb{Z}_+$. Note that in this option, slight changes of the economic MPC framework are required. First, the terminal constraint (3.21f) is replaced with (3.25f). Second, the prediction horizon is extended to cover the period of the cost and process dynamics. This is done in order to avoid the use of a precomputed trajectory. Note that (3.21f) is a sort of generalised terminal constraint associated to a periodic trajectory that results from the same prediction model used in problem rather than to a precomputed trajectory. This makes the system find its own new optimal orbit if the operational conditions (prices or demands) change.

Option B: Inclusion of a pseudo-reference to track

$$\min_{\mathbf{u}_k x_0^s, \mathbf{u}_k^s} \sum_{j=0}^{T-1} \gamma_{\mathrm{E}}\, \ell(k+j, x_j^s, u_j^s) + \sum_{i=0}^{N-1} \gamma_{\mathrm{T}} \left(\|x_{k+i|k} - x_i^s\|_{Q_x}^2 + \|u_{k+i|k} - u_i^s\|_{Q_u}^2 \right), \tag{3.26a}$$

subject to (3.21b)-(3.21e) $\forall i \in \mathbb{Z}_{[0,N]}$ and

$$x_{j+1}^s = Ax_j^s + Bu_j^s + B_d \bar{d}_{\lfloor k+j \rfloor_T}, \qquad \forall j \in \mathbb{Z}_{[0,T-1]} \tag{3.26b}$$

$$E_u u_j^s + E_d \bar{d}_{\lfloor k+j \rfloor_T} = 0, \qquad \forall j \in \mathbb{Z}_{[0,T-1]} \tag{3.26c}$$

$$(x_j^s, u_j^s) \in \mathbb{X}_r \times \mathbb{U}_r, \qquad \forall j \in \mathbb{Z}_{[0,T-1]} \tag{3.26d}$$

$$x_T^s = x_0^s, \tag{3.26e}$$

$$x_{k+N|k} = x_N^s, \tag{3.26f}$$

$$x_{k|k} = x_k, \tag{3.26g}$$

with $Q_x \succ 0$, $Q_u \succ 0$ and the prediction horizon N selected in such a way that $N \leq T$, being $T \in \mathbb{Z}_{\geq 1}$ the period of the time-varying parameters. Moreover, $\gamma_{\mathrm{E}} \in \mathbb{R}_+$ and $\gamma_{\mathrm{T}} \in \mathbb{R}_+$ are scalars introduced to establish a trade-off between economic and tracking performance. Recall that $\mathbb{X}_r$ and $\mathbb{U}_r$ are tightened constraint sets that are used here to avoid active constraints at the optimal pseudo-reference. Note that the constraints (3.26b)-(3.26e) imposed on the pseudo-reference model are identical to the constraints of periodic scheduling problem (3.17), and guarantee that this trajectory is admissible and periodic. The terminal constraint (3.26f) forces the predicted trajectory of the real system to reach the pseudo-reference in N steps.

This controller has the following interesting theoretical and practical properties that are proved and discussed in detail in [106]:

(i) The controller guarantees convergence of the closed-loop system to the optimal trajectory $(\bar{\mathbf{x}}^\star, \bar{\mathbf{u}}^\star)$ without requiring its a priori computation by a dynamic RTO for the given periodic economic parameters.

(ii) The proposed optimisation problem is recursively feasible for the nominal periodic demand, even if the economic cost function changes.

(iii) The domain of attraction of the proposed controller is in general larger than the domain associated with the standard economic MPC approach.

(iv) The closed-loop system is input-to-state stable with respect to additive disturbances, whenever the evolution of the system is admissible. This implies that the closed-loop system is robust to small variations of the demand vector $\bar{d}$.

Remark 3.3. *The economic MPC controllers associated with the optimisation problems* (3.4.2) *and* (3.26) *are not enforcing convergence to the precomputed optimal trajectory obtained in* (3.17)*. Instead, they are meant to retain feasibility under possible changes of the economic parameters in the cost function and to find new optimal stable trajectories for the current conditions. To do so, certain assumptions are still required such as (weak) controllability and (strict) convexity of the cost function.*

3.4.3 Numerical Results

This section presents the results of applying the economic MPC approaches described in § 3.4.2 to the small-size sector example (see Figure 2.5) of the drinking water network case study described in § 2.4. The FHOP of each strategy is properly adapted to incorporate the additional safety constraint and slack variables used in (2.23). The sampling time is $\Delta t = 3600$ s. The simulation horizon is sixteen days ($N_s = 384$ hours) for each strategy. The weights of the aggregate user-defined cost function are $\gamma_1 = 100$, $\gamma_2 = 1$, and $\gamma_3 = 10$. For the tracking terms, the weighting matrices Q_x and Q_u are set up arbitrarily as identity matrices of proper dimensions (although other weights can be freely use depending on the desired response). The prediction horizon has been selected as $N = 24$ hours due to the periodicity of both water demands and electricity prices (i.e., $T = 24$ hours). For the hierarchical controller, the upper layer is executed every 24 hours as usually done in water distribution scheduling, while the lower layer runs in an hourly basis as in the other economic MPC strategies. The initial common state for all simulations is $x_0 = [160.44, 646.23, 633.89]^\top$ in m^3 and the security threshold is $s = [42, 18.0, 270]^\top$ in m^3. The formulation of the optimisation problems and the closed-loop simulations have been carried out using YALMIP Toolbox, CPLEX solver and Matlab R2012b (64 bits), running in a PC Intel Core E8600 at 3.33GHz with 8GB of RAM.

The closed-loop performance of each controller has been assessed using the key performance indicators (KPIs) described in § 2.4.4. Results are summarised in Table 3.1, where the hierarchical MPC, the standard economic MPC, and both economic MPC options A and B for changing patterns are compared and labeled HEMPC, EMPC,

Table 3.1: Comparison of controller performance

Controller	KPI_E (e.u.)	$\mathrm{KPI}_{\Delta U}$ $(\mathrm{m}^3/\mathrm{s})^2$
Ideal scheduling (Problem (3.17))	28.6056	2.06×10^{-3}
$\mathrm{HEMPC}_{(1)}$	28.4347	2.14×10^{-3}
$\mathrm{HEMPC}_{(2)}$	28.6114	2.08×10^{-3}
$\mathrm{EMPC}_{(1)}$	28.4124	8.89×10^{-7}
$\mathrm{EMPC}_{(2)}$	28.6080	8.89×10^{-7}
$\mathrm{EMPCT\text{-}A}_{(1)}$	28.4124	8.89×10^{-7}
$\mathrm{EMPCT\text{-}A}_{(2)}$	28.6041	8.89×10^{-7}
$\mathrm{EMPCT\text{-}B}_{(1)@\{\gamma_{\mathrm{O}}=1,\ \gamma_{\mathrm{T}}=1\}}$	28.5165	2.17×10^{-3}
$\mathrm{EMPCT\text{-}B}_{(1)@\{\gamma_{\mathrm{O}}=1,\ \gamma_{\mathrm{T}}=10\}}$	28.4493	2.35×10^{-3}
$\mathrm{EMPCT\text{-}B}_{(1)@\{\gamma_{\mathrm{O}}=1,\ \gamma_{\mathrm{T}}=100\}}$	28.4178	2.34×10^{-3}
$\mathrm{EMPCT\text{-}B}_{(1)@\{\gamma_{\mathrm{O}}=10,\ \gamma_{\mathrm{T}}=1\}}$	28.8128	2.29×10^{-3}
$\mathrm{EMPCT\text{-}B}_{(2)@\{\gamma_{\mathrm{O}}=1,\ \gamma_{\mathrm{T}}=100\}}$	28.6120	2.43×10^{-3}

e.u.: economic units.

Subindex (1) indicates non-periodic behaviour enforced while (2) indicates that the periodic constraint is enforced.

EMPCT-A and EMPCT-B, respectively. The sub-index (1) indicates that the controller is not including the terminal constraint, while sub-index (2) indicates that the terminal periodic constraint is enforced. The safety indicator has been omitted in Table 3.1 given that, for all simulated scenarios and strategies, $\mathrm{KPI}_S = 0$, which means that all of the MPC controllers decided not to use water from the safety stocks for the given periodic demand. Note that for each strategy the enforcement of terminal constraints implies an increment of the economic cost. This decrease in performance is the price for gaining in stability.

Furthermore, Table 3.2 discloses details of the production and operational costs related to each strategy starting from a non-optimal state $x_0 = [92.45, 905.82, 504.14]^\top$ in m^3, and compares the daily average economic performance of the controllers enforcing their corresponding periodic terminal constraints. For the standard $\mathrm{EMPC}_{(2)}$, the terminal constraint is set up in relation to a pre-calculated optimal cycle obtained from

Table 3.2: Comparison of daily average costs of EMPC strategies

Controller	Water Cost (e.u./day)	Electric Cost (e.u./day)	Daily Cost (e.u./day)
EMPC$_{(2)}$	577.24	110.04	687.28
HEMPC$_{(2)}$	610.02	134.13	744.15
EMPCT-A$_{(2)}$	577.79	109.56	687.35
EMPCT-B$_{(2)}$	577.75	109.79	687.54

e.u.: economic units

(3.17). In the HEMPC$_{(2)}$, the reference trajectory is computed by the upper layer every 24 hours. For controllers EMPCT-A$_{(2)}$ and EMPCT-B$_{(2)}$, no pre-calculated trajectory is needed. It can be seen how the HEMPC$_{(2)}$ cost degrades notoriously the performance in comparison with the other MPC strategies due to the time-scale separation in its layers. Even when feasibility issues were not found for any of the strategies in this case study, these results reaffirm the current tendency of improving the economic performance by migrating to one-layer economic MPC controllers which are robust to changes in the cost function.

To further highlight the performance of controllers EMPCT-A and EMPCT-B, which cope with changing economic criterion, the price parameter of the economic term in (2.22) has been affected switching the price profile at different time steps but keeping the same period, see Figure 3.1. As it can be seen in Figure 3.2 for the evolution of the states, both controllers maintain the recursive feasibility and stabilise at similar trajectories.

Even so, the approach in (3.26), which includes a pseudo-reference and tracking terms, presents a slightly higher cost with respect to the approach in (3.4.2). This behaviour might be due to the regularisation terms that decrease the economic performance if design parameters (i.e., γ_E, γ_T, Q_x and Q_u) are not properly tuned. A possible enhancement could be to include the economic cost function also in the tracking term of the EMPCT-B. Further simulations and numerical results of this latter controller for this case study and for a four tank benchmark example can be found in [106].

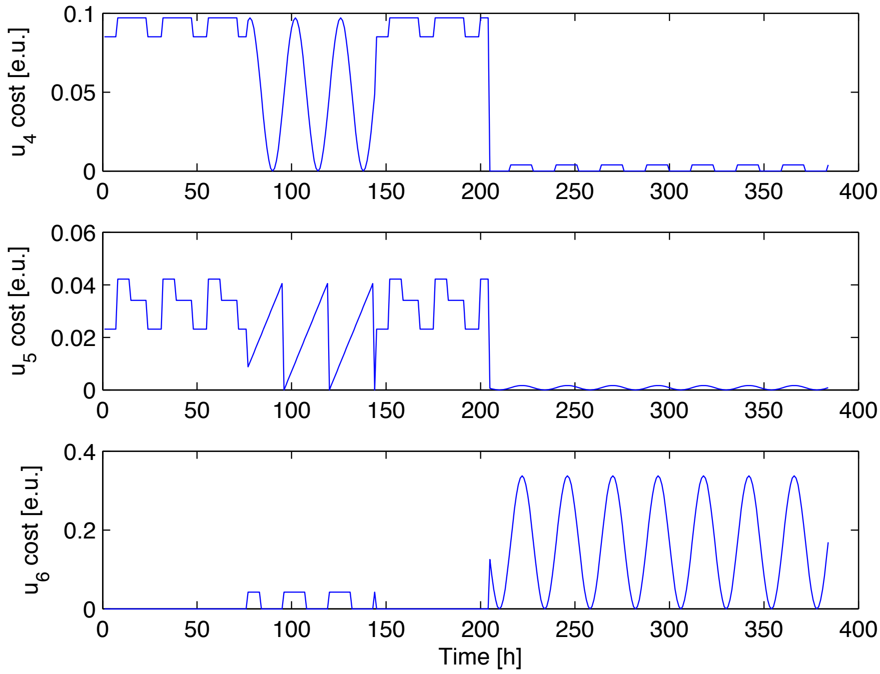


Figure 3.1: Price-of-use for actuators u_4, u_5 and u_6 in economic units (e.u.)

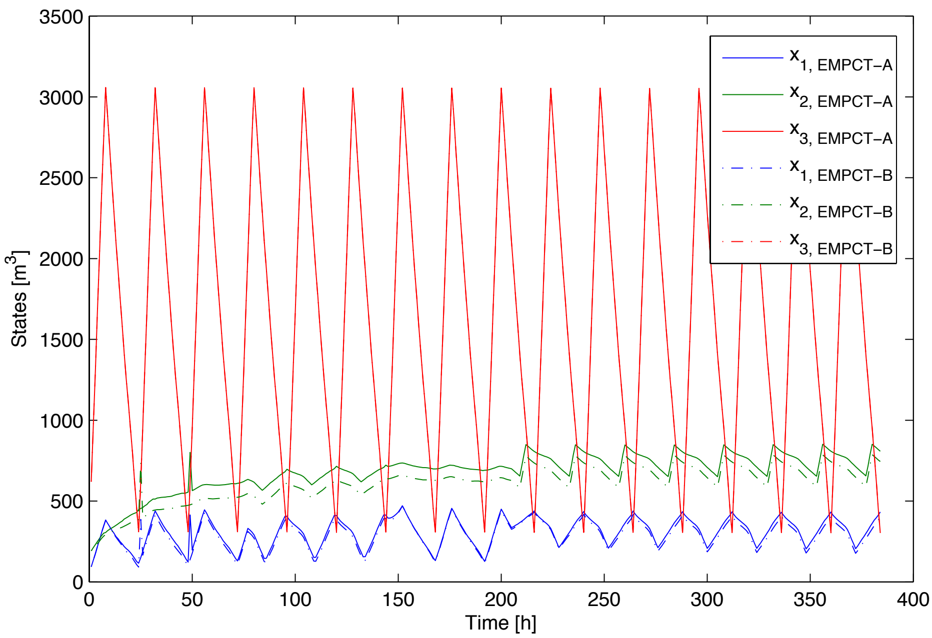


Figure 3.2: Evolution of some states under varying economic parameters

3.5 Economic MPC for Periodic Systems

Previous stabilising strategies are based on a periodically time-varying terminal equality constraint MPC formulation, in which the system is stabilised by forcing the terminal state to a point of the best periodic orbit. Nevertheless, as highlighted in [6] for the time-invariant case, a terminal equality constraint may result to be quite restrictive and should be removed or replaced with a terminal penalty and a terminal region constraint. Therefore, the main contribution of this section is the extension and generalisation of previous results addressing periodic economic operation of time-invariant systems with periodic costs to the case of periodically time-varying systems. Specifically, a terminal penalty/region economic MPC formulation for general linear periodic systems is proposed here. For this, the methodology proposed in [27], based on linear matrix inequalities (LMIs), is incorporated into the economic MPC framework to design the required periodically time-varying terminal penalty functions, terminal control laws and terminal ellipsoidal regions. The resultant conditions are less conservative than those of [194], thus leading to an improved size of the feasible set of initial conditions and a possibly improved closed-loop performance.

Consider a class of dynamic time-varying affine system described by

$$x_{k+1} = f(k, x_k, u_k) := A_k x_k + B_k u_k + w_k, \tag{3.27}$$

where $x_k \in \mathbb{R}^n$ and $u_k \in \mathbb{R}^m$ represent the system state and input vectors at current time step $k \in \mathbb{Z}_+$, respectively. State measurements are assumed to be available at each time step. The vector $w_k \in \mathbb{W} \subseteq \mathbb{R}^p$ represents a known and bounded (possibly time-varying) exogenous input. Matrices A_k and B_k are assumed to be known for each time step k.

The system is subject to (possibly time-varying) equality and inequality constraints given in the form of a convex closed polyhedral set, which is defined as

$$\mathbb{Y}_k := \{(x, u) \in \mathbb{R}^{n+m} \mid a_{j,k}x + b_{j,k}u \leq h_{j,k}, \forall j \in \mathbb{Z}_{[1,r]}\}, \ \forall k \in \mathbb{Z}_+, \tag{3.28}$$

where $r \in \mathbb{Z}_{\geq 1}$ is the number of hyperplanes and $a_{j,k} \in \mathbb{R}^{1\times n}$ and $b_{j,k} \in \mathbb{R}^{1\times m}$ for all $j \in \mathbb{Z}_{[1,r]}$ and $k \in \mathbb{Z}_+$. Denote by $\mathbb{X}_k \subseteq \mathbb{R}^n$ and $\mathbb{U}_k \subset \mathbb{R}^m$ the projections of $\mathbb{Y}_k$ at each time step k on the state and input domains, respectively. The performance of the system is measured by a possibly time-varying economic stage cost function $\ell : \mathbb{Z}_+ \times \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}_+$.

In what follows, this section focuses on a subclass of time-varying systems, namely *periodically time-varying* systems.

Definition 3.6. *System* (3.27) *is called* T-periodic *if there exists a* $T \in \mathbb{Z}_{\geq 1}$ *such that for all* $(k, x, u) \in \mathbb{Z}_{\geq 0} \times \mathbb{R}^n \times \mathbb{R}^m$ *it holds that* $f(k, x, u) = f(k + T, x, u)$*. The smallest such* T *is called* period *of system* (3.27)*.*

Assumption 3.5 (Properties of constraint sets)**.** *The constraint set is periodically-time varying, i.e.,* $\mathbb{Y}_{k+T} = \mathbb{Y}_k$*, and satisfies in addition* $\mathbb{Y}_k \subseteq \mathcal{C}$ *for all* $k \in \mathbb{Z}_+$ *and some compact set* $\mathcal{C}$ *containing the origin.*

Assumption 3.6 (Periodicity and continuity of functions)**.** *The functions* f *and* ℓ *are continuous on* $\mathbb{Y}_k$ *for all* $k \in \mathbb{Z}_+$ *and* T*-periodic, i.e.,* $f(k, \cdot, \cdot) = f(k + T, \cdot, \cdot)$ *and* $\ell(k, \cdot, \cdot) = \ell(k + T, \cdot, \cdot)$*.*

From (3.27), Assumption 3.6 implies that $A_{k+T} = A_k$, $B_{k+T} = B_k$, and $w_{k+T} = w_k$ for all $k \in \mathbb{Z}_+$. Note that a time-invariant system is a T-periodic system with period $T = 1$.

3.5.1 Economically Optimal Scheduling for Periodic Systems

Similarly to § 3.4.1, the optimal scheduling for the system (3.27) satisfying Assumptions 3.5 and 3.6, with known periodic sequence $\mathbf{w}_T := \{w_i\}_{i \in \mathbb{Z}_{[0,T-1]}}$ and known economic parameter $\mathbf{p}_T := \{p_i\}_{i \in \mathbb{Z}_{[0,T-1]}}$, is given by the following optimisation problem:

$$V_T^0(k, \mathbf{w}_T, \mathbf{p}_T) = \min_{\bar{x}_0, \bar{\mathbf{u}}} \sum_{i=0}^{T-1} \ell(k + i, \bar{x}_i, \bar{u}_i), \tag{3.29a}$$

subject to

$$\bar{x}_{i+1} = f(k + i, \bar{x}_i, \bar{u}_i), \qquad \forall i \in \mathbb{Z}_{[0,T-1]} \tag{3.29b}$$

$$(\bar{x}_i, \bar{u}_i) \in \mathbb{Y}_{k+i}, \qquad \forall i \in \mathbb{Z}_{[0,T-1]} \tag{3.29c}$$

$$\bar{x}_0 = \bar{x}_T, \tag{3.29d}$$

from which the optimal state and input periodic trajectories can be constructed as $\bar{\mathbf{x}}^\star := \{\bar{x}_i^\star\}_{i \in \mathbb{Z}_{[0,T-1]}}$ and $\bar{\mathbf{u}}^\star := \{\bar{u}_i^\star\}_{i \in \mathbb{Z}_{[0,T-1]}}$, respectively. Each $w_{\lfloor k+i \rfloor_T}$ element is the affine term corresponding to the function $f(k + i, \cdot, \cdot)$, and each $p_{\lfloor k+i \rfloor_T}$ element defines the cost $\ell(k + i, \cdot, \cdot)$. Recall that from optimality and periodicity of the solution, it holds $V_T^0(0, \mathbf{w}_T, \mathbf{p}_T) = V_T^0(k, \mathbf{w}_T, \mathbf{p}_T)$ for all $k \in \mathbb{Z}_+$. Hence, the infinite horizon state and input trajectories can be defined from the solution of (3.29) with $k = 0$, as $\hat{\mathbf{x}}_\infty^\star := \{\hat{x}_i^\star = \bar{x}_{\lfloor i \rfloor_T}^\star\}_{i \in \mathbb{Z}_{[0,\infty)}}$ and $\hat{\mathbf{u}}_\infty^\star := \{\hat{u}_i^\star = \bar{u}_{\lfloor i \rfloor_T}^\star\}_{i \in \mathbb{Z}_{[0,\infty)}}$. In this way, the best T-periodic orbit is given by $\mathcal{X}_T(\bar{\mathbf{d}}_T, \mathbf{p}_T) := \bigcup_{i=0}^{T-1} \{\bar{x}_i^\star\}$.

3.5.2 Economic MPC with Periodic Terminal Penalty and Region

In this section, an economic MPC controller with terminal penalty/region formulation is proposed, which extends the results in [6] for the case of T-periodic systems with possibly periodically time-varying economic cost functions. The proposed formulation also satisfies the inequality (3.22), i.e., the closed-loop system under non-steady operation outperforms the asymptotic average performance of the optimal trajectory $(\hat{\mathbf{x}}_\infty^\star, \hat{\mathbf{u}}_\infty^\star)$ given by (3.29). The periodic terminal point constraint setting described in § 3.4.2 is a particular case of the economic MPC presented in this chapter. The proposed economic MPC problem is defined as follows:

$\mathcal{P}_N(k, x_k, \mathbf{w}_T, \mathbf{p}_T)$:

$$\min_{\mathbf{u}_k} V_N(k, x_k, \mathbf{u}_k) = \sum_{i=0}^{N-1} \ell(k+i, x_{k+i|k}, u_{k+i|k}) + V_f(k+N, x_{k+N|k}) \tag{3.30a}$$

subject to:

$$x_{k+i+1|k} = f(k+i, x_{k+i|k}, u_{k+i|k}), \qquad \forall i \in \mathbb{Z}_{[0,N-1]} \tag{3.30b}$$

$$(x_{k+i|k}, u_{k+i|k}) \in \mathbb{Y}_{k+i}, \qquad \forall i \in \mathbb{Z}_{[0,N-1]} \tag{3.30c}$$

$$x_{k+N|k} \in \mathbb{X}_f(k+N, \hat{x}_{k+N}^\star), \tag{3.30d}$$

$$x_{k|k} = x_k, \tag{3.30e}$$

where function $V_f : \mathbb{Z}_{\geq 0} \times \mathbb{R}^n \to \mathbb{R}_+$ is a time-varying penalty on the terminal state, and the set $\mathbb{X}_f(k+N, \hat{x}_{k+N}^\star) \subseteq \mathbb{X}_{k+N}$ is a time-varying compact terminal region containing the target state $\hat{x}_{k+N}^\star$ in its interior, which is defined as $\hat{x}_{k+N}^\star := \bar{x}_{\lfloor k+N \rfloor_T}^\star$, where the term in the right hand side of the equality comes from solving (3.29) with $k = 0$.

The control law is derived from the receding horizon policy, i.e.,

$$u_k = \kappa_N(k, x_k) := u^\star(k|k), \tag{3.31}$$

and the closed-loop system evolution is given by

$$x_{k+1} = f(k, x_k, \kappa_N(k, x_k)). \tag{3.32}$$

The set of all control sequences $\mathbf{u}_k$ satisfying the state, input, and terminal constraints of $\mathbb{P}_N(k, x_k, \mathbf{w}_T, \mathbf{p}_T)$ is denoted as

$$\mathcal{U}_N(k, x_k) := \{\mathbf{u}_k \mid (x_k, \mathbf{u}_k) \in \mathcal{F}_N(k)\} \tag{3.33}$$

in which, for each $k \in \mathbb{Z}_+$, $\mathcal{F}_N(k) \subset \mathbb{R}^n \times \mathbb{R}^{Nm}$ is a compact set defined by

$$\mathcal{F}_N(k) := \{(x_k, \mathbf{u}_k) \mid x_{k|k} = x_k,\ x_{k+i+1|k} = f(k+i, x_{k+i|k}, u_{k+i|k}), \tag{3.34}$$

$$(x_{k+i|k}, u_{k+i|k}) \in \mathbb{Y}_{k+i},\ \forall i \in \mathbb{Z}_{[0,N-1]},$$

$$x_{k+N|k} \in \mathbb{X}_f(k+N, \hat{x}^\star_{k+N})\}. \tag{3.35}$$

The time-varying domain of admissible states is defined as the projection of $\mathcal{F}_N(k)$ onto the space $\mathbb{R}^n$, i.e.,

$$\mathcal{X}_N(k) := \{x_k \in \mathbb{R}^n \mid \exists \mathbf{u}_k \text{ such that } (x_k, \mathbf{u}_k) \in \mathcal{F}_N(k)\}. \tag{3.36}$$

Let $\{\mathbb{S}_t\}_{t \in \mathbb{Z}_{[0,T-1]}}$ denote a sequence of sets with $\mathbb{S}_t \subseteq \mathbb{X}_t$ for all $t \in \mathbb{Z}_{[0,T-1]}$, $T \in \mathbb{Z}_+$. Then, the following definitions are introduced.

Definition 3.7. *The sequence* $\{\mathbb{S}_t\}_{t \in \mathbb{Z}_{[0,T-1]}}$ *is called* periodically positive invariant (PPI) *for an autonomous system of the form* $x_{k+1} = f(k, x_k)$ *if for each* $k \in \mathbb{Z}_+$ *it holds that* $x_k \in \mathbb{S}_{\lfloor k \rfloor_T}$ *implies* $x_{k+1} \in \mathbb{S}_{\lfloor k+1 \rfloor_T}$.

Definition 3.8. *System* (3.27) *is called* strictly dissipative *with respect to a T-periodic* supply rate function $s : \mathbb{Z}_+ \times \mathbb{X}_k \times \mathbb{U}_k \to \mathbb{R}$ *if there exists a T-periodic* storage function $\lambda : \mathbb{Z}_+ \times \mathbb{X}_k \to \mathbb{R}_{\geq 0}$, *and a $\mathcal{K}_\infty$ function $\rho(\cdot)$ such that the following inequality holds for all $(x, u) \in \mathbb{Y}_k$ and $k \in \mathbb{Z}_+$:*

$$s(k, x, u) + \lambda(k, x) - \lambda(k+1, f(k, x, u)) \geq \rho(\|x - \hat{x}^\star_k\|). \tag{3.37}$$

This latter definition is similar to Definition 3.5, but applied to the general case of T-periodic linear systems with possibly periodically-time varying constraints.

Assumption 3.7 (Strict dissipativity)**.** *System* (3.27) *is strictly dissipative with respect to the supply rate defined as* $s(k, x, u) := \ell(k, x, u) - \ell(k, \hat{x}^\star_k, \hat{u}^\star_k)$.

Assumption 3.8 (Continuity of terminal penalty and storage functions)**.** *The terminal penalty function $V_f(k,\cdot)$ is T-periodic and continuous on the terminal compact set $\mathbb{X}_f(k,\hat{x}_k^\star)$ for all k. The T-periodic storage function $\lambda(k,\cdot)$ is continuous on $\mathbb{Y}_k$ for all k.*

Assumption 3.9 (Periodic positive invariance)**.** *There exists a T-periodic sequence of convex compact terminal regions $\{\mathbb{X}_f(i,\hat{x}_i^\star)\}_{i\in\mathbb{Z}_{[0,T-1]}}$ with each $\mathbb{X}_f(i,\hat{x}_i^\star)\subseteq\mathbb{X}_i$ containing the point $\hat{x}_i^\star$ in its interior, a periodic auxiliary control law $\kappa_f:\mathbb{Z}_+\times\mathbb{R}^n\to\mathbb{R}^m$, and a periodic terminal cost function $V_f:\mathbb{Z}_+\times\mathbb{R}^n\to\mathbb{R}_+$, such that the following holds for all $k\in\mathbb{Z}_+$ and all $x\in\mathbb{X}_f(k,\hat{x}_k^\star)$:*

$$V_f(k+1,f(k,x,\kappa_f(k,x)))\leq V_f(k,x)-\ell(k,x,\kappa_f(k,x))+\ell(k,\hat{x}_k^\star,\hat{u}_k^\star), \tag{3.38a}$$
$$f(k,x,\kappa_f(k,x))\in\mathbb{X}_f(k+1,\hat{x}_{k+1}^\star), \tag{3.38b}$$
$$(x,\kappa_f(k,x))\in\mathbb{Y}_k. \tag{3.38c}$$

Note that (3.38a) implicitly requires the set $\mathbb{X}_f(k,\hat{x}_k^\star)$ for all $k\in\mathbb{Z}_+$ to be periodically invariant under the periodic control law $\kappa_f(k,x)$. Hence, $x\in\mathbb{X}_f(k,\hat{x}_k^\star)$ implies $f(k,x,\kappa_f(k,x))\in\mathbb{X}_f(k+1,\hat{x}_{k+1}^\star)$ for all k.

3.5.3 Analysis of the Average Performance

To analyse the performance of system (3.27) controlled by the proposed quasi-infinite horizon economic MPC approach with periodic terminal penalty/region, denote first by $V_N^0(k,x_k,\mathbf{u}_k)$ the optimal value of (3.30a). If Assumption 3.7 is not satisfied, asymptotic convergence to the optimal periodic trajectories $(\hat{\mathbf{x}}_\infty,\hat{\mathbf{u}}_\infty)$ is not generally to be expected, nevertheless, the control law (3.31) induces an asymptotic average performance that is not worse than that of operating the system at the optimal periodic trajectories.

Theorem 3.2. *Consider the economic MPC formulation of problem* (3.30) *with a given period $T\in\mathbb{Z}_{\geq 1}$. If Assumptions 3.5, 3.6, 3.8 and 3.9 hold, the asymptotic average performance of the closed-loop system* (3.32) *is better than the performance of the optimal periodic trajectories derived from* (3.29)*, i.e.,*

$$\limsup_{M\to+\infty}\frac{\sum_{k=0}^{M}\ell(k,x_k,u(k))}{M+1}\leq\frac{\sum_{k=0}^{T-1}\ell(k,\hat{x}_k^\star,\hat{u}_k^\star)}{T}. \tag{3.39}$$

Proof. This result follows from the combination of preliminary results on economic MPC with periodic terminal equality constraint [7] and economic MPC with fixed terminal region constraint [6]. Assume that $\mathcal{P}_N(k,x_k,\mathbf{w}_T,\mathbf{p}_T)$ has a feasible solution for the current state $x_k\in\mathcal{X}_N(k)$, which gives optimal input and state sequences denoted respectively as $\mathbf{u}_k^\star = \{u^\star_{k+i|k}\}_{i\in\mathbb{Z}_{[0,N-1]}}$ and $\mathbf{x}_k^\star = \{x^\star_{k+i|k}\}_{i\in\mathbb{Z}_{[0,N]}}$. Choose a candidate input sequence and its associated state sequence admissible in $\mathcal{F}_N(k+1)$ for the next time step, as follows:

$$\tilde{\mathbf{u}}_{k+1} = \{u^\star_{k+1|k},\dots,u^\star_{k+N-1|k},\kappa_f(k+N,x^\star_{k+N|k})\},$$
$$\tilde{\mathbf{x}}_{k+1} = \{x^\star_{k+1|k},\dots,x^\star_{k+N|k},x^\star_{k+N+1|k}\},$$

where $x^\star_{k+N+1|k} = f(k+N,x^\star_{k+N|k},\kappa_f(k+N,x^\star_{k+N|k}))$. Due to the terminal constraint (3.30d) and the periodic invariance property stated in Assumption 3.9, it holds $x^\star_{k+N+1|k} \in \mathbb{X}_f(k+N+1,\hat{x}^\star_{k+N})$. Moreover, the cost (3.30a) evaluated along these feasible candidate state/input sequences is given by

$$\begin{aligned} V_N(k+1,x^\star_{k+1|k},\tilde{\mathbf{u}}_{k+1}) &= \sum_{i=1}^{N-1}\ell(k+i,x^\star_{k+i|k},u^\star_{k+i|k}) + \ell(k+N,x^\star_{k+N|k},\kappa_f(k+N,x^\star_{k+N|k})) \\ &\quad + V_f(k+N+1,x^\star_{k+N+1|k}) \\ &= V_N^0(k,x_k) - \ell(k,x_k,u^\star_{k|k}) + \ell(k+N,x^\star_{k+N|k},\kappa_f(k+N,x^\star_{k+N|k})) \\ &\quad - V_f(k+N,x^\star_{k+N|k}) + V_f(k+N+1,x^\star_{k+N+1|k}). \end{aligned}$$

From (3.38a) in Assumption 3.9, it follows that

$$V_N(k+1,x^\star_{k+1|k},\tilde{\mathbf{u}}_{k+1}) \le V_N^0(k,x_k) + \ell(k,\hat{x}^\star_k,\hat{u}^\star_k) - \ell(k,x_k,u^\star_{k|k}).$$

By optimality, $V_N^0(k+1,x_{k+1}) \le V_N(k+1,x^\star_{k+1|k},\tilde{\mathbf{u}}_{k+1})$. Therefore, for all states $x_k \in \mathcal{X}_N(k)$, it holds that

$$V_N^0(k+1,x_{k+1}) - V_N^0(k,x_k) \le \ell(k,\hat{x}^\star_k,\hat{u}^\star_k) - \ell(k,x_k,u^\star_{k|k}). \tag{3.40}$$

Taking averages on both sides of (3.40) gives

$$0 = \liminf_{M\to+\infty} \frac{\sum_{k=0}^M V_N^0(k+1,x_{k+1}) - V_N^0(k,x_k)}{M+1} \tag{3.41a}$$
$$\le \liminf_{M\to+\infty} \frac{\sum_{k=0}^M \ell(k,\hat{x}^\star_k,\hat{u}^\star_k) - \ell(k,x_k,u_k)}{M+1} \tag{3.41b}$$
$$= \frac{\sum_{k=0}^{T-1} \ell(k,\hat{x}^\star_k,\hat{u}^\star_k)}{T} - \limsup_{M\to+\infty} \frac{\sum_{k=0}^M \ell(k,x_k,u_k)}{M+1}. \tag{3.41c}$$

The left-hand side equality of (3.41) comes from Assumptions 3.5, 3.6 and 3.8, which imply that $V_N^0(k+1,x_{k+1}) - V_N^0(k,x_k)$ is bounded. The right-hand side equality of (3.41) comes from the fact that the pair $(\hat{x}^\star_k,\hat{u}^\star_k)$ is T-periodic for all k, then, the infinite horizon average cost is equal to the average cost of a single period (see [7, Theorem 4]). Rearranging, one obtains the desired inequality (3.39), which completes the proof. □

3.5.4 Lyapunov Nominal Stability

It has been already shown that the closed-loop asymptotic average performance of the T-periodic system is at least as good as the one of the best T-periodic orbit (as formalised in § 3.5.3), but in general, the closed-loop system might not converge to this orbit but possibly some other (better) oscillating (or even a non-periodic) behaviour is encountered. However, it was shown in [75] that periodic dissipativity implies that the system is optimally operated at the best T-periodic orbit $\mathcal{X}_T(\bar{\mathbf{d}}_T, \mathbf{p}_T)$ obtained from (3.29). Hence, it will be shown in the following how the proposed terminal penalty/region based economic MPC controller guarantees that the closed-loop system converges to $\mathcal{X}_T(\bar{\mathbf{d}}_T, \mathbf{p}_T)$.

In order to analyse the asymptotic stability of the closed-loop system, the following periodically time-varying *rotated* stage and terminal costs are considered:

$$L(k, x, u) := \ell(k, x, u) - \ell(k, \hat{x}_k^\star, \hat{u}_k^\star) + \lambda(k, x) - \lambda(k+1, f(k, x, u)), \tag{3.42}$$

$$\bar{V}_f(k, x) := V_f(k, x) - V_f(k, \hat{x}_k^\star) + \lambda(k, x) - \lambda(k, \hat{x}_k^\star). \tag{3.43}$$

Remark 3.4. *It can be assumed without loss of generality that $V_f(k, \hat{x}_k^\star) = 0$ and $\lambda(k, \hat{x}_k^\star) = 0$ for all $k \in \mathbb{Z}_+$.* ♢

With the above rotated cost, the following auxiliary optimal control problem is introduced:

$\bar{\mathcal{P}}_N(k, x_k, \mathbf{w}_T, \mathbf{p}_T)$:

$$\min_{\mathbf{u}_k} \bar{V}_N(k, x_k, \mathbf{u}_k) := \sum_{i=0}^{N-1} L(k+i, x_{k+i|k}, u_{k+i|k}) + \bar{V}_f(k+N, x_{k+N|k}), \tag{3.44}$$

subject to (3.30b)–(3.30e). Note that the constraints in problem $\bar{\mathcal{P}}_N(k, x_k, \mathbf{w}_T, \mathbf{p}_T)$ are the same as the ones in the original problem $\mathcal{P}_N(k, x_k, \mathbf{w}_T, \mathbf{p}_T)$. Therefore, both problems have an identical feasible set $\mathcal{F}_N(k)$ for all $k \in \mathbb{Z}_+$.

Lemma 3.1 (Equivalence of solutions)**.** *Let Assumptions 3.5, 3.6, 3.8 and 3.9 hold. The solution of the auxiliary problem $\bar{\mathcal{P}}_N(k, x_k, \mathbf{w}_T, \mathbf{p}_T)$ is identical to the solution of the original problem $\mathcal{P}_N(k, x_k, \mathbf{w}_T, \mathbf{p}_T)$.*

Proof. Note that both problems only differ in the cost function. Hence, expanding the rotated cost function yields

$$
\begin{aligned}
\bar{V}_N(k, x_k, \mathbf{u}_k) &= \sum_{i=0}^{N-1} L(k+i, x_{k+i|k}, u_{k+i|k}) + \bar{V}_f(k+N, x_{k+N|k}) \\
&= \sum_{i=0}^{N-1} \ell(k+i, x_{k+i|k}, u_{k+i|k}) - \ell(k+i, \hat{x}^\star_{k+i}, \hat{u}^\star_{k+i}) \\
&\quad + \lambda(k+i, x_{k+i|k}) - \lambda(k+i+1, f(k+i, x_{k+i|k}, u_{k+i|k})) \\
&\quad + V_f(k+N, x_{k+N|k}) - V_f(k+N, \hat{x}^\star_{k+N}) \\
&\quad + \lambda(k+N, x_{k+N|k}) - \lambda(k+N, \hat{x}^\star_{k+N}) \\
&= V_N(k, x_k, \mathbf{u}_k) + \lambda(k, x_{k|k}) - V_f(k+N, \hat{x}^\star_{k+N}) \\
&\quad - \lambda(k+N, x_{k+N|k}) + \lambda(k+N, x_{k+N|k}) - \lambda(k+N, \hat{x}^\star_{k+N}) \\
&\quad - \sum_{i=0}^{N-1} \ell(k+i, \hat{x}^\star_{k+i}, \hat{u}^\star_{k+i}).
\end{aligned}
$$

Moreover, from Remark 3.4, it is finally obtained that

$$
\bar{V}_N(k, x_k, \mathbf{u}_k) = V_N(k, x_k, \mathbf{u}_k) + \lambda(k, x_k) - \sum_{i=0}^{N-1} \ell(k+i, \hat{x}^\star_{k+i}, \hat{u}^\star_{k+i}). \tag{3.45}
$$

Since $\lambda(k, x_k)$ and $\sum_{i=0}^{N-1} \ell(k+i, \hat{x}^\star_{k+i}, \hat{u}^\star_{k+i})$ are independent of the decision variable $\mathbf{u}_k$ for a given initial state $x_k \in \mathbb{X}_k$, the cost functions $V_N(k, x_k, \mathbf{u}_k)$ and $\bar{V}_N(k, x_k, \mathbf{u}_k)$ differ only by a constant. Therefore, $\bar{\mathcal{P}}_N(k, x_k, \mathbf{w}_T, \mathbf{p}_T)$ and $\mathcal{P}_N(k, x_k, \mathbf{w}_T, \mathbf{p}_T)$ have identical solutions at all time steps $k \in \mathbb{Z}_+$. $\square$

In addition to the equivalence of the problems, it can be seen that the modified terminal cost inherits the basic stability condition of the original terminal cost.

Lemma 3.2 (Stability condition of the modified costs)**.** *Let Assumption 3.9 hold. The modified costs L and $\bar{V}_f$ satisfy the following property for all $k \in \mathbb{Z}_+$ and all $x \in \mathbb{X}_f(k, \hat{x}^\star_k)$:*

$$
\bar{V}_f(k+1, f(k, x, \kappa_f(k, x))) \leq \bar{V}_f(k, x) - L(k, x, \kappa_f(k, x)). \tag{3.46}
$$

Proof. Similarly to the proof of [6, Lemma 9], the desired inequality comes from adding to both sides of (3.43) the term $\lambda(k, x) + \lambda(k+1, f(k, x, \kappa_f(k, x)))$, rearranging and considering Remark 3.4. $\square$

Lemma 3.3 (MPC cost is less than terminal cost)**.** *Let Assumptions 3.5, 3.6, 3.8 and 3.9 hold, and denote by* $\bar{V}_N^0(k,x_k)$ *the optimal solution to* (3.44) *at time step* $k \in \mathbb{Z}_+$*. Then,*

$$\bar{V}_N^0(k,x_k) \leq \bar{V}_f(k,x_k), \quad \forall x_k \in \mathbb{X}_f(k,\hat{x}_k^\star), \quad \forall k \in \mathbb{Z}_+. \tag{3.47}$$

Proof. From Assumption 3.9, there exists a control law $\kappa_f(k,x_k) \in \mathbb{U}_k$ such that $f(k,x_k,\kappa_f(k,x_k)) \in \mathbb{X}_f(k+1,\hat{x}_{k+1}^\star)$ for all $x_k \in \mathbb{X}_f(k,\hat{x}_k^\star)$ and all $k \in \mathbb{Z}_+$. Due to the periodic positive invariance of the sequence of terminal regions, every $\kappa_f(t,x_t))$ for time steps $t > k$ is a suboptimal but feasible control action that keeps the state within the feasible set. Therefore, for all $k \in \mathbb{Z}_+$ and all $x_k \in \mathbb{X}_f(k,\hat{x}_k^\star)$ it follows by optimality that

$$\begin{aligned}\bar{V}_N^0(k,x_k) &\leq \sum_{i=0}^{N-1} L(k+i,x_{k+i},\kappa_f(k+i,x_{k+i})) + \bar{V}_f(k+N,x_{k+N}) \\ &= \bar{V}_f(k,x_k) + \sum_{i=0}^{N-1}\big(L(k+i,x_{k+i},\kappa_f(k+i,x_{k+i})) \\ &\qquad + \bar{V}_f(k+i+1,x_{k+i+1}) - \bar{V}_f(k+i,x_{k+i})\big).\end{aligned} \tag{3.48}$$

Then, applying (3.46) consecutively to the terms in the last summation of (3.48) leads to (3.47) and the claim is proved. □

Lemma 3.4 (Bounds on positive definite functions [89])**.** *Let* $\rho(x): \mathcal{A} \to \mathbb{R}_{\geq 0}$ *be a positive definite function defined on a compact set* $\mathcal{A}$*, i.e., zero at zero and strictly positive on* $x \neq 0$*. Then, there exists function* $\alpha_1, \alpha_2 \in \mathcal{K}$*, such that*

$$\alpha_1(x) \leq \rho(x) \leq \alpha_2(x), \quad \forall x \in \mathcal{A}, \quad k \in \mathbb{Z}_+. \tag{3.49}$$

Lemma 3.5 (Bounds on modified stage and terminal costs)**.** *Let Assumptions 3.5 to 3.9 hold, and let* α_1 *and* $\tilde{\alpha}_2$ *be* $\mathcal{K}_\infty$ *functions. The rotated stage cost* L *and terminal cost* $\bar{V}_f$ *satisfy, for all* $k \in \mathbb{Z}_+$*, the following inequalities:*

$$L(k,x_k,u_k) \geq \alpha_1(\|x_k - \hat{x}_k^\star\|), \quad \forall x_k \in \mathcal{X}_N(k),\ \forall u_k \in \mathbb{U}_k, \tag{3.50}$$

$$\alpha_1(\|x_k - \hat{x}_k^\star\|) \leq \bar{V}_f(k,x_k) \leq \tilde{\alpha}_2(\|x_k - \hat{x}_k^\star\|), \quad \forall x_k \in \mathbb{X}_f(k,\hat{x}_k^\star). \tag{3.51}$$

Proof. From (3.37), (3.42) and Assumption 3.7, it holds that $L(k,x_k,u_k) \geq \rho(\|x_k - \hat{x}_k^\star\|)$ for all $(x_k,u_k) \in \mathbb{Y}_k$, which in addition to Lemma 3.4, leads to (3.50). Consider now a trajectory starting in the terminal region, that is, $x_k \in \mathbb{X}_f(k,\hat{x}_k^\star)$, and driven by the terminal controller k_f. Following the line of arguments in [6, Lemma 11 and 12], it can be shown from (3.46) and (3.50) that $\bar{V}_f(k,x_k) \geq \sum_{k=0}^{+\infty} L(k,x_k,\kappa_f(k,x_k))$. Hence, from

(3.50) it follows that $\bar{V}_f(k, x_k) \geq \alpha_1(\|x_k - \hat{x}_k^\star\|) \geq 0$ for all $k \in \mathbb{Z}_+$. In addition, from Assumption 3.8, $\bar{V}_f(k, x_k)$ is locally bounded and by definition $\bar{V}_f(k, \hat{x}_k^\star) = 0$, thus, it can be upperbounded by a class $\mathcal{K}_\infty$ function, i.e., $\bar{V}_f(k, x_k) \leq \tilde{\alpha}_2(\|x_k - \hat{x}_k^\star\|)$ for all $x_k \in \mathbb{X}_f(k, \hat{x}_k^\star)$ and all $k \in \mathbb{Z}_{\geq 0}$. □

Theorem 3.3 (Lyapunov asymptotic stability with periodic terminal penalty/region)**.** *Consider a T-periodic system and let Assumptions 3.5 to 3.9 hold. Let $\mathcal{X}_T(\mathbf{w}_T, \mathbf{p}_T)$ be the best feasible T-periodic orbit of the system obtained by solving (3.29). Then, the economically optimal trajectory $(\hat{\mathbf{x}}_\infty, \hat{\mathbf{u}}_\infty)$ is asymptotically stable for all feasible initial states $x_0 \in \mathcal{X}_N(0)$ and the closed-loop system (3.32) converges to $\mathcal{X}_T(\mathbf{w}_T, \mathbf{p}_T)$. The periodically time-varying Lyapunov function is $\bar{V}_N^0(k, x_k)$, and satisfies*

$$\bar{V}_N^0(k, x_k) \geq \alpha_1(\|x_k - \hat{x}_k^\star\|), \tag{3.52}$$

$$\bar{V}_N^0(k, x_k) \leq \alpha_2(\|x_k - \hat{x}_k^\star\|), \tag{3.53}$$

$$\bar{V}_N^0(k+1, x_{k+1}) - \bar{V}_N^0(k, x_k) \leq -\alpha_1(\|x_k - \hat{x}_k^\star\|), \tag{3.54}$$

$\forall x_k \in \mathcal{X}_N(k)$, $k \in \mathbb{Z}_{\geq 0}$, with α_1 and α_2 being class $\mathcal{K}_\infty$ functions.

Proof. Consider the optimal modified cost function for $x_k \in \mathcal{X}_N(k)$, i.e.,

$$\bar{V}_N^0(k, x_k) = \sum_{i=0}^{N-1} L(k+i, x_{k+i|k}^\star, u_{k+i|k}^\star) + \bar{V}_f(k+N, x_{k+N|k}^\star).$$

The lower bound imposed by inequality (3.52) follows directly from Lemma 3.5. The upper bound in (3.53) follows from Lemma 3.3, Lemma 3.5 and Proposition 2 in [152]. Finally, condition (3.54) can be proved following the same analysis used in the proof of Theorem 3.2 for the original cost. Specifically, from Assumption 3.9 and Lemma 3.2, it follows for the rotated cost function (3.44) that

$$\bar{V}_N(k+1, x_{k+1|k}^\star, \tilde{\mathbf{u}}_{k+1}) \leq \bar{V}_N^0(k, x_k) - L(k, x_k, u_{k|k}^\star). \tag{3.55}$$

By optimality, $\bar{V}_N^0(k+1, x_{k+1}, \tilde{\mathbf{u}}_{k+1}) \leq \bar{V}_N(k+1, x_{k+1|k}^\star)$, hence, from (3.55) and Lemma 3.5 it holds that

$$\bar{V}_N^0(k+1, x_{k+1}) - \bar{V}_N^0(k, x_k) \leq -\alpha_1(\|x_k - \hat{x}_k^\star\|), \quad \forall x_k \in \mathcal{X}_N(k), \tag{3.56}$$

which completes the proof. □

3.5.5 Computation of the Terminal Ingredients

To completely characterise the proposed stabilising economic MPC controller, it is necessary to compute the terminal ingredients of $\mathcal{P}_N(k, x, \mathbf{w}_T, \mathbf{p}_T)$, in such a way that Assumption 3.9 is satisfied. To do so, a systematic procedure is presented next, which builds on the results reported in [6] for time-invariant systems, where a fixed terminal region around the economically optimal steady-state was used. Here, to cover linear periodically time-varying systems, these previous results are combined with ideas from [27, Section 4], where terminal sets and quadratic cost functions were computed for periodic systems in a stabilising MPC context.

Consider system (3.27) subject to (3.28) and let Assumptions 3.5 and 3.6 hold, i.e., $A_{k+T} = A_k$, $B_{k+T} = B_k$ and $w_{k+T} = w_k$, for all $k \in \mathbb{Z}_+$ and period $T \in \mathbb{Z}_+$. The matrices A_k, B_k and the vector w_k are assumed to be known for $k \in \mathbb{Z}_{[0,T-1]}$, and by periodicity also known for any $k \geq T$.

Assumption 3.10. *There exist T-periodic matrices $K_k \in \mathbb{R}^{n\times m}$, $k \in \mathbb{Z}_{[0,T-1]}$, such that the matrices $\tilde{A}_k := A_k + B_k K_k$ are Schur (also periodic).*

The computation of the terminal ingredients of (3.30) requires in addition the existence of a terminal control law, which is here chosen as follows:

$$\kappa_f(k, x) := K_k(x - \hat{x}_k^\star) + \hat{u}_k^\star, \tag{3.57}$$

where K_k is the periodically time-varying feedback gain satisfying Assumption 3.10, and the pair $(\hat{x}_k^\star, \hat{u}_k^\star)$ are elements of the economically optimal periodic state and input trajectories obtained from (3.29).

The terminal regions are proposed here to be ellipsoidal-level sets associated to quadratic periodically time-varying functions of the form

$$V(k, x) := \frac{1}{2}(x - \hat{x}_k^\star)^\top P_k (x - \hat{x}_k^\star), \tag{3.58}$$

where $P_k = P_{k+T} \in \mathbb{S}_{++}^n$ for all $k \in \mathbb{Z}_+$. Thus, the terminal regions are periodically time-varying ellipsoids centred around the nominal best periodic trajectory and are defined as

$$\mathbb{X}_f(k, \hat{x}_k^\star) := \{x \in \mathbb{R}^n \mid (x - \hat{x}_k^\star)^\top P_k (x - \hat{x}_k^\star) \leq \beta\}, \tag{3.59}$$

where $\beta \in \mathbb{R}_+$. The periodic matrices P_k and K_k and the scalar β must ensure that the state and input constraints are always satisfied under the use of the terminal controller (3.57), i.e., $x_k \in \mathbb{X}_f(k, \hat{x}_k^\star) \subset \mathbb{X}_k$ and $\kappa_f(k, x_k) \in \mathbb{U}_k$ for all $k \in \mathbb{Z}_+$.

In the following, the procedure in [6, Section 4.1] is properly modified to derive a suitable terminal function V_f for the periodically time-varying case. First, assume that the economic costs $\ell(k, \cdot, \cdot)$ are twice continuously differentiable for all $k \in \mathbb{Z}_+$ and define T-periodic functions of the form

$$\bar{\ell}(k, x) := \ell(k, x, \kappa_f(k, x)) - \ell(k, \hat{x}_k^\star, \hat{u}_k^\star), \quad k \in \mathbb{Z}_{[0,T-1]}. \tag{3.60}$$

The ideal terminal penalty function V_f satisfying (3.38a) is the infinite horizon cost for the terminal controller κ_f, which is given by $V_f(k, x) = \sum_{i=0}^{\infty} \bar{\ell}(k + i, x_{k+i})$ and can be explicitly determined for special cases like quadratic costs. In the sequel it is proposed how to analytically compute such terminal function.

From adaptation of [6, Lemma 22], it follows that one can select T matrices $Q_k \in \mathbb{S}_{++}^n$ (possibly time-invariant) such that $Q_k - \bar{\ell}_{xx}(k, x) \succcurlyeq 0$ for all $x \in \mathbb{X}_k$, $(\hat{x}_k^\star, \hat{u}_k^\star) \in \mathbb{Y}_k$ and $k \in \mathbb{Z}_{[0,T-1]}$ (and by periodicity for all $k \in \mathbb{Z}_+$). Moreover, the quadratic cost function

$$\ell_q(k, x) := \frac{1}{2}(x - \hat{x}_k^\star)^\top Q_k (x - \hat{x}_k^\star) + q_k^\top (x - \hat{x}_k^\star), \tag{3.61}$$

with $q_k := \bar{\ell}_x(k, \hat{x}_k^\star, \hat{u}_k^\star)$ for all $k \in \mathbb{Z}_+$, is such that for all $x \in \mathbb{X}_k$ the inequality $\ell_q(k, x) \geq \bar{\ell}(k, x) + (1/2)(x - \hat{x}_k^\star)^\top (x - \hat{x}_k^\star)$ holds. In this way, a candidate terminal penalty function is given by

$$V_f(k, x) := \sum_{i=0}^{\infty} \ell_q(k + i, x_{k+i}), \tag{3.62}$$

with $x_k = x$ and $x_{k+i+1} = A_{k+i} x_{k+i} + B_{k+i}(K_{k+i}(x_{k+i} - \hat{x}_{k+i}^\star) + \hat{u}_{k+i}^\star) + w_{k+i}$, for all $i \in \mathbb{Z}_+$. To obtain an explicit expression of (3.62) recall that the optimal trajectory obtained in (3.29) satisfies $\hat{x}_{k+1}^\star = A_k \hat{x}_k^\star + B_k \hat{u}_k^\star + w_k$. From (3.57) and Assumption 3.10, the error dynamics are given by

$$(x_{k+1} - \hat{x}_{k+1}^\star) = \tilde{A}_k (x_k - \hat{x}_k^\star), \quad \forall k \in \mathbb{Z}_+. \tag{3.63}$$

The so-called monodromy matrix of system (3.63) with period T, is given by

$$\Psi_k := \prod_{i=0}^{T-1} \tilde{A}_{k+i}, \quad \forall k \in \mathbb{Z}_+. \tag{3.64}$$

This matrix is periodic, i.e., $\Psi_{k+T} = \Psi_k$ for all $k \in \mathbb{Z}_+$ due to periodicity of each matrix $\tilde{A}_k$. From, (3.61), (3.63) and (3.64), the terminal penalty function (3.62) can be written as follows:

$$
\begin{aligned}
V_f(k,x) &= \frac{1}{2}\sum_{j=0}^{T-1}(x_{k+j}-\hat{x}^\star_{k+j})^\top \left(\sum_{i=0}^{\infty}(\Psi^i_{k+j})^\top Q_{k+j}\Psi^i_{k+j}\right)(x_{k+j}-\hat{x}^\star_{k+j}) \\
&\quad + \sum_{j=0}^{T-1}\bar{\ell}_x(k+j,\hat{x}^\star_{k+j},\hat{u}^\star_{k+j})^\top \sum_{i=0}^{\infty}\Psi^i_{k+j}(x_{k+j}-\hat{x}^\star_{k+j}) \\
&= \frac{1}{2}\sum_{j=0}^{T-1}(x_{k+j}-\hat{x}^\star_{k+j})^\top P_{k+j}(x_{k+j}-\hat{x}^\star_{k+j}) + p^\top_{k+j}(x_{k+j}-\hat{x}^\star_{k+j}),
\end{aligned} \tag{3.65}
$$

with $x_k = x$ and $x_{k+j+1} = A_{k+j}x_{k+j} + B_{k+j}(K_{k+j}(x_{k+j}-\hat{x}^\star_{k+j}) + \hat{u}^\star_{k+j}) + w_{k+j}$, for all $j \in \mathbb{Z}_{[0,T-1]}$. For the given k and each $j \in \mathbb{Z}_{[0,T-1]}$, matrices P_{k+j} are the solutions to the discrete Lyapunov equations $\Psi^\top_{k+j}P_{k+j}\Psi_{k+j} - P_{k+j} = -Q_{k+j}$, while $p^\top_{k+j} = \bar{\ell}_x(k+j,\hat{x}^\star_{k+j},\hat{u}^\star_{k+j})^\top(I_n - \Psi_{k+j})^{-1}$. From (3.62), it follows directly that the candidate function V_f satisfies condition (3.38a) of Assumption 3.9. In fact, from periodicity assumptions and proper manipulation of (3.62)–(3.65), it can be derived the following equality:

$$
\begin{aligned}
&V_f(k+1, f(k,x,\kappa_f(k,x))) - V_f(k,x) \\
&\quad = \frac{1}{2}(x-\hat{x}^\star_k)^\top(\Psi^\top_k P_k\Psi_k - P_k)(x-\hat{x}^\star_k) - \bar{\ell}_x(k,\hat{x}^\star_k,\hat{u}^\star_k)^\top(x-\hat{x}^\star_k).
\end{aligned} \tag{3.66}
$$

Therefore, (3.62) and (3.66) yield the following Lyapunov equations

$$
\Psi^\top_k P_k\Psi_k - P_k = -Q_k, \quad \forall k \in \mathbb{Z}_{[0,T-1]}. \tag{3.67}
$$

It is important to remark that matrices P_k and K_k must satisfy in addition the following condition:

$$
\tilde{A}^\top_k P_{k+1}\tilde{A}_k - P_k \preceq 0, \quad \forall k \in \mathbb{Z}_{[0,T-1]} \tag{3.68}
$$

with $P_T = P_0$. This is required to satisfy the periodic positive invariance condition (3.38b). Furthermore, for all $x \in \mathbb{X}_f(k,\hat{x}^\star_k) \subseteq \mathbb{X}_k$ the periodic feedback gains K_k involved in the terminal controller $\kappa_f(k,x)$ have to guarantee (3.38c) for all k.

From the above, it can be seen that the explicit characterisation of the candidate function requires the computation of the T-periodic matrices P_k and K_k, with $k \in$

$\mathbb{Z}_{[0,T-1]}$. The main difficulty to do so is that the monodromy matrices defined in (3.64) lead to non-linear matrix inequalities that cannot be straightforwardly solved. To cope with this difficulty, a predictive controller relying on the online solution of a semi-definite programming problem was proposed in [25]. However, the approach lies within the (non-economic) stabilising predictive control framework and can be computationally burdensome for generalised flow-based network problems.

Despite the aforementioned difficulty for solving (3.67) and (3.68), it is still possible to find a set of matrices fulfilling Assumption 3.9. To do so, relax condition (3.67) and consider instead $\Psi_k^\top P_k \Psi_k - P_k \preceq -Q_k$ for all $k \in \mathbb{Z}_{[0,T-1]}$. Following [26, Remark 19], this latter condition and (3.68) can be satisfied if there exist matrices P_k and K_k such that $P_T = P_0$ and $\tilde{A}_k^\top P_{k+1} \tilde{A}_k - P_k \preceq -Q_k$ holds for all $k \in \mathbb{Z}_{[0,T-1]}$. This claim is formalised with the following result.

Theorem 3.4. *Consider the system $x_{k+1} = f(k, x_k, \kappa_f(k, x_k))$ satisfying Assumption 3.6 with period $T \in \mathbb{Z}_{\geq 1}$, control law (3.57) and the pair $(\hat{x}_k^\star, \hat{u}_k^\star) \in \mathbb{Y}_k$ for each $k \in \mathbb{Z}_{[0,T-1]}$. Let $X_k \in \mathbb{S}_{++}^n$, $Y_k \in \mathbb{R}^{m\times n}$ and $\beta \in \mathbb{R}_+$ be decision variables, and solve*

$$\max_{X_k \succ 0,\, Y_k \in \mathbb{R}^{m\times n},\, \beta\in\mathbb{R}_+} \; -\log\det(X_0) \tag{3.69a}$$

subject to

$$\begin{bmatrix} X_k & \star & \star \\ A_k X_k + B_k Y_k & X_{k+1} & \star \\ Q_k^{\frac{1}{2}} X_k & 0_{n\times n} & \beta I_n \end{bmatrix} \succeq 0, \tag{3.69b}$$

$$\begin{bmatrix} (h_{j,k} - (a_{j,k}\hat{x}_k^\star + b_{j,k}\hat{y}_k^\star))^2 & \star \\ X_k a_{j,k}^\top + Y_k^\top b_{j,k}^\top & X_k \end{bmatrix} \succeq 0, \tag{3.69c}$$

$\forall k \in \mathbb{Z}_{[0,T-1]}$, $j \in \mathbb{Z}_{[1,r]}$, with $X_T = X_0$. Let $X_{opt,k}$, $Y_{opt,k}$, and β_{opt} denote the optimal solution to (3.69). Set

$$P_k := X_{opt,k}^{-1}\beta_{opt}, \quad K_k := Y_k X_k^{-1}, \quad \forall k \in \mathbb{Z}_{[0,T-1]}. \tag{3.70}$$

Further, set $P_T = P_0$ and $K_T = K_0$. If problem (3.69) can be solved, then Assumption 3.9 is satisfied.

Proof. First, it will be shown that the solution of (3.69) renders the terminal sets defined in (3.59) to be periodically time-varying positively invariant for the system $x_{k+1} =$

$f(k, x_k, \kappa_f(k, x_k))$ with κ_f as defined in (3.57). To do so, note that for $k \in \mathbb{Z}_+$ the requirement $x_k \in \mathbb{X}_f(k, \hat{x}_k^\star)$ is equivalent to the quadratic functional condition

$$F_1 = (x_k - \hat{x}_k^\star)^\top P_k (x_k - \hat{x}_k^\star) - \beta \leq 0.$$

In a similar way, the requirement that $x_{k+1} \in \mathbb{X}_f(k+1, \hat{x}_{k+1}^\star)$ is equivalent to

$$F_0 = (x_{k+1} - \hat{x}_{k+1}^\star)^\top P_{k+1} (x_{k+1} - \hat{x}_{k+1}^\star) - \beta \leq 0.$$

By Lemma D.1, the requirement that $x_k \in \mathbb{X}_f(k, \hat{x}_k^\star)$ implies $x_{k+1} \in \mathbb{X}_f(k+1, \hat{x}_{k+1}^\star)$, is equivalent to the existence of $\zeta_k > 0$, such that

$$(x_{k+1} - \hat{x}_{k+1}^\star)^\top P_{k+1} (x_{k+1} - \hat{x}_{k+1}^\star) - \beta - \zeta_k ((x_k - \hat{x}_k^\star)^\top P_k (x_k - \hat{x}_k^\star) - \beta) \leq 0. \quad (3.71)$$

From (3.63), the above inequality can be rewritten as a quadratic functional of $(x_k - \hat{x}_k^\star)$ for all $k \in \mathbb{Z}_+$. Hence, by Lemma D.3, an equivalent linear matrix inequality condition can be obtained, i.e.,

$$\begin{bmatrix} \tilde{A}_k^\top P_{k+1} \tilde{A}_k - \zeta_k P_k & 0 \\ 0 & \zeta_k - \beta \end{bmatrix} \preceq 0.$$

The above inequality can be decoupled to obtain, $0 < \zeta_k \leq \beta$, and

$$\tilde{A}_k^\top P_{k+1} \tilde{A}_k - \zeta_k P_k \preceq 0. \quad (3.72)$$

As reviewed in [25, Chapter 3], there exists a ζ_k such that (3.72) is equivalent to

$$\tilde{A}_k^\top P_{k+1} \tilde{A}_k - P_k \preceq -Q_k. \quad (3.73)$$

By retrieving X_k and Y_k according to (3.70), applying the Schur complement to (3.69b), and pre- and post-multiplying the result with P_k, it can be shown that (3.69b) is equivalent to (3.73) with $k \in \mathbb{Z}_{[0,T-1]}$. Since $Q_k \succ 0$ for all $k \in \mathbb{Z}_+$, then condition (3.68) is satisfied and the sequence formed by the terminal sets $\{\mathbb{X}_f(k, \hat{x}_k^\star)\}_{k \in \mathbb{Z}_{[0,T-1]}}$ is periodically positive invariant for the closed-loop system $x_{k+1} = f(k, x_k, \kappa_f(k, x_k))$.

The terminal sets satisfy state and input constraints for all $k \in \mathbb{Z}_+$. This can be shown by retrieving X_k and Y_k according to (3.70) and applying the Schur complement to (3.69c), which gives

$$(a_{j,k} + b_{j,k} K_k)(\beta P_k^{-1})(a_{j,k} + b_{j,k} K_k)^\top \leq (h_{j,k} - a_{j,k} \hat{x}_k^\star - b_{j,k} \hat{u}_k^\star)^2, \quad (3.74)$$

for all $j \in \mathbb{Z}_{[1,r]}$ and all $k \in \mathbb{Z}_+$. This inequality satisfies the necessary condition for an ellipsoid to be contained in a polyhedron, see [27, Lemma 1], thus, (3.74) guarantees that $\mathbb{X}_f(k, \hat{x}_k^\star) \subset \mathbb{X}_k$, $\forall k \in \mathbb{Z}_+$. Consequently, if $x_k \in \mathbb{X}_f(k, \hat{x}_k^\star)$ then $(x_k, \kappa(k, x_k)) \in \mathbb{Y}_k$ for all $k \in \mathbb{Z}_+$.

It only remains to prove (3.38a). Then, by pre- and post-multiplying (3.73) with $(x_k - \hat{x}_k^\star)^\top$ and $(x_k - \hat{x}_k^\star)$, respectively, it follows that

$$(x_{k+1} - \hat{x}_{k+1}^\star)^\top P_{k+1}(x_{k+1} - \hat{x}_{k+1}^\star) - (x_k - \hat{x}_k^\star)^\top P_k(x_k - \hat{x}_k^\star) \leq -(x_k - \hat{x}_k^\star)^\top Q_k(x_k - \hat{x}_k^\star), \tag{3.75}$$

for all $k \in \mathbb{Z}_+$. Summing up (3.75) from $k = 0$ to $k = T-1$, and using (3.63) and (3.64), yields

$$(x_k - \hat{x}_k^\star)^\top (\Psi_k^\top P_k \Psi_k - P_k)(x_k - \hat{x}_k^\star) \leq -\sum_{k=0}^{T-1} (x_k - \hat{x}_k^\star)^\top Q_k(x_k - \hat{x}_k^\star). \tag{3.76}$$

Multiplying this latter inequality by (1/2) and adding to both of its sides the term $-\bar{\ell}_x(k, \hat{x}_k^\star, \hat{u}_k^\star)^\top (x - \hat{x}_k^\star)$, lead to

$$\begin{aligned}
&\frac{1}{2}(x_k - \hat{x}_k^\star)^\top (\Psi_k^\top P_k \Psi_k - P_k)(x_k - \hat{x}_k^\star) - \bar{\ell}_x(k, \hat{x}_k^\star, \hat{u}_k^\star)^\top (x - \hat{x}_k^\star) \\
&\qquad \leq -\frac{1}{2}\sum_{k=0}^{T-1} (x_k - \hat{x}_k^\star)^\top Q_k(x_k - \hat{x}_k^\star) - \bar{\ell}_x(k, \hat{x}_k^\star, \hat{u}_k^\star)^\top (x - \hat{x}_k^\star) \\
&\qquad \leq -\frac{1}{2}(x_k - \hat{x}_k^\star)^\top Q_k(x_k - \hat{x}_k^\star) - \bar{\ell}_x(k, \hat{x}_k^\star, \hat{u}_k^\star)^\top (x - \hat{x}_k^\star) \\
&\qquad \leq \bar{\ell}(k, x_k) = \ell(k, x_k, \kappa_f(k, x_k)) - \ell(k, \hat{x}_k^\star, \hat{u}_k^\star),
\end{aligned} \tag{3.77}$$

for all $k \in \mathbb{Z}_+$. The second inequality follows from the positive definiteness of Q_k, while the last inequality comes from the definition of $\ell_q(k, x_k)$ in (3.61). Then, from (3.65) and (3.77), condition (3.38a) is satisfied. Consequently, the terminal ingredients obtained by solving (3.69) fulfil Assumption 3.9 and the claim of Theorem 3.4 is proved. □

Remark 3.5. *The economic MPC setting based on a periodically time-varying terminal equality constraint [194] is a particular case of the approach introduced in this chapter, and it can be recovered by setting* $V_f(x_{k+N|k}) = 0$, $\mathbb{X}_f(k, x(\lfloor k \rfloor_T)) = \{\hat{x}_k^\star\}$ *and* $\kappa_f(k, x_k) = u_p^\star(\lfloor k \rfloor_T)$ *for all* $k \in \mathbb{Z}_{\geq 0}$ *and given period* $T \in \mathbb{Z}_{\geq 1}$. *Nevertheless, it is known that the closed-loop performance achieved with a terminal penalty/region MPC formulation is at least as good as the asymptotic performance associated with the terminal equality constraint setting.* ◊

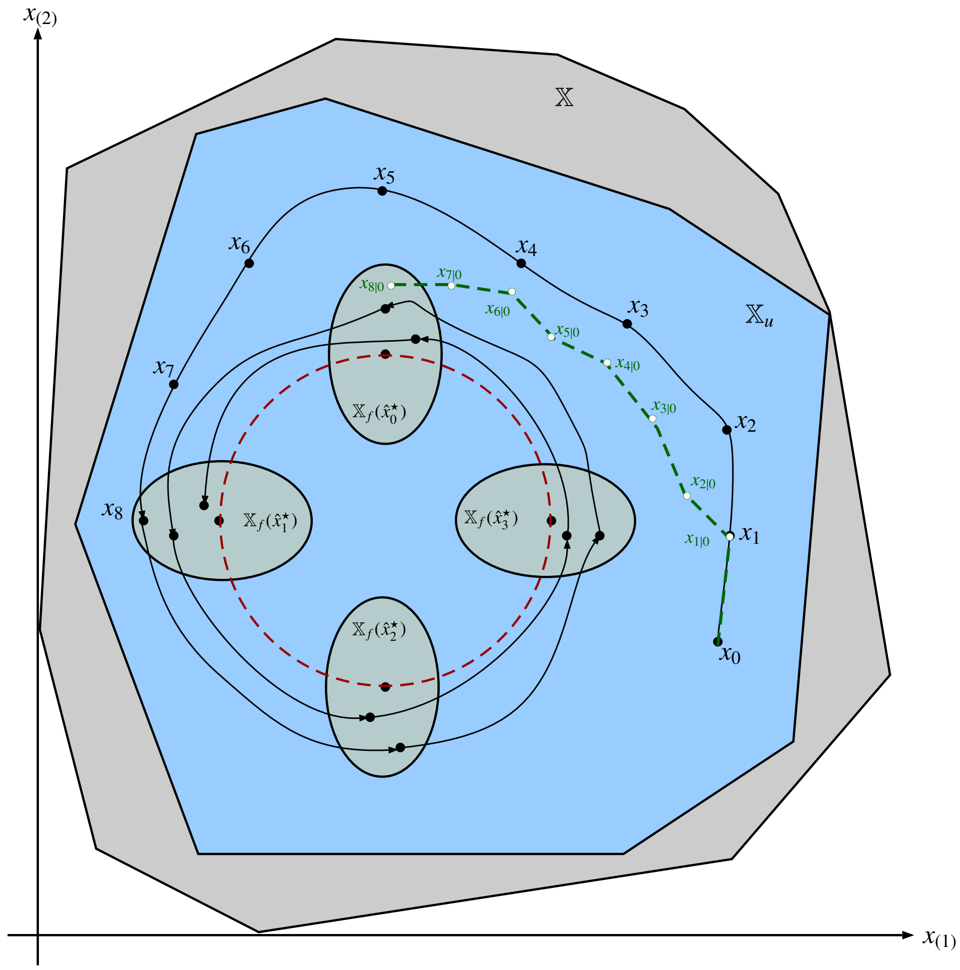


Figure 3.3: Graphical illustration of the proposed economic MPC with periodic terminal region for the case $n = 2$, $N = 8$, $T = 4$. optimal periodic orbit (dashed red), predicted state trajectory (dashed green), closed-loop state trajectory (solid black).

The concept of the proposed economic MPC with periodic terminal region is illustrated in Figure 3.3 for the exemplary case of a two-dimensional system with prediction horizon $N = 8$, period $T = 4$ and invariant state and input constraint sets. Define the set $\mathbb{X}_u \subseteq \mathbb{X}$ as follows: $\mathbb{X}_u := \{x \in \mathbb{X} \mid \kappa_f(k,x) \in \mathbb{U},\ \forall k\}$ with $\kappa_f(k,x)$ chosen with the form in (3.57). Then, the terminal state of the predicted state trajectory will lie in the corresponding terminal region $\mathbb{X}_f(\cdot)$, which depends on the time step when the prediction is performed. For instance, in the graphical illustration, at initial time $k = 0$, the predicted terminal state $x_{8|0}$ lies in the set around the optimal periodic reference $\hat{x}_0$ since $\lfloor k + N \rfloor_T = 0$. Once the closed-loop state trajectory reaches one of the periodic terminal sets, the terminal control law can render the state towards the desired optimal orbit exponentially. It is worth highlighting that the closed-loop state trajectory does not necessarily converge monotonically to the optimal periodic orbit at each subsequent step but in each period. Therefore, the periodically time-varying terminal region might lead to a larger domain of attraction.

3.5.6 Numerical Results

This section presents the results of applying the proposed terminal penalty/region economic MPC formulation also to the small-size sector example (see Figure 2.5) of the drinking water network case study described in § 2.4. The off-line design of the periodically time-varying terminal penalties and terminal regions as well as the on-line MPC optimisations have been carried out on a Macbook Pro Intel Core 2 Duo at 2.4GHz with 4GB of RAM using MATLAB R2010b (64 bits), the YALMIP Toolbox and the MOSEK 7.0 solver.

Again, the nominal control-oriented model of the DWN is described by the following discrete-time linear model:

$$x_{k+1} = Ax_k + Bu_k + B_d d_k, \quad k \in \mathbb{Z}_{\geq 0}, \quad x_0 = \bar{x}, \tag{3.78}$$

where $x_k \in \mathbb{R}^3$ is the state vector given in m^3, which represents the volume of water stored in the tanks, $u_k \in \mathbb{R}^6$ is the input vector given in m^3/s and represents the water flow through the actuators (valves and pumps), and $d_k \in \mathbb{R}^4$ is a known disturbance vector given in m^3/s, which is related to non-stationary and cyclic nominal water demands with known period $T = 24$ hours, i.e., $d_{k+T} = d_k$ for all $k \in \mathbb{Z}_+$. Moreover, $\bar{x} \in \mathbb{R}^n$ is the initial state, and matrices A, B, B_d are time-invariant of compatible dimensions dictated by the network topology. In this case, even when matrices A, B and B_d in (3.78) are time-invariant, the DWN still evolves periodically due to the cyclic behaviour of water demands.

The system is subject to state and input constraints due to physical limits and operational restrictions. To fit with the proposed formulation, the constraint set is defined as $\mathbb{Y}_k := \mathbb{X}_k \times \mathbb{U}_k$, with

$$\mathbb{X}_k = \mathbb{X} := \{x \in \mathbb{R}^3 : x_{\min} \leq x \leq x_{\max}\}, \quad \forall k \in \mathbb{Z}_+ \tag{3.79a}$$

$$\mathbb{U}_k = \{u \in \mathbb{R}^6 : u_{\min} \leq u \leq u_{\max},\ Eu = -E_d d_k\}, \quad \forall k \in \mathbb{Z}_+ \tag{3.79b}$$

where $x_{\min}$ and $x_{\max}$, given in m^3, are vectors in $\mathbb{R}^3$ related to the minimum and maximum volume of water that tanks are able to store, and similarly, $u_{\min}$ and $u_{\max}$, given in m^3/s, are vectors in $\mathbb{R}^6$ related to the minimum and maximum allowable water flow through actuators. Note that (3.79b) is periodically time-varying due to the presence of the periodic demand in the inputs equality constraint that arises from static relations (i.e., mass balance at junction nodes) within the network. Moreover, $E \in \mathbb{R}^{2\times 6}$ and $E_d \in \mathbb{R}^{2\times 4}$ are time-invariant matrices.

The economic stage cost is given by the multi-objective function denoted by

$$\ell_M(k, x_k, u_k) := \gamma_1 \ell_{E,k} + \gamma_2 \ell_{\Delta,k} + \gamma_3 \ell_{S,k}, \tag{3.80}$$

where γ_1, γ_2, γ_3 are positive prioritisation weights and the functions $\ell_{E,k}$, $\ell_{\Delta,k}$ and $\ell_{S,k}$ are the same used in (2.22). Moreover, as discussed in § 3.5.4, convergent behaviour of the system to the optimal orbit (if desired) can be ensured if the system is strictly dissipative (see Definition 3.5). Given that (3.80) is not dissipative with respect to the optimal orbit, asymptotic stability can be enforced by adding sufficiently convex regularisation terms to the cost function as proved in [7, Theorem 3]. A common practice is to use a tracking term, which for this periodic case may be defined as follows:

$$\ell_T(k, x_k, u_k) := \|x_k - \hat{x}_k^\star\|_Q^2 + \|u_k - \hat{u}_k^\star\|_R^2, \tag{3.81}$$

with the penalties $Q \in \mathbb{S}_{++}^n$ and $R \in \mathbb{S}_{++}^m$ chosen to achieve strong convexity of the stage cost function and positive definiteness with respect to the optimal periodic orbit, sufficing for strict dissipativity.

In order to compare the economic profit achievable with the pure economic MPC strategy and that achievable with the stabilising version that incorporates tracking terms, the following weighted sum is defined:

$$\ell(k, x_k, u_k) := \theta \ell_M(k, x_k, u_k) + (1-\theta)\ell_T(k, x_k, u_k), \tag{3.82}$$

where $\theta \in [0, 1]$ is a parameter that expresses the manager's relative importance of transient profit and convergence to the optimal orbit. Setting $\theta = 1$ gives the pure economic MPC formulation with cost function (3.80), while $\theta = 0$ gives the standard tracking MPC formulation with cost function (3.81).

Table 3.3: Open-loop average cost of MPC strategies

MPC strategy	/ Terminal condition	Average cost (e.u.)	% Gain
Tracking MPC	/ periodic terminal point constraint	25.0219	-
Economic MPC	/ periodic terminal point constraint	25.0201	+ 0.0072
Economic MPC	/ periodic terminal penalty/region	21.2425	+15.1043

e.u.: economic units

The assessment of the proposed controller has been carried out by solving (3.30) equipped with the stage cost (3.82), a sampling time of $\Delta t = 3600$ s and a prediction horizon equal to the period of the water demands and the electricity prices, i.e., $N = T = 24$ hours. The weights of the cost function (3.80) are $\gamma_1 = 100$, $\gamma_2 = 1$, and $\gamma_3 = 10$, while the state and input penalties in (3.81) are chosen to be $Q = 10^{-6} I_n$ and $R = 10 I_m$ to ensure strong convexity of (3.82). Results are specified for three MPC controllers, i.e., a standard MPC that tracks the state and input trajectories obtained in (3.29), an economic MPC with periodically time-varying terminal equality constraint (see, e.g., [7] and [194]), and the proposed economic MPC with periodically time-varying terminal penalty and terminal region.

Table 3.3 shows the average performance results of the open-loop optimisation, i.e., $\sum_{k=0}^{N-1} \ell(k, x_k, u_k)/N$ with prediction horizon $N = 24$ hours. The initial state is a point near to the best periodic orbit computed in (3.29). It can be noticed that the economic MPC setting with periodically time-varying terminal penalty and region is able to improve the performance by 15.10% of the tracking MPC setting, which has the same cost as the optimal orbit. The enhancement achieved in this test with the terminal equality constraint based economic MPC is negligible for this simulation, i.e., 0.007%, since the initial state is quite near to the optimal reference.

Table 3.4 shows the closed-loop results over a simulation horizon of four days ($M = 96$ hours) with a common initial condition $x_0 = [141, 288, 930]^\top$ in m^3 and a safety threshold $s = [42, 18, 270]^\top$ in m^3. The aforementioned table compares the average economic cost, i.e., $\sum_{k=0}^{M} \ell(k, x_k, u_k)/(M+1)$, and the transient cost defined as $\sum_{k=0}^{\infty} \ell(k, x_k, u_k) - \ell(k, \hat{x}_k^\star, \hat{u}_k^\star)$, where lower values imply better economic performance. It can be noticed that the loss of economic performance occurs when moving from the extreme of a pure economic MPC formulation ($\theta = 1$), to the extreme of a pure tracking MPC ($\theta = 0$).

Table 3.4: Closed-loop performance of periodic economic MPC strategies

θ	Terminal Region		Terminal Constraint	
	Average cost	Transient cost	Average cost	Transient cost
1.00	25.0313	-	26.3124	-
0.75	26.2925	122.1548	26.2957	122.4663
0.50	26.2932	122.2157	26.2958	122.4630
0.25	26.2941	122.3026	26.2958	122.4629
0.00	26.4380	136.1194	26.2995	122.8267

All costs are in economic units (e.u.)

Greater values of θ leads to faster convergence and less economic profits, which is the price to gain in stability. In fact, in the tracking extreme, the terminal penalty/region approach did not outperform the cost of the terminal point constraint approach. This behaviour may be expected since the terminal penalty is not optimally designed for the pure tracking MPC scheme. Nevertheless, the results show that when operating with $\theta > 0$, the proposed formulation with periodically time-varying terminal region and terminal penalty may outperform the performance achievable with a periodically time-varying terminal equality constraint.

3.6 Summary

In this chapter, the potential of economic MPC for control of generalised flow-based networks has been verified on a proof of concept case study. A multi-objective cost function was considered and different economic MPC formulations were analysed and extended for controlling a network under nominal demands, which were considered periodic. In the first part of the chapter, initial conditions to guarantee feasibility of the MPC optimisation problem were derived under a max-min paradigm. Later recursive feasibility and stabilising economic MPC controllers based on terminal equality constraint formulations were discussed. Especially, the single-layer economic MPC approaches resulted to be of great utility for the control of flow networks due to their capacity to cope with changes in the economic parameters of the cost function. In the last part of the chapter, an economic MPC formulation based on periodically time-varying terminal ingredients (terminal penalty, terminal region and terminal control law) has been suggested. The contribution of this latter formulation was the extension and generalisation of previous results addressing periodic economic operation of time-invariant systems to the case of linear periodic systems. It has been shown that when using a terminal region constraint instead of a terminal equality constraint, the closed-loop average performance of a periodic economic MPC algorithm can be enhanced. The methodologies were verified with a drinking water network example, considering a periodic multi-objective cost function.

Chapter 4

Reliability-based MPC of Generalised Flow-based Networks

As reviewed in previous chapters, the management criteria in generalised flow-based networks often involve a trade-off between operational costs and reliability assurance. Thus, if cost-reliability optimisation is desired, the acceptability of risk should be considered by the decision maker. In this way, to enable the incorporation of risk perception into the controller design, the approaches introduced in previous chapters have to be enhanced with a more robust strategy, capable to take into account stochastic properties of critical parameters that directly affect the overall reliability of a given network, such as the uncertainty related with the uncontrolled flows and the health of actuators.

This chapter presents an MPC strategy that assures reliability in generalised flow-based networks given a customer service level, a forecasting demand and a degradation rate of actuators. The underlying idea concerns an MPC controller with capabilities to dynamically allocate minimal safety volumes of the transported commodity in each storage node to face uncertainties, and to distribute control effort within actuators to extend their useful life and improve overall system reliability.

4.1 Introduction

The control strategy addressed in this chapter is based on a multilayer (hierarchical) control system structure enhanced with forecasting demand and actuators health estimation modules (see Figure 4.1). The hierarchical architecture has been widely used in

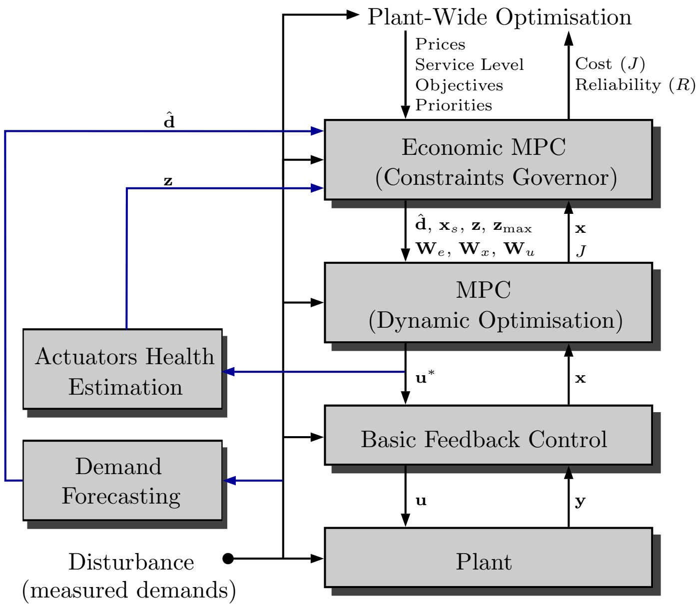


Figure 4.1: Reliability-based MPC structure

process control with satisfactory results, optimising economic profits when disturbances are slowly varying, see e.g., [100, 174]. In several generalised flow-based networks (e.g., water networks, electric power networks), these disturbances often follow a pattern in a daily basis and can be well forecasted for an hourly sampling time, which makes the hierarchical structure suitable to optimise targets for the policies of the operational level.

The proposed control architecture has two layers, combining an upper Economic Optimisation Layer (EOL) and a lower Optimal Feedback-Control Layer (OFCL). The EOL is a strategic layer that deals with the adjustment of targets, bounds and tuning weights for the control problem, taking into account economic cost functions. In this layer, the minimal volume required in each storage node and the allowed health degradation of actuators are computed dynamically to assure a predefined service level. The OFCL is a tactical layer that executes a dynamic optimisation within an MPC algorithm to translate strategic policies into desired control actions.

4.2 Safety Stocks Allocation Policy

There is often the need of guaranteeing a safety stock in each storage node of a generalised flow-based network in order to decrease the probability of stock-outs (when a node has insufficient resources to satisfy either external demands or the flow requested by other intermediate nodes) due to possible uncertainties in the network. As discussed in § 1.2.1, stock allocation problems have been addressed before in the literature of supply chain management, where solutions are mainly based on inventory planning strategies that incorporate, within deterministic formulations, safety mechanisms to cope with randomness and risks associated to networks operation (see also [40]). Most techniques from inventory management suppose a hierarchical and descendant flow of products, even in multi-stage multi-echelon schemes, in a way that predicted safety stock changes are easily communicated backwards in order to support availability of quantities when they are needed [88]. Nevertheless, this behaviour is not true in real large-scale generalised flow-based networks since a meshed topology with multi-directional flows between nodes prevails instead of spread tree configurations.

To circumvent the aforementioned difficulties and to determine the amount of safety stocks, an inventory planning strategy is addressed here to enrich previous control approaches with replenishment policies. Consider the system (2.10) and let the disturbance vector d satisfy Assumption 2.8. Hence, the goal is to dynamically allocate a minimal amount of flow volume $s_k \in \mathbb{X}$ in each storage node at time step $k \in \mathbb{Z}_+$ to avoid stock-outs. To do so, the EOL first estimates future flows for a short-term prediction horizon $N_s \in \mathbb{Z}_+$, as follows:

$$\mathbf{u}_{s,k} = \arg\min_{\mathbf{u}_{s,k}} \sum_{i=0}^{N_s-1} \ell(k+i, x_{k+i|k}, u_{s,k+i|k}), \tag{4.1a}$$

subject to:

$$x_{k+i+1|k} = Ax_{k+i|k} + Bu_{k+i|k} + B_d\hat{d}_{k+i|k}, \quad \forall i \in \mathbb{Z}_{[0,N_s-1]} \tag{4.1b}$$

$$E_u u_{k+i|k} + E_d\hat{d}_{k+i|k} = 0, \quad \forall i \in \mathbb{Z}_{[0,N_s-1]} \tag{4.1c}$$

$$x_{k+i|k} \in \mathbb{X}, \quad \forall i \in \mathbb{Z}_{[1,N_s]} \tag{4.1d}$$

$$u_{k+i|k} \in \mathbb{U}, \quad \forall i \in \mathbb{Z}_{[0,N_s-1]} \tag{4.1e}$$

$$x_{k|k} = x_k. \tag{4.1f}$$

The above optimisation problem uses the same model structure as in (2.14) but using demand forecasts $\hat{d}$. The resultant sequence of estimated flows $\mathbf{u}_{s,k} := \{u_{s,k+i|k}\}_{i\in\mathbb{Z}_{[0,N_s-1]}}$, allows to virtually decouple storage nodes and to estimate at time step $k \in \mathbb{Z}_+$ their average forecast net demand $\hat{d}_{\text{avg},k} \in \mathbb{D}$ as follows:

$$\hat{d}_{\text{avg},k} = \frac{\sum_{i=0}^{N_s-1} \hat{d}_{\text{net},k+i|k}}{N_s - 1}, \tag{4.2}$$

where $\hat{d}_{\text{net},k+i|k} := |B_{\text{out}} u_{s,k+i|k} + B_p \hat{d}_{k+i|k}|$, $i \in \mathbb{Z}_{[0,N_s-1]}$ is the forecast net demand consisting of the term $B_{\text{out}} \hat{u}_{s,k+i|k}$ representing the estimated endogenous demands (the outflows of storage nodes imposed by neighbouring nodes) and the term $B_p \hat{d}_{k+i|k}$ representing the estimated exogenous (customer) demands, both for a given prediction step $i \in \mathbb{Z}_{[0,N_s]}$ starting from the current time step k. Now, even if the net demand is predicted with a strong confidence level in a short horizon N_s, it is usually required to store safety stocks $y_k \in \mathbb{R}_+^n$ at each time step $k \in \mathbb{Z}_+$ to face flow uncertainties. The amount of safety stocks is related with the stochastic nature of demands and lead times (the time from the moment a supply requirement is placed to the step it is received), and consequently with the quality of the forecasting model. Hence, the safety stock is given by

$$y_k = \Phi^{-1}(\gamma)\sigma_k, \tag{4.3}$$

where $\Phi^{-1}(\cdot)$ is the inverse cumulative normal distribution, $\gamma \in (0, 100)\%$ is the desired customer service level (percentage of customers that do not experience a stock-out) and $\sigma_k := \left[\sigma_{(1),k}, \ldots, \sigma_{(n),k}\right]^\top \in \mathbb{R}^n$ is the vector of total forecast deviations, where each j-th deviation is given by

$$\sigma_{(j),k} := \sqrt{\sigma_{d(j),k}^2 \, \tau_{(j),k} + \sigma_{\tau(j),k}^2 \, \hat{d}_{\text{avg}(j),k}}$$

for all $j \in \mathbb{Z}_{[1,n]}$. At each time step k the individual total deviation takes into account the forecast deviations of the individual net demands and lead times, denoted respectively as $\sigma_{d(j),k}$ and $\sigma_{\tau(j),k}$. In this way, the vector $s_k \in \mathbb{R}_+^n$ of commodity base-stocks is computed as follows:

$$s_{(j),k} = \tau_{(j),k}\hat{d}_{\text{avg}(j),k} + y_{(j),k}, \quad \forall j \in \mathbb{Z}_1^n, \tag{4.4a}$$

$$s_k := \left[s_{(1),k}, \ldots, s_{(n),k}\right]^\top. \tag{4.4b}$$

The base-stock s_k is introduced in the MPC design of § 2.3.2 as a soft constraint to lead the state of storage nodes to be greater than such stocks (when possible) and to let the system employ safety stocks y_k to face uncertainties (when needed) but penalising in the MPC cost function the used amount of safety $\xi_k \in \mathbb{R}^n_+$. This soft constraint is expressed as $x_k \geq s_k - \xi_k$ and the stage cost function is now $\tilde{\ell}(k, x_k, u_k) := \ell(k, x_k, u_k) + \|\xi_k\|^2$.

Remark 4.1. *This strategy deals specifically with storage node reliability (assuming their faulty behaviour as the inability to satisfy their own demands), which is affected by both the capacity and reliability of the elements supplying water to them. If the supply capacity is less than the average demand, no tank will be probably large enough to provide a sustained service.* $\diamondsuit$

4.3 Actuator Health Management Policy

Unless some damage mitigating policy is adopted to ensure the availability of actuators for a given maintenance horizon, their inherent degradation could compromise the overall service reliability of the network. Therefore, system safety can be enhanced by taking into account the health of the components explicitly in the controller design. Several models have been proposed in literature to describe reliability and ageing of actuators under nominal operation, see [71, 76, 102] for a review. Nevertheless, as pointed in [92, 111], a realistic health measurement should also include the trend of actuators ageing according to the variation of the operating conditions. Rates of degradation can be assumed constant for some equipment but others present a highly variable and non-linear rate depending on the degradation mechanism and the local conditions [188].

For the sake of simplicity, the linear proportional degradation model presented in [143] and its uniform rationing heuristic is adopted in this chapter to estimate and manage the health of the actuators. The approach considers the health condition of each actuator being described by a wear process with rate associated to the exerted control effort as follows:

$$z_{k+1} = z_k + \varphi|u_k|, \tag{4.5}$$

where $z_k \in \mathbb{R}^m$ denotes the state of cumulative degradation of actuators at time step k and $\varphi := \mathrm{diag}(\psi_1, \ldots, \psi_m)$ is a diagonal matrix of constant degradation coefficients $\psi_i \in \mathbb{R}$, $i \in \mathbb{Z}_{[1,m]}$, associated with the m actuators.

Degradation of each actuator will accumulate until the element reaches a state in which it will not perform its function with an acceptable level. At this time step, it can be considered that the actuator operation may be compromising the network supply service unless the demands result reachable from other redundant flow paths or a fault tolerant mechanism is activated. Therefore, instead of incurring into a failure that requires corrective control actions, a preventive strategy can be implemented to improve overall system reliability by guaranteeing that each actuator remains available until the step of a programmed maintenance intervention.

To circumvent the system availability problem, an obvious approach is to constrain the accumulated degradation of actuators at each time step to remain below a safe threshold until a predefined maintenance horizon is reached. Here, the health management is considered to be ruled by the constraints proposed in [143], which are:

$$z_{k+N|k} \leq z_{\max,k}, \tag{4.6}$$

$$z_{\max,k} := z_k + N\frac{z_{\text{tresh}} - z_k}{M+N-k}, \tag{4.7}$$

where $N \in \mathbb{Z}_+$ is the prediction horizon used also for prognosis, $\delta_z \in (0,1)$ is the risk acceptability level, $z_{\max,k} \in \mathbb{R}^m$ is the vector of maximum accumulated degradation of actuators allowed for the time step k, and $z_{\text{tresh}} \in \mathbb{R}^m$ is the vector of thresholds for the terminal degradation at a maintenance horizon $M \in \mathbb{Z}_+$. Notice that (4.6) restricts the predicted accumulated degradation of actuators' health at N-steps ahead from the current time step k, and the right side of (4.7) is a uniform rationing of the remaining allowed degradation $(z_{\text{tresh}} - z_k)$ that is updated at each time step according to the applied control actions and ensures that $z_k \leq z_{\text{tresh}}$ for $k = M$.

Remark 4.2. *Despite the inherent relation, a degraded state is not the same as a faulty state, see [81]. In fact, under nominal conditions of operation, degradation always precedes failure. When a component is degraded, maintenance actions should be executed to improve its performance to acceptable levels, but when the component is faulty, repairing actions are needed to restore its functionality.* ◊

Keeping in mind the difference between degraded and faulty states, it can be noticed that the strategy for uniform rationing of degradation should be complemented with other safety mechanisms to incorporate the remaining useful life of the actuators on the basis of their reliability and keep them available as long as possible. Accordingly, here the improvement of the safety and reliability of a generalised flow-based network is proposed using a smarter control allocation policy following the results of [91] and the proportional hazard model reported in [186]. The main idea is to add to the economic cost function a penalisation on control actions, which is weighted with a matrix $W_u \in \mathbb{R}_+^{m\times m}$ that depends directly on the actuators reliability. This strategy leads to a smart use of actuators, minimising the frequency of unscheduled downtimes and related costs.

Consider that the actuators' reliability can be estimated for the variable operating conditions with the following modified exponential distribution:

$$R_{i,k} = \exp\left(-\lambda_i^0 \exp\left(\beta_i \|u_{i,0:k}\|_2^2\right)\right), \quad \forall i \in \mathbb{Z}_{[1,m]}, \tag{4.8}$$

where $\lambda_i^0 \in \mathbb{R}_+$ is a coefficient representing the nominal failure rate of the i-th actuator, $\beta_i = \left(t_M\left(u_{i,\max} - u_{i,\min}\right)\right)^{-1} \in \mathbb{R}$ is a shape parameter of the actuator failure for a expected life $t_M \in \mathbb{Z}_+$, and $\exp(\beta_i\|u_{i,0:k}\|_2^2) \in \mathbb{R}_+$ with $u_{i,0:k} = [u_{i,0}, \ldots, u_{i,k}]^\top$ is the load function that modifies the failure rate according to the energy of the control actions applied from the initial time until the time step k. From (4.8), it follows that the cumulative probability of failure rate can be written as $F_{i,k} = 1 - R_{i,k}$. Hence, the optimal control actions computed by the predictive controller can be distributed among actuators in a way that components with larger accumulated damage are relieved. This can be achieved by adding to the original cost function a weighted term for the suppression of control moves, i.e., $\|\Delta u_k\|_{W_{u,k}}^2$, in which the weighting matrix is given by

$$W_{u,k} = \mathrm{diag}\left(w_1, w_2, \ldots, w_m\right), \tag{4.9}$$

where $w_{i,k} = F_{i,k} = 1 - R_{i,k}$ for $i \in \mathbb{Z}_{[1,m]}$. Notice that the weighting matrix is re-computed on-line at each time step k to take into account the variation of the control actions and actuators' reliability. Hence, this weighting strategy leads to improve system availability, i.e., to retain the operability of the network elements for longer times.

4.4 Reliability-based Economic MPC Problem

After discussing reliability aspects of storage and supply infrastructure, next the setting of the proposed reliability-based economic MPC (RB-MPC) controller is shown, which incorporates into its optimisation problem both the dynamic safety stocks policy and the actuator-health management policy, in order to improve the flow supply service level in a given network, facing uncertainty in the demands and equipment wear. The approach is based on a bi-level optimisation problem, in which the first level solves problem (4.1) and the second level solves the following problem at each time step k:

$$\min_{\mathbf{u}_k,\boldsymbol{\xi}_k} \sum_{i=0}^{N-1} \ell(k+i, x_{k+i|k}, u_{k+i|k}) + \|\xi_{k+i|k}\|^2_{W_s} + \|\Delta u_{k+i|k}\|^2_{W_{u,k}} \tag{4.10a}$$

subject to:

$$x_{k+i+1|k} = Ax_{k+i|k} + Bu_{k+i|k} + B_d\hat{d}_{k+i|k}, \qquad \forall i \in \mathbb{Z}_{[0,N-1]} \tag{4.10b}$$

$$z_{k+i+1|k} = z_{k+i|k} + \varphi|u_{k+i|k}|, \qquad \forall i \in \mathbb{Z}_{[0,N-1]}, \tag{4.10c}$$

$$E_u u_{k+i|k} + E_d\hat{d}_{k+i|k} = 0, \qquad \forall i \in \mathbb{Z}_{[0,N-1]} \tag{4.10d}$$

$$x_{k+i|k} \in \mathbb{X}, \qquad \forall i \in \mathbb{Z}_{[1,N]} \tag{4.10e}$$

$$u_{k+i|k} \in \mathbb{U}, \qquad \forall i \in \mathbb{Z}_{[0,N-1]} \tag{4.10f}$$

$$\xi_{k+i|k} \geq s_k - x_{k+i|k}, \qquad \forall i \in \mathbb{Z}_{[0,N]} \tag{4.10g}$$

$$\xi_{k+i|k} \geq 0, \qquad \forall i \in \mathbb{Z}_{[0,N]} \tag{4.10h}$$

$$z_{k+N|k} \leq z_{\max,k}, \tag{4.10i}$$

$$\Delta u_{k+i|k} = u_{k+i|k} - u_{k+i-1|k}, \tag{4.10j}$$

$$(x_{k|k}, z_{k|k}, u_{k-1|k}) = (x_k, z_k, u_{k-1}), \tag{4.10k}$$

where $\mathbf{u}_k = \{u_{k+i|k}\}_{i\in\mathbb{Z}_{[0,N-1]}}$ and $\boldsymbol{\xi}_k = \{\xi_{k+i|k}\}_{i\in\mathbb{Z}_{[0,N-1]}}$ are the decision variables with $\mathbf{u}_k$ being the controlled flows and $\boldsymbol{\xi}_k$ the level of violation of the safety constraint, $\hat{d}_{k+i|k} \in \mathbb{D}$ is the predicted demand obtained at the current time step k for i-steps ahead, $\varphi \in \mathbb{R}^m$ is the vector of actuators degradation coefficients, $s_k \in \mathbb{X}$ is the vector of base-stocks obtained at time k and given by (4.4), $z_{\max,k} \in \mathbb{R}^m$ is the maximum accumulated degradation of actuators allowed for the time step k. The weight $W_s \in \mathbb{R}^{n\times n}_+$ is used to manage the acceptability level for safety constraint violation and $W_{u,k} \in \mathbb{R}^{m\times m}_+$ is the reliability-based weighting matrix introduced to relieve the actuators with larger accumulated degradation.

Remark 4.3. *The core of the RB-MPC approach relies on the dynamic handling of constraints in order to trade-off reliability and economic optimisation to obtain a enhanced robust performance. Note that the worse the demand forecasting and actuators degradation models are, the stricter the constraints and the more conservative control policy will be. The RB-MPC gives just an enhancement of robustness, neither robust stability nor robust feasibility are guaranteed for this scheme.* ◊

4.5 Numerical Results

This section presents the results of applying the RB-MPC approach to the full model (see Figure 2.3) of the Barcelona drinking water network described in § 2.4. Simulations have been done under the same real demand scenarios, initial conditions and computational resources of Section 2.5. The matrix φ of degradation coefficients involved in the actuators health degradation model (4.5) is set up with the individual coefficients $\psi_i = 1.3459 \times 10^{-4}$, if the i^{th} actuator is a pump, or $\psi_i = 2.9496 \times 10^{-5}$ if the actuator is a valve. A maintenance horizon of six months ($M = 4320$ hours) is used to manage the actuators degradation and a supply service level of $\gamma = 95\%$ is required. These nominal degradation values are based on typical failure rates as those used in [5], and on the analysis developed in [81] about the relation between failure coefficients and degradation coefficients. The forecast demand involved in the proposed safety stocks allocation policy is based on a forecasting module proposed later on Chapter 6, which predicts future water demands for the selected case study according to past demands and weather conditions (i.e., temperature and relative humidity). The optimisation problem is given by

$$\min_{\mathbf{u}_k, \boldsymbol{\xi}_k} V_N(k, x_k, \mathbf{u}_k, \boldsymbol{\xi}_k), \tag{4.11}$$

subject to (4.10b)–(4.10k). The cost function $V_N(k, x_k, \mathbf{u}_k, \boldsymbol{\xi}_k)$ is given by (2.22) but setting the weighting matrix $W_{\Delta u} = W_{u,k}$ and $W_{u,k}$ obtained from (4.9).

In order to assess the control enhancements developed in this chapter, two control strategies have been simulated:

- RB-MPC$_{(1)}$: implements only the dynamic safety stock policy in order to study the impact of considering solely the service reliability in tanks, i.e., solve problem (4.11) neglecting constraints (4.10i) and without the reliability-based weight.
- RB-MPC$_{(2)}$: implements the full approach, i.e., considering (4.11) with both the safety stocks allocation policy and the prognostics and health management (PHM) of actuators.

Table 4.1 summarises the performance of each of these RB-MPC strategies, according to the KPIs defined in Section 2.4.4. Taking into account that the lower the value of

Table 4.1: Key performance indicators for RB-MPC

Controller	KPI_E (e.u.)	KPI_S (m^3)	$\text{KPI}_{\Delta U}$ $(m^3/s)^2$
RB-MPC$_{(1)}$	2383.97	1987.75	0.9024
RB-MPC$_{(2)}$	2569.59	3029.94	2.1023

e.u.: economic units

Table 4.2: Water and electric cost comparison of RB-MPC strategies

	MPC Approach	Water Cost (e.u.)	Electric Cost (e.u.)	Total Cost (e.u.)
Day 1	RB-MPC$_{(1)}$	23176.05	29068.32	52244.37
	RB-MPC$_{(2)}$	34601.19	18472.19	53073.38
Day 2	RB-MPC$_{(1)}$	30304.53	29803.52	60108.05
	RB-MPC$_{(2)}$	47613.86	20120.06	67733.93
Day 3	RB-MPC$_{(1)}$	28487.43	29686.35	58173.78
	RB-MPC$_{(2)}$	42258.51	19805.85	62064.37
Day 4	RB-MPC$_{(1)}$	29059.73	29764.94	58824.67
	RB-MPC$_{(2)}$	43818.31	19991.01	63809.32

e.u.: economic units

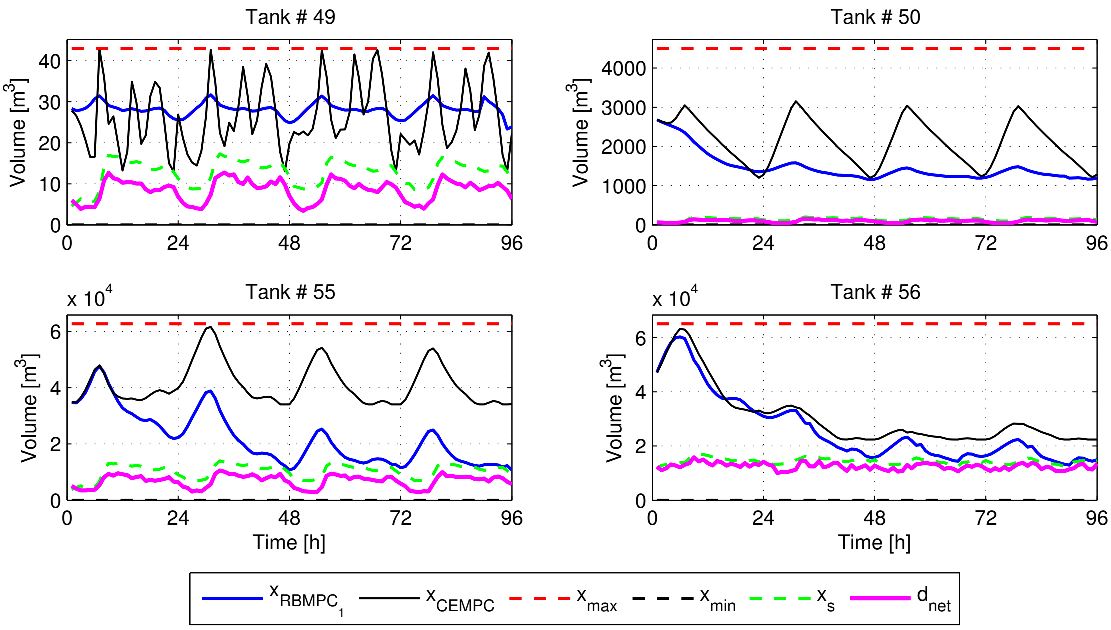
the KPIs the better the performance of the controller is, it might be hastily concluded that the RB-MPC$_{(1)}$ strategy has the best results. Nevertheless, the higher values in the RB-MPC$_{(2)}$ strategy are due to the smart distribution of the control effort to extend or guarantee the availability of the system. The actuators health management in RB-MPC$_{(2)}$ requires to distribute the load between redundant actuators according to their level of degradation no matter if water has to be supplied from more expensive sources or if its necessary to store more water. This fact might decrease the electric cost and increase the water cost. Table 4.2 shows in detail the economic costs per day of including reliability aspects explicitly in the MPC law. The results reflect the expected conflictive relation between economic optimisation and reliability.

Figure 4.2 shows the hourly excursion of water in four tanks of the Barcelona DWN. The plots compare the results of both RB-MPC strategies with the replenishment behaviour obtained with the CE-MPC approach. Notice in Figure 4.2(a) that the solely inclusion of the dynamic safety stocks, computed as proposed in § 4.2, improves the water management in the DWN by widening the water excursion range. The proper selection of the safety volume constraint with the RB-MPC$_{(1)}$ smooths the behaviour of tanks and the required pumping scheme. Figure 4.2(b) shows the replenishment of tanks when including also the PHM policy. The plots show again a relaxed behaviour and a wider excursion of water in tanks. The difference between the RB-MPC$_{(2)}$, the RB-MPC$_{(1)}$ and the CE-MPC, is that the former will tend to store larger amounts of water in tanks (if possible) when the associated actuators are degrading too fast.

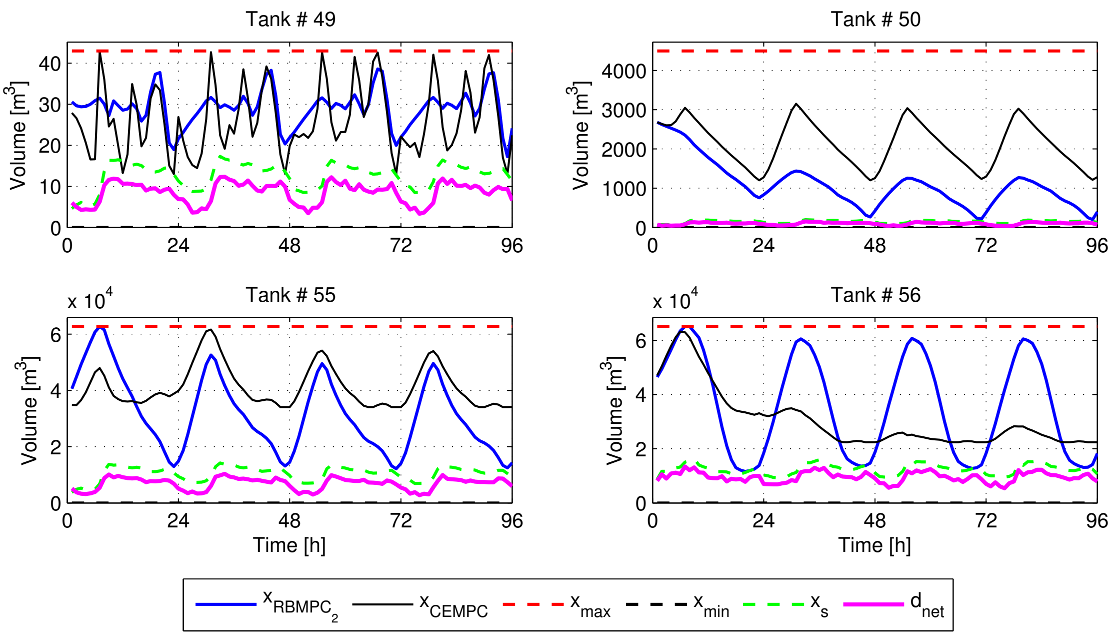
On other hand, Figure 4.3 shows a comparison of the control actions computed with the RB-MPC strategies (under nominal degradation conditions) and those computed with the CE-MPC approach. As expected, all of the controllers decide to activate the actuators with variable electric cost (i.e., pumps) when the electric tariff is cheaper. Of course, if it is necessary to satisfy the associated water requirement, some elements (e.g., Actuator 87) must operate even in high electric cost periods. The RB-MPC$_{(1)}$ strategy computes similar patterns of control actions to those of the computed by the CE-MPC. Instead, the RB-MPC$_{(2)}$ differs in the way that it tends to select control actions that decelerate (if possible) the wear of the actuators. It can be seen that specially pumps, which are rotative elements with higher degradation coefficients and higher electric costs, are less used (see e.g., Actuator 71) with this latter strategy if it is not a critical element in a reachability analysis.

To analyse the advantages of incorporating actuators reliability in the MPC law, Figure 4.4 shows four redundant actuators operated with the RB-MPC$_{(2)}$ strategy. Simulations were done for three different degradation coefficients of one of the plotted actuators in order to simulate possible faulty behaviours. It can be seen how the controller decreases the use of an actuator if its health condition is getting worse. The results highlight the robustness and the benefits of the health-aware capability of the RB-MPC approach, specially to manage large DWN.

Figure 4.5 shows in detail the effect of the health degradation on the reliability of Actuator 49, and how the RB-MPC strategy uses this information to smartly tune the move suppression of the mentioned actuator. An increment in the degradation of the element causes a decrement of its reliability, unless the maintenance horizon is reached and the health condition improved or renewed.



(a) Replenishment of storage tanks with RB-MPC$_{(1)}$



(b) Replenishment of storage tanks with RB-MPC$_{(2)}$

Figure 4.2: Management of storage of water with the RB-MPC strategies

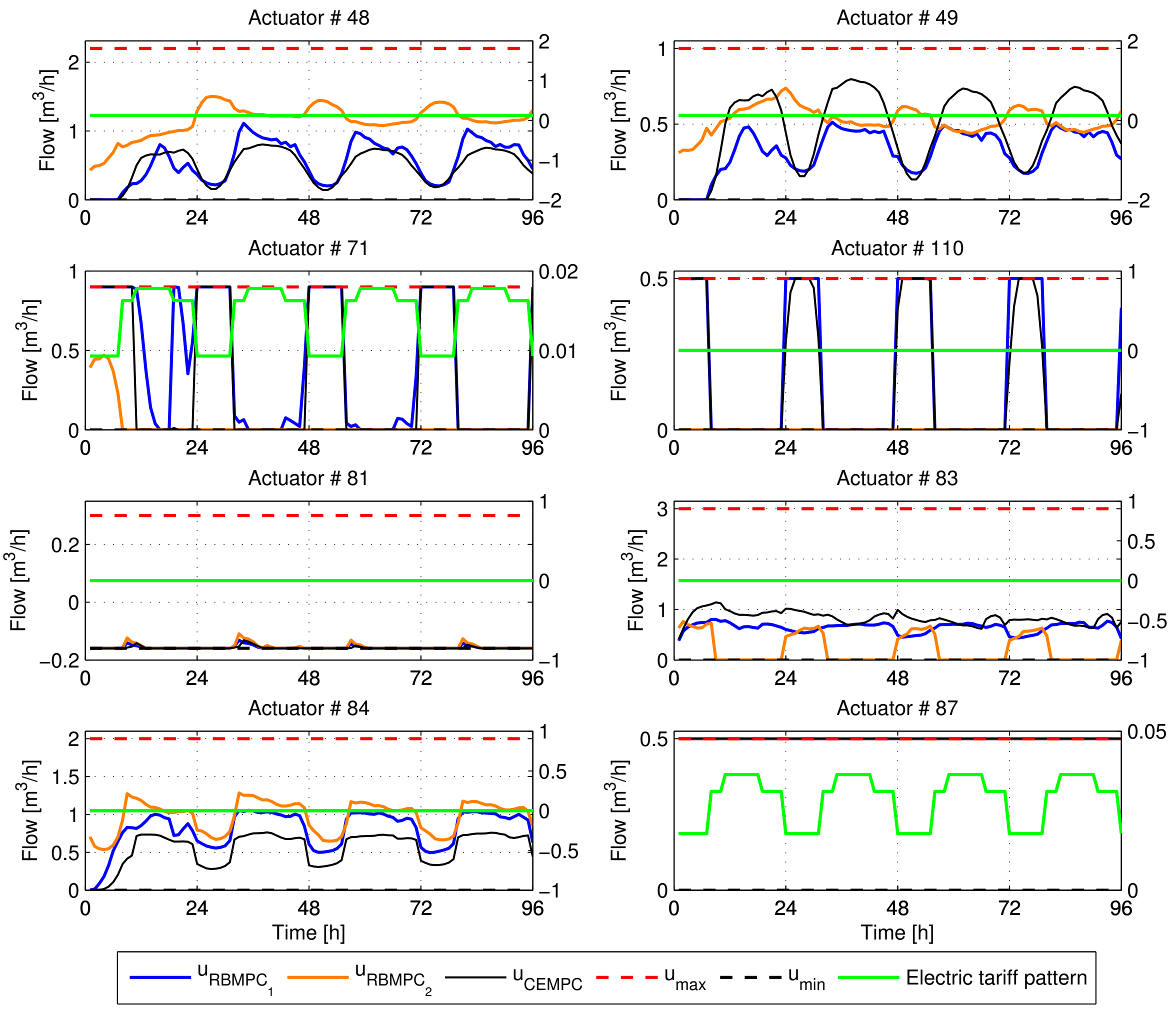


Figure 4.3: Management of actuators with the RB-MPC strategies

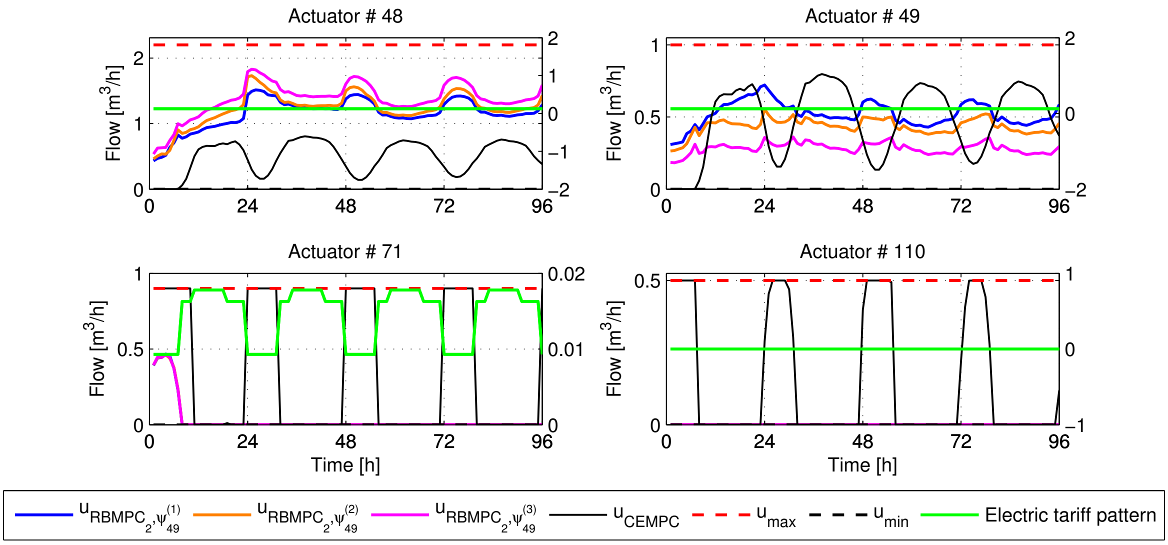


Figure 4.4: Control actions for a sample of redundant actuators using RB-MPC$_{(2)}$ strategy, varying the degradation coefficient of Actuator 49, i.e., $\psi_{49}^{(1)} = 2.95 \times 10^{-5}$, $\psi_{49}^{(2)} = 1.5 \times 10^{-4}$ and $\psi_{49}^{(3)} = 6 \times 10^{-4}$

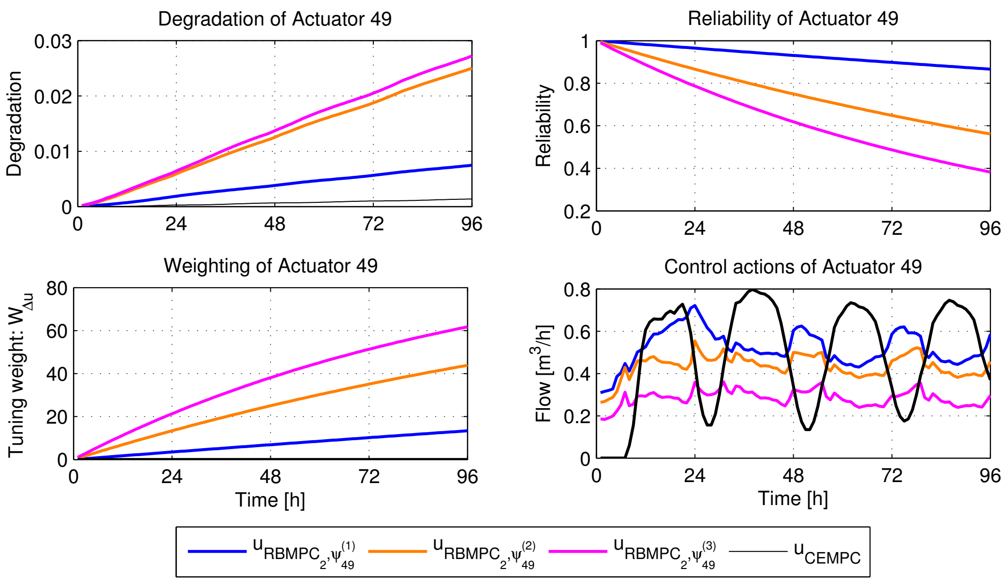


Figure 4.5: Example of the smart tuning strategy within the RB-MPC$_{(2)}$ strategy

The level of estimated reliability, depends on both time of use and load conditions of the actuator. A reliability loss implies an increment in the tuning weight of the actuators.

If the topology of the system arises feasibility problems when applying the RB-MPC$_{(2)}$ strategy, constraint (4.10i) can be softened by adding a slack variable to one of the sides of the inequality and penalising it in the MPC cost function. Moreover, if the manager considers that the economic cost of applying the full RB-MPC approach to operate the DWN is expensive, then, instead of using a hard constraint for the actuators degradation (which is the main cause that increases the cost of the solution), constraint (4.10i) can be neglected and problem (4.11) solve only considering the safety stock allocation policy in combination with the reliability-based weighting strategy. This will still incorporate tanks and actuators reliability in the control law but without guarantee of reaching the maintenance horizon.

4.6 Summary

This chapter has shown through a real case study the effectiveness of the proposed RB-MPC strategy, which enhances the CE-MPC described in § 2.4.3 by incorporating dynamic planning of safety stocks and actuators health monitoring, to assure reliability in the flow supply and to minimise operational costs for a given customer service level. The EOL allows to efficiently solve the non-linear problems and the tuning of strategic targets such as minimum storage volumes and maximum degradation of actuators, before the MPC algorithm executes. This simplifies the inherent optimisation by maintaining the dynamic model and constraints in the linear domain. It is important to remark that uncertainty is considered stationary within the prediction horizon and consequently the safety constraint keeps a uniform back-off of demand. The core of the approach relies on the quality of the forecasting demand. An increment in the forecasting error leads to require greater amounts of safety stocks, causing a reduction of the available capacity in the storage nodes to perform optimal excursions, which in turn increases operational costs. The safety mechanism involved in the strategy does not propagate uncertainties along the entire prediction horizon. Classic robust techniques assume bounded uncertainties and solve a worst-case problem, what leads to an unnecessary conservatism that detriments economic objectives. Therefore, a probabilistic handling of constraints might be beneficial. Future avenues for research on reliability-based controllers could be: multi-period analysis with different replenishment cycle for each storage node, distributed control of the network with pooling risk analysis and actuators ageing models enriched with the effect of maintenance quality and health-aware control.

Chapter 5

Stochastic MPC for Robustness in Generalised Flow-based Networks

The control of generalised flow-based networks is a challenging problem due to their size and exposure to uncertain influences such as flow demands and other external phenomena affecting their operation. In this chapter, two different stochastic programming approaches are proposed for optimising network flows under uncertainty: chance-constrained model predictive control and tree-based model predictive control. Under the former approach, disturbances are modelled as stochastic variables with a non-stationary description, unbounded support and quasi-concave probabilistic distribution. A deterministic equivalent of the related stochastic problem is formulated by using Boole's inequality and a uniform allocation of risk. In the latter approach, demand is modelled as a disturbance rooted tree where branches are formed by the most probable evolutions of the demand. In both approaches, MPC is used to optimise the expectation of the operational cost of the disturbed system.

5.1 Introduction

In generalised flow-based networks, a common purpose is the achievement of the highest level of consumer satisfaction and service quality in line with the prevailing regulatory framework, whilst making best use of available resources. Hence, networks must be reliable and resilient while being subject to constraints and to continuously varying conditions with both deterministic and probabilistic nature. Customer behaviour determines the transport and storage operations within the network, and flow demands can vary in both the long and the short term, often presenting time-based patterns in some applications. Therefore, a better understanding and forecasting of demands will improve both modelling and control of flow-based networks.

This chapter focuses on the way that uncertainty can be faced in the control of generalised flow-based networks from an MPC framework. The simplest way to do this is by ignoring the explicit influence of disturbances or using their expected value as done in Chapters 2 and 3. Unfortunately, this approach may lead to poor control performance or frequent constraint violations. In Chapter 4, a reliability-based MPC was proposed to handle demand uncertainty by means of a (heuristic) safety stock allocation policy, which takes into account short-term demand predictions but without propagating uncertainty along the prediction horizon. As discussed in [34], alternative approaches of MPC for stochastic systems are based on *min-max* MPC, *tube-based* MPC, and *stochastic* MPC. The first two consider disturbances to be unmeasured but bounded in a predefined set, which is more conservative and reduces the control performance due to the worst-case nature of the schemes. On the other hand, stochastic MPC considers a more realistic description of uncertainty, which leads to less conservative control approaches at the expense of a more complex modelling of the disturbances. The stochastic approach has a mature theory in the field of optimisation [31], but a renewed attention has been given to the stochastic programming techniques as powerful tools for control design, see, e.g., [32] and references therein.

From the wide range of stochastic MPC methods, this chapter specialises on *scenario tree-based MPC* (TB-MPC) and *chance-constrained MPC* (CC-MPC). Regarding TB-MPC, see, e.g., [147] and [107], uncertainty is addressed by considering simultaneously a set of possible disturbance scenarios modelled as a rooted tree, which branches along the prediction horizon. On the other hand, CC-MPC [161] is a stochastic control strategy that describes robustness in terms of probabilistic (chance) constraints, which require that the probability of violation of any operational requirement or physical constraint is below a prescribed value, representing the notion of reliability or risk of the system. By setting this value properly, the operator/user can trade conservatism against performance. Relevant works that address the CC-MPC approach in water systems can be found in [65, 137] and references therein. The main contribution of this chapter is then the design and assessment of CC-MPC and TB-MPC controllers for the operational management of generalised flow-based networks, discussing their advantages and weakness in the sense of applicability and performance. The particular case study is related to the water network described in § 2.4.

5.2 Problem Formulation

Consider a generalised flow-based network being described by the difference-algebraic equation (2.10) and satisfying Assumptions 2.5 to 2.8. The system is subject to state and input constraints considered here in the form of convex polyhedra defined as

$$\mathbb{X} \coloneqq \{x \in \mathbb{R}^n \mid Gx \leq g\}, \tag{5.1a}$$

$$\mathbb{U} \coloneqq \{u \in \mathbb{R}^m \mid Hu \leq h\}, \tag{5.1b}$$

where $G \in \mathbb{R}^{r_x \times n}$, $g \in \mathbb{R}^{r_x}$, $H \in \mathbb{R}^{r_u \times m}$, $h \in \mathbb{R}^{r_u}$, being $r_x \in \mathbb{Z}_+$ and $r_u \in \mathbb{Z}_+$ the number of state and input constraints, respectively.

Notice in (2.10b) that a subset of controlled flows are directly related with a subset of uncontrolled flows. Hence, it is clear that u does not take values in $\mathbb{R}^m$ but in a linear variety as shown in (3.2). This latter observation, in addition to Assumption 2.5 and Interpretation 3.1, can be exploited to develop an affine parametrisation of control variables in terms of a minimum set of disturbances as shown in Appendix B, mapping control problems to a space with a smaller decision vector and with less computational burden due to the elimination of the equality constraints. Thus, the system (2.10) can be rewritten as

$$x_{k+1} = Ax_k + \tilde{B}\tilde{u}_k + \tilde{B}_d d_k, \tag{5.2}$$

and the input constraint (5.1b) replaced with a time-varying restricted set defined as

$$\tilde{\mathbb{U}}_k \coloneqq \{\tilde{u} \in \mathbb{R}^{m-q} \,|\, H\tilde{P}\tilde{M}_1\tilde{u} \leq h - H\tilde{P}\tilde{M}_2 d_k\}, \tag{5.3}$$

which is non-empty if the dominance conditions in (3.9) hold.

The control goal is again to minimise a convex (possibly multi-objective) stage cost $\ell(k,x,\tilde{u}) : \mathbb{Z}_+ \times \mathbb{X} \times \tilde{\mathbb{U}}_k \to \mathbb{R}_+$, which might bear any functional relationship to the economics of the system operation. Let $x_k \in \mathbb{X}$ be the current state and let $\mathbf{d}_k = \{d_{k+i}\}_{i\in\mathbb{Z}_{[0,N-1]}}$ be the sequence of disturbances over a given prediction horizon $N \in \mathbb{Z}_{\geq 1}$. The first element of this sequence is measured, while the rest of the elements are estimates of future disturbances computed by an exogenous system and available at each time step $k \in \mathbb{Z}_+$. Hence, the MPC controller design is based on the solution of the following finite horizon optimisation problem:

$$\min_{\tilde{\mathbf{u}}_k=\{\tilde{u}_{k+i|k}\}_{i\in\mathbb{Z}_{[0,N-1]}}} \sum_{i=0}^{N-1} \ell(k+i, x_{k+i|k}, \tilde{u}_{k+i|k}), \tag{5.4a}$$

subject to:

$$x_{k+i+1|k} = Ax_{k+i|k} + \tilde{B}\tilde{u}_{k+i|k} + \tilde{B}_d d_{k+i}, \qquad \forall i \in \mathbb{Z}_{[0,N-1]} \tag{5.4b}$$

$$x_{k+i|k} \in \mathbb{X}, \qquad \forall i \in \mathbb{Z}_{[1,N]} \tag{5.4c}$$

$$\tilde{u}_{k+i|k} \in \tilde{\mathbb{U}}_{k+i}, \qquad \forall i \in \mathbb{Z}_{[0,N-1]} \tag{5.4d}$$

$$x_{k|k} = x_k. \tag{5.4e}$$

Assuming that (5.4) is feasible, i.e., there exists a non-empty sequence $\tilde{\mathbf{u}}_k$, then the receding horizon philosophy commands to apply the control action

$$u_k = \kappa_N(k, x_k, \mathbf{d}_k) = \tilde{u}^*_{k|k}. \tag{5.5}$$

This procedure is repeated at each time step k, using the current measurements of states and disturbances and the most recent forecast of these latter over the next future horizon.

Due to the stochastic nature of future disturbances, the prediction model (5.4b) involves exogenous additive uncertainty, which might cause that the compliance of state constraints for a given control input cannot be ensured. Therefore, uncertainty has to be represented in such a way that its effect on present decision making can properly be taken into account. To do so, stochastic modelling based on data analysis, probability distributions, disturbance scenarios, among others, and the use of stochastic programming may allow to establish a trade-off between robustness and performance. In the sequel, two stochastic MPC strategies are proposed for their application on network flows control.

5.3 Chance-Constrained MPC

Since the optimal solution to (5.4) does not always imply feasibility of the real system, it is appropriate to relax the original constraints in (5.4c) with probabilistic statements in the form of the so-called *chance constraints*. In this way, state constraints are required to be satisfied with a predefined probability to manage the reliability of the system. Considering the form of the state constraint set $\mathbb{X}$, there are two types of chance constraints according to the definitions below.

Definition 5.1 (Joint chance constraint). *A (linear) state joint chance constraint is of the form*

$$\mathbb{P}[G_{(j)}x \leq g_{(j)}\, , \forall j \in \mathbb{Z}_{[1,r_x]}] \geq 1 - \delta_x, \tag{5.6}$$

where $\mathbb{P}$ denotes the probability operator, $\delta_x \in (0,1)$ is the risk acceptability level of constraint violation for the states, and $G_{(j)}$ and $g_{(j)}$ denote the j-th row of G and g, respectively. This requires that all rows j have to be jointly fulfilled with probability $1 - \delta_x$ at least.

Definition 5.2 (Individual chance constraint). *A (linear) state individual chance constraint is of the form*

$$\mathbb{P}[G_{(j)}x \leq g_{(j)}] \geq 1 - \delta_{x,j}, \quad \forall j \in \mathbb{Z}_{[1,r_x]}, \tag{5.7}$$

which requires that each j-th row of the inequality has to be fulfilled individually with probability $1 - \delta_{x,j}$ at least, where $\delta_{x,j} \in (0,1)$.

Both forms of constraints are useful to measure risks, hence, their selection depends on the application. All chance-constrained models require prior knowledge of the acceptable risk δ_x associated with the constraints. A lower risk acceptability implies a harder constraint. This chapter is concerned with the use of joint chance constraints since they can express better the management of the overall reliability in a generalised flow-based network. In general, joint chance constraints lack from analytic expressions due to the involved multivariate probability distribution. Nevertheless, sampling-based methods, numeric integration, and convex analytic approximations exist, see e.g., [31] and references therein. Here, (5.6) is approximated following the results in [125, 144] by upper bounding the joint constraint and assuming a uniform distribution of the joint risk among a set of *individual chance constraints* that are later transformed into equivalent deterministic constraints under Assumption 5.1.

Assumption 5.1. *Each demand in $d \in \mathbb{R}^p$ follows a log-concave univariate distribution, whose stochastic description is known.*

Given the dynamic model in (5.2), the stochastic nature of the demand vector d makes the state vector $x \in \mathbb{R}^n$ to be also a stochastic variable. Then, let the cumulative distribution function of the constraint be denoted as

$$F_{Gx}(g) \coloneqq \mathbb{P}\left[\left\{G_{(1)}x \leq g_{(1)}, \ldots, G_{(r_x)}x \leq g_{(r_x)}\right\}\right]. \tag{5.8}$$

Defining the events $C_j \coloneqq \left\{G_{(j)}x \leq g_{(j)}\right\}$ for all $j \in \mathbb{Z}_1^{r_x}$, and denoting their complements as $C_j^c \coloneqq \left\{G_{(j)}x > g_{(j)}\right\}$, then it follows that

$$F_{Gx}(g) = \mathbb{P}\left[C_1 \cap \ldots \cap C_{r_x}\right] \tag{5.9a}$$
$$= \mathbb{P}\left[(C_1^c \cup \ldots \cup C_{r_x}^c)^c\right] \tag{5.9b}$$
$$= 1 - \mathbb{P}\left[(C_1^c \cup \ldots \cup C_{r_x}^c)\right] \geq 1 - \delta_x. \tag{5.9c}$$

Taking advantage of the *union bound*, Boole's inequality allows to bound the probability of the second term in the left-hand side of (5.9c), stating that for a countable set of events, the probability that at least one event happens is not higher than the sum of the individual probabilities [144]. This yields

$$\mathbb{P}\left[\bigcup_{j=1}^{r_x} C_j^c\right] \leq \sum_{j=1}^{r_x} \mathbb{P}\left[C_j^c\right]. \tag{5.10}$$

Applying (5.10) to the inequality in (5.9c), it follows that

$$\sum_{j=1}^{r_x} \mathbb{P}\left[C_j^c\right] \leq \delta_x \Leftrightarrow \sum_{j=1}^{r_x} \left(1 - \mathbb{P}\left[C_j\right]\right) \leq \delta_x. \tag{5.11}$$

At this point, a set of constraints arise from previous result as sufficient conditions to enforce the joint chance constraint (5.6), by allocating the joint risk δ_x in separate individual risks denoted by $\delta_{x,j}$, $j \in \mathbb{Z}_1^{r_x}$. These constraints are:

$$\mathbb{P}\left[C_j\right] \geq 1 - \delta_{x,j}, \quad \forall j \in \mathbb{Z}_1^{r_x}, \tag{5.12}$$
$$\sum_{j=1}^{r_x} \delta_{x,j} \leq \delta_x, \tag{5.13}$$
$$0 \leq \delta_{x,j} \leq 1, \tag{5.14}$$

where (5.12) forms the set of r_x resultant individual chance constraints, which bounds the probability that each inequality of the receding horizon problem may fail; and (5.13) and (5.14) are conditions imposed to bound the new single risks in such a way that the joint risk bound is not violated. Any solution that satisfies the above constraints, is guaranteed to satisfy (5.6). As done in [125], assigning, e.g., a fixed and equal value of risk to each individual constraint, i.e., $\delta_{x,j} = \delta_x / r_x$ for all $j \in \mathbb{Z}_{[1,r_x]}$, then (5.13) and (5.14) are satisfied.

Remark 5.1. *The single risks $\delta_{x,j}$, $j \in \mathbb{Z}_{[1,r_x]}$, might be considered as new decision variables to be optimised, see e.g., [133]. This should improve the performance but at the cost of more computational burden due to the greater complexity and dimensionality of the optimisation task. Therefore, as generalised flow-based networks are often large-scale systems, the uniform risk allocation policy is adopted to avoid overloading of the optimisation problem.* $\diamondsuit$

After decomposing the joint constraints into a set of individual constraints, the *deterministic equivalent* of each separate constraint may be used given that the probabilistic statements are not suitable for algebraic solution. Such deterministic equivalents might be obtained following the results in [37]. Assuming a known (or approximated) quasi-concave probabilistic distribution function for the effect of the stochastic disturbance in the dynamic model (5.2), it follows that

$$\begin{aligned} \mathbb{P}\left[G_{(j)} x_{k+1} \leq g_{(j)}\right] \geq 1 - \delta_{x,j} &\Leftrightarrow F_{G_{(j)}\tilde{B}_d d_k}(g_{(j)} - G_{(j)}(A x_k + \tilde{B}\tilde{u}_k)) \geq 1 - \delta_{x,j} \\ &\Leftrightarrow G_{(j)}(A x_k + \tilde{B}\tilde{u}_k) \leq g_{(j)} - F^{-1}_{G_{(j)}\tilde{B}_d d_k}(1 - \delta_{x,j}), \end{aligned} \tag{5.15}$$

for all $j \in \mathbb{Z}_{[1,r_x]}$, where $F_{G_{(j)}\tilde{B}_d d_k}(\cdot)$ and $F^{-1}_{G_{(j)}\tilde{B}_d d_k}(\cdot)$ are the cumulative distribution and the left-quantile function of $G_{(j)}\tilde{B}_d d_k$, respectively. Hence, the original state constraint set $\mathbb{X}$ is contracted by the effect of the r_x deterministic equivalents in (5.15) and replaced by the stochastic feasibility set given by

$$\begin{aligned} \mathbb{X}_{s,k} := \{x_k \in \mathbb{R}^n \,| \exists \tilde{u}_k \in \bar{\mathbb{U}}_k, \text{ such that} \\ G_{(j)}(A x_k + \tilde{B}\tilde{u}_k) \leq g_{(j)} - F^{-1}_{G_{(j)}\tilde{B}_d d_k}(1 - \delta_{x,j}), \, \forall j \in \mathbb{Z}_{[1,r_x]}\}, \end{aligned}$$

for all $k \in \mathbb{Z}_+$. From convexity of $G_{(j)} x_{k+1} \leq g_{(j)}$ and Assumption 5.1, it follows that the set $\mathbb{X}_{s,k}$ is convex when non-empty for all $\delta_{x,j} \in (0,1)$ [86] and most quasi-concave distribution function. For some particular distributions, e.g., Gaussian, convexity is retained for $\delta_{x,j} \in (0, 0.5]$

In this way, the reformulated predictive controller solves the following deterministic equivalent optimisation problem for the expectation $\mathbb{E}[\cdot]$ of the cost function in (5.4a):

$$\min_{\tilde{\mathbf{u}}_k} \sum_{i=0}^{N-1} \mathbb{E}[\ell(k+i, x_{k+i|k}, \tilde{u}_{k+i|k})], \tag{5.16a}$$

subject to:

$$x_{k+i+1|k} = Ax_{k+i|k} + \tilde{B}\tilde{u}_{k+i|k} + \tilde{B}_d \bar{d}_{k+i}, \quad \forall i \in \mathbb{Z}_{[0,N-1]} \tag{5.16b}$$

$$G_{(j)}(Ax_{k+i|k} + \tilde{B}\tilde{u}_{k+i|k}) \leq g_{(j)} - z_{k,j}(\delta_x), \quad \forall i \in \mathbb{Z}_{[0,N-1]}, \forall j \in \mathbb{Z}_{[1,r_x]} \tag{5.16c}$$

$$\tilde{u}_{k+i|k} \in \tilde{\mathbb{U}}_{k+i}, \quad \forall i \in \mathbb{Z}_{[0,N-1]} \tag{5.16d}$$

$$x_{k|k} = x_k, \tag{5.16e}$$

where $\tilde{\mathbf{u}}_k = \{\tilde{u}_{k+i|k}\}_{i \in \mathbb{Z}_{[0,N-1]}}$ is the sequence of controlled flows, $\bar{d}_{k+i}$, is the expected future demands computed at time step $k \in \mathbb{Z}_+$ for i-steps ahead, $i \in \mathbb{Z}_{[0,N-1]}$, $n_c \in \mathbb{Z}_{\geq 1}$ is the number of total individual state constraints along the prediction horizon, i.e., $n_c = r_x N$ and $z_{k,j}(\delta_x) := F^{-1}_{G_{(j)}\tilde{B}_d d_{k+i}}\left(1 - \frac{\delta_x}{n_c}\right)$. Since n_c does not only depend on the number of state constraints r_x but also on the value of N, the decomposition of the original joint chance constraint within the MPC algorithm could lead to a large number of constraints. This fact reinforces the use of a fixed risk distribution policy for generalised flow-based network control problems, in order to avoid the addition of a large number of new decision variables to be optimised.

Remark 5.2. *It turns out that most (not all) probability distribution functions used in different applications, e.g., uniform, Gaussian, logistic, Chi-squared, Gamma, Beta, log-normal, Weibull, Dirichlet, Wishart, among others, share the property of being log-concave. Then, their corresponding quantile function can be computed off-line for a given risk acceptability level and used within the MPC convex optimisation.* $\diamondsuit$

Conservatism

The approach presented above to derive deterministic equivalents of joint chance constraints gives a conservative approximation of the original stochastic problem. The impact of the conservatism on the quality of the solution is discussed below.

Consider again that the satisfaction of each individual constraint is an event C_i, $i \in \mathbb{Z}_{[1,n_c]}$. A joint chance constraint requires that the conjunction of all the individual constraints is satisfied with a desired probability level $1 - \delta_x$, i.e.,

$$\mathbb{P}\left[\bigcap_{i=1}^{n_c} C_i\right] \geq 1 - \delta_x. \tag{5.17}$$

Under the assumption that each individual constraint is probabilistically independent, the probability of the joint constraint, considering the uniform risk allocation policy, is given by

$$\begin{aligned}\mathbb{P}\left[\bigcap_{i=1}^{n_c} C_i\right] &= \prod_{i=1}^{n_c} \mathbb{P}\left[C_i\right] \\ &= \prod_{i=1}^{n_c} \left(1-\delta_{x,i}\right) \\ &= \left(1-\frac{\delta_x}{n_c}\right)^{n_c}. \end{aligned} \tag{5.18}$$

Taking into account that using Boole's inequality to upper bound the joint constraint leads to (5.12), (5.13) and (5.14), it follows that

$$\left(1-\frac{\delta_x}{n_c}\right)^{n_c} \geq (1-\delta_x) \Leftrightarrow \delta_x \geq 1-\left(1-\frac{\delta_x}{n_c}\right)^{n_c}. \tag{5.19}$$

In this way, the approximated conservatism $\tilde{\Delta}$ introduced by the CC-MPC approach with individual constraints presented in this chapter is given by

$$\tilde{\Delta} = \delta - \left(1-\left(1-\frac{\delta_x}{n_c}\right)^{n_c}\right). \tag{5.20}$$

Remark 5.3. *The level of conservatism, without assumptions on the independence of events, can be derived by using the inclusion-exclusion principle for the union of finite events, E_i, $\forall i \in \mathbb{Z}_{[1,n_c]}$, which asserts the following equality:*

$$\begin{aligned}\mathbb{P}\left[\bigcup_{i=1}^{n_c} E_i\right] &= \sum_{i=1}^{n_c} \mathbb{P}\left[E_i\right] \\ &\quad - \sum_{1\leq i<j\leq n_c} \mathbb{P}\left[E_i \cap E_j\right] \\ &\quad + \sum_{1\leq i<j<k\leq n_c} \mathbb{P}\left[E_i \cap E_j \cap E_k\right] \\ &\quad - \cdots + (-1)^{n_c-1}\,\mathbb{P}\left[\bigcap_{i=1}^{n_c} E_i\right]. \end{aligned} \tag{5.21}$$

Defining $E_i \coloneqq C_i^c$, and subtracting (5.21) from (5.10), it follows that the conservatism *is given by*

$$\Delta = \sum_{1\leq i<j\leq n_c} \mathbb{P}\left[C_i^c \cap C_j^c\right] - \sum_{1\leq i<j<k\leq n_c} \mathbb{P}\left[C_i^c \cap C_j^c \cap C_k^c\right] + \cdots - (-1)^{n_c-1}\,\mathbb{P}\left[\bigcap_{i=1}^{n_c} C_i^c\right]. \tag{5.22}$$

To evaluate the resulting expression requires the knowledge of the conditional probability of the events, which might be even impractical to obtain. Hence, (5.20) is used as an approximated indicator. ♢

5.4 Tree-based MPC

The deterministic equivalent CC-MPC proposed before might be still conservative if the probabilistic distributions of the stochastic variables are not well characterised or do not have a log-concave form. Therefore, this section presents the TB-MPC strategy that relies on scenario-trees to approximate the original problem, dropping Assumption 5.1. The approach followed by TB-MPC is based on modelling the possible scenarios of the disturbances as a rooted tree (see Figure 5.1 right). This means that all the scenarios start from the same measured disturbance value. From that point, the scenarios must remain equal until the point in which they diverge from each other, which is called a bifurcation point. Each node of the tree has a unique parent and can have many children. The total number of children at the last stage corresponds to the total number of scenarios. The probability of a scenario is the product of probabilities of each node in that scenario.

Notice that before a bifurcation point, the evolution followed by the disturbance cannot be anticipated because different evolutions are possible. For this reason, the controller has to calculate control actions that are valid for all the scenarios in the branch. Once the bifurcation point has been reached, the uncertainty is solved and the controller can calculate specific control actions for the scenarios in each of the new branches. Hence, the outcome of TB-MPC is not a single sequence of control actions, but a tree with the same structure of that of the disturbances. As in standard MPC, only the first element of this tree is applied (the root) and the problem is repeated in a receding horizon fashion.

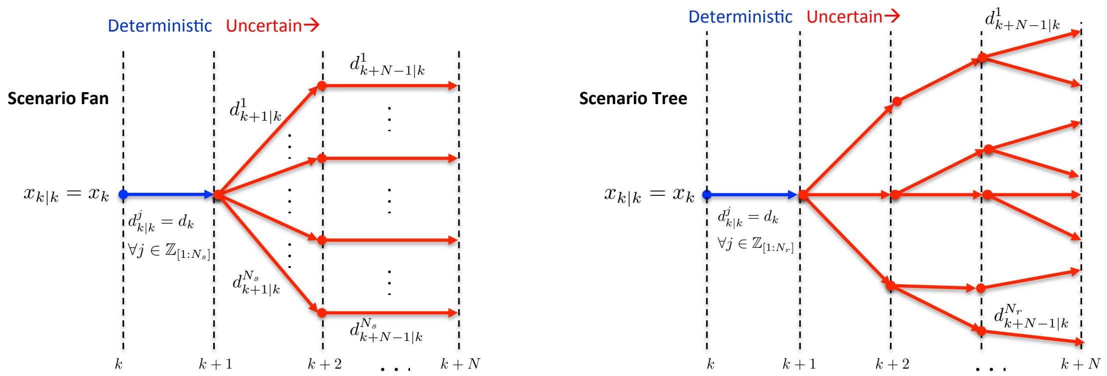


Figure 5.1: Reduction of a disturbance fan (left) of equally probable scenarios into a rooted scenario-tree (right).

In generalised flow-based networks the uncertainty is generally introduced by the unpredictable behaviour of consumers. Therefore, a proper demand modelling is required to achieve an acceptable supply service level. For the case study of this thesis, the reader is referred to [156], where the authors presented a detailed comparison of different forecasting models. Once a model is selected, it has to be calibrated and then used to generate a large number of possible demand scenarios by Monte Carlo sampling for a given prediction horizon $N \in \mathbb{Z}_{\geq 1}$. For the CC-MPC approach, the mean demand path is used, while for the TB-MPC approach a set of scenarios is selected. The size of this set is here computed following the bound proposed in [159], which takes into account the desired risk acceptability level. A large number of scenarios might improve the robustness of the TB-MPC approach but at the cost of additional computational burden and economic performance losses. Hence, a trade-off must be achieved between performance and computational burden. To this end, a representative subset of scenarios may be chosen using scenario reduction algorithms. In this paper, the backward reduction algorithm proposed in [79] is used to reduce a specified initial fan of $N_s \in \mathbb{Z}_{\geq 1}$ equally probable scenarios into a rooted tree of $N_r << N_s$ scenarios, see Figure 5.1

The easiest way to understand the optimisation problem that has to be solved in TB-MPC is to solve as many instances of Problem (5.4) as the number N_r of considered scenarios, but formally it is a multi-stage stochastic program and solved as a big optimisation for all the scenarios. Due to the increasing uncertainty, it is necessary to include non-anticipativity constraints [155] in the MPC formulation so that the calculated input sequence is always ready to face any possible future bifurcation in the tree. More specifically, if $\mathbf{d}_k^a = \{d_{k|k}^a, d_{k+1|k}^a, \dots, d_{k+N|k}^a\}$ and $\mathbf{d}_k^b = \{d_{k|k}^b, d_{k+1|k}^b, \dots, d_{k+N|k}^b\}$ are two disturbance sequences corresponding respectively to certain forecast scenarios $a, b \in \mathbb{Z}_{[1,N_r]}$, then the non-anticipativity constraint $\tilde{u}_{k+i|k}^a = \tilde{u}_{k+i|k}^b$ has to be satisfied for any $i \in \mathbb{Z}_{[0,N]}$ whenever $d_{k+i|k}^a = d_{k+i|k}^b$ in order to guarantee that for all $j \in \mathbb{Z}_{[1,N_r]}$ the input sequences $\tilde{\mathbf{u}}^j = \{\tilde{u}_{k+i|k}^j\}_{i\in\mathbb{Z}_{[0,N-1]}}$ form a tree with the same structure of that of the disturbances.

In this way, the TB-MPC controller has to solve the following optimisation problem at each time step $k \in \mathbb{Z}_+$, accounting for the N_r demand scenarios, each with probability $p_j \in (0, 1]$ satisfying $\sum_{j=1}^{N_r} p_j = 1$:

$$\min_{\tilde{\mathbf{u}}_k^j} \sum_{j=1}^{N_r} p_j \left(\sum_{i=0}^{N-1} \ell(k+i, x_{k+i|k}^j, \tilde{u}_{k+i|k}^j) \right), \tag{5.23a}$$

subject to:

$$x_{k+i+1|k}^j = Ax_{k+i|k}^j + \tilde{B}\tilde{u}_{k+i|k}^j + \tilde{B}_d d_{k+i}^j, \qquad \forall i \in \mathbb{Z}_{[0,N-1]}, \forall j \in \mathbb{Z}_{[1,N_r]}, \tag{5.23b}$$

$$x_{k+i+1|k}^j \in \mathbb{X}, \qquad \forall i \in \mathbb{Z}_{[0,N-1]}, \forall j \in \mathbb{Z}_{[1,N_r]}, \tag{5.23c}$$

$$\tilde{u}_{k+i|k}^j \in \tilde{\mathbb{U}}_{k+i}^j, \qquad \forall i \in \mathbb{Z}_{[0,N-1]}, \forall j \in \mathbb{Z}_{[1,N_r]}, \tag{5.23d}$$

$$x_{k|k}^j = x_k, \quad d_{k|k}^j = d_k, \qquad \forall j \in \mathbb{Z}_{[1,N_r]}, \tag{5.23e}$$

$$\tilde{u}_{k+i|k}^a = \tilde{u}_{k+i|k}^b \ \text{ if } \ d_{k+i|k}^a = d_{k+i|k}^b, \qquad \forall i \in \mathbb{Z}_{[0,N-1]}, \forall a, b \in \mathbb{Z}_{[1,N_r]}. \tag{5.23f}$$

where $\tilde{\mathbb{U}}_{k+i}^j := \{\tilde{u}^j \in \mathbb{R}^{m-q} \,|\, H\tilde{P}\tilde{M}_1\tilde{u}^j \leq h - H\tilde{P}\tilde{M}_2 d_{k+i}^j\}$.

Remark 5.4. *The number of scenarios used to build the rooted tree should be determined regarding the computational capacity and the probability of risk that the manager is willing to accept.* $\diamondsuit$

5.5 Numerical Results

Performance Comparison on a Small-Scale System

This section discusses the results of applying the deterministic equivalent CC-MPC and the TB-MPC to the Barcelona DWN case study, described previously in § 2.4. The DWN is considered as a stochastic constrained system subject to deterministic hard constraints on the control inputs and linear joint chance constraints on the states. The source of uncertainty in the system is assumed to be the forecasting error of the water demands. Therefore, regular forecasting of a vast number of univariate time series is an essential task to design the proposed controllers for the operational management of the DWN. The water demand characterisation, modelling and forecasting method for the selected case study is detailed in Appendix A.

The stochastic control problem of the DWN is stated as follows:

$$\min_{\tilde{\mathbf{u}}_k} \sum_{i=0}^{N-1} \mathbb{E}\left[\gamma_1 \ell_E(k+i, x_{k+i|k}, \tilde{u}_{k+i|k}) + \gamma_2 \ell_\Delta(\Delta\tilde{u}_{k+i|k})\right], \tag{5.24a}$$

subject to:

$$x_{k+i+1|k} = Ax_{k+i|k} + \tilde{B}\tilde{u}_{k+i|k} + \tilde{B}_d d_{k+i|k}, \tag{5.24b}$$

$$\mathbb{P}\left[x_{k+i+1|k} \geq x_{\min}\right] \geq 1 - \frac{\delta_x}{2}, \tag{5.24c}$$

$$\mathbb{P}\left[x_{k+i+1|k} \leq x_{\max}\right] \geq 1 - \frac{\delta_x}{2}, \tag{5.24d}$$

$$\mathbb{P}\left[x_{k+i+1|k} \geq d_{\text{net},k+i+1|k}\right] \geq 1 - \delta_s, \tag{5.24e}$$

$$\tilde{u}_{k+i|k} \in \tilde{\mathbb{U}}_{k+i}, \tag{5.24f}$$

$$d_{\text{net},k+i+1|k} = -(\tilde{B}_{\text{out}}(\tilde{P}\tilde{M}_1\tilde{u}_{k+i|k} + \tilde{P}\tilde{M}_2 d_{k+i|k}) + \tilde{B}_d d_{k+i+1|k}), \tag{5.24g}$$

$$\Delta\tilde{u}_{k+i|k} = \tilde{u}_{k+i|k} - \tilde{u}_{k+i-1|k}, \tag{5.24h}$$

$$(x_{k|k}, \tilde{u}_{k-1|k}) = (x_k, \tilde{u}_{k-1}), \tag{5.24i}$$

for all $i \in \mathbb{Z}_{[0,N-1]}$, where $\ell_E(k+i, x_{k+i|k}, \tilde{u}_{k+i|k}) := c_{u,k+i}^\top W_e\, \tilde{u}_k \Delta t$ captures the process economics with $c_{u,k+i} \in \mathbb{R}^m$ being a known periodically time-varying price of electric tariff, and $\ell_\Delta(\Delta\tilde{u}_{k+i|k}) := \|\tilde{P}\tilde{M}_1\Delta\tilde{u}_{k+i|k} + \tilde{P}\tilde{M}_2\Delta d_{k+i|k}\|^2_{W_{\Delta\tilde{u}}}$ is a control move suppression term aiming to enforce a smooth operation. Moreover, $\delta_x, \delta_s \in (0,1)$ are the accepted maximum risk levels for the state constraints and the safety constraint (5.24e), respectively. The objectives are traded-off with the scalar weights $\gamma_1, \gamma_2 \in \mathbb{R}_+$, while the elements of the decision vector are prioritised by the weighting matrices $W_e, W_{\Delta\tilde{u}} \in \mathbb{S}^m_{++}$. The service reliability goal (i.e., demand satisfaction) is enforced by the constraints (5.24e) and (5.24g). In this latter constraint, $d_{\text{net},k+i+1|k} \in \mathbb{R}^n$ is a vector of net demands above which is desired to keep the reservoirs to avoid stock-outs. The $\tilde{B}_{\text{out}}(\tilde{P}\tilde{M}_1\tilde{u}_{k+i|k} + \tilde{P}\tilde{M}_2 d_{k+i|k})$ component represents the current prediction step endogenous demand, i.e., the outflow of the tanks caused by water requirements from neighbouring tanks or nodes, and the $\tilde{B}_d d_{k+i+1|k}$ component denotes the exogenous (customer) demands of tanks for the next prediction step.

The open-loop feed-forward uncertainty in the DWN can be modelled by the relationship between predicted states and predicted disturbances, see (C.2) and (C.3). In the dynamic model (5.2) of the DWN, randomness is directly described by the uncertainty of customer demands, which can be estimated from historical data. Figure A.1 shows the histogram of a specific water demand node in the Barcelona DWN for the same time step in different days during year 2007. It can be seen in the envelope of the histogram, that the uncertain demand obeys a probabilistic distribution close to a Gaussian distribution (red curve). This behaviour is shared by the rest of the demand nodes of the network. Hence, Assumption 2.8 holds and a tractable safe approximation of (5.24) can be derived following § 5.3. In this way, the joint chance constraints (5.24c)–(5.24e) are transformed into deterministic equivalent constraints as shown in Appendix C for the particular case of Gaussian distributions.

The optimisation problem associated with the deterministic equivalent CC-MPC for the selected application is stated as follows for a given sequence $\bar{\mathbf{d}}_k = \{\bar{d}_{k+i|k}\}_{i\in\mathbb{Z}_{[0,N-1]}}$ of forecasted demands:

$$\min_{\tilde{\mathbf{u}}_k,\boldsymbol{\xi}_k} \sum_{i=0}^{N-1} \mathbb{E}\left[\gamma_1\ell_E(k+i,\bar{x}_{k+i|k},\tilde{u}_{k+i|k}) + \gamma_2\ell_\Delta(\Delta\tilde{u}_{k+i|k}) + \gamma_3\ell_S(\xi_{k+i|k})\right], \tag{5.25a}$$

subject to:

$$\bar{x}_{k+i+1|k} = A\bar{x}_{k+i|k} + \tilde{B}\tilde{u}_{k+i|k} + \tilde{B}_d\bar{d}_{k+i|k}, \tag{5.25b}$$
$$\bar{x}_{(j),k+i+1|k} \geq x_{\min(j)} + \Phi^{-1}\left(1-\frac{\delta_x}{2nN}\right)\Sigma^{1/2}_{x_{(j)},k+i+1|k}, \tag{5.25c}$$
$$\bar{x}_{(j),k+i+1|k} \leq x_{\max(j)} - \Phi^{-1}\left(1-\frac{\delta_x}{2nN}\right)\Sigma^{1/2}_{x_{(j)},k+i+1|k}, \tag{5.25d}$$
$$\bar{x}_{(j),k+i+1|k} \geq \bar{d}_{\mathrm{net}(j),k+i+1|k} + \Phi^{-1}\left(1-\frac{\delta_s}{nN}\right)\Sigma^{1/2}_{d_{\mathrm{net}(j)},k+i+1|k} - \xi_{(j),k+i|k}, \tag{5.25e}$$
$$\xi_{k+i|k} \geq 0, \tag{5.25f}$$
$$\tilde{u}_{k+i|k} \in \tilde{\mathbb{U}}_{k+i}, \tag{5.25g}$$
$$\bar{d}_{\mathrm{net},k+i+1|k} = -(\tilde{B}_{\mathrm{out}}\left(\tilde{P}\tilde{M}_1\tilde{u}_{k+i|k} + \tilde{P}\tilde{M}_2\bar{d}_{k+i|k}\right) + \tilde{B}_d\bar{d}_{k+i+1|k}), \tag{5.25h}$$
$$\Delta\tilde{u}_{k+i|k} = \tilde{u}_{k+i|k} - \tilde{u}_{k+i-1|k}, \tag{5.25i}$$
$$(\bar{x}_{k|k},\tilde{u}_{k-1|k}) = (x_k,\tilde{u}_{k-1}), \tag{5.25j}$$

for all $i \in \mathbb{Z}_{[0,N-1]}$ and all $j \in \mathbb{Z}_{[1,n]}$, where $\tilde{\mathbf{u}}_k = \{\tilde{u}_{k+i|k}\}$ and $\boldsymbol{\xi}_k = \{\xi_{k+i|k}\}$ are the decision variables. The vectors $\bar{x}$ and $\bar{d}$ denote the mean of the random state and demand variables, respectively. Moreover, Φ^{-1} is the left-quantile function of the Gaussian distribution, and $\bar{x}_{(j)}$ and $\Sigma_{x_{(j)}}$ denote respectively the mean and variance of the j-th row of the state vector, which are obtained as described in Appendix A. Notice that Problem (5.25) includes the additional objective $\ell_S(\xi_{k+i|k}) := \|\xi_{k+i|k}\|^2_{W_s}$ with $W_s \in \mathbb{S}^n_{++}$, and the additional constraint (5.25f), which are related to the safety operational goal. These elements appear due to the safety deterministic equivalent soft constraint (5.25e) introduced with the slack decision variable $\xi \in \mathbb{R}^n$ to allow the trade-off between safety, economic and smoothness objectives. Constraints (5.25c) and (5.25d) can be softened in the same way to guarantee recursive feasibility of the optimisation problem if uncertainty is too large. For a strongly feasible stochastic MPC approach using closed-loop predictions by means of an affine disturbance parametrisation of the control inputs, the reader is referred to [96].

The enforcement of the chance constraints enhances the robustness of the MPC controller by causing an optimal back-off from the nominal deterministic constraints as a risk averse mechanism to face the non-stationary uncertainty involved in the prediction model of the MPC. The states are forced to move away from their limits before the disturbances have chance to cause constraint violation. The $\Phi^{-1}(\cdot)$ terms represent safety factors for each constraint, and specially in (5.25e), it denotes the optimal safety stock of storage tanks.

Problem (5.16) may be casted as a second-order cone programming problem. However, state uncertainty is a function of the disturbances only and is not a function of the decision variables of the optimisation problem. Therefore, the variance terms in each deterministic equivalent can be forecasted prior to the solution of the optimisation problem to include them as known parameters in the MPC formulation. This simplification results in a set of linear constraints and the optimisation remains as a quadratic programming (QP) problem, which can be efficiently solved.

The optimisation problem associated with the scenario tree-based MPC approach is stated as follows for all $i \in \mathbb{Z}_{[0,N-1]}$ and all $j \in \mathbb{Z}_{[1,N_r]}$:

$$\min_{\tilde{\mathbf{u}}_k^j, \boldsymbol{\xi}_k^j} \sum_{j=1}^{N_r} p_j \sum_{i=0}^{N-1} \gamma_1 \ell_E(k+i, x_{k+i|k}^j, \tilde{u}_{k+i|k}^j) + \gamma_2 \ell_\Delta(\Delta \tilde{u}_{k+i|k}^j) + \gamma_3 \ell_S(\xi_{k+i|k}^j), \tag{5.26a}$$

subject to:

$$x_{k+i+1|k}^j = A x_{k+i|k}^j + \tilde{B} \tilde{u}_{k+i|k}^j + \tilde{B}_d d_{k+i|k}^j, \tag{5.26b}$$
$$(x_{k+i+1|k}^j, \tilde{u}_{k+i|k}^j, \xi_{k+i|k}^j) \in \mathbb{X} \times \tilde{\mathbb{U}}_{k+i}^j \times \mathbb{R}_+^n, \tag{5.26c}$$
$$x_{k+i+1|k}^j \geq d_{\mathrm{net},k+i+1|k}^j - \xi_{k+i|k}^j, \tag{5.26d}$$
$$d_{\mathrm{net},k+i+1|k}^j = -(\tilde{B}_{\mathrm{out}} \left(\tilde{P} \tilde{M}_1 \tilde{u}_{k+i|k}^j + \tilde{P} \tilde{M}_2 d_{k+i|k}^j \right) + \tilde{B}_d d_{k+i+1|k}^j), \tag{5.26e}$$
$$\Delta \tilde{u}_{k+i|k}^j = \tilde{u}_{k+i|k}^j - \tilde{u}_{k+i-1|k}^j, \tag{5.26f}$$
$$(x_{k|k}^j, \tilde{u}_{k-1|k}^j, d_{k|k}^j) = (x_k, \tilde{u}_{k-1}, d_k), \tag{5.26g}$$
$$\tilde{u}_{k+i|k}^a = \tilde{u}_{k+i|k}^b \ \text{ if } \ d_{k+i|k}^a = d_{k+i|k}^b \quad \forall a, b \in \mathbb{Z}_{[1,N_r]}. \tag{5.26h}$$

Table 5.1 summarises the results of applying the deterministic equivalent CC-MPC and the TB-MPC to the *sector model* (see Figure 2.5) of the Barcelona drinking water network (DWN) described in § 2.4. The formulation of the optimisation problems and the closed-loop simulations have been carried out using YALMIP Toolbox, CPLEX solver and Matlab R2012b (64 bits), running in a PC Intel Core E8600 at 3.33GHz with 8GB of RAM. Simulations have been carried out over a time period of eight days, i.e., $n_s = 192$ hours, with a sampling time of one hour. Applied demand scenarios were taken from historical data of the Barcelona DWN. The weights of the multi-objective cost function are $\gamma_1 = 100$, $\gamma_2 = 1$, and $\gamma_3 = 10$. The prediction horizon is selected as $N = 24$ hours due to the periodicity of demands. The key performance indicators used to assess the aforementioned controllers are defined as follows:

$$\mathrm{KPI}_1 \triangleq \frac{24}{n_s + 1} \sum_{k=0}^{n_s} \gamma_1 \ell_E(k, x_k, \tilde{u}_k) + \gamma_2 \ell_\Delta(\Delta \tilde{u}_k) + \gamma_3 \ell_S(\xi_k), \tag{5.27a}$$
$$\mathrm{KPI}_2 \triangleq | \left\{ k \in \mathbb{Z}_1^{n_s} \mid x_k < -B_p d_k \right\} |, \tag{5.27b}$$
$$\mathrm{KPI}_3 \triangleq \sum_{k=1}^{n_s} \sum_{i=1}^{n_x} \max\{0, -B_{p(i)} d_k - x_{k(i)}\}, \tag{5.27c}$$
$$\mathrm{KPI}_4 \triangleq \frac{1}{n_s} \sum_{k=1}^{n_s} t_k, \tag{5.27d}$$

Table 5.1: Assessment of CC-MPC and TB-MPC applied to the sector model of the DWN case study.

CC-MPC					TB-MPC					
δ_x	KPI_1	KPI_2	KPI_3	KPI_4	KPI_1	KPI_2	KPI_3	KPI_4	N_r	N_s
					58397.14	0	0	0.94	5	
0.3	58535.80	0	0	1.25	58280.69	1	0.51	1.61	10	19
					58279.95	1	4.16	2.37	14	
					58482.14	3	0.18	1.18	7	
0.2	58541.19	0	0	1.21	58903.63	0	0	2.33	14	29
					58452.41	0	0	4.05	21	
					58610.32	0	0	2.57	14	
0.1	58558.29	0	0	1.25	58630.20	0	0	6.65	29	59
					58656.56	1	0.18	13.47	44	
					-	-	-	-	149	
0.01	58612.28	0	0	1.25	-	-	-	-	299	599
					-	-	-	-	449	
					-	-	-	-	1499	
0.001	58667.85	0	0	1.25	-	-	-	-	2999	5999
					-	-	-	-	4499	

where KPI_1 is the average daily multi-objective cost, KPI_2 is the number of time steps where the stored water goes below the demanded volume (for this, $|\cdot|$ denotes the cardinal of a set of elements), KPI_3 is the accumulated volume of water demand that was not satisfied over the simulation horizon, and KPI_4 is the average time in seconds required to solve the MPC problem at each time step $k \in \mathbb{Z}_{[0,n_s]}$. For the CC-MPC approach, the effect of considering different levels of joint risk acceptability was analysed using $\delta_x \in \{0.3, 0.2, 0.1, 0.01, 0.001\}$ and $\delta_s = \delta_x$. Regarding the TB-MPC approach, different sizes for the initial set of scenarios were considered, i.e., $N_s \in \{19, 28, 59, 599, 5999\}$. The size of this initial set was computed following the bound proposed in [159] taking into account the risk levels involved in the chance constraints. This initial set was reduced later by a factor of 0.25, 0.50, and 0.75 to obtain different rooted trees with N_r scenarios.

As shown in Table 5.1, the different CC-MPC scenarios highlight that reliability and control performance are conflicting objectives, i.e., the inclusion of safety mechanisms in the controller increases the reliability of the DWN in terms of demand satisfaction, but also the cost of its operation. The main advantage of the CC-MPC is its formal methodology, which leads to obtain optimal safety constraints that tackle uncertainties and allow to achieve a specified global service level in the DWN. Moreover, the robustness of the deterministic equivalent CC-MPC approach is achieved with a low computational burden given that the only extra load (comparing with a nominal formulation) is the computation of the stochastic characteristics of disturbances propagated in the prediction horizon. In this way, the deterministic equivalent CC-MPC approach is suitable for real-time control (RTC) of large-scale DWNs.

Regarding the TB-MPC approach, numeric results in Table 5.1 show that considering higher N_s increments the stage cost while reducing the volume of unsatisfied water demand. Nevertheless, this latter observation is not applicable for the different N_r cases within a same N_s. This might be influenced by the quality of the information that remains after the scenario generation and reduction algorithms that affect the robustness of the approach and will be subject of further research. The main drawback of the TB-MPC approach is the solution average time and the computational burden. In this case study, the implementation for all cases taking $N_s \in \{599, 5999\}$ was not possible due to memory issues. Hence, some simplification assumptions as those used in [107] or parallel computing techniques might be useful.

Performance Assessment of CC-MPC on a Large-Scale System

Previous results showed that both CC-MPC and TB-MPC have similar performance under high levels of risk acceptability. Nevertheless, when requiring small risk levels ($\delta_x < 0.1$), CC-MPC retains tractability of the FHOP with low complexity, while the TB-MPC suffers the curse of dimensionality. Therefore, in the following only the performance of the CC-MPC approach is assessed on the *full model* of the Barcelona DWN (see Figure 2.3). The tuning of the controller parameters is the same as in the previous simulations.

In order to further evaluate the proposed CC-MPC scheme, results are compared with the baseline CE-MPC approach discussed in § 2.4.3, which assumes predictions of demands as certain. In these simulations, the CE-MPC strategy has been set up to allow the volume of water in tanks to decrease until the predicted volume of future net demands, which is set as a hard constraint but ignoring the influence of uncertainty. Contrary, the CC-MPC strategy considers and propagates the uncertainty of forecasted demands explicitly in the MPC design and, as a consequence, involves a robust handling of constraints. Again, to analyse the effect of the risk level (δ_x) in this CC-MPC strategy when considering large-scale systems, different scenarios have been simulated for acceptable joint risks of 50%, 40%, 30%, 20%, 10%, 5% and 1%. Table 5.2 summarises the numeric results of the aforementioned controllers through different key performance indicators (KPIs), which are defined below:

$$\mathrm{KPI}_E \coloneqq \frac{1}{n_s+1}\sum_{k=0}^{n_s} c_{u,k}^{\top}\tilde{u}_k\Delta t, \tag{5.28a}$$

$$\mathrm{KPI}_{\Delta U} \coloneqq \frac{1}{n_s+1}\sum_{i=1}^{m}\sum_{k=0}^{n_s}\left(\Delta\tilde{u}_{(i),k}\right)^2, \tag{5.28b}$$

$$\mathrm{KPI}_S \coloneqq \sum_{i=1}^{n}\sum_{k=0}^{n_s}\max\left\{0, s_{(i),k}-x_{(i),k}\right\}, \tag{5.28c}$$

$$\mathrm{KPI}_D \coloneqq \sum_{i=1}^{n}\sum_{k=0}^{n_s}\max\left\{0, d_{\mathrm{net}(i),k}-x_{(i),k}\right\}, \tag{5.28d}$$

$$\mathrm{KPI}_R \coloneqq \frac{\sum_{i=1}^{n}\sum_{k=1}^{n_s} s_{(i),k}}{\sum_{i=1}^{n}\sum_{k=1}^{n_s} x_{(i),k}}\times 100\%, \tag{5.28e}$$

$$\mathrm{KPI}_O \coloneqq t_{\mathrm{opt},k}, \tag{5.28f}$$

where KPI_E is the average economic performance of the DWN operation, $\mathrm{KPI}_{\Delta U}$ measures the smoothness of the control actions, KPI_S is the amount of water used from safety stocks, KPI_D is the volume of water demand that is not satisfied over the simulation period, KPI_R is the average percentage of safety volume that is contained in the real water volume, and KPI_O determines the difficulty to solve the optimisation tasks involved in each strategy by accounting $t_{\mathrm{opt},k}$ as the average time that takes to solve the corresponding MPC optimisation problem. The CE-MPC controller has been tuned with a safety stock for each tank equal to its net exogenous demand, i.e., $s_k = d_{\mathrm{net},k}$. Therefore, the KPI_S results to be equal to the KPI_D as should be expected given their definitions. In the case of the CC-MPC controller, s_k has been set equal to the right hand of (5.25e). The formulation of the optimisation problems and the closed-loop simulations have been carried out using YALMIP Toolbox, CPLEX solver and Matlab R2012b (64 bits), running in a PC Intel Core E8600 at 3.33GHz with 8GB of RAM.

Regarding the comparison of the KPI_S between the CE-MPC and the CC-MPC controllers, the results in Table 5.2 present greater values for the CC-MPC cases. This

Table 5.2: Comparison of the MPC strategies applied to the Barcelona DWN

Controller	KPI_E (e.u.)	$\text{KPI}_{\Delta U}$ $(\text{m}^3/\text{s})^2$	KPI_S (m^3)	KPI_D (m^3)	KPI_R (%)	KPI_O (s)
CE-MPC	2297.02	2.3586	3.8886	3.8886	19.41	4.82
$\text{CC-MPC}_{@50\%}$	2486.40	1.0747	695.54	0	27.79	4.72
$\text{CC-MPC}_{@40\%}$	2487.77	1.0767	750.06	0	27.86	4.83
$\text{CC-MPC}_{@30\%}$	2489.31	1.0795	819.82	0	27.95	4.79
$\text{CC-MPC}_{@20\%}$	2491.61	1.0835	920.36	0	28.07	4.71
$\text{CC-MPC}_{@10\%}$	2496.23	1.0964	1101.7	0	28.18	4.70
$\text{CC-MPC}_{@5\%}$	2500.52	1.1012	1298.9	0	28.18	4.89
$\text{CC-MPC}_{@1\%}$	2509.89	1.1131	1759.4	0	28.43	4.86

e.u.: economic units

trend is also an expected behaviour given that reducing the risk probability generates a larger back-off of the demand satisfaction constraint, i.e., more safety stock is stored to address demand uncertainty. This latter fact, in addition with the tuning of the multi-objective cost function, leads to higher KPI_S (but lower or null KPI_D) if this is required by the real demand scenario in order to guarantee a service level. It can be also observed that CE-MPC is the cheapest control strategy (lower KPI_E) but the less reliable one given that the certainty equivalence assumption leads to unsatisfied demands (higher KPI_D), especially when the water volume in the tank is close to the expected demand. Thus, the CE-MPC controller has a higher risk of failure for the supply of drinking water.

The different CC-MPC scenarios (those of varying the risk acceptability level) have shown that reliability and economic performance are conflicting objectives that have to reach a trade-off, i.e., the inclusion of safety mechanisms in the controller increases the reliability of the DWN in terms of demand satisfaction (see Figure 5.2), but also the economic cost of its operation. The main advantage of the CC-MPC is its formal methodology that leads to obtain optimal dynamic constraints that tackle uncertainties with a minimum cost to achieve also a global service level of the DWN. Table 5.2 shows a smooth degradation of the economic performance under the CC-MPC when varying the risk within a wide range of acceptability levels. Therefore, the CC-MPC approach addressed in this chapter is a suitable mean to compute the proper amount of safety and

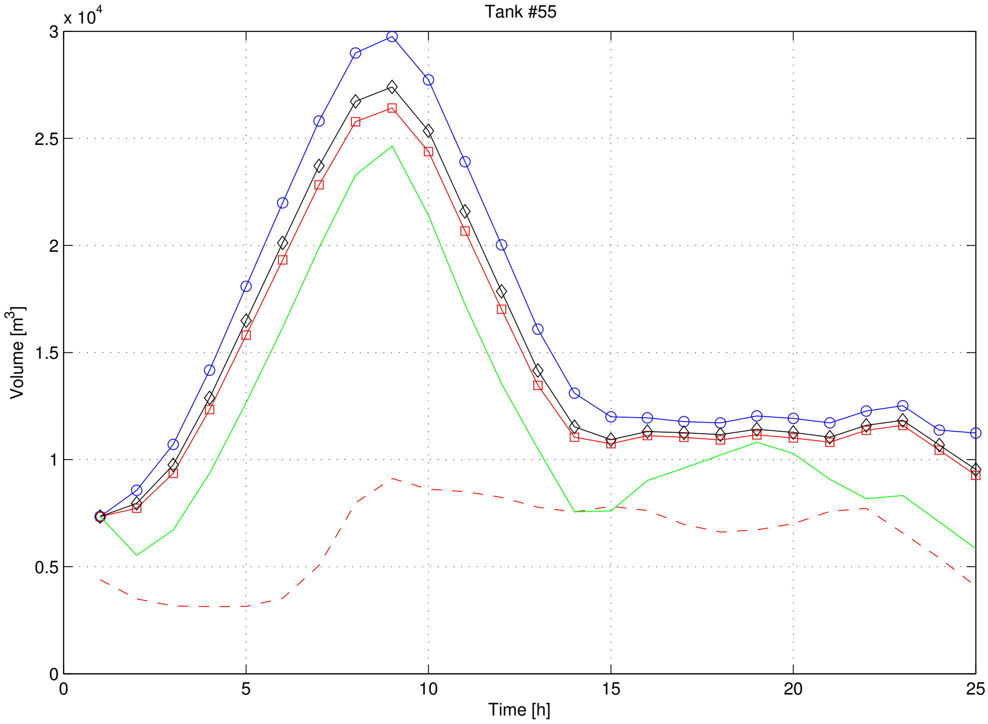


Figure 5.2: Comparison of the robustness in the management of water storage in a sample of tanks of the Barcelona DWN: (blue circle) CC-MPC$_{1\%}$, (black diamond) CC-MPC$_{20\%}$, (red square) CC-MPC$_{50\%}$, (solid green) CE-MPC, (dashed red) Net demand.

the proper control actions to assure a desired service level. Notice that the computational burden (KPI$_O$) of the CC-MPC is similar to the CE-MPC given that the complexity of the optimisation problem is not altered, i.e., the number of constraints and decision variables remain the same. The only extra load that might be added is the computation of the variance of the disturbances propagated in the prediction horizon. Consequently, the CC-MPC approach is suitable for RTC of the Barcelona DWN.

Table 5.3 discloses details of the average production and operational costs related to each strategy. Comparing the CE-MPC controller with the CC-MPC$_{@5\%}$ controller (requiring a reliability of 95%), it can be noticed that the dynamic safety stocks resulting within the stochastic approach might lead to an increase of the operational cost, especially in the electric cost, mainly due to the extra amount of water that is needed to be moved through the network and allocated in tanks to guarantee that the water supply will be feasible with a certain probability for future disturbance realisations.

Table 5.3: Comparison of daily average economic costs of MPC strategies

Controller	Water Cost (e.u./day)	Electric Cost (e.u./day)	Daily Average Cost (e.u./day)
CE-MPC	23015.42	27195.31	50210.73
CC-MPC$_{@5\%}$	22980.34	28514.71	51495.05

e.u.: economic units

In the sequel the CC-MPC approach is further simulated for a different initial condition and compared under two different settings to assess the effect of actuators health. The MPC settings are:

- CC-MPC$_{(1)}$: It is based in the optimisation problem (5.25), i.e., it considers only the uncertainty related to exogenous water demands.
- CC-MPC$_{(2)}$: It incorporates additionally the actuator health management policy described in § 4.3, i.e., it considers both the uncertainty of forecast demands and the uncertainty of the actuator degradation model. In this setting, the optimisation problem (5.25) is modified by including constraint (4.10c) and a probabilistic form of constraint (4.10i) with an associated risk level denoted as $\delta_z \in (0,1)$. This latter chance constraint is approximated following § 5.3, what gives a deterministic equivalent constraint $z_{(r),k+N|k} \leq z_{\max(r),k} - \Phi^{-1}\left(1 - \frac{\delta_z}{mN}\right) \|\Sigma_\eta^{1/2} I_{(r)}^\top\|_2$ for all $r \in \mathbb{Z}_{[1,m]}$, where $\eta \in \mathbb{R}^{n_u}$ is a random vector of noise incorporated in the actuator health degradation process defined in § 4.3, whose components lie in a normal distribution $\mathcal{N}(0, \Sigma_\eta)$. Recall that the actuator degradation management policy also adapts on-line the weight $W_{\Delta u}$ in the cost function according to the reliability of the actuators.

Table 5.4 summarises the performance of each CC-MPC strategy according to economic, safety and smoothness indicators (see § 2.4.4). As expected, the CC-MPC$_{(2)}$ has a higher KPI$_E$ than the CC-MPC$_{(1)}$. The reason of this result is the same as in the RB-MPC$_{(2)}$, i.e., the inclusion of actuators degradation constraints leads to control actions that sacrifice (if necessary) economic performance in order to guarantee the availability of actuators for a given maintenance horizon. The rationing of actuators degradation also leads to increase the smoothness (KPI$_{\Delta U}$), especially due to the operation of pumps

Table 5.4: Key performance indicators for CC-MPC

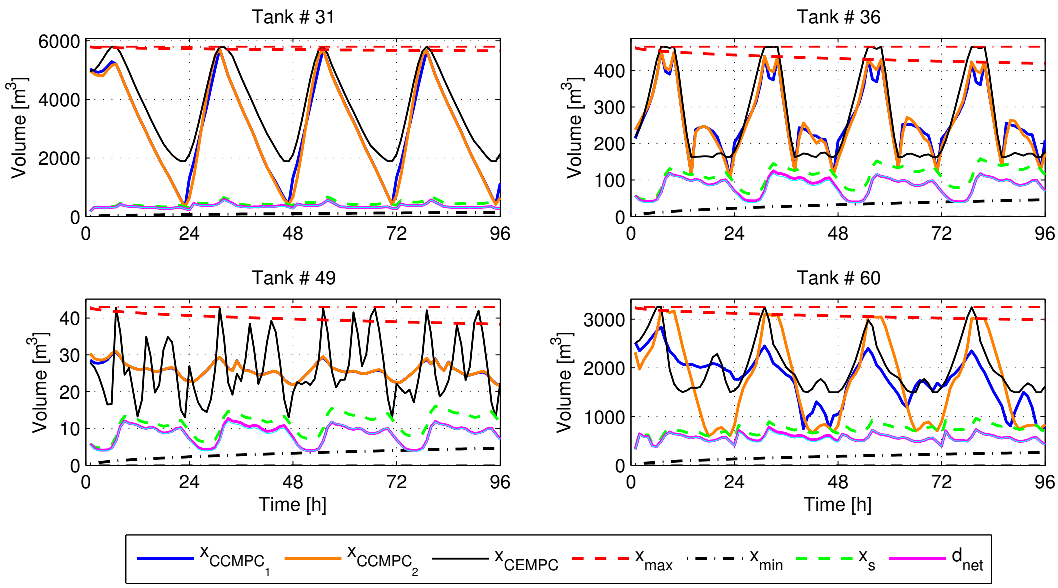
Controller	KPI_E (e.u.)	KPI_S (m^3)	$\mathrm{KPI}_{\Delta U}$ $(\mathrm{m}^3/\mathrm{s})^2$
CC-MPC$_{(1)}$	2390.57	9421.46	1.0223
CC-MPC$_{(2)}$	2761.48	3364.82	2.8664

e.u.: economic units

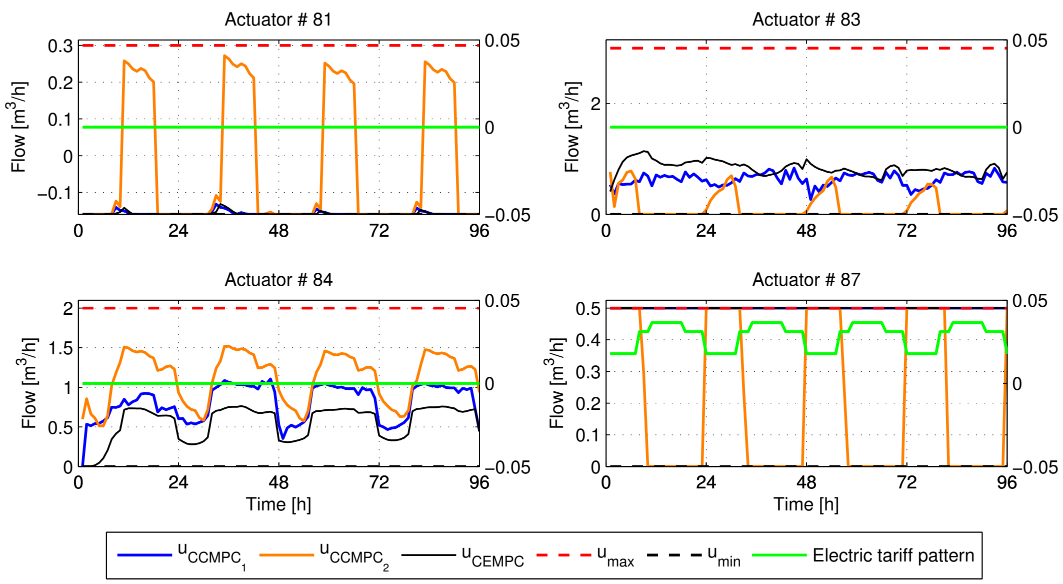
associated with Tank 55 to Tank 63 in the bottom-right part of the DWN diagram (see Figure 2.3). With the CC-MPC$_{(1)}$ the volume of water in the aforementioned tanks are managed near the safety constraints without complete replenishments, while with the CC-MPC$_{(2)}$ the excursion of water is cyclic within the full range of operation. The actuators health management policy forces to cycle the operation of several pumps instead of keeping some of them always active, and therefore require to exploit the full capacity of the related tanks. The safety performance indicator (KPI_S) is drastically higher in the CC-MPC$_{(1)}$; the reason is that the volume in tanks tends to remain more time in the limit of constraints, what leads to increase the frequency of violation of safety thresholds. Figure 5.3 illustrates the mentioned behaviour of the system. In general, chance constraints cause an optimal back-off from real constraints as a risk averse mechanism to face the non-stationary uncertainty involved in the prediction of states.

Figure 5.4 shows the accumulated degradation of a set of redundant actuators. Notice how the CC-MPC$_{(2)}$ smartly decides to decrease the rate of degradation of Actuator 87 (a pump) by distributing the control effort between the other three plotted actuators (which are valves that have lower coefficients of degradation) according to their flow capacity. This behaviour is equivalent to the one obtained with the RB-MPC$_{(2)}$ in Chapter 4, the difference is that the chance constrained approach narrows the maximum level of degradation allowed at each time step according to the uncertainty in the health prediction model of actuators. The wear process with the CC-MPC$_{(1)}$ is neglected, compromising the reliability of the supply infrastructure even if safety stocks are optimally computed for a reliable service. This latter observation also applies for the RB-MPC$_{(1)}$ and the CE-MPC approaches discussed in previous chapters.

Table 5.5 details the water production and electricity costs of each CC-MPC strategy. The CC-MPC$_{(1)}$ has very similar costs to those of the baseline CE-MPC, but with the benefit of a better handling of constraints, automatic computation of safety stocks and



(a) Management of water storage with the CC-MPC strategies



(b) Management of actuators with the CC-MPC strategies

Figure 5.3: Operation of the Barcelona DWN with the CC-MPC strategies

management of risk near to the output bounds. On the other hand, the CC-MPC$_{(2)}$ achieves a notorious improvement in electric costs but at the expense of increasing stored volumes of water (no matter the expensive the source could be) and consequently water

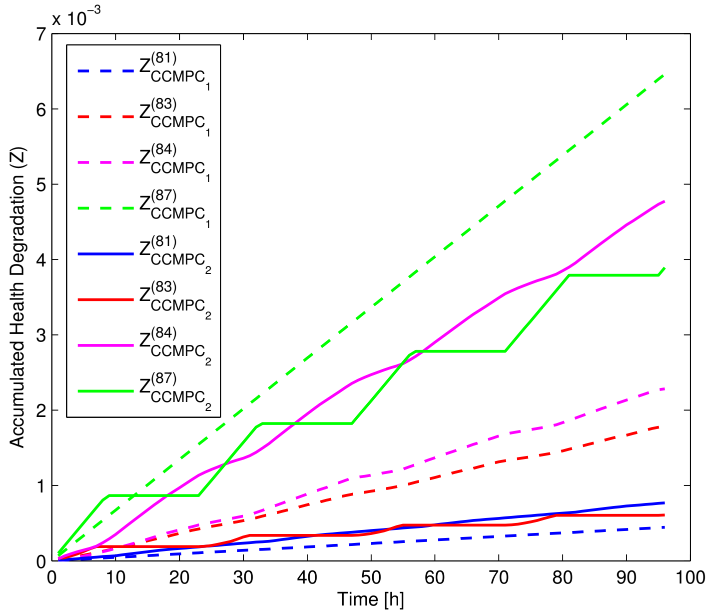


Figure 5.4: Degradation of a set of redundant actuators under the CC-MPC strategies

costs. Nevertheless, the full approach has the capability to manage actuators health, and of course, such enhancement implies a higher total cost (if based only on water and electric prices without considering corrective maintenance costs that probably will appear if the degradation management is not active to guarantee actuator availability).

The conservatism of reformulating the pure stochastic MPC into the tractable deterministic equivalent CC-MPC is shown in Table 5.6 for the respective chance constraints associated with the state bounds, safety levels and degradation management, with different levels of maximum joint risk. Notice that the conservatism increases when the risk level increases but remains constant despite the variation of the number of individual constraints. Hence, the goodness of the approximation using the Boole's inequality is not affected; neither by the number of decision variables, nor by the prediction horizon.

An important aspect in any MPC controller is the handling of constraints. In the Barcelona DWN, manipulated variables can always be kept within bounds by the controller, but output constraints, which are subject to measured and/or unmeasured uncertainties, must be controlled in advance. Neither the baseline CE-MPC approach described in Chapter 2 nor the economic MPC approaches in Chapter 3 consider un-

Table 5.5: Water and electric cost comparison of CC-MPC strategies

	MPC Approach	Water Cost (e.u.)	Electric Cost (e.u.)	Total Cost (e.u.)
Day 1	CC-MPC$_{(1)}$	21937.42	29074.87	51012.30
	CC-MPC$_{(2)}$	45248.32	12176.02	57424.34
Day 2	CC-MPC$_{(1)}$	31300.76	29869.56	61170.32
	CC-MPC$_{(2)}$	58707.65	13380.02	72087.68
Day 3	CC-MPC$_{(1)}$	28958.00	29783.34	58741.34
	CC-MPC$_{(2)}$	55131.50	13227.99	68359.50
Day 4	CC-MPC$_{(1)}$	28630.73	29939.65	58570.38
	CC-MPC$_{(2)}$	53629.68	13600.91	67230.60

e.u.: economic units

certainty explicitly in the controller and might require on-line tuning to ensure an appropriate robust performance, as addressed with the learning-based tuning strategy in Chapter 6. The other MPC strategies developed so far in this thesis, i.e., RB-MPC, CC-MPC and TB-MPC, focus on robust performance of the DWN, where flow demand constitutes the main source of uncertainty. Both the RB-MPC and the CC-MPC strategies enhance the robustness of the baseline CE-MPC by performing a dynamic handling of constraints while keeping tractability of the optimisation problems even for the large-scale model of the case study. Instead, the TB-MPC introduces robustness by considering a single optimisation problem with multiple disturbance scenarios, what precludes the tractability and limits the applicability of the technique to small-scale systems.

Figure 5.5 shows the mechanism that the RB-MPC and the CC-MPC use to guarantee a service level in the DWN and to avoid the violation of real output constraints due to uncertainty. The plot shows the response of both controllers for a forecasted demand with confidence levels of 80% and 95%. Notice that both approaches dynamically generate a back-off of original constraints. The characteristics of the safety mechanism observed in each controller are the following:

- RB-MPC: This controller uses the original output bounds, but computes a dynamic soft constraint (i.e., the base stock) to guarantee a desired service level. It can be seen in the evolution of the base stock that the uncertainty is considered stationary within the MPC algorithm, i.e., the constraint keeps a uniform back-off

Table 5.6: Conservatism of the Deterministic Equivalent CC-MPC

Joint Chance Constraint	Number of Individual Constraints	Joint Risk	Conservatism of Approximation
State Hard Bounds	3024	0.001	4.9967×10^{-7}
		0.01	4.9817×10^{-5}
		0.03	4.4539×10^{-4}
		0.05	1.2290×10^{-3}
		0.1	4.8359×10^{-3}
Safety Constraint	1512	0.001	4.9950×10^{-7}
		0.01	4.9801×10^{-5}
		0.03	4.4524×10^{-4}
		0.05	1.2286×10^{-3}
		0.1	4.8344×10^{-3}
Degradation Constraint	114	0.001	4.9965×10^{-7}
		0.01	4.9816×10^{-5}
		0.03	4.4537×10^{-4}
		0.05	1.2290×10^{-3}
		0.1	4.8358×10^{-3}

of demand, whose amount represents the safety stock along the prediction horizon. The stochastic description of demands, used to define this soft constraint, is computed a posteriori, based on the sample mean and sample deviation (see § 4.2), before each MPC execution.

- CC-MPC: This controller incorporates robustness by replacing the deterministic constraints with chance constraints. In this approach, every constraint that involves random variables is adjusted, hence, either the base stock, the hard bounds of the states and the terminal constraint of actuators degradation are dynamically managed by the CC-MPC controller. The level of observed back-off is variable and depends on the volatility of the forecasted demand at each prediction step (see § 5.3). In this approach, a prediction model of the stochastic properties of disturbances is used in parallel with the MPC model.

An important observation regarding the handling of constraints by both RB-MPC and CC-MPC controllers is the inherent relation between the service level in the RB-MPC and the joint risk level in the CC-MPC. Despite being defined under different

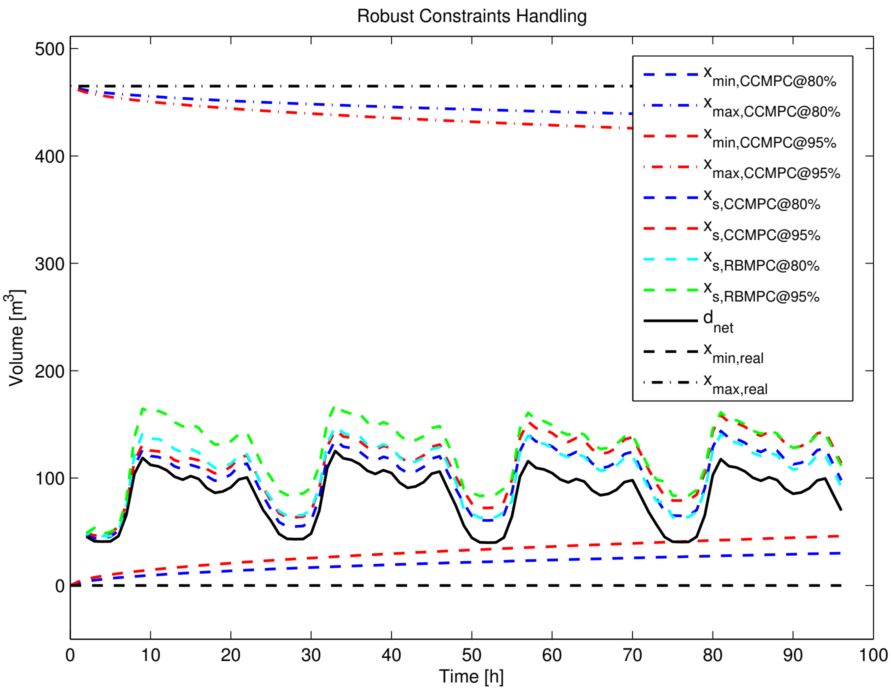


Figure 5.5: Risk averse mechanism using RB-MPC and CC-MPC

philosophies, both parameters represent a measure of reliability for the DWN function. The relation between them is $\gamma = (1 - \delta)$. Nevertheless, Figure 5.5 shows that the dynamic safety stock computed by the RB-MPC controller is more conservative than the one computed by the CC-MPC, which increases according to the forecast error along the prediction horizon. This fact highlights the importance of a suitable forecasting model and the effect of explicitly propagate uncertainty within the MPC model. In general, decreasing the value of the service level, e.g., from 95% to 80% (equivalent to increase the value of the risk level from 5% to 20%), causes a reduction of the safety stock and leads the base stock closer to the demand pattern, which means that, due to the demand uncertainty, the probability of not meeting the customer requirements increases.

To further analyse and highlight the benefits of the CC-MPC approach, a numeric comparison with respect to the RB-MPC controller is shown in Table 5.7. A lower KPI value represents better performance result. Two additional KPIs are considered. The first new indicator accounts for the degradation of actuators and it is defined as $\text{KPI}_Z = \frac{1}{n_s} \sum_{i=1}^{m} \sum_{k=1}^{n_u} z_{(i),k}$, where m is the number of actuators, $z_{(i),k}$ is the accu-

Table 5.7: Comparison of controllers performance

Controller	KPI_E (e.u.)	KPI_S (m^3)	$\text{KPI}_{\Delta U}$ $(m^3/s)^2$	KPI_Z	KPI_V	KPI_O (s)	CPU Time (s)
CE-MPC	2442.97	0.18011	0.8419	0.1374	2245	1.83	202.37
$\text{RB-MPC}_{(1)@95\%}$	2383.97	1987.75	0.9024	0.1377	1596	10.22	884.10
$\text{RB-MPC}_{(2)@95\%}$	2569.59	3029.94	2.1023	0.1098	1699	9.18	892.34
$\text{RB-MPC}_{(1)@80\%}$	2373.44	991.68	0.8265	0.1376	1775	9.01	878.19
$\text{RB-MPC}_{(2)@80\%}$	2560.72	1625.29	2.0665	0.1187	1761	9.17	891.38
$\text{CC-MPC}_{(1)@5\%}$	2390.57	9421.46	1.0223	0.1373	1822	2.65	624.36
$\text{CC-MPC}_{(2)@5\%}$	2761.48	3364.82	2.8664	0.1270	1710	2.50	603.91
$\text{CC-MPC}_{(1)@20\%}$	2362.64	710.22	1.1556	0.1374	1960	2.41	603.61
$\text{CC-MPC}_{(2)@20\%}$	2560.36	4946.13	2.2038	0.1076	1715	2.67	629.35

e.u.: economic units

mulated health degradation of the i-th actuator at time step k, and n_s is the simulation horizon of the assessment. The second new indicator measures the number of safety constraint violations that have been occurred during the simulation and it is defined as $\text{KPI}_V = \sum_{k=1}^{n_s} n_{v,k}$, where $n_{v,k}$ is the number of tanks that required the use of its safety stock at time step k. Simulations have been carried out using $\gamma = \{80, 95\}\%$ for the RB-MPC and $\delta = \{5, 20\}\%$ for the CC-MPC. In addition, Table 5.8 and Figure 5.6 disclose details of the production and operational costs related to each strategy, which are the primary objectives for managers. Furthermore, Table 5.9 summarises the capabilities handled by each controller. This qualitative information complements the quantitative evaluation of the strategies in order to highlight the benefits of the MPC designs.

After reviewing the performance indicators and capabilities, it can be reaffirmed that the robust MPC strategies developed in this thesis outperform the CE-MPC controller, which may have low values in most of the KPIs but with any guarantee of reliability, robust or probabilistic feasibility. Despite to have the lowest KPI_S, the baseline approach is the one that presents the highest number of soft constraints violations, what means that the safety thresholds might be overestimated (as observed in several tanks in the DWN), causing more oscillations in the excursion of water, or keeping states near the threshold with easiness to activate the constraints in the controller. Therefore, the

Table 5.8: Comparison of daily average costs of MPC strategies

MPC Approach	Water Average Cost (e.u./day)	Electric Average Cost (e.u./day)	Daily Average Cost (e.u./day)
CE-MPC	29037.21	29594.14	58631.35
RB-MPC$_{(1)}$	27756.93	29580.78	57337.72
RB-MPC$_{(2)}$	42072.97	19597.27	61670.25
CC-MPC$_{(1)}$	27706.72	29666.85	57373.58
CC-MPC$_{(2)}$	53179.29	13096.23	66275.53

e.u.: economic units

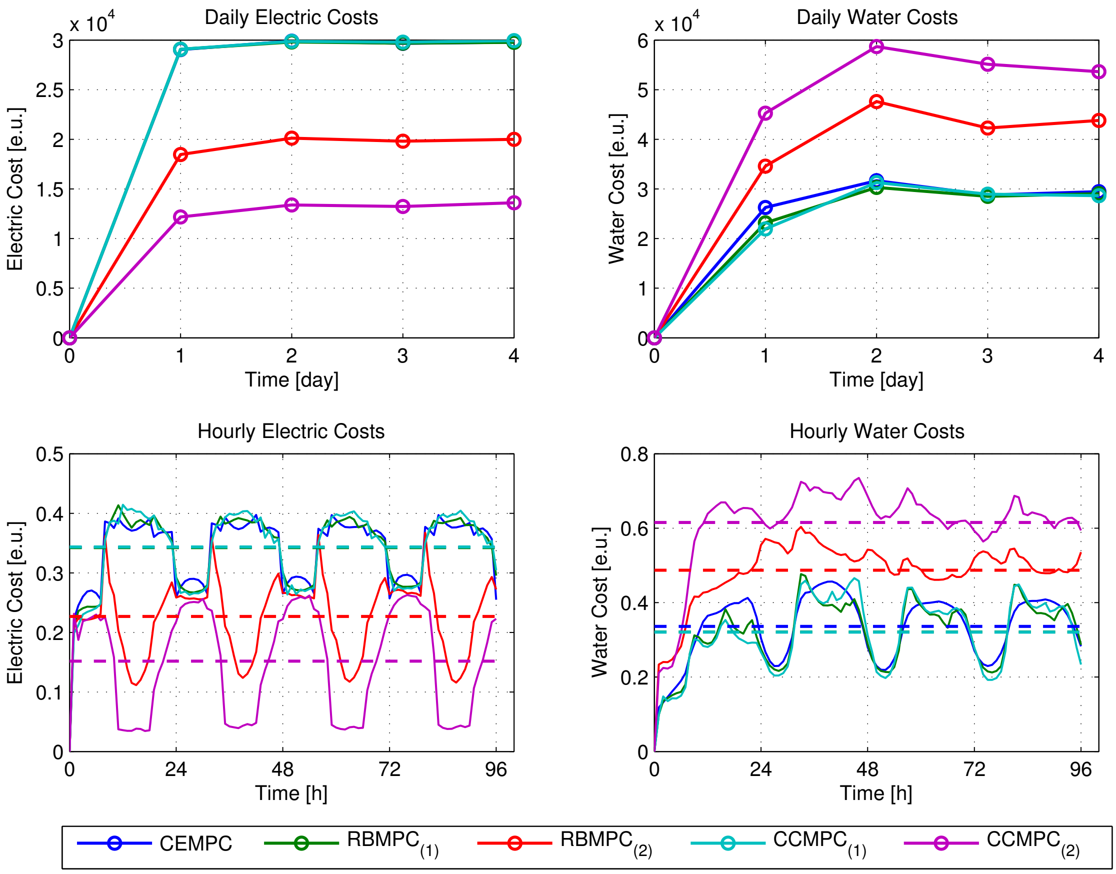


Figure 5.6: Comparison of hourly and daily costs

baseline CE-MPC approach, with fixed and empirical safety stocks, limits the economic optimisation. Instead, the strategies RB-MPC$_{(1)}$ and CC-MPC$_{(1)}$, reached the lowest KPI$_E$ (in both 80% and 95% risk levels) by incorporating robust and optimal safety

Table 5.9: Comparison of capabilities handled by each controller

Controller	Dynamic Safety Stocks	Dynamic Output Bounds	Actuators PHM	Smart Tuning
CE-MPC				
RB-MPC$_{(1)}$	$\surd$			
RB-MPC$_{(2)}$	$\surd$		$\surd$	$\surd$
CC-MPC$_{(1)}$	$\surd$	$\surd$		
CC-MPC$_{(2)}$	$\surd$	$\surd$	$\surd$	$\surd$

$\surd$: *handled*

stocks to face demand uncertainty with minimum storage of water. These robust approaches have lower KPI$_V$, i.e., they reduce the number of violations of the base stocks, but increase the amount of safety stocks used to meet demands (higher KPI$_S$). This is an expected behaviour due to the policy of minimum storage behind the computation of the base stocks, which prefers to use the safety stocks instead of keeping more volume of water than the required. The lower cost of water in the Barcelona DWN (see Figure 5.6), comparing the CE-MPC with the RB-MPC$_{(1)}$ and CC-MPC$_{(1)}$, reinforces this observation. The main disadvantage of these cheaper controllers is that control actions are computed based on economic criteria and accounting for tanks reliability but not for actuators reliability. This fact leads to higher values of the KPI$_Z$, i.e., the controllers overexploit those actuators that have lower operational costs, accelerating their wear and compromising the service reliability.

In order to manage the overall system reliability, the RB-MPC$_{(2)}$ and CC-MPC$_{(2)}$ controllers incorporate actuators health models and restrict their maximal cumulative degradation at each time step to ensure their proper functioning until a maintenance horizon is reached. As seen in Table 5.8, the ability to compute control actions for an efficient management of actuator reliability implies an important reduction of the electric costs, but at expense of an increment of the KPI$_E$ due to the higher water cost, of the KPI$_{\Delta U}$ due to the distribution of control effort that avoids (if possible) constant control actions that could cause an imbalance degradation of actuators, and of the KPI$_S$ due to the narrowing of constraints. It is important to point out that the two RB-MPC controllers have greater KPI$_O$ than the CC-MPC controllers. The reason is that the former ones have to solve a bi-level optimisation problem on-line, compared with the CC-MPC controllers that just require to solve a single optimisation problem. Nonetheless, all the compared controllers are suitable for RTC considering the sampling time in the DWN is one hour. Looking at the results discussed before, the CC-MPC strategy should be preferred given their tractability for large-scale systems (as shown with the Barcelona DWN case study) and its ability to handle probabilistic and deterministic constraints.

5.6 Summary

In this chapter, two stochastic control approaches have been assessed to deal with the management of generalised flow-based networks. Both the CC-MPC and the TB-MPC approaches focused on robust economic performance under additive disturbances (unbounded and stationary or non-stationary) and avoid relying on heuristic fixed safety volumes such as those used in the CE-MPC or the RB-MPC schemes proposed in the previous chapters, which results in better economic performance. According to the results obtained for the considered case study, both techniques showed a relatively similar performance. However, it seems clear that CC-MPC is more appropriate when requiring a low probability of constraint violation, since the use of TB-MPC demands the inclusion of a higher number of scenarios, which may be an issue for the application of the latter to large-scale networks. The analytical approximation of joint chance constraints based on their decomposition into individual chance constraints, these latter bounded by means of Boole's inequality, has shown to be suitable for large networks regarding that the conservatism involved is not affected neither by the number of the inequalities nor the prediction horizon of the MPC. The level of resultant back-off is variable and depends on the volatility of the forecasted demand at each prediction step and the suitability of the probabilistic distribution used to model uncertainty. The presence of unbounded disturbances in the system precludes the guarantee of robust feasibility with these schemes. Hence, the approaches proposed in this chapter are based on a service-level guarantee and a probabilistic feasibility. The case study shows that the CC-MPC is suitable for the operational guidance of large-scale networks due to its robustness, flexibility, modest computational requirements, and ability to include risk considerations directly in the decision-making process. Even when the CC-MPC increased the operational costs by around 2.5%, it allowed to improve service reliability by more than 90% when comparing with a CE-MPC setting.

Future research will be directed to incorporate parametric uncertainty and unmeasured disturbances in the model. In addition, future work should include a more detailed study regarding the number of scenarios contained in the tree. Likewise, distributed computation could be used in order to relieve the scaling problems of TB-MPC when the number of scenarios is too high. Moreover, it is of interest to extend the results and develop decentralised/distributed stochastic MPC controllers for large-scale complex flow networks.

Chapter 6

Learning-based Tuning of Supervisory MPC for Generalised Flow-based Networks

This chapter proposes a synergy of artificial intelligence, supply chain theory and automatic control to devise an adaptive and robust MPC controller for the management of generalised flow-based networks. Here, a Learning-based Supervisory MPC (LB-MPC) strategy with a hierarchical multilayer structure is developed, where soft-computing techniques (i.e., neural networks and fuzzy logic) are used not to approximate an MPC controller but to tune its design parameters on-line by learning the expectation of the performance. In contrast to general tuning approaches, the LB-MPC method presented in this chapter not only adapts the weights of the multi-objective optimisation problem that takes place within the MPC controller but also the prediction horizon according to the current situation of the plant. Furthermore, operational output constraints are dynamic and governed according to the non-stationary uncertainty of the system disturbances to assure service reliability while optimising economic resources. The proposed control scheme is a quasi-explicit MPC approach with low on-line computational burden, because most of the heavy computations are converted into non-linear explicit modules using neural networks. The benefits, i.e., flexibility and reliability of this LB-MPC controller as a decision-support tool are shown in this chapter through the real case study of § 2.4.

6.1 Control System Structure

The control strategy addressed in this chapter is based on a multilayer (hierarchical) control system structure. The hierarchical architecture has been frequently used in process control with satisfactory results optimising economic profits when disturbances are slowly varying [174]. In several generalised flow-based network applications and particularly in the case study addressed in this thesis, disturbances often follow a pattern in a daily basis and can be well predicted for an hourly sampling time, which makes the hierarchical structure suitable to optimise targets for the policies of the direct control level. Therefore, the controller proposed in this chapter is based on a three-layer structure (see Figure 6.1). First, a Learning and Planning layer (LPL) determines on-line strategic dynamic safety constraints, economically optimal state references and demand forecasts, all of them by means of artificial neural networks (ANNs). Secondly, a Supervision and Adaptation layer (SAL) implements a fuzzy rule-based inference system (FIS) to continually adjust the parameters of an MPC controller, which computes, along the receding horizon approach, the optimal controlled flows according to the current status of the plant. Finally, a Basic Feedback Control layer (BFCL), commonly based on PID controllers or any other tracking-oriented controller, is responsible for real-time operation of the system and has direct access to the manipulated variables. The BFCL is not addressed in this chapter because it is assumed that the design problem of each local regulatory controller is already solved by the operators of the system. Hence, as a common practice in the design of hierarchical controllers, perfect reference tracking of the control loops at the lower layer is assumed in this chapter.

6.2 Learning and Planning Layer

Classical hierarchical MPC structures exhibit a high computational burden when setpoint optimisation, governing of constraints and forecasting of disturbances, are required to be executed at the same frequency that the control problem is solved. Sequential optimisations are often required for the planning of actions in top layers but many parameters and non-linear functions cause these tasks to be intractable for on-line tuning of controllers, especially in large-scale systems as generalised flow-based networks. Therefore,

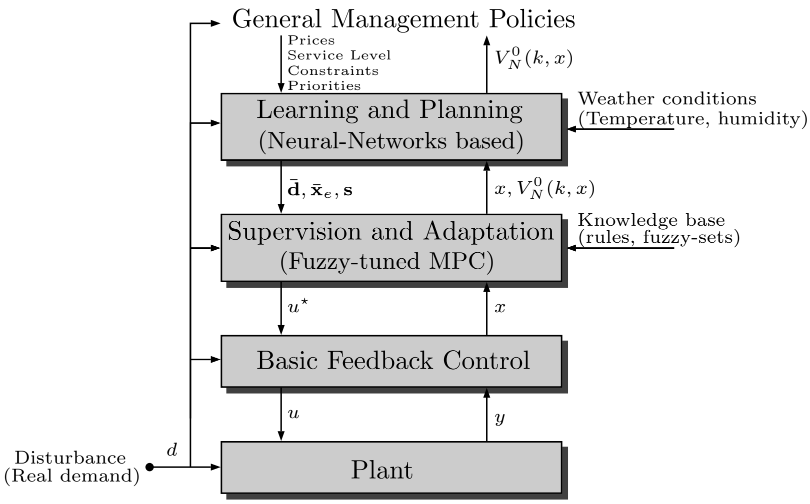


Figure 6.1: Multilayer hierarchical intelligent control architecture

this chapter proposes a planning layer that uses ANNs in order to reduce the computational burden of optimisation-based approaches.

ANNs have a remarkable ability to derive meaning from complicated or imprecise data and can be employed to extract patterns and detect trends that are too complex to be noticed by other computational techniques. Specifically, this chapter uses multi-layer perceptron (MLP) neural models, which as proved in [44, 61] can represent continuous functions to any degree of accuracy, with at least one hidden layer, provided that the number of neural units is sufficiently large. MLP is considered a universal approximator and has been efficiently used for on-line predictive control due to its natural capability for storing and generalising experience-based knowledge, see e.g., [60, 98, 141]. As illustrated in Figure 6.2, a basic MLP consists of one input layer containing a vector of $N \in \mathbb{Z}_{\geq 1}$ predictor variable values $(z_1, \ldots, z_N)^\top$, one or more hidden layers with $S \in \mathbb{Z}_{\geq 1}$ active neurons, and one output layer with $L \in \mathbb{Z}_{\geq 1}$ neurons. Learning is a process through which free parameters (i.e., synaptic weights w_{ji}^h and w_{kj}^o, and bias levels b_j^h and b_k^o, with $i \in \mathbb{Z}_{[1,N]}$, $j \in \mathbb{Z}_{[1,S]}$, $k \in \mathbb{Z}_{[1,L]}$) of an ANN are adapted through a continuous process of stimulation by the environment in which the network is embedded. In this chapter, three MLP supervised training structures (see Figure 6.3) with one hidden layer are devised in order to forecast flow demands and to plan future base-stocks and economically optimal references. Tan-sigmoid and linear activation functions are used for the hidden and the output layers, respectively, following the results presented in

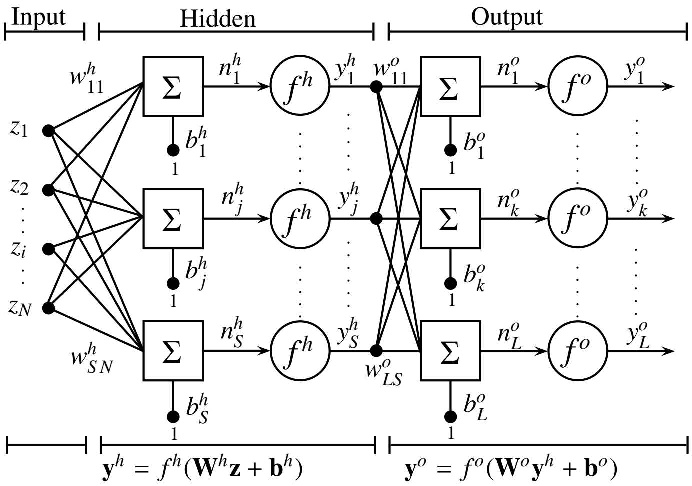


Figure 6.2: Feed-forward Neural Network diagram

[192]. The training patterns (i.e., 8640) used in each ANN are divided randomly with a ratio of 60%, 20% and 20%, to form training, validation and testing sets, respectively. Networks with different numbers of active neurons have been trained off-line in order to find the one with the smallest test data cost. The training of the ANNs is done with the Gauss-Newton approximation to Bayesian regularisation (GNBR) algorithm developed in [46], which avoids over-fitting and improves the generalisation ability of the neural models. This training method performs Bayesian regularisation employing the Levenberg-Marquardt back-propagation algorithm, which ensures that the ANNs provide accurate output values for inputs not represented in the calibration set of the neural network and imposes constraints to get smaller weights and a smoother ANN behaviour.

6.2.1 ANN for Demand Forecasting

This module focuses on the problem of flow demand forecasting for real-time operation of a given generalised flow-based network. An hourly consumption data analysis is proposed here for training an MLP (see Figure 6.3 top) with 100 neurons in the hidden layer. For the particular DWN case study (see § 2.4), the inputs to the neural model are chosen based on the suggestions in [11] and correlation analysis, i.e., considering consumption data and meteorological variables such as temperature T and air relative humidity H. Principal component analysis (PCA) preprocessing is applied to the training patterns

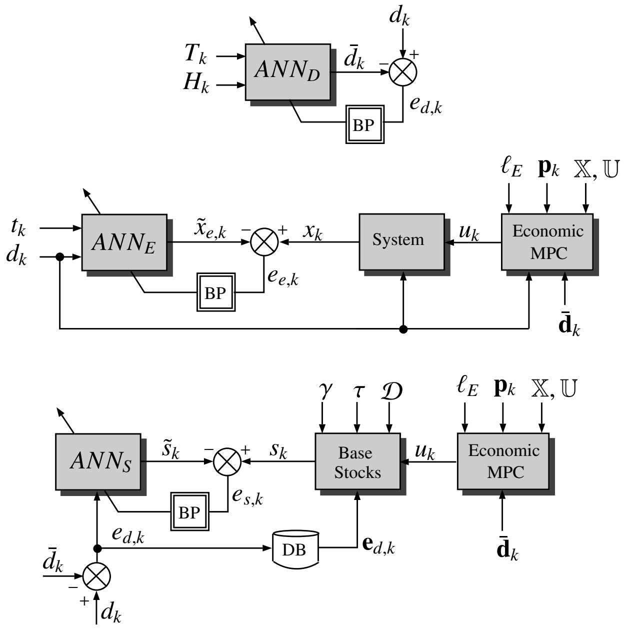


Figure 6.3: ANNs training diagrams for Demand Forecasting (top), Economic trajectory (middle) and Base-stocks setting (bottom)

in order to reduce the dimension of the input vectors and to obtain uncorrelated values that facilitate the learning process.

6.2.2 ANN for Optimal Economic Trajectory

The economically optimal state trajectory for each flow storage unit is the one obtained considering only the economic stage cost term $\ell_E : \mathbb{Z}_+ \times \mathbb{X} \times \mathbb{U} \to \mathbb{R}_+$ of a possibly multi-objective cost function related to the operation of the network flows, solving often a linear programming constrained optimisation problem on-line. Again, for the particular case study in § 2.4, and with the intention to reduce computational effort, an MLP with 50 neurons in the hidden layer is trained off-line to emulate the economic MPC controller. A non-linear explicit approximated model of the optimal operation of the system is obtained, which is later used to compute at each time step $k \in \mathbb{Z}_+$ the economic trajectory $\tilde{\mathbf{x}}_{e,k} = \{\tilde{x}_{e,k+i}\}_{i\in\mathbb{Z}_{[0,N]}}$ for a given prediction horizon $N \in \mathbb{Z}_{\geq 1}$, taking into account the known economic parameter $\mathbf{p}_k$ involved in the stage cost ℓ_E, the constraint sets and the predicted demand sequence $\bar{\mathbf{d}}_k$ as inputs (see Fig. 6.3 middle).

6.2.3 ANN for Dynamical Safety Volumes

In order to include the safety stock management policy proposed in § 4.2, this chapter uses an ANN to avoid the on-line optimisations that are required to virtually decouple the dynamic states of a generalised flow-based network for the calculation of the base-stock vector in (4.4). Here, the non-linear explicit approximated model of a trained MLP dynamically computes the safe base-stock vector $s_k \in \mathbb{R}^n$ for each $k \in \mathbb{Z}_+$ (see Figure 6.3 bottom). For the particular case study in § 2.4, an MLP with 50 neurons in the hidden layer is trained with input-output patterns that are generated using historical records of demand forecasting errors $e_{d,k} \in \mathbb{R}^p$ as inputs, while using safety inventories in $\tilde{s}_k \in \mathbb{R}^n$ as outputs. The target s_k of this ANN has been calculated following § 4.2 with a service level $\gamma = 95\%$ and a lead time $\tau = 4$ hours.

Remark 6.1. *The number of hidden layers and neuron units depend upon the complexity of the problem and the available computational resources, i.e., both are design parameters.* ♢

Remark 6.2. *Neural models may not adapt to all characteristics of the environment, especially when there are uncertainties and no feedback correction mechanisms. The performance of ANNs in any application will be as adequate as the scenarios and the quality of the training data set are. This lack of guarantees is the reason why this chapter enhances a constrained MPC controller with neuro-learning for tuning purposes instead of implementing a pure neuro-control approach.* ♢

6.3 Supervision and Adaptation Layer

Self-tuning on-line algorithms for MPC of large-scale flow networks is not a widely reported topic in literature. Most of the tuning strategies for the inherent multi-objective optimisation problems take into account the exploration of the complete Pareto frontier to choose a non-dominated solution in line with the management objectives. As shown in [178] for a water network application, one of the goals behind the Pareto frontier applied to the predictive control of a generalised flow-based network should be to find a direct relation between the weights of the solution points and the flow demands, which often are assumed periodic as discussed in Chapter 3. Nevertheless, the computation of the Pareto frontier for a given system and cost function could be cumbersome. Therefore, to reduce computational complexity, this chapter presents an adaptation scheme, similar to the one in [4]. Here, a fuzzy logic module interacts with an MPC controller by

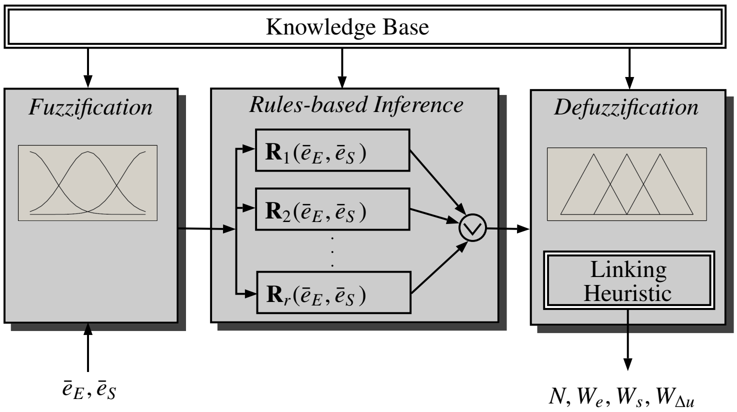


Figure 6.4: Fuzzy rules-based MPC tuner diagram

automatically adjusting the tuning parameters of the controller based on the supervision of output feedback and measured disturbances.

6.3.1 Reasoning Mechanism

A knowledge-based tuner for the MPC controller is proposed in the sequel. The tuning mechanism consists in a fuzzy inference system (FIS) that involves: (i) a fuzzyfication interface, (ii) an a priori rule base, (iii) a defuzzyfication interface, and (iv) a linking heuristic (see Figure 6.4). The design of each of these elements is application-dependent and should be set up after a phase of experimentation and understanding of the effect that the selected tuning parameters have on the given generalised flow-based network. Hence, the methodology is next explained for the case study of § 2.4.

First, a *fuzzyfication* phase converts the crisp values of both the normalised economic state error defined as $\bar{e}_{E,k} := |x_k - \tilde{x}_{e,k}|/\tilde{x}_{e,k} \in \mathbb{R}^n$ and the normalised safety volume state error defined as $\bar{e}_{S,k} := (x_k - s_k)/s_k \in \mathbb{R}^n$, into fuzzy values. Indeed, to each input of the supervisor n_{ifs} input fuzzy sets are associated and labelled with n_{ifs} linguistic input variables and described by specific Gaussian membership functions with values in the interval $[0, 1]$. The universe of discourse for the input $\bar{\mathbf{e}}_E$ is: small (S), medium (M), large (L); and for the input $\bar{\mathbf{e}}_S$ it is: large-negative (LN), medium-negative (MN), small-negative (SM), very-small-negative (VSN), very-small-positive (VSP), small-positive (SP), medium-positive (MP), large-positive (LP).

Table 6.1: Fuzzy-logic rules for LB-MPC.

Rule	Inputs		Outputs			
	$\bar{e}_{E,k}$	$\bar{e}_{S,k}$	$\tilde{N}_k$	$\tilde{W}_{e,k}$	$\tilde{W}_{s,k}$	$\tilde{W}_{\Delta u,k}$
1	L	LP	L	L	VS	VS
2	L	MP	L	L	VS	S
3	L	SP	L	M	VS	M
4	L	VSP	L	S	S	L
5	M	LP	M	L	VS	VS
6	M	MP	M	L	VS	S
7	M	SP	M	M	VS	M
8	M	VSP	M	S	S	L
9	S	LP	S	L	VS	VS
10	S	MP	S	M	VS	S
11	S	SP	S	M	VS	M
12	S	VSP	S	S	S	L
13	any	LN	L	VS	L	VS
14	any	MN	L	VS	L	VS
15	any	SN	L	VS	L	VS
16	any	VSN	L	VS	L	VS

The fuzzy-logic supervisor involves expert knowledge using the Mamdani's *fuzzy inference system*, which applies the set of linguistic *rules* presented in Table 6.1 to the fuzzy inputs in order to evaluate the supervisor fuzzy outputs. The inference table associates n_{ofs} output fuzzy sets (n_{ofs} linguistic output variables), described by specific trapezoidal membership functions, to the supervisor's crisp output variables $\tilde{N}_k, \tilde{W}_{e,k}, \tilde{W}_{s,k}, \tilde{W}_{\Delta u,k} \in \mathbb{R}^n$, $k \in \mathbb{Z}_+$. The universe of discourse for output $\tilde{N}_k$ is: small (S), medium (M), large (L); and for the outputs $\tilde{W}_{e,k}$, $\tilde{W}_{s,k}$, $\tilde{W}_{\Delta u,k}$ is: very small (VS), small (S), medium (M), large (L). The *min-max inference* method is used for the evaluation of the fuzzy rules contribution, and the *gravity center* method is used in the *defuzzyfication* phase. For a detailed explanation on fuzzy inference reasoning, the reader could refer to [185]. Note that the outputs of the FIS are in $\mathbb{R}^n$ because they are computed for every state in the network. Therefore, it is further required to transform the resulting weights $\tilde{W}_{e,k}$ and $\tilde{W}_{\Delta u,k}$ into an actuator base in $\mathbb{R}^{m\times m}$ and the results for $\tilde{N}_k$ into an integer value $N \in \mathbb{Z}_{\geq 1}$. The weight $\tilde{W}_{s,k}$ is already related to states but it has to be transformed into a diagonal matrix in $\mathbb{R}^{n\times n}$.

6.3.2 Linking Heuristic

To obtain the quantitative value of the tuned parameters in the corresponding domain for the MPC problem stated in § 2.4.3, an *associative heuristic* is proposed in Algorithm 2, where the argument of sub-index $(\cdot)$ represents a specific row of the associated variable. This heuristic is based on the topology of the generalised flow-based network.

Algorithm 2 Linking Heuristic

procedure LinkingHeuristic($\tilde{W}_{e,k}, \tilde{W}_{s,k}, \tilde{W}_{\Delta u,k}, \tilde{N}_k$)
  $m :=$ number of actuators
  $n :=$ number of tanks
  $w_e \leftarrow 0_{m\times 1}$
  $w_{\Delta u} \leftarrow 0_{m\times 1}$
  for $\{i = 1 \rightarrow m\}$ **do**
    if $\{u_{(i)}$ connects *junction–junction* or *storage–demand*$\}$ **then**
      $w_{e(i)} \leftarrow 0$
      $w_{\Delta u(i)} \leftarrow 0$
    else
      if $\{u_{(i)}$ connects *storage*$_{(a)}$–*storage*$_{(b)}$, with $a, b \in \mathbb{R}^n\}$ **then**
        $w_{e(i)} \leftarrow \max\{\tilde{W}_{e(a),k}, \tilde{W}_{e(b),k}\}$
        $w_{\Delta u(i)} \leftarrow \max\{\tilde{W}_{\Delta u(a),k}, \tilde{W}_{\Delta(b),k}\}$
      else
        if $\{u_{(i)}$ connects *storage*$_{(a)}$–*junction* , with $a \in \mathbb{R}^n\}$ **then**
          $w_{e(i)} \leftarrow \tilde{W}_{e(a),k}$
          $w_{\Delta u(i)} \leftarrow \tilde{W}_{\Delta u(a),k}$
        end if
      end if
    end if
  end for
  $W_e \leftarrow \mathbb{R}^{m\times m}$ (diagonal matrix whose diagonal is the vector w_e)
  $W_{\Delta u} \leftarrow \mathbb{R}^{m\times m}$ (diagonal matrix whose diagonal is the vector w_u)
  $W_s \leftarrow \mathbb{R}^{n\times n}$ (diagonal matrix whose diagonal is the vector $\tilde{W}_{s,k}$)
  $N \leftarrow \max\{\tilde{N}_k\}$
  return $N, W_e, W_s, W_{\Delta u}$
end procedure

Remark 6.3. *Most of the tuning guidelines reported in literature assume weighting matrices with equal elements in the diagonal. The learning-based tuning approach presented in this thesis allows to adapt on-line individual elements of the cost function weighting matrices giving more degrees of freedom to the managers of the network.* ◊

Table 6.2: Performance of the ANNs (MAE%)

Neurons	$\mathrm{ANN_d}$	$\mathrm{ANN_e}$	$\mathrm{ANN_s}$
10	8.8299	629.4652	3.5512
20	7.1257	87.2827	2.0423
30	10.9843	89.3765	1.5513
40	7.8918	1.4455	1.4124
50	8.6294	1.4098	1.0376
60	11.2109	1.4263	1.0323
70	11.3223	1.4788	1.0206
80	7.4649	–	–
90	14.0614	–	–
100	6.6945	–	–
110	14.8369	–	–

6.4 Numerical Results

In this section, simulation results of the LB-MPC approach applied to the *aggregate* model (see Figure 2.4) of the DWN case study described in § 2.4 are presented. All the simulations have considered a time period of four days (96 hours). The selected sampling time is one hour. Simulations have been carried out using the CPLEX solver of the TOMLAB 7.6 optimisation package, together with the Fuzzy Logic Toolbox and the Neural Network Toolbox of Matlab R2010b (64 bits). The computer used to run the simulations is a PC Intel Core E8600 running both cores at 3.33GHz with 8GB of RAM.

6.4.1 Demand Forecasting and States Planning with ANNs

Table 6.2 shows the mean absolute percentage error (MAE%) of the ANNs described in § 6.2, that were trained using the function `trainrb` included in the Neural Network Toolbox of Matlab. The final architecture of each ANN was selected by comparing the performance of experiments varying the number of neurons in the hidden layer. The selected number of neurons were 100, 50 and 50 for the ANN_d, ANN_e, and ANN_s, respectively. Forecasting of demand with the ANN_d is based on water consumption and meteorological (temperature and air relative humidity) records available from AGBAR Company and from the Servei de Meteorologia de Catalunya[1], respectively. The results

[1] http://www.meteo.cat

show the prediction of water demand is reliable and the magnitude of the forecasting error is not a reason to reject the obtained model. In fact, these minimal discrepancies are reflected in an increase of the safety volume. In contrast to ANN_d, the ANN_e for the economic optimal trajectory and the ANN_s for the safety volume trajectory perform with higher accuracy as it could be expected, because both MLPs are trained with the solution of QP problems instead of experimental driven data as in the forecasting demand case. The advantage of all of the neural models used in this chapter is that the time invested in processing data and training the ANNs will be gained in the on-line solving process once they are accurately validated and tested.

6.4.2 Fuzzy Tuning of MPC Parameters

In most of the results presented in literature for control of DWNs, tuning is focused on the weighting matrices with no adaptation schemes. Nevertheless, for large-scale systems, an efficient selection of the prediction horizon is demanded since the size and complexity of the optimisation problem is based mainly on this parameter. Thus, the methodology proposed in this chapter adapts the prediction horizon according to a trade-off between risk and economic cost. The selected tuning parameters for problem (2.23) are the prediction horizon N and the weights W_e, W_s, $W_{\Delta u}$. These parameters were computed and adapted by the FIS described in § 6.3 using the Fuzzy Logic Toolbox of Matlab. The evolution of such parameters is shown in Figure 6.5, while the histograms of their values are shown in Figure 6.6. These results were obtained for an unmeasured random disturbance of at most 20% of the nominal demand pattern.

6.4.3 LB-MPC Controller for DWN

The proposed learning-based approach has been implemented for the tuning of the constrained MPC described in § 2.4.3 to operate the Barcelona DWN. Results have been compared with two other strategies. The controllers are the following ones:

- MPCo: the baseline MPC approach introduced in § 2.4.3, which uses a fixed prediction horizon ($N = 24$ hours), constant water base-stocks and constant tuning weights for the prioritisation of management objectives.

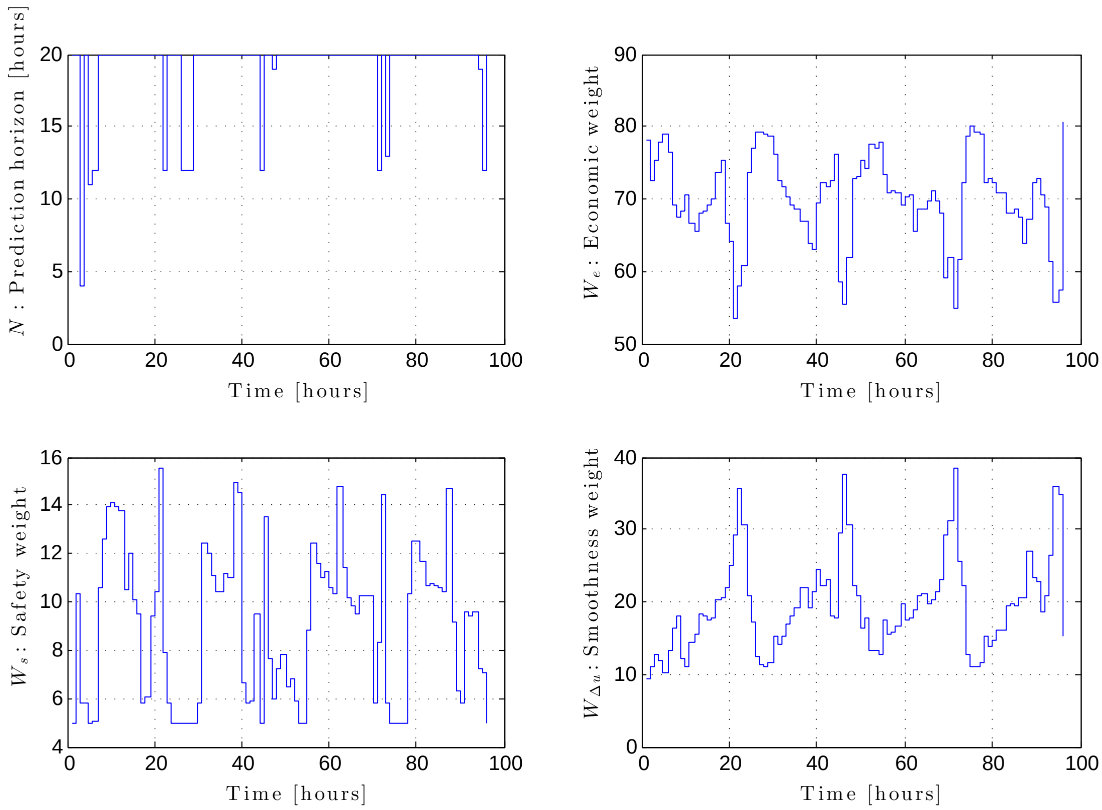


Figure 6.5: Evolution of tuning parameters for the LB-MPC strategy

- MPCss: the bi-level RB-MPC approach described in Chapter 4, but implementing only the dynamic optimisation of safety stocks following Section 4.2. It considers fixed prediction horizon and fixed weights as well.
- LB-MPC: the learning-based MPC approach addressed in this chapter, consisting in the same problem (2.23) but with adaptive prediction horizon, tuning weights and safety stocks.

Table 6.3 shows the specific performance indicators (as defined in § 2.4.4) that were used to assess the aforementioned controllers over the simulation period ($N_s = 96$ hours). Recall that KPI_E is the total economic cost of the DWN operation, KPI_S is the accumulation of all safety level violations and $\mathrm{KPI}_{\Delta U}$ is the accumulated RMS of control variations. Simulations show that static MPC design parameters (safety stocks, tuning weights and prediction horizon) are a drawback for the management of network flows,

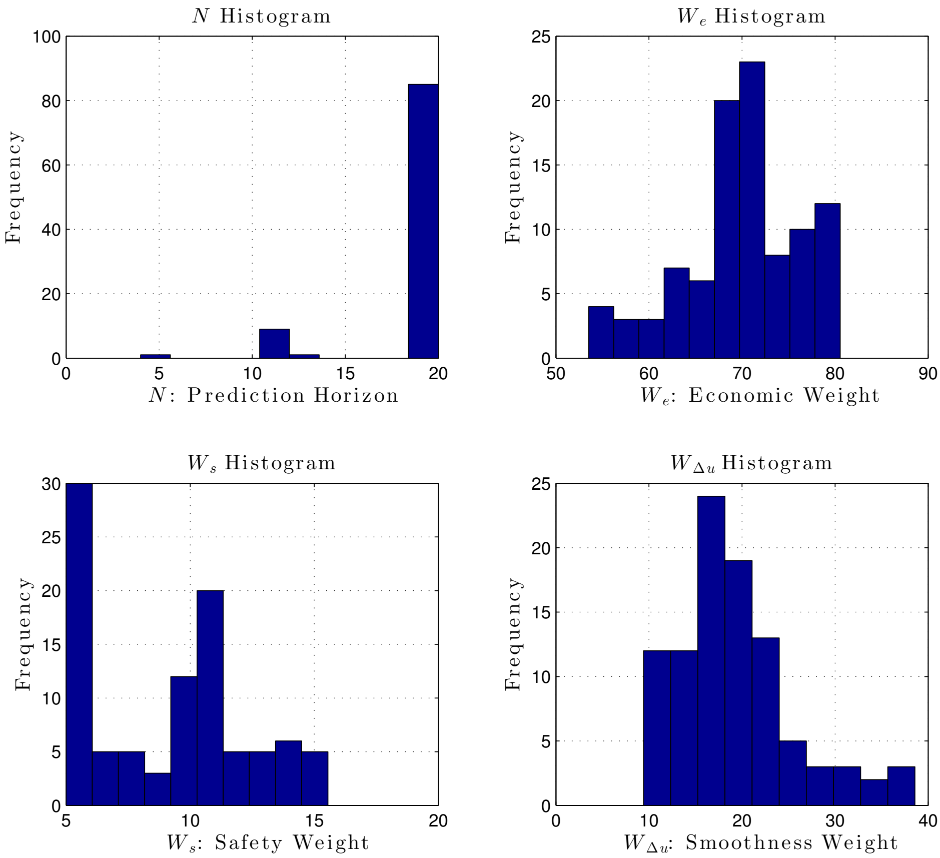


Figure 6.6: Histograms of tuning parameters for the LB-MPC strategy

because under uncertain disturbances, the fixed value of these parameters might cause an undesired restriction of the solution space that degrades the economic performance, and/or increment the risk of constraint violation. Figure 6.7 shows the excursion of water in a tank of a representative part of the Barcelona DWN, whose behaviour is representative of most storages in the DWN. It can be seen that all of the compared MPC controllers keep the volume in tank within the hard and soft constraints satisfying also the net demand along the simulation horizon but with differences in the computational time (see Table 6.3) and the management of safety stocks that impacts the aforementioned KPIs. As expected, the safety stocks of the LB-MPC controller tend to reproduce the ones computed with the MPCss controller because this latter was used to train the ANN that predicts the safety level in the LB-MPC.

Table 6.3: Key performance indicators for the different approaches

Controller	KPI_E (10^3) (e.u.)	KPI_S (m^3)	$\text{KPI}_{\Delta U}$ $(m^3/s)^2$	CPU time (s)
MPCo	183.74	28.8022	0.1318	142.01
MPCss	176.77	5.0295	0.1340	286.17
LB-MPC	178.99	5.2138	0.1172	132.91

e.u.: economic units

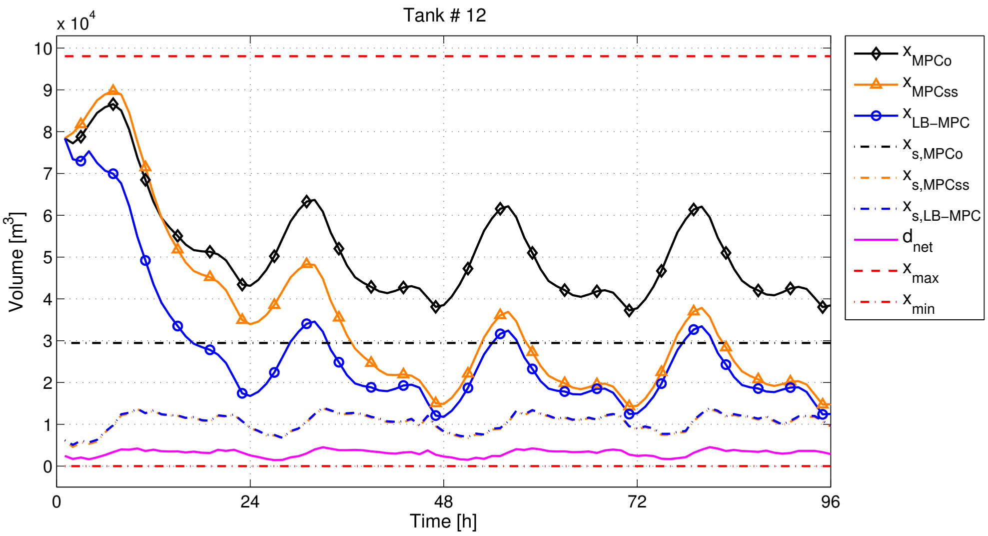


Figure 6.7: Dynamic variation of tanks volumes for the different approaches

The MPCo controller presents the highest economic cost due to the conservative and static safety volumes that limits the economic optimisation. In general, this approach does not guarantee optimal results for any condition because the safety is fixed heuristically without taking into account demand variations. Instead, the MPCss controller has a certain robustness to disturbances by optimising the dynamic safety stocks (see Figure 6.7) in accordance to the deviation of the forecasting error. The MPCss presents the best economic performance but the highest computational effort (see Table 6.3) since it involves more on-line optimisation problems to set those safety stocks.

Results show that the LB-MPC controller outperforms the previous strategies. It presents similar results to the MPCss controller for the economic, safety and smoothness indicators but reduces the computational burden (see Table 6.3). In addition, Figure 6.7 and Figure 6.8 show that, despite the similar safety stocks of the MPCss controller and

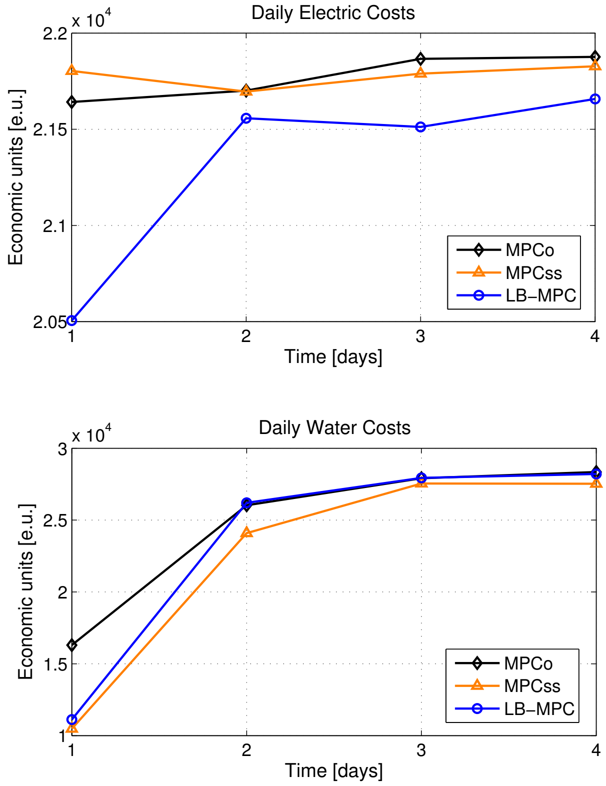


Figure 6.8: Comparison of the daily electric and water costs for the different approaches

the LB-MPC controller, this latter makes better use of hydraulic and economic resources due to its flexibility to self-adapt the parameters of the controller if the operational conditions change (see Figure 6.6). This capability helps managers to deal with demand uncertainty and prediction errors in an optimal and economic way, guaranteeing the desired safety and service level with less volume of water and less electric energy.

Furthermore, Figure 6.8 details that even when the MPCss approach has a lower value than the LB-MPC approach for the economic performance indicator (see Table 6.3), which integrates water costs and electric costs, the LB-MPC presents a lower cost in the electric component due to the adaptation of weights by means of the AI techniques used in this application. From an operational point of view, this difference in costs implies that the LB-MPC controller decides to take water from more expensive sources in order to further reduce the electrical costs by pumping more, with respect to the other approaches, when the electric tariff is cheaper.

6.5 Summary

This chapter has presented a multilayer MPC approach with self-tuning capabilities for the efficient management of network flows based on soft-computing techniques. The approach has been applied to the aggregate model of the Barcelona DWN obtaining important improvements in the computation time towards on-line implementation for large-scale systems. The selected tuning parameters of the MPC problem were the prediction horizon and the weighting matrices of the multi-objective cost function. The main advantage of the fuzzy tuner is its ability to adapt every element independently, which is a difficult task in analytical approaches due their lack of intuitiveness for multi-variable large-scale systems. The controller also tunes the set-points based on inventory management theory, enriching the controller design with reliability aspects to assure a customer service level under disturbances uncertainty. Further research will be conducted in using other ANNs structures and in reinforcement learning algorithms with two main directions: (i) to implement intelligent distributed MPC of generalised-flow based networks where shared variables are negotiated using learning techniques, and (ii) to adapt and improve the presented fuzzy inference system.

Part III

Distributed Economic MPC Schemes for Network Management

Chapter 7

Multi-Layer Non-Iterative Distributed Economic MPC

This chapter proposes a multi-layer non-iterative distributed economic MPC approach for its application to large-scale generalised flow-based networks. The topology of the controller is structured in two layers. First, an upper layer exploits the periodic nature of the demands to compute a sequence of flow economic prices that minimises a set of global objectives that are used to influence local controllers. This layer works with a sampling time corresponding to the period of the flow demands. Second, a lower layer formed by local MPC controllers operating with a higher sampling frequency is in charge of computing, in a sequential way, the flow references for the system actuators in order to satisfy a set of local objectives.

7.1 Introduction

The control schemes proposed in previous chapters have shown the potential applicability of *centralised* MPC for economic scheduling-control of network flows. Nevertheless, as illustrated with the case study used along this thesis, generalised flow-based networks are generally systems comprised of multiple subsystems and/or large-scale systems with communication constraints. Thus, the centralisation of decisions in a single MPC-based agent could be disadvantageous for the reliability of the network operation and the maintenance of the monolithic prediction model. These issues have received a lot of attention from the control research community during the last years. As reviewed in § 1.2.5, several *non-centralised* control strategies have been already proposed in the literature, where either large-scale systems are partitioned into subsystems with individual control agents or a plant-wide optimisation problem is distributed in a set of smaller optimisation problems that are usually coordinated by a master problem. The importance of *system partitioning* and/or *distributed optimisation* has already been noticed in classic references addressing decentralised control of large-scale systems [108, 167] and the decomposition of mathematical programming problems [42]. For distributing the centralised MPC optimisation problem, several analytic methods exist, e.g., Dantzig-Wolfe decomposition, Bender's decomposition, optimality condition decomposition, among other dual or primal decomposition techniques. These analytic decompositions rely strongly on the form of both the constraints and the objective function, and are specialised to particular problem structures that might not cover many real large-scale generalised flow-based networks. Therefore, system partitioning by means of graph theory is also used to cope with large-scale networks. Basically, the partitioning of a flow-based network consists in choosing subsets of the global variables to be assigned to different local agents that are in charge of controlling individual partitions/subsystems. Such subsets might be joint or disjoint, depending on the non-centralised control approach to be applied. In general, the optimal decomposition of networks is an open problem and is out of the scope of this thesis (nonetheless, some approaches have been proposed so far, see, e.g., [84, 87, 116, 128]). Particularly, this chapter addresses a large-scale network as a system-of-systems instead of analytically decomposing the global optimisation problem; the corresponding partitions will be assumed given from now on (as often done in the literature).

It has been demonstrated in [148] that exchanging only interaction information (even iteratively) among the local controllers is not enough to guarantee closed-loop stability and/or optimal plant-wide performance due to their competitive behaviour. Hence, for economically optimal operation (or to reduce sub-optimality) of the network, cooperation between local controllers must be induced. This latter can be achieved, e.g., by means of cooperative, coordinated or hierarchical MPC schemes, which incorporate negotiation/coordination mechanisms to approach the centralised solution. A crucial issue in all these non-centralised control schemes is that of guaranteeing recursive feasibility of the optimisation problem, specially when addressing dynamically coupled subsystems.

Among the non-centralised MPC schemes that have been proposed in the literature (see e.g., [123] and references therein), one important criterion of classification is the exchange of information between local agents (e.g., predicted trajectories, prices or dual variables), which in general can be either local or global. On the one hand, there are schemes that use local information and iterative communication to improve performance, guaranteeing feasibility mostly only upon convergence to the global optimal solution. To cope with feasibility losses (e.g., due to early termination of the iterative algorithm) other non-iterative distributed MPC schemes consider the shared variables as local disturbances and rely on the design of (possibly over-conservative) robust local controllers, guaranteeing feasibility of the network at the expense of a worse economic performance. On the other hand, there exists several cooperative approaches inspired in [170], which exchange global information and ensure recursive feasibility of the optimisation problem (even with non-iterative communication) by using centralised prediction models. Generally, these cooperative schemes converge asymptotically to the central optimum under certain structural assumptions, e.g., sparse couplings.

Most of the available non-centralised MPC algorithms were proposed to control systems operating under a standard (tracking) cost functions and only few cooperative (iterative) distributed economic MPC schemes have been recently published (see e.g., [50, 101]). Differently, this chapter proposes a non-iterative multi-layer distributed economic MPC (ML-DMPC) approach for its application to generalised flow-based networks. This approach is based on a temporal and functional decomposition of the centralised economic scheduling-control problem discussed in Chapter 3. The architecture of the proposed ML-DMPC controller lies in the class of hierarchical systems [114]. Specifically, the controller comprises two layers that operate at different time scales and interact to fulfil a set $\mathcal{O}$ of desired control objectives. In a top-down hierarchy, the control structure has a centralised coordinator in the upper layer and a set of local distributed MPC controllers in the lower layer.

Contrary to the standard coordinated distributed control structures [114], where the local controllers use local information and communicate iteratively only with the coordinator to reconstruct the centralised performance, the proposed ML-DMPC scheme considers non-iterative and hierarchical-like neighbour-to-neighbour communication between the local controllers and the coordinator is used to influence (also non-iteratively) the overall performance trough economic *intervention parameters*. The ML-DMPC controller aims to improve the performance of a decentralised MPC strategy (but still being globally sub-optimal) and to guarantee recursive feasibility of the related distributed algorithm.

7.2 Problem Formulation

In § 2.3.1, a method to obtain the monolithic state-space model of a given generalised flow-based network graph was described. Once the control-oriented model is stated, it is important to determine the objective of performing the partitioning of the physical network depending on the control strategy to be followed. For large-scale network flow problems, the partitioning of the system gains sense from the point of view of modularity of the control architecture and the reduction of computational burden. In any case, the way the network elements are interconnected is a key factor for performing the partitioning and control of the overall network since it determines the type of couplings between subsystems and consequently the complexity and rationality of the control strategy.

In the sequel, the overall system (2.10) is assumed to be decomposed in a set of $M \in \mathbb{Z}_{\geq 1}$ dynamically coupled non-overlapping subsystems denoted by $\mathcal{S}_i$, $i \in \mathbb{Z}_{[1,M]}$. The number M of subsystems is generally a tuning parameter. In this chapter, a two-stage decomposition is performed. In the first stage, a reachability analysis is used to define a set of subsystems that can be supplied only by one source each. These resultant subsystems are here called *anchored subsystems* and are denoted as $\mathcal{S}_i$, $i \in \mathbb{Z}_{[1,r]}$, where $r \leq M$, is the number of flow sources in the network. The remaining elements of the network are grouped in a subsystem denoted as $\tilde{\mathcal{S}}$, which is supplied by the cross-border outflows of the anchored subsystems. Such flows are considered as *pseudo-sources* of $\tilde{\mathcal{S}}$. In the second stage of the decomposition, subsystem $\tilde{\mathcal{S}}$ is later subdivided into $M - r$ subsystems by means of the graph-based partitioning algorithm proposed in [128]. This algorithm aims at decomposing $\tilde{\mathcal{S}}$ and its corresponding directed graph into sub-graphs, in such a way that all resultant partitions have nearly the same number of vertices and a hierarchical/sequential solution order can be stated. This feature balances computations and allows minimising communications/interactions between the local controllers involved. Note that another set of pseudo-sources may appear after the

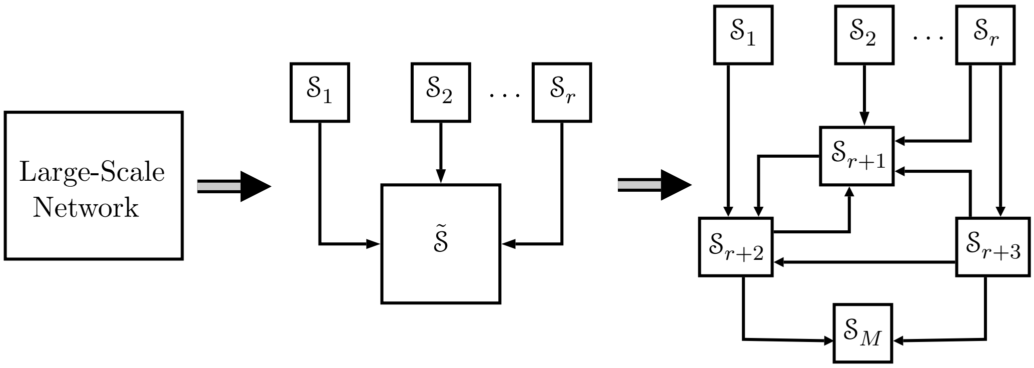


Figure 7.1: Decomposition of a network with r sources into M subsystems

decomposition of $\tilde{\mathcal{S}}$ and, contrary to the first stage of decomposition, each subsystem may have both entering and leaving cross-border flows depending on the interconnections of the resultant $\mathcal{S}_i$ subsystems, $i \in \mathbb{Z}_{[r+1,M]}$. A sketch of the overall decomposition process is depicted in Figure 7.1.

Particularly, due to Assumption 2.6, this thesis considers only input-coupled dynamics and input-coupled constraints. Then, each subsystem can be described by the following discrete-time linear model:

$$\begin{cases} x_{k+1}^{[i]} = A_{ii}x_k^{[i]} + B_{ii}u_k^{[i]} + B_{d,ii}d_k^{[i]} + \sum\limits_{\substack{j=1\\ j\neq i}}^{M} B_{ij}u_k^{[j]}, & (7.1a)\\ 0 = E_{u,ii}u_k^{[i]} + E_{d,ii}d_k^{[i]} + \sum\limits_{\substack{j=1\\ j\neq i}}^{M} E_{u,ij}u_k^{[j]}, & (7.1b)\end{cases}$$

for all $k \in \mathbb{Z}_+$ and $i, j \in \mathbb{Z}_{[1,M]}$, where $x_k^{[i]} \in \mathbb{R}^{n_i}$, $u_k^{[i]} \in \mathbb{R}^{m_i}$ and $d_k^{[i]} \in \mathbb{R}^{p_i}$ are respectively the local state, input and demand vectors of subsystem $\mathcal{S}_i$, $i \in \mathbb{Z}_{[1,M]}$. Local matrices are given by the topology of each subsystem, with $A_{ii} = I_{n_i}$, $B_{ii} \in \mathbb{R}^{n_i \times m_i}$, $B_{d,ii} \in \mathbb{R}^{n_i \times p_i}$, $B_{ij} \in \mathbb{R}^{n_i \times m_j}$, $E_{u,ii} \in \mathbb{R}^{q_i \times m_i}$, $E_{d,ii} \in \mathbb{R}^{q_i \times p_i}$ and $E_{u,ij} \in \mathbb{R}^{q_i \times m_i}$ for all $i, j \in \mathbb{Z}_{[1,M]}$. The decomposition assures that $\sum_{i=1}^{M} n_i = n$, $\sum_{i=1}^{M} m_i = m$, $\sum_{i=1}^{M} p_i = p$ and $\sum_{i=1}^{M} q_i = q$ for all $n_i, m_i, p_i, q_i \in \mathbb{Z}_{\geq 1}$. Similarly, the global constraint sets $\mathbb{X}$, $\mathbb{U}$ and $\mathbb{D}$ are decomposed to give place to a set of local constraints defined by:

$$x_k^{[i]} \in \mathbb{X}_i := \{x^{[i]} \in \mathbb{R}^{n_i} \mid 0 \leq x^{[i]} \leq x_{\max}^{[i]}\}, \tag{7.2a}$$
$$u_k^{[i]} \in \mathbb{U}_i := \{u^{[i]} \in \mathbb{R}^{m_i} \mid 0 \leq u^{[i]} \leq u_{\max}^{[i]}\}, \tag{7.2b}$$
$$d_k^{[i]} \in \mathbb{D}_i := \{d^{[i]} \in \mathbb{R}^{p_i} \mid 0 \leq d^{[i]} \leq d_{\max}^{[i]}\}. \tag{7.2c}$$

Definition 7.1 (Neighbour and neighbourhood)**.** *A subsystem $\mathcal{S}_j$ is defined as a* neighbour *of subsystem $\mathcal{S}_i$ if and only if $B_{ij} \neq 0$ or $E_{u,ij} \neq 0$, $j \in \mathbb{Z}_{[1,M]}$, $j \neq i$. Hence, the* neighbourhood *of $\mathcal{S}_i$ is defined as $\mathcal{N}_i := \{j \in \mathbb{Z}_{[1,M]} \mid B_{ij} \neq 0$ or $E_{u,ij} \neq 0,\ j \neq i\}$.*

Remark 7.1. *Note that the overall system model can be obtained by the composition of the above M subsystems, as follows:*

$$\begin{cases} x_{k+1} = Ax_k + Bu_k + B_d d_k, \\ \quad\ \ 0 = E_u u_k + E_d d_k, \end{cases}$$

where the vectors and matrices are now a permutation of the original ones, with

$$x_k = \begin{bmatrix} x_k^{[1]} \\ \vdots \\ x_k^{[M]} \end{bmatrix}, \quad u_k = \begin{bmatrix} u_k^{[1]} \\ \vdots \\ u_k^{[M]} \end{bmatrix}, \quad d_k = \begin{bmatrix} d_k^{[1]} \\ \vdots \\ d_k^{[M]} \end{bmatrix}, \tag{7.3}$$

and

$$A = \begin{bmatrix} I_{n_1} & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & I_{n_M} \end{bmatrix}, \qquad B = \begin{bmatrix} B_{11} & \dots & B_{1M} \\ \vdots & \ddots & \vdots \\ B_{M1} & \dots & B_{MM} \end{bmatrix},$$
$$B_d = \begin{bmatrix} B_{d,ii} & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & B_{d,MM} \end{bmatrix}, \quad E_u = \begin{bmatrix} E_{u,11} & \dots & E_{u,1M} \\ \vdots & \ddots & \vdots \\ E_{u,M1} & \dots & E_{u,MM} \end{bmatrix},$$
$$E_d = \begin{bmatrix} E_{d,ii} & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & E_{d,MM} \end{bmatrix}.$$

Moreover, since the dynamic and static nodes of the network were decomposed into M disjoint subsets, it follows that the global constraint sets can be recovered as Cartesian products, i.e.,

$$\mathbb{X} = \prod_{i=1}^{M} \mathbb{X}_i, \quad \mathbb{U} = \prod_{i=1}^{M} \mathbb{U}_i, \quad \mathbb{D} = \prod_{i=1}^{M} \mathbb{D}_i. \tag{7.4}$$

$\Diamond$

Before getting through the design of the ML-DMPC strategy, the following preliminary assumptions related to the overall system are stated.

Assumption 7.1. *All sinks have a periodic flow request (with period $T \in \mathbb{Z}_{\geq 1}$) that can be supplied by at least one flow source through at least one flow path*[1].

Assumption 7.2. *The required control objectives can be grouped in a set $\mathcal{O} = \mathcal{O}_l \cup \mathcal{O}_g$, which is a composition of a set $\mathcal{O}_l$ of local control objectives and a set $\mathcal{O}_g$ of* global *control objectives. Moreover, $m_l \triangleq |\mathcal{O}_l|$, $m_g \triangleq |\mathcal{O}_g|$, and hence $m_l + m_g = |\mathcal{O}|$.*

Assumption 7.2 allows to rewrite a centralised general economic stage cost function $\ell : \mathbb{Z}_+ \times \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}_+$ in the following form:

$$\ell(k, x_k, u_k) = \sum_{g=1}^{m_g} \gamma_g \ell_g(k, x_k, u_k) + \sum_{l=1}^{m_l} \gamma_l \ell_l(k, x_k, u_k), \tag{7.5}$$

where $\gamma_g, \gamma_l \in \mathbb{R}_+$ are scalar weights that prioritise, within the overall cost function, each global and local control objective, particularly represented by convex functions $\ell_g : \mathbb{Z}_+ \times \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}_+$ and $\ell_l : \mathbb{Z}_+ \times \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}_+$, respectively. Hence, from (7.1), (7.2) and Remark 7.1, the centralised MPC optimisation problem with stage cost (7.5) and prediction horizon N can be rewritten as follows:

$$\min_{\mathbf{u}_k} \sum_{t=0}^{N-1} \left(\sum_{g=1}^{m_g} \gamma_g \ell_g(k, x_{k+t|k}, u_{k+t|k}) + \sum_{l=1}^{m_l} \gamma_l \ell_l(k, x_{k+t|k}, u_{k+t|k}) \right), \tag{7.6a}$$

subject to:

$$x^{[i]}_{k+t+1|k} = A_{ii} x^{[i]}_{k+t|k} + B_{ii} u^{[i]}_{k+t|k} + B_{d,ii} d^{[i]}_{k+t|k} + \sum_{\substack{j=1 \\ j \neq i}}^{M} B_{ij} u^{[j]}_{k+t|k}, \tag{7.6b}$$

$$0 = E_{u,ii} u^{[i]}_{k+t|k} + E_{d,ii} d^{[i]}_{k+t|k} + \sum_{\substack{j=1 \\ j \neq i}}^{M} E_{u,ij} u^{[j]}_{k+t|k}, \tag{7.6c}$$

$$(x^{[i]}_{k+t+1|k}, u^{[i]}_{k+t|k}) \in \mathbb{X}_i \times \mathbb{U}_i, \tag{7.6d}$$

$$x^{[i]}_{k|k} = x^{[i]}_k, \tag{7.6e}$$

for all $i \in \mathbb{Z}_{[1,M]}$ and all $t \in \mathbb{Z}_{[0,N-1]}$. The aggregate state and input vectors in the cost function are given by $x_{k+t|k} = (x^{[1]^\top}_{k+t|k}, \ldots, x^{[M]^\top}_{k+t|k})^\top$, $u_{k+t|k} = (u^{[1]^\top}_{k+t|k}, \ldots, u^{[M]^\top}_{k+t|k})^\top$, respectively. The decision variable is the input sequence $\mathbf{u}_k = \{u_{k+t|k}\}_{t \in \mathbb{Z}_{0,N-1}}$.

[1] A flow path is an ordered sequence of arcs, which may connect sources, intermediate nodes and sinks.

Thus, the goal of the ML-DMPC approach proposed in this chapter is that of solving (7.6) in a distributed fashion in order to cope with the aforementioned disadvantages of a centralised controller. To do so, a set $\mathcal{C} := \{C_1, \ldots, C_M\}$ of local controllers, their communication network and a coordination mechanism are designed in the following to properly address the effect of couplings between subsystems and to take into account Assumption 7.2.

7.3 Description of the Approach

The whole ML-DMPC set-up consists on: (i) an *upper* layer in charge of achieving the global objectives by solving a centralised optimisation problem with a sampling time Δt_1, and (ii) a lower layer comprising a set of distributed MPC agents that compute the references for the system actuators in order to satisfy the local objectives. This latter layer operates with a sampling time Δt_2 ($\Delta t_2 \leq \Delta t_1$). The local controllers solve their associated optimisation problem in a hierarchical/sequential fashion and exchange (non-iteratively) in a neighbour-to-neighbour communication strategy the predicted sequence of the inputs affecting neighbouring subsystems. The upper layer influences the operation of the lower layer by projecting global economic information into the local agents, specifically by modifying the prices/weights of the flow arcs that are shared among the subsystems arising in the lower layer. Figure 7.2 shows the proposed control structure. The ML-DMPC scheme leads to a suboptimal plant-wide performance but with the advantage of a tractable implementation due to a hierarchical/sequential communication approach that avoids negotiations among local controllers. In what follows, a formal description of the two optimisation layers involved in the ML-DMPC approach and their interaction is given.

7.3.1 Lower Optimisation Layer

Once the network partitioning is performed and the M local models are obtained, it only remains to distribute the original centralised economic MPC problem among the local controllers C_i, considering the given management policies and constraints.

In order to simplify the notation, let rewrite the interaction-oriented local models in

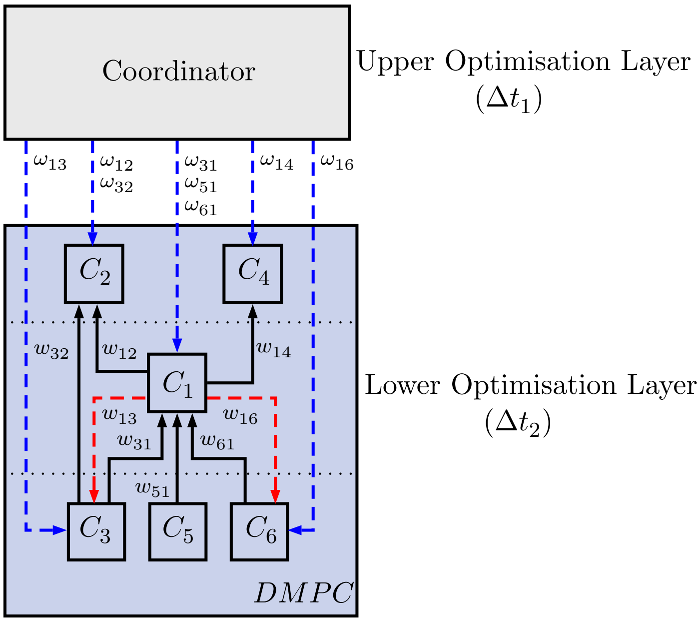


Figure 7.2: ML-DMPC control architecture

the following more compact form:

$$\begin{cases} x_{k+1}^{[i]} = A_{ii}x_k^{[i]} + B_{ii}u_k^{[i]} + B_{d,ii}d_k^{[i]} + \bar{B}_i w_k^{[i]} & (7.7a) \\ \quad\; 0 = E_{u,ii}u_k^{[i]} + E_{d,ii}d_k^{[i]} + \bar{E}_i w_k^{[i]}, & (7.7b) \end{cases}$$

for all $i \in \mathbb{Z}_{[1,M]}$, where $w_k^{[i]} := (w_{i_1,k}^\top, \dots, w_{i_{|\mathcal{N}_i|},k}^\top)^\top \in \mathbb{W}_i$ is a vector stacking the flows decided by the controllers of neighbours of subsystem $\mathcal{S}_i$, being $\{i_1, \dots, i_{|\mathcal{N}_i|}\}$ an ordered sequence of the indices contained in the set $\mathcal{N}_i$ (that is, $i_1 < \dots < i_{|\mathcal{N}_i|}$) and $w_{j,k} := T_{w_j}^\top u_k^{[j]}$ for all $j \in \mathcal{N}_i$. In the definition of each $w_{j,k}$, the matrix $T_{w_j} \in \mathbb{R}^{m_j \times m_{ij}}$ ($T_{w_j}^\top T_{w_j} = I_{m_{ij}}$) is such that it collects the m_{ij} ($m_{ij} < m_j$) columns of the identity matrix of order m_j, corresponding to the indices of the rows of $\tilde{u}_k^{[j]} \in \mathbb{R}^{n_j}$ related to the controlled flows decided by the controller C_j and affecting subsystem $\mathcal{S}_i$. Moreover, matrices $\bar{B}_i$ and $\bar{E}_i$ are suitably defined to represent the effect of $w_k^{[i]}$ on the local state vector $x_k^{[i]}$, and the set $\mathbb{W}_i$ is obtained appropriately from $\mathbb{U}_i$. In the sequel, every subsystem $\mathcal{S}_j$ that impose an outflow $w_{j,k}$ to a subsystem $\mathcal{S}_i$ will be considered as a *virtual sink* of $\mathcal{S}_i$.

Interpretation 7.1. *At any time step $k \in \mathbb{Z}_+$ when the controlled flow $u_k^{[i]}$ is computed, the controller C_i has knowledge of the state $x_k^{[i]}$ and the demands $d_k^{[i]}$ and $w_k^{[i]}$ imposed by* *the local and virtual sinks, respectively. Future demands $d_{k+t}^{[i]}$ and $w_{k+t}^{[i]}$ might be unknown for all $t \in \mathbb{Z}_{\geq 1}$ and can take arbitrary values in $\mathbb{D}_i$ and $\mathbb{W}_i$, respectively. Nevertheless, the controller C_i has also knowledge of the N-step sequences of both the local and virtual demand expectations.*

Each controller C_i will be in charge of deciding only the network flows corresponding to subsystem $\mathcal{S}_i$ by using local and neighbouring information under Interpretation 7.1. In this chapter, the local problems are defined in such a way that each of them considers a local stage cost function but with a structure similar to the one in (7.5). Specifically, the stage cost function related to each C_i is written as

$$\ell_i(k, x_k^{[i]}, u_k^{[i]}) = \sum_{g=1}^{m_g} \hat{\gamma}_{g,i} \hat{\ell}_{g,i}(k, x_k^{[i]}, u_k^{[i]}) + \sum_{l=1}^{m_l} \gamma_{l,i} \ell_{l,i}(k, x_k^{[i]}, u_k^{[i]}), \tag{7.8}$$

where each $\hat{\ell}_{g,i}$, $g \in \mathbb{Z}_{[1,m_g]}$, corresponds to the g-th global control objective properly expressed and weighted with a suitable $\hat{\gamma}_{g,i} \in \mathbb{R}_+$ in order to influence controllers C_i to improve plant-wide performance. Moreover, each $\ell_{l,i}$ is assumed to be the corresponding part of the separable local objectives ℓ_l, $l \in \mathbb{Z}_{[1,m_l]}$, related to the subsystem $\mathcal{S}_i$.

For each subsystem $\mathcal{S}_i$, a portion of control authority is removed by its neighbours and added to its local uncertainty in a max-min sense due to the local knowledge considered in Interpretation 7.1. Hence, before fully devising the distributed MPC controllers operating in the lower layer, the following definition (adjusted from [13, Definition 4.1] for the max-min case) is introduced.

Definition 7.2. *Denote a given* network decomposition *with $\Delta = \{\mathcal{S}_i\}_{i \in \mathbb{Z}_{1,M}}$ and let $\mathcal{C}_\infty^{\mathcal{S}_i}$ be the maximal max-min robust control invariant set for subsystem $\mathcal{S}_i$. Then, the* decentralised max-min robust control invariant set *for the overall system* (2.10) *subject to constraints* (2.11) *and decomposed into Δ is given by $\mathcal{C}_\infty^\Delta = \prod_{i=1}^M \mathcal{C}_\infty^{\mathcal{S}_i}$.*

For a given network decomposition Δ and local sets $\mathbb{X}_i$, $\mathbb{U}_i$, $\mathbb{D}_i$ and $\mathbb{W}_i$, $i \in \mathbb{Z}_{[1,M]}$, each maximal max-min robust control invariant set $\mathcal{C}_\infty^{\mathcal{S}_i}$ (see Definition 3.4) can be explicitly computed as done in § 3.3 for the overall network. Note that such sets $\mathcal{C}_\infty^{\mathcal{S}_i}$ may result to be empty for a given Δ (consequently $\mathcal{C}_\infty^\Delta = \emptyset$), which implies that there is no guarantee that a decentralised control strategy will be feasibility for all times. In such a case, the sets $\mathbb{U}_i$ (accordingly $\mathbb{W}_i$), $i \in \mathbb{Z}_{[1,M]}$, should be properly modified to make the decentralised design of $\mathcal{C}_\infty^\Delta$ possible, see e.g., [13].

Assumption 7.3. *The local constraint sets arising for a given network decomposition* $\Delta = \{\mathcal{S}_i\}_{i \in \mathbb{Z}_{1,M}}$ *are such that*

$$B_{d,ii}\mathbb{D}_i \oplus \bar{B}_i \mathbb{W}_i \subseteq -B_{ii}\mathbb{U}_i \quad \textit{and} \quad E_{d,ii}\mathbb{D}_i \oplus \bar{E}_i \mathbb{W}_i \subseteq -E_{u,ii}\mathbb{U}_i,$$

for all $\mathcal{S}_i \in \Delta$*. Hence,* $\mathcal{C}_\infty^{\mathcal{S}_i} := \big((\mathbb{X}_i \oplus (-B_{ii}\mathbb{U}_i)) \ominus \big(B_{d,ii}\mathbb{D}_i \oplus \bar{E}_i\mathbb{W}_i\big)\big) \cap \mathbb{X}_i \neq \emptyset$.

Even when Assumption 7.3 holds and $\mathcal{C}_\infty^{\Delta}$ exists, the algebraic equation (7.7b) for each local model acts as a coupling constraint that forbids the design of non-iterative distributed controllers with parallel solution of the local optimisation problems. Thus, the distributed MPC algorithm considered in the lower layer of the proposed ML-DMPC approach consists in a non-iterative communication-based MPC design that builds on the hierarchical-like decentralised MPC approach reported in [127]. The strategy proposed here also follows a hierarchical sequence of solution but considering conditions to deal with the existence of bidirectional complicating flows between neighbour subsystems. The optimisation problem to be solved in the lower layer of the ML-DMPC by each local controller C_i, $i \in \mathbb{Z}_{[1,M]}$, with sampling time Δt_2, is defined as follows:

$$\min_{\mathbf{u}_k} \sum_{t=0}^{N-1} \left(\sum_{g=1}^{m_g} \hat{\gamma}_{g,i}\hat{\ell}_{g,i}(k, x^{[i]}_{k+t|k}, u^{[i]}_{k+t|k}) + \sum_{l=1}^{m_l} \gamma_{l,i}\ell_{l,i}(k, x^{[i]}_{k+t|k}, u^{[i]}_{k+t|k}) \right), \tag{7.9a}$$

subject to:

$$x^{[i]}_{k+t+1|k} = A_{ii}x^{[i]}_{k+t|k} + B_{ii}u^{[i]}_{k+t|k} + B_{d,ii}d^{[i]}_{k+t|k} + \bar{B}_i w^{[i]}_{k+t|k}, \forall t \in \mathbb{Z}_{[0,N-1]} \tag{7.9b}$$
$$0 = E_{u,ii}u^{[i]}_{k+t|k} + E_{d,ii}d^{[i]}_{k+t|k} + \bar{E}_i w^{[i]}_{k+t|k}, \quad \forall t \in \mathbb{Z}_{[0,N-1]} \tag{7.9c}$$
$$x^{[i]}_{k+1|k} \in \mathcal{C}_\infty^{\mathcal{S}_i}, \tag{7.9d}$$
$$x^{[i]}_{k+t|k} \in \mathbb{X}_i, \quad \forall t \in \mathbb{Z}_{[2,N]} \tag{7.9e}$$
$$u^{[i]}_{k+t|k} \in \mathbb{U}_i, \quad \forall t \in \mathbb{Z}_{[0,N-1]} \tag{7.9f}$$
$$u^{[i]}_{(r),k|k} = u^{[i]\star}_{(r),k+1|k-1}, \quad \forall r \in \mathbb{I}_u \tag{7.9g}$$
$$x^{[i]}_{k|k} = x^{[i]}_k, \tag{7.9h}$$

where $\mathbb{I}_u \subset \mathbb{Z}_+$ is a set containing the indices of all the rows of vector $u^{[i]}_k$ related to the inputs decided locally by C_i but affecting neighbours whose controllers C_j are located in higher levels of the pre-defined hierarchy of solution. Moreover, $u^{[i]\star}_{(r),k+1|k-1}$ denotes the

r row of the optimal local input vector $u^{[i]}$ computed at time step $k-1$ for the predicted step $k+1$.

Comparing with the algorithms in [127, 131], problem (7.9) has two subtle but important differences:

(i) The incorporation of (7.9d) as a robustness constraint that enforces the predicted state to lie within the maximal max-min robust control invariant set at the first prediction step.

(ii) The incorporation of (7.9f), restricting those components of the first control action that are decided locally but affect neighbouring subsystems whose controllers are located at higher levels of the solution hierarchy.

As demonstrated in [90, Chapter 6] for a min-max interpretation in a standard centralised MPC controller, the robustness constraint (7.9d) leads to a robust strongly feasible MPC algorithm. Nonetheless, this constraint by its own cannot guarantee recursive feasibility of the overall distributed MPC solution sequence because $\mathcal{C}_\infty^{\mathcal{S}_i}$ is computed under Interpretation 7.1, which requires that each controller C_i knows at least the first demand value of its local and virtual sinks (i.e., $d_k^{[i]}$ and $w_k^{[i]}$ when solving at k). This latter requirement is not fulfilled if controllers C_i are allowed to freely optimise their full input vector without considering their effect in the hierarchical sequence of solution of the non-iterative ML-DMPC approach. To exemplify this observation, assume that a controller C_j optimises the flow of a complicating arc affecting a subsystem $\mathcal{S}_i$ whose controller C_i has already solved the i-th problem in the solution sequence. Then, the trajectory obtained by C_j could be infeasible (specially due to the equality coupling constraint (7.9c)) for $\mathcal{S}_i$ since $w_k^{[i]}$ might be changed and C_i does not have the chance to recompute its solution. Hence, constraint (7.9f) results to be an extra necessary condition to satisfy Interpretation 7.1 and retain feasibility of the overall sequence of local problems.

7.3.2 Upper Optimisation Layer

The fulfilment of a global objective from a local point of view often implies information from the entire network, but this is lost when the system partitioning is performed.

Therefore, it is necessary to figure out how to induce cooperation among the set of distributed controllers, considering all the control objectives belonging to $\mathcal{O}$ in a suitable way.

As previously mentioned in § 1.2.5 and § 7.1, one common way to improve overall closed-loop performance of a decentralised/distributed control scheme is to incorporate a supervisor controller or coordinator on top of the local controllers. Two frequently used coordination methods are the *goal coordination* and the *interaction prediction coordination* (cf., [114]). The fundamental idea behind these approaches is to have independent subproblems containing certain coordinating parameters (e.g., Lagrange multipliers, co-state variables, pseudo-variables, etc.) in addition to the local decision variables. In both coordination methods, duality theory is used as a standard to construct an equivalent two-level problem to the primal (centralised) optimisation problem. Within such framework, the coordinating parameters are updated iteratively by the coordinator based on the local solutions until an optimal solution to the overall system is achieved (cf. [42, 114]). Feasibility of these coordinated control strategies is guarantee only upon convergence.

Contrary to the common methods, the upper optimisation layer of the ML-DMPC approach proposed in this chapter is not focused on reconstructing the centralised optimal solution in an iterative manner but to improve the economic performance of the local MPC controllers by intervening in their decision process with a low frequency of intervention. Specifically, this upper layer influences the local solutions by computing, in a non-iterative way, the weight $\omega \in \mathbb{R}^{n_\omega}$ (being n_ω the number of arcs interconnecting the subsystems) related to the pseudo-sources discussed in § 7.2 that appears after the selected network decomposition method (see Figure 7.1). The weights in ω will affect the first term in the local cost function (7.9a) of each controller C_i, $i \in \mathbb{Z}_{[1,M]}$. Therefore, to compute ω, a centralised optimisation problem based on a temporal and functional decomposition of the network is stated in the upper layer of the ML-DMPC by considering: (i) a static model of the whole network, and (ii) a cost function that only takes into account the global control objectives associated to the system.

The proposed upper optimisation layer works with a sampling time $\Delta t_1 = T$, where $T \in \mathbb{Z}_{\geq 1}$ corresponds to the period of the periodic flow requested by local sinks (see Assumption 7.1). Thus, when looking at the volume evolution of storage elements, they show a similar behaviour as the flow to the sinks, i.e., volumes might also show a periodic behaviour with period T. For this reason, when modelling the network with sampling time Δt_1, it can be assumed that volumes do not change. From now on, sub-index c is used to differentiate the temporal scale of the model in the upper layer to that of the lower layer (e.g., $x_{c,k}$ denotes the state at the coordinator level at time step k with sampling time Δt_1). Hence, storage nodes behave as static nodes in this layer and the network dynamic model (2.10a) becomes a stationary model, i.e., $x_{c,k} = A_c x_{c,k} + B_c u_{c,k} + B_{d,c} d_{c,k}$.

Having the stationary model considered by the coordinator and the functional

$$\ell_{up}(k, x_{c,k}, u_{c,k}) := \sum_{g=1}^{m_g} \gamma_{g,c} \ell_{g,c}(x_{c,k}, u_{c,k}), \tag{7.10}$$

the upper layer optimisation problem is here proposed to be formulated for a generalised flow-based network as the search of the economically optimal path flows from sources nodes to sink nodes.

Definition 7.3 (Directed path). *Given a directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{A})$ of a network, a directed path is an ordered sequence of nodes $v_1, v_2, \ldots, v_n$ in which there is an arc (i, j) pointing from each node i in the sequence to its successor node j in the sequence, that is, $\{(v_1, v_2)(v_2, v_3), \ldots, (v_{n-1}, v_n)\}$.*

To mathematically and systematically find all flow paths in a given network this chapter follows the methodology in [38, Appendix A], which exploits the information contained in the node-arc incidence matrix of the network directed graph to construct the path-arc matrix for the given sources and sinks. The description of such algorithm is omitted here for brevity and the reader is referred to the aforementioned reference. Once the path-arc matrix is obtained, a constrained optimisation problem can be stated to minimise (7.10) in terms of path flows, which are denoted here as $u_p \in \mathbb{R}^{n_p}$ with n_p the number of possible paths. Hence, the coordinator solves in the upper layer of the ML-DMPC, an optimisation problem with the following structure:

$$\min_{u_p} \hat{\ell}_{up}(x_{c,k}, u_{p,k}) \tag{7.11a}$$

subject to:

$$A_p u_{p,k} \leq b_{p,k}, \tag{7.11b}$$

$$A_{eq} u_{p,k} = b_{eq,k}, \tag{7.11c}$$

where function $\hat{\ell}_{up}$ is equivalent to (7.10) but properly expressed in terms of the path flows $u_{p,k}$ by using the graph path-arc matrix. Moreover, constraint (7.11b) is used to consider the physical bounds of each actuator involved in each path, while constraint (7.11c) is used to enforce satisfaction of demands $d_{c,k}$. Matrices A_p and A_{eq} and vectors b_p and b_{eq} are defined accordingly to the considered bounds and balance constraints.

Throughout this thesis it has been assumed that the flow at each arc of the network is driven by an actuator. Therefore, by using the optimal solution of problem (7.11) and the information contained in the path-arc matrix of the overall network, it is possible to compute the accumulated cost incurred in traversing all the paths that reach the intermediate nodes from which the arcs interconnecting the M subsystems depart. This accumulated cost information, in addition to Assumption 7.1, allow to define the weight ω as a coordinating economic parameter. This weight is used by the coordinator to project, into the cost function of each local controller C_i, the economic impact (from a global point of view) that each subsystem $\mathcal{S}_i$ will suffer when requesting flow from its neighbour subsystems.

In network flow problems, the global objectives are often given as a composition of economic linear cost functions. In such a case, the value of ω can be obtained by following Algorithm 3. Note that Assumption 7.1 and the temporal scale selected for the upper layer make (7.11) independent of the state. Furthermore, the weight ω is more an intervention parameter than a coordination variable since the upper layer does not use any feedback information from the local controllers allocated at the lower layer.

7.3.3 ML-DMPC Algorithm

The sharing of information between the two layers of the proposed ML-DMPC approach depends on the nature and features of each application. For the case considered in this chapter (i.e., periodic demands), the interaction is unidirectional from the upper optimisation layer to the lower optimisation layer. Once the optimisation problem related to the upper layer is solved, the resultant parameters are properly updated for each optimisation problem behind each C_i, $i \in \mathbb{Z}_{[1,M]}$. This updating is performed with a periodicity Δt_1 to consider possible changes in the periodic pattern of demands. In fact, if a given application involves an agreement of pre-defined demands to be satisfied, the optimisation problem of the upper layer needs to be executed only once at the beginning

Algorithm 3 Computation of the economic intervention parameter ω

1: Compute the path-arc matrix of the network graph, denoted here by $R_p \in \mathbb{R}^{n_p \times m}$.
2: Define a matrix $C_p \in \mathbb{R}^{n_p \times m}$ with the same structure of matrix R_p but containing in each matrix element the unitary flow cost of each actuator in each possible path.
3: Identify all the arcs interconnecting subsystems $\mathcal{S}_i$, $i \in \mathbb{Z}_{[1,M]}$, and denote with $n_{u_s} \in \mathbb{Z}_+$ the number of such arcs, called from now on as complicating arcs.
4: Solve problem (7.11) and identify from the optimal solution all the paths in which each complicating arc participates, and denote by $n_{p_j} \in \mathbb{Z}_+$, $j \in \mathbb{Z}_{[1,n_{u_s}]}$, the numbers of such paths.
5: Define a set of matrices $T_{s_j} \in \mathbb{R}^{n_p \times n_{p_j}}$, $j \in \mathbb{Z}_{[1,n_{u_s}]}$, satisfying $T_{s_j}^\top T_{s_j} = I_{n_{p_j}}$, each of them collecting the n_{p_j} columns of the identity matrix of order n_p.
6: Define a set of matrices $R_{p_j} := T_{s_j}^\top R_p$ and $C_{p_j} := T_{s_j}^\top C_p$ for all $j \in \mathbb{Z}_{[1,n_{u_s}]}$.
7: From the sequential order of the directed paths involved in each matrix R_{p_j}, define a set of matrices $\tilde{R}_{p_j}$ whose elements will be the same as the ones in matrices R_{p_j} for all the positions related to the sequential arcs that reach the complicating arcs (these latter included) in each path, and zero in those matrix elements related to the successor arcs.
8: Define the vector $\omega := (\omega_1, \dots, \omega_{n_{u_s}})^\top$, with each of its components computed as

$$\omega_j = \frac{\mathbb{1}_{n_{u_s}}^\top \left(\left(C_{p_j} \circ R_{p_j} \right) \circ \tilde{R}_{p_j} \right)^\top T_{s_j}^\top u_{p,k}^\star}{\left[R_{p_j}^\top T_{s_j}^\top u_{p,k}^\star \right]_{(r_j)}}, \quad \forall j \in \mathbb{Z}_{[1,n_{u_s}]}$$

where $\mathbb{1}_{n_{u_s}}$ denotes an all-ones column vector of length n_{u_s}, the operator $(\circ)$ indicates the Hadamard product of matrices and $[\cdot]_{(r_j)}$ is the r_j row of the vector in the brackets with r_j being the position of the associated j-th complicating arc in the input vector $u_{c,k}$. Then, ω_j represents a unitary cost per flow unit.

of the operation. In general, the computation time that the upper layer spends is quite low with respect to the computation time of the lower layer. This fact is due to the difference in the nature of the models handled by each layer and the interactions given by the distributed MPC controllers as well as their amount and disposition within the defined hierarchy. Algorithm 4 collects the main steps of the proposed ML-DMPC approach. The computation time spent by the scheme corresponds with the sum of maximum times of each hierarchical level of controllers.

One important property desired in the design of any MPC strategy is recursive feasibility. In the following, it is shown that the proposed ML-DMPC algorithm remains

Algorithm 4 Non-iterative Multi-Layer Distributed Economic MPC

1: **Initialisation:** Set $k = 0$, establish an arbitrary weight ω in the upper layer and send that information to every local controller C_i, $i \in \mathbb{Z}_{[1,M]}$. For each current local state $x_k^{[i]}$ and local demand sequence $\mathbf{d}_k^{[i]} = \{d_k^{[i]}, \bar{d}_{k+1|k}^{[i]}, \ldots, \bar{d}_{k+N-1|k}^{[i]}\}$, find for all subsystems $\mathcal{S}_i$ a feasible (not necessarily optimal) pair of state and input sequences $(\mathbf{x}_k^{[i]} = \{x_{k+t|k}\}_{t\in\mathbb{Z}_{[0,N]}}, \mathbf{u}_k^{[i]} = \{u_{k+t|k}\}_{t\in\mathbb{Z}_{[0,N-1]}})$. Apply $u_{k|k}^{[i]}$ in every subsystem and transmit each $\mathbf{u}_k^{[i]}$ to the controllers of the corresponding neighbours of each $\mathcal{S}_i$.

2: **Collecting of information:** After receiving all the neighbour trajectories $\mathbf{u}_k^{[j]}$, $j \in \mathcal{N}_i$, each controller C_i builds the trajectory $\mathbf{w}_k^{[i]} = \{w_{k+t|k}^{[i]}\}_{\in\mathbb{Z}_{[0,N-1]}}$, differencing between shared inputs to be imposed by controllers arranged in higher levels of hierarchy and shared inputs planned by controllers arranged in the same or lower levels of hierarchy. These imposed and planned input trajectories are formed locally as $\mathbf{w}_{a,k}^{[i]} = \{w_{a,k|k}^{[i]\star}, \ldots, w_{a,k+N-1|k}^{[i]\star}\}$ and $\mathbf{w}_{a,k}^{[i]} = \{w_{b,k+1|k-1}^{[i]\star}, \ldots, w_{b,k+N-1|k}^{[i]\star}, w_{b,k+1|k-1}^{[i]\star}\}$, respectively, and it is assumed that $w_{k+t|k}^{[i]} = (w_{a,k}^{[i]\top}, w_{b,k}^{[i]\top})^\top$. At each sampling time, obtain $x_k^{[i]}$ and $\mathbf{d}_k^{[i]}$ for each subsystem $\mathcal{S}_i$.

3: **Solution of local problems**: Solve each optimisation problem (7.9) following a predefined hierarchical sequence.

4: **Implementation of control action:** Each local controller C_i applies $\kappa_i(x_k^{[i]}, \mathbf{u}_k^{[i]}, \mathbf{d}_k^{[i]}, \mathbf{w}_k^{[i]}) = u_{k|k}^{[i]\star}$ to the associated subsystem $\mathcal{S}_i$. Transmit each $\mathbf{u}_k^{[i]}$ to the controllers of the corresponding neighbours of each $\mathcal{S}_i$.

5: **Updating of the economic intervention parameter:** If $\lfloor k \rfloor_{\Delta_1} \in \mathbb{Z}_+$, then solve problem (7.11) for the current $\mathbf{d}_k$ and update ω following Algorithm 3. Send the new weight to each local controller C_i. Otherwise, go to step 5.

6: Increment k and go to step 2.

feasible for all times if initial feasibility is assumed. The guarantee of feasibility of the approach is unrelated to optimality of the distributed solution.

Theorem 7.1. *Let Assumptions 7.1 to 7.3 hold and suppose that an initial feasible solution in Step 1 of Algorithm 4 exists. Then, each local MPC problem (7.9) solved in Step 3 of Algorithm 4 is robust strongly feasible for each subsystem $\mathcal{S}_i \in \Delta$.*

Proof: The proof is by induction, showing that feasibility at time step k implies feasibility at time step $k+1$. Let $x_k^{[i]}$ be a feasible initial condition for each local problem (7.9) and assume that there exists a pair of feasible (not necessarily optimal) state-input trajectories given by $(\mathbf{x}_k^{[i]}, \mathbf{u}_k^{[i]})$ for each subsystem $\mathcal{S}_i \in \Delta$. Consider now the hierarchical flow of the solution at the next time step $k+1$. Since each subsystem applied previously the first control action of the initial feasible trajectory $\mathbf{u}_k^{[i]}$, it follows then that $x_{k+1}^{[i]} =$

$x^{[i]}_{k+1|k}$ and from constraint (7.9d) it holds that $x^{[i]}_{k+1} \in \mathcal{C}^{\mathcal{S}_i}_{\infty}$ for all $i \in \mathbb{Z}_{[1,M]}$. Since $\mathcal{C}^{\mathcal{S}_i}_{\infty} \neq \emptyset$ by Assumption 7.3, it follows from the invariance property of $\mathcal{C}^{\mathcal{S}_i}_{\infty}$ (see Definitions 3.3 and 3.4 but adjusted to consider $\mathbf{w}^{[i]}_k$ as an additional local disturbance) that for all $(x^{[i]}_{k+1}, \mathbf{d}^{[i]}_{k+1}, \mathbf{w}^{[i]}_{k+1}) \in \mathcal{C}^{\mathcal{S}_i}_{\infty} \times \mathbb{D}^N_i \times \mathbb{W}^N_i$, there exists a control sequence $\mathbf{u}^{[i]}_{k+1} \in \mathbb{U}^N_i$ such that the constraints in problem (7.9) are satisfied at time step $k+1$ for all $i \in \mathbb{Z}_{[1,M]}$. This latter claim holds only under Interpretation 7.1, that is, if and only if each controller C_i knows at least the first demand value of its local and virtual sinks ($d^{[i]}_{k+1}$ and $w^{[i]}_{k+1}$ when solving at $k+1$). Such requirement is guaranteed by means of constraint (7.9f), which is feasible by the assumption of existence of any initial feasible trajectory $\mathbf{u}_k$. Therefore, all the local problems solved sequentially by controllers C_i are feasible at $k+1$. Feasibility for all times follows then by induction over k and the assumption of initial feasibility. Consequently, the ML-DMPC approach is strongly feasible and the claim is proved. □

7.4 Numerical Results

In order to evaluate the effectiveness of the proposed ML-DMPC approach, the case study related to the full model of the Barcelona DWN is used (see details in § 2.4). In such network, the set $\mathcal{O}_g$ of global control objectives is formed only by the cost function (2.21a), while the set $\mathcal{O}_l$ of local control objectives is formed by the cost functions (2.21b) and (2.21c). The overall network is assumed to be decomposed in six subsystems ($\Delta = \{\mathcal{S}_1, \ldots, \mathcal{S}_6\}$), which are non-overlapping, output-decentralised and input-coupled (see § 7.4). The model and constraints of each subsystem $\mathcal{S}_i$ are obtained following § 7.2. The controller C_i of each subsystem $\mathcal{S}_i$ uses the following local multi-objective stage cost in its optimisation problem:

$$\ell_i(k, x^{[i]}_k, u^{[i]}_k) = \hat{\gamma}_{1,i}\hat{\ell}_{E,i}(x^{[i]}_k, u^{[i]}_k; c^{[i]}_{u,k}) + \gamma_{2,i}\ell_{\Delta,i}(\Delta u^{[i]}_k) + \gamma_{3,i}\ell_{S,i}(\xi^{[i]}_k; x^{[i]}_k, s^{[i]}_k), \tag{7.12}$$

where functions $\hat{\ell}_{E,i}$, $\ell_{\Delta,i}$ and $\ell_{S,i}$ are the local economic, safety and smoothness objectives for subsystems $\mathcal{S}_i$ (see § 2.4.2 for details on the centralised cost and § 7.3.1 for the derivation of the local costs). Moreover, $\hat{\gamma}_{1,i}$, $\gamma_{2,i}$, and $\gamma_{3,i}$ are positive scalar weights to prioritise each objective in the aggregate local cost function. Each local MPC controller operates with a sampling time $\Delta t_2 = 1$ hour and a prediction horizon $N = 24$ hours. The weight $\hat{\gamma}_{1,i}$ and the internal economic parameters of each function $\hat{\ell}_{E,i}$, $i \in \mathbb{Z}_{[1,6]}$, are

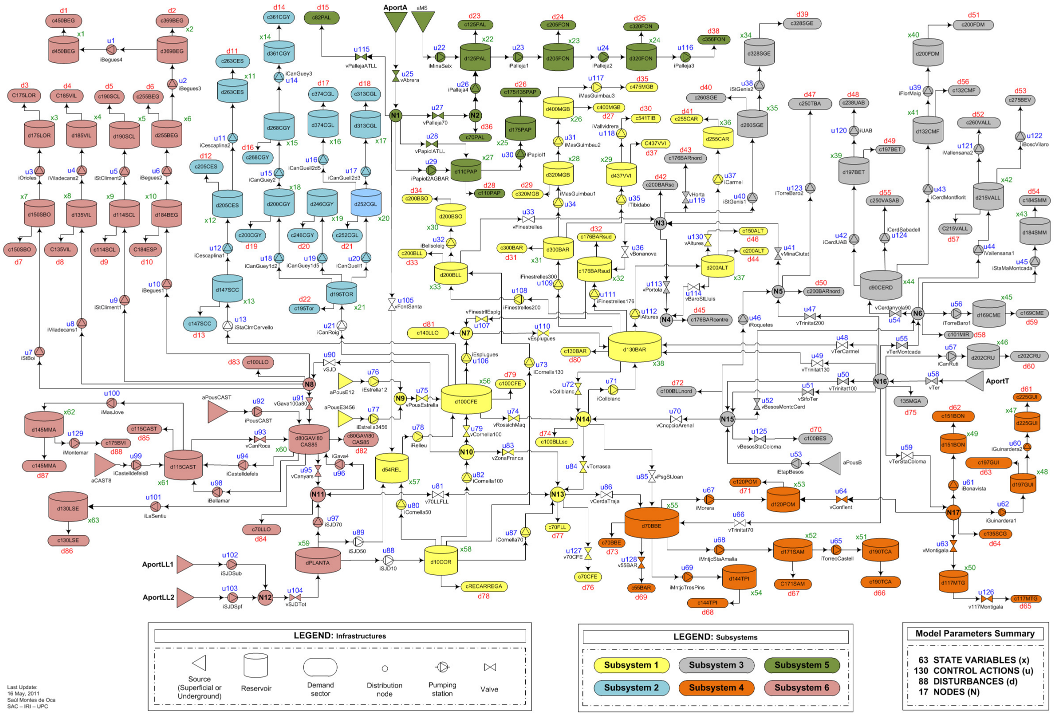


Figure 7.3: Partition of the Barcelona DWN

modified by the upper optimisation layer, placing properly each element of the intervention parameter ω (see Algorithm 3) in the local cost of the corresponding complicated arcs. The cost function used in the upper optimisation layer is given by:

$$\ell_{up}(k, x_{c,k}, u_{c,k}) = \ell_{E,c}(x_{c,k}, u_{c,k}), \tag{7.13}$$

which is derived from (2.21a) but expressed in a temporal scale of days (i.e., $\Delta t_1 = 24$ hours). The constraints and the rest of the parameters involved in the optimisation problems (i.e., water demands, economic prices of water end electricity, safety thresholds, etc.) are set up according to § 2.4.

Figure 7.4 shows, in a more compact way, the resulting subsystems and the important couplings between them including their direction. Instead of neglecting the effect of these shared links as classic pure decentralised control schemes do, the ML-DMPC approach applied to the aforementioned case study has the control architecture shown in Figure 7.2.

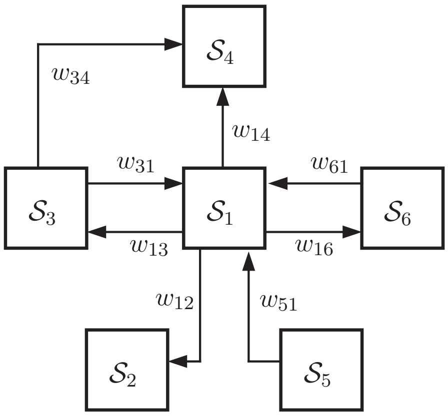


Figure 7.4: Network subsystems $\mathcal{S}_i$ and their shared connections w_{ij}

The results obtained by applying the ML-DMPC (Algorithm 4) are compared with those of applying a centralised MPC (CMPC) approach and a decentralised MPC (DMPC) strategy proposed in [128]. The formulation of the optimisation problems and the closed-loop simulations have been carried out using YALMIP Toolbox, CPLEX solver and Matlab R2012b (64 bits), running in a PC Intel Core E8600 at 3.33GHz with 8GB of RAM. All of the results were obtained for a simulation horizon of 72 hours with data of the real network, and are summarised in Table 7.1 in terms of computational burden and of economic cost as a global management performance indicator. For each MPC approach, the computation time (in seconds) and the water, electric and total cost in economic units (e.u.), are detailed. It can be noticed that an increment of nearly 30% of the total costs of operation occurs when using the non-multilevel hierarchical DMPC strategy with respect to the CMPC baseline. Despite the lower electric costs, the loss of performance in the overall cost is due to the specialised behaviour of local MPC controllers to solve their own optimisation problems without knowing the real water-supply cost of using shared resources with the neighbours. In contrast, the ML-DMPC outperforms the DMPC results by including the bi-level optimisation, which allows to propagate the water cost of sources related with neighbour subsystems to the shared links thanks to the daily centralised control level. With this ML-DMPC approach, the level of sub-optimality is acceptable comparing with the CMPC strategy, i.e., total costs are very similar, but the computational burden is reduced.

Table 7.1: Performance comparisons

Controller	Water Cost (e.u.)	Electric Cost (e.u.)	Total Cost (e.u.)	CPU time (s)
CMPC	93.01	90.31	183.33	1143
DMPC	205.55	34.58	240.13	537
ML-DMPC	97.11	87.53	184.64	540

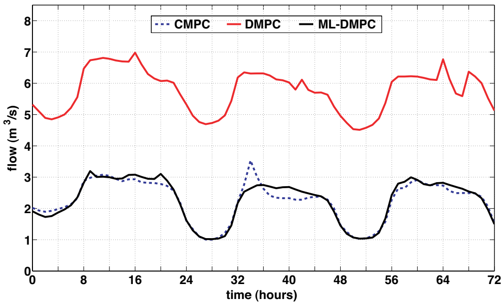
e.u.: economic units

For this particular application, the computation time of the three approaches is able to satisfy the real-time constraint since the control sampling time is 1 hour. Thus, the main motivation for using ML-DMPC is the scalability and easy adaptability of the sub-models if network changes, as well as the modularity of the control policy that leads to face some malfunction/fault without stopping the overall supervisory MPC strategy.

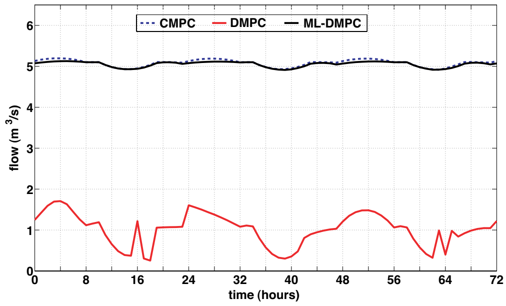
Due the difference of price between water sources and the impact of electric costs on the overall economic performance, the CMPC and ML-DMPC strategies decide to use more water from the Llobregat source despite the consequent pumping of more water through the network (see Figure 7.5), thus achieving a lower total cost, while the hierarchical DMPC decides to exploit in each subsystem its own water source (which could be expensive) and minimise the pumping operation cost. Figure 7.6 shows in detail the evolution of water cost and electric cost, respectively. These results confirm the improvement obtained by including an upper optimisation layer to coordinate the local MPC controllers, which copes with the lack of communication of a pure decentralised approach.

7.5 Summary

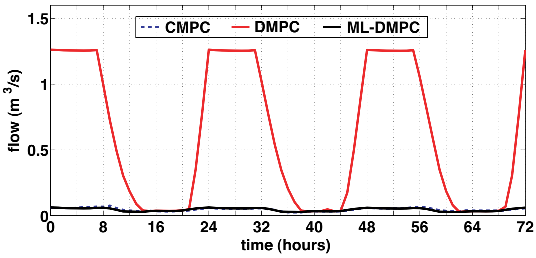
This chapter proposed a non-iterative multi-layer distributed economic MPC approach for large-scale generalised flow-based networks. The control architecture consists in two optimisation layers. The upper layer, working with a larger time scale, is in charge of improving the global performance (in general related to an optimal economic cost) by influencing a set of distributed MPC controllers by means of an intervention economic parameter. These distributed controllers are hierarchically arranged in a lower optimisation layer and are in charge of determining the set-point of the flow actuators to satisfy



(a) Ter source

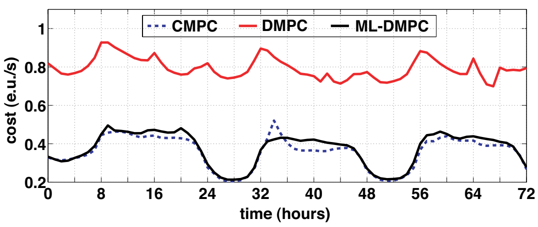


(b) Llobregat source

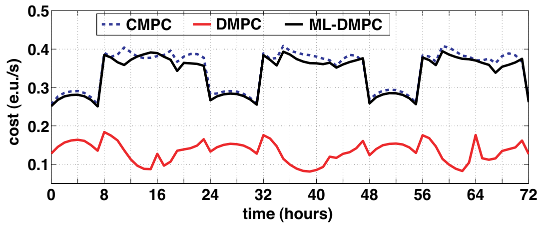


(c) Abrera source

Figure 7.5: Total flow per water source in the Barcelona DWN



(a) Water cost



(b) Electric cost

Figure 7.6: Economic costs of the three MPC strategies

the local management/control objectives. The system decomposition is based on graph partitioning theory. Results obtained on selected simulation scenarios have shown the effectiveness of the control strategy in terms of system modularity, reduced computational burden and, at the same time, reduced loss of performance in contrast to a CMPC strategy and a hierarchical-like DMPC strategy. Additionally, it has been proved that the proposed approach results in a strongly feasible distributed MPC algorithm. For clarity of presentation, in Algorithm 4 it was required that each subsystem calculates its input trajectory at each time step in a hierarchical and sequential order. However, the algorithm works in the same way if non-neighboring systems located in the same level of hierarchy solve their problems in parallel. Future work will be focused on finding stability conditions under the framework of economic MPC, and also on improving the mechanism of coordination to avoid the requirement of plant-wide information in the upper layer of the ML-DMPC approach.

Chapter 8

Distributed Economic MPC with Global Communication

In this chapter, a cooperative distributed economic MPC approach for the control of generalised flow-based networks is discussed. The core of the approach relies on solving the centralised economic MPC formulation proposed in § 3.5 for periodically time-varying systems but in a distributed way, following the algorithm proposed in [101]. All the local controllers cooperate to improve the overall network performance without requiring a coordination layer. Instead, the controllers use both the global cost function and the monolithic model of the network to optimise their local control actions but considering the impact of their strategy on all the subsystems composing the network. To do so, a global (all-to-all) communication strategy is used. Pareto optimality is not possible to achieve due to non-sparse coupling constraints but asymptotic convergence to a Nash equilibrium can be guaranteed.

8.1 Introduction

Throughout this thesis, the importance of developing economic MPC strategies to make optimal use of system resources has been discussed, especially when the main control objectives are not regulation or tracking but profitability, reliability or any general multi-objective function. Particularly, Chapter 3 discussed several economic MPC formulations of interest for generalised flow-based networks driven by periodic demands and, more generally, for periodically time-varying networks. Such formulations were based on the centralisation of decisions in a single controller, which from a computational and organisational point of view is not appealing. Consequently, Chapter 7 proposed a non-iterative distributed economic MPC scheme with low computational requirements. Such scheme (labelled ML-DMPC) uses an upper coordination layer to influence the decisions of a set of non-cooperative local controllers in order to improve the decentralised performance. Nevertheless, the improvement seems to be limited by the simplifications made in the design of the coordinator and by the low frequency of its intervention. Despite the lack of performance bounds for the ML-DMPC approach, it could be thought that the overall closed-loop performance with such a non-iterative algorithm will improve further if the coordinator intervenes with the same sampling time as the local controllers. Since the coordinator strategy is based on a monolithic model and a linear programming problem, increasing its frequency to that of the local controllers would result in a centralised decision process.

In generalised flow-based network applications, objections to centralised MPC are often not due to the computational burden of the algorithms but about modularity and scalability of the control architecture. Moreover, the maintenance of a monolithic model could also not be a problematic issue in some networks whose critical infrastructures can remain unchanged for long periods of time (e.g., municipal water systems, oil transport networks, etc.). Therefore, if a centralised model is available, a more appealing control scheme could be designed following the cooperative distributed MPC framework proposed in [170] for a standard tracking cost function. Contrary to the ML-DMPC and other distributed MPC strategies reviewed in § 1.2.5, the aforementioned cooperative distributed control approach does not require a coordination layer. Instead, the subsystems need to share models and objective functions with each other, but deciding only their own control actions based on the input predictions of the others. Although the algorithm relies on an iterative exchange of local solutions to improve the performance, it does not depend on achieving optimality to guarantee recursive feasibility. This latter is an important feature of the approach, since the operation of the system is not compromised for any possibly early termination of the parallel optimisation problems. The advantages of cooperative distributed MPC and its design techniques have been recently extended in [101] to the case of cooperative economic MPC for linear time-invariant systems with convex objectives. In such an approach, asymptotic convergence to a set of Nash equilibria is guaranteed.

The contribution of this chapter is to extend the analysis of cooperative economic MPC presented in [101] to the case of periodically time-varying generalised flow-based networks. Thus, this chapter relies strongly on the theoretical framework developed previously in § 3.5 for periodic systems.

8.2 Problem Formulation

As discussed in § 8.1, the cooperative distributed MPC framework requires generally that the subsystems share their models, constraints and objective functions with each other (in addition to the exchange of their input plans). In this chapter, a discrete-time linear composite model derived from Remark 7.1 is used to represent generalised flow-based networks formed by M coupled subsystems, but modified for the case of periodically time-varying systems (see Definition 3.6). Specifically, each local controller C_i, $i \in \mathbb{Z}_{[1,M]}$ is equipped with the following plant-wide model:

$$\begin{cases} x_{k+1} = A_k x_k + B_{i,k} u_k^{[i]} + \sum_{\substack{j=1 \\ j \neq i}}^{M} B_{j,k} u_k^{[j]} + B_{d,k} d_k, & \text{(8.1a)} \\ \quad\quad 0 = E_{i,k} u_k^{[i]} + \sum_{\substack{j=1 \\ j \neq i}}^{M} E_{j,k} u_k^{[j]} + E_{d,k} d_k, & \text{(8.1b)} \end{cases}$$

where, for all $k \in \mathbb{Z}_+$, the global state, local input and global demand vectors satisfy point-wise constraints of the form $x_k \in \mathbb{X} \subset \mathbb{R}^n$, $u_k^{[i]} \in \mathbb{U}_i \subset \mathbb{R}^{m_i}$ for all $i \in \mathbb{Z}_{[1,M]}$ and $d_k \in \mathbb{D} \subset \mathbb{R}^p$, respectively. It is assumed that all the sets are compact and that the local input sets are disjoint and satisfy $\mathbb{U} = \prod_{i=1}^{M} \mathbb{U}_i$. Moreover, the demand vector and all the matrices are assumed to be T-periodic and known for each time step k, that is, $d_k = d_{k+T}$, $A_k = A_{k+T}$, $B_{i,k} = B_{i,k+T}$, $B_{d,k} = B_{d,k+T}$, $E_{u,k} = E_{u,k+T}$ and $E_{d,k} = E_{d,k+T}$, with $T \in \mathbb{Z}_{\geq 1}$ the period of the system.

Similarly to Remark 7.1, the global vectors of the composite model (8.1) are a permutation of the original ones, i.e.,

$$x_k = \begin{bmatrix} x_k^{[1]} \\ \vdots \\ x_k^{[M]} \end{bmatrix}, \quad u_k = \begin{bmatrix} u_k^{[1]} \\ \vdots \\ u_k^{[M]} \end{bmatrix}, \quad d_k = \begin{bmatrix} d_k^{[1]} \\ \vdots \\ d_k^{[M]} \end{bmatrix}. \tag{8.2}$$

Therefore, matrices $B_{i,k}$ and $E_{i,k}$ in (8.1) are given by the columns of matrix B_k, which is similar to the invariant matrix B in Remark 7.1, i.e.,

$$B_{i,k} = \begin{bmatrix} B_{1i,k} \\ \vdots \\ B_{Mi,k} \end{bmatrix}, \quad E_{i,k} = \begin{bmatrix} E_{1i,k} \\ \vdots \\ E_{Mi,k} \end{bmatrix}, \quad \forall i \in \mathbb{Z}_{[1,M]}, \tag{8.3}$$

where block matrices $B_{ji,k}$ and $E_{ji,k}$, $j \in \mathbb{Z}_{[1,M]}$, describe the effect that the input vector $u_k^{[i]}$ of subsystem $\mathcal{S}_i$ has on all the subsystems $\mathcal{S}_j$.

For notational simplicity and readability, and especially to fit with the supporting theoretical framework developed previously in § 3.5, the subsequent content considers that each controller C_i uses a plant-wide model represented more compactly as a class of constrained dynamic time-varying affine system, redefined here as follows:

$$x_{k+1} = f(k, x_k, u_k) \coloneqq A_k x_k + \begin{bmatrix} B_{1,k} & \dots & B_{M,k} \end{bmatrix} \begin{bmatrix} u_k^{[1]} \\ \vdots \\ u_k^{[M]} \end{bmatrix} + B_{d,k} d_k \quad \forall k \in \mathbb{Z}_+. \tag{8.4}$$

Note that model (8.4) does not include (8.1b), because such algebraic condition is now included in the redefined overall constraint set, which is time-varying due to the non-stationary demand equality and inequality constraints given in the form of a convex closed polyhedral set, which is here defined as

$$\mathbb{Y}_k \coloneqq \left\{ (x, u) \in \mathbb{X} \times \prod_{i=1}^{M} \mathbb{U}_i \mid \begin{bmatrix} E_{1,k} & \dots & E_{M,k} \end{bmatrix} \begin{bmatrix} u^{[1]} \\ \vdots \\ u^{[M]} \end{bmatrix} + E_{d,k} d_k = 0 \right\} \quad \forall k \in \mathbb{Z}_+. \tag{8.5}$$

In order to induce cooperation between the local controllers, each of them is equipped with the same cost function used in the centralised economic MPC approach proposed in § 3.5.2 for the periodically time-varying case. However, each controller C_i, $i \in \mathbb{Z}_{[1,M]}$, can adjust only the inputs under its control authority for the corresponding subsystem $\mathcal{S}_i$. The rest of the elements of the composite input vector in the i-th local problem are assumed to be fixed parameters that are determined by subsystems $\mathcal{S}_j$, $j \in \mathbb{Z}_{[1,M]}, j \neq i$. It is important to highlight that this cooperative scheme requires further that all the local controllers have to be synchronised to update simultaneously the global state, input, and demand vectors.

8.3 Cooperative Distributed Economic MPC Formulation based on a Periodic Terminal Penalty and Region

The optimisation problem considered in this section for the cooperative economic MPC of generalised flow-based networks is built on the periodic terminal penalty/region formulation proposed in (3.30). Therefore, the design of the MPC strategy depends on Assumptions 3.5 to 3.9, which are supposed to hold in the sequel. Specifically, each distributed controller C_i, $i \in \mathbb{Z}_{[1,M]}$, solves the following optimisation problem:

$\mathcal{P}_N^{[i]}(k, x_k, \mathbf{d}_T, \mathbf{p}_T)$:

$$\min_{\mathbf{u}_k^{[i]}} V_N(k, x_k, \mathbf{u}_k) = \sum_{i=0}^{N-1} \ell(k+i, x_{k+i|k}, u_{k+i|k}) + V_f(k+N, x_{k+N|k}), \tag{8.6a}$$

subject to:

$$x_{k+i+1|k} = f(k+i, x_{k+i|k}, u_{k+i|k}), \qquad \forall i \in \mathbb{Z}_{[0,T-1]} \tag{8.6b}$$

$$(x_{k+i|k}, u_{k+i|k}) \in \mathbb{Y}_{k+i}, \qquad \forall i \in \mathbb{Z}_{[0,T-1]} \tag{8.6c}$$

$$x_{k+N|k} \in \mathbb{X}_f(k+N, \hat{x}^\star_{k+N}), \tag{8.6d}$$

$$x_{k|k} = x_k, \tag{8.6e}$$

$$\mathbf{u}_k = \begin{bmatrix} \mathbf{u}^{[1]} \\ \vdots \\ \mathbf{u}^{[M]} \end{bmatrix}, \tag{8.6f}$$

$$\mathbf{u}_k^{[j]} = \mathbf{u}_k^{[j],p}, \qquad \forall j \in \mathbb{Z}_{[1,M]} \setminus \{i\} \tag{8.6g}$$

where $\mathbf{u}_k^{[i]} = \{u_{k+t|k}^{[i]}\}_{t \in \mathbb{Z}_{[0,N-1]}}$ is the decision vector and $\mathbf{u}_k^{[j],p} = \{u_{k+t|k}^{[j],p}\}_{t \in \mathbb{Z}_{[0,N-1]}}$ for all $j \in \mathbb{Z}_{[1,M]} \setminus \{i\}$ is the current input sequence computed and transmitted by the j-th subsystems at the p-th iteration. Moreover, $\mathbf{d}_T = \{d_{k+t}^{[i]}\}_{t \in \mathbb{Z}_{[0,N-1]}}$ is a known T-periodic demand sequence involved in the definition of function f and $\mathbf{p}_T = \{p_{k+t}\}_{t \in \mathbb{Z}_{[0,N-1]}}$ is a known T-periodic economic parameter defining the time varying nature of the cost. Additionally, $\ell : \mathbb{Z}_+ \times \mathbb{R}^n \times \mathbb{R}^{m_1} \times \ldots \times \mathbb{R}^{m_M} \to \mathbb{R}_+$ is a convex periodic economic stage cost function, $V_f : \mathbb{Z}_+ \times \mathbb{R}^n \to \mathbb{R}_+$ is a periodic function that penalises the terminal state, the set $\mathbb{X}_f(k+N, \hat{x}^\star_{k+N}) \subseteq \mathbb{X}$ is a time-varying compact terminal region containing the optimal periodic target state $\hat{x}^\star_{k+N}$ in its interior, which comes from solving (3.29) with $k = 0$. The terminal ingredients of the optimisation problem (8.6) are computed following § 3.5.5.

Remark 8.1. *A cooperative distributed economic MPC formulation with terminal equality constraint can be recast from* (8.6) *by setting the terminal cost* $V_f(k+N, x_{k+N|k}) = 0$ *and* $\mathbb{X}_f(k+N, \hat{x}^\star_{k+N}) = \{\hat{x}^\star_{k+N}\}$. ◊

The cooperative distributed economic MPC design relies merely on a set of local optimisation problems emulating the structure of a centralised economic MPC formulation, whose theoretical results (i.e., recursive feasibility, asymptotic average performance and stability) rely on convexity assumptions and the existence of feasible (sub-optimal) shifted candidate solutions (see [101, 170]). Hence, it could be expected that the cooperation between controllers in a distributed formulation for periodically time-varying generalised flow-based networks leads to heritage the benefits of the terminal penalty/region based economic MPC approach discussed in § 3.5.

In the sequel, consider (with some abuse of notation) that the vector $\mathbf{u}_k$ of input sequences can be denoted as $(\mathbf{u}_k^{[1]}, \mathbf{u}_k^{[2]}, \ldots, \mathbf{u}_k^{[M]})$. Thus, Algorithm 5 summarises the principle of operation of the cooperative distributed economic MPC strategy. The inner loop of Algorithm 5 is based on iterative parallel optimisation of the Gauss-Jacobi type. For convex problems, this Gauss-Jacobi routine has the property that it generates feasible iterates with non-increasing objective function values [170].

It is remarkable that, contrary to the distributed ML-DMPC approach proposed in Chapter 7, the cooperative distributed economic MPC discussed above does not include a coordination layer. In fact, subsystems just need to interact among themselves to transfer information regarding their predicted inputs under a global communication strategy. There is no minimum number of iterations required in the algorithm to ensure recursive feasibility. Subsystems can choose to stop after any number of iterations, and even then, stability of the closed-loop system can also be guaranteed.

Properties of the Cooperative Distributed Economic MPC approach

Three important properties arise from the inner loop of Algorithm 5, which were previously proved for the non-periodic cooperative economic MPC strategy suggested in [101].

1. **Recursive feasibility.** Given the feasibility set $\mathcal{F}_N(k)$ defined as in (3.34) and any feasible initial condition $(x_k, (\mathbf{u}_k^{[1]}, \mathbf{u}_k^{[2]}, \ldots, \mathbf{u}_k^{[M]})^p) \in \mathcal{F}_N(k)$ for some $p \in \mathbb{Z}_+$,

then the pair obtained from the same current state and any future input iterate obtained as specified in Step 10 of Algorithm 5 is also feasible. That is, $(x_k, (\mathbf{u}_k^{[1]}, \mathbf{u}_k^{[2]}, \ldots, \mathbf{u}_k^{[M]})^{p+i}) \in \mathcal{F}_N(k)$ for all $i \in \mathbb{Z}_+$. This follows from convexity of the set $\mathbb{U}$ and the fact that any convex combination of states and input sequences in $\mathcal{F}_N(k)$ also belong to the set.

2. **Convergence.** The cost $V_N(k, x_k, \mathbf{u}_k^p)$ decreases on each iteration and is convergent as $p \to \infty$. This property can be shown from the monotonicity of the cost, which follows according to:

$$\begin{aligned} V_N(k, x_k, \mathbf{u}_k^{p+1}) &= V_N\left(k, x_k, \sum_{i=1}^{M} \alpha_i \left(\mathbf{u}_k^{[1],p}, \ldots, \mathbf{u}_k^{[i]\star}, \ldots, \mathbf{u}_k^{[M],p}\right)\right) \\ &\leq \sum_{i=1}^{M} \alpha_i V_N\left(k, x_k, \left(\mathbf{u}_k^{[1],p}, \ldots, \mathbf{u}_k^{[i]\star}, \ldots, \mathbf{u}_k^{[M],p}\right)\right) \\ &\leq \sum_{i=1}^{M} \alpha_i V_N\left(k, x_k, (\mathbf{u}_k^{[1],p}, \ldots, \mathbf{u}_k^{[i],p}, \ldots, \mathbf{u}_k^{[M],p})\right) \\ &= V_N\left(k, x_k, \mathbf{u}_k^p\right). \end{aligned}$$

The first equality follows from Step 10 of Algorithm 5. The first inequality follows from convexity of the function V_N, while the second inequality follows from optimality of $\mathbf{u}^{[i]\star}$, $i \in \mathbb{Z}_{[1,M]}$. The last equality follows from the condition of the convex combination of weights α_i, i.e., $\sum_{i=1}^{M} \alpha_i = 1$. Because the cost is lower bounded, it converges.

3. **Optimality.** The iteration $(\mathbf{u}_k^{[1],p}, \ldots, \mathbf{u}_k^{[i],p}, \ldots, \mathbf{u}_k^{[M],p})$ converges to the set of Nash equilibria as $p \to \infty$ and not to a Pareto (centralised) solution. This means that the iterated cost ends up in deadlock situations where, for a set of strategies $(\hat{\mathbf{u}}_k^{[1]}, \ldots, \hat{\mathbf{u}}_k^{[M]})$, it holds

$$V_N(k, x_k, (\tilde{\mathbf{u}}_k^{[1]}, \ldots, \tilde{\mathbf{u}}_k^{[M]})) \leq V_N(k, x_k, (\tilde{\mathbf{u}}_k^{[1]}, \ldots, \tilde{\mathbf{u}}_k^{[i-1]}, \tilde{\mathbf{u}}_k^{[i]}, \tilde{\mathbf{u}}_k^{[i+1]}, \ldots, \tilde{\mathbf{u}}_k^{[M]})),$$

for all $\mathbf{u}_k^{[i]}$. This property was formally proved in [101].

Algorithm 5 Cooperative Distributed Economic MPC with Terminal Penalty

Inputs: Current state x_0, initial feasible (not necessarily optimal) sequence $\tilde{\mathbf{u}}_0$, network decomposition Δ, periodic sequences $\mathbf{d}_T$ and $\mathbf{p}_T$, periodic matrices $A_k, B_k, B_{d,k}, E_{u,k}, E_{d,k}$, $p_{\max} \geq 1$, $\alpha_i \in (0,1)$ such that $\sum_{i=1}^{M} \alpha_i = 1$.
Output: Closed-loop trajectories (x_k, u_k), $k \in \mathbb{Z}_{\geq 1}$
Set $k \leftarrow 0$
while $k \geq 0$ **do**
  Set $p \leftarrow 0$, $x \leftarrow x_k$
  $\mathbf{u}_k^{[i],p} \leftarrow \tilde{\mathbf{u}}_k^{[i]}$ for all $i \in \mathbb{Z}_{[1,M]}$
  Transmit the inputs $\mathbf{u}^{[i]}$ from current subsystem to the other rest of the subsystems
  while $p < \bar{p}$ **do**
    Solve problem (8.6) to obtain $\mathbf{u}_k^{[i]\star}$ for all $i \in \mathbb{Z}_{[1,M]}$
    Set $\mathbf{u}_k^{[i],p+1} \leftarrow \alpha_i \mathbf{u}_k^{[i],p} + (1-\alpha_i)\mathbf{u}_k^{[i]\star}$ for all $i \in \mathbb{Z}_{[1,M]}$
    Set $p \leftarrow p+1$
  end while
  Set $\mathbf{u}_k \leftarrow (\mathbf{u}_k^{[1],p}, \mathbf{u}_k^{[2],p}, \ldots, \mathbf{u}_k^{[M],p})$ and obtain $x_{k+N|k} \leftarrow \phi(N; x_k, \mathbf{u}_k, \mathbf{d}_T)$
  Obtain $u_+ := (u_{1+}, u_{2+}, \ldots, u_{M+}) \leftarrow \kappa_f(k+N, x_{k+N|k})$
  Compute next warm start $\tilde{\mathbf{u}}_{k+1}^{[i]} = (u_{k+1|k}^{[i],p}, u_{k+2|k}^{[i],p}, \ldots, u_{i+})$ for all $i \in \mathbb{Z}_{[1,M]}$
  Set input as $u_k = (u_{k|k}^{[1],p}, u_{k|k}^{[2],p}, \ldots, u_{k|k}^{[M],p})$
  Apply input u_k to the system to obtain x_{k+1}
  $k \leftarrow k+1$
end while

In a generalised flow-based network applications, the difficulty to converge to the solution of the centralised problem by means of the cooperative distributed economic MPC is mainly due to two causes, which are: (i) the input-coupled constraints arising from (8.1b) and, (ii) the couplings induced by the terminal constraint (8.6d) used in the proposed approach. For the case when only sparse input-coupled constraints are present, a method to recover the Pareto optimality in a standard tracking MPC scheme has been proposed in [170]. In the setting related to (8.6), such method can also be used to improve performance and ameliorate the numerical conditioning of the equality input-coupled constraints involved in (8.5). However, since the terminal state constraint is strongly coupled, convergence to Pareto optimality is still not attainable.

Regarding the outer loop of Algorithm 5, recursive feasibility follows from the terminal equality constraint (8.6d) and the existence of a suboptimal but feasible candidate solution for the next time step (obtained from Steps 14 and 15 of Algorithm 5). This latter warm start, in addition to Assumptions 3.5 to 3.9, allow to establish the following result.

Theorem 8.1 (Stability). *Consider a T-periodic generalised flow-based network described in the form of (8.4) subject to (8.5) and let Assumptions 3.5 to 3.9 hold. Let $\mathcal{X}_T(\mathbf{w}_T, \mathbf{p}_T)$ be the best feasible T-periodic orbit of the system obtained by solving (3.29). Then, $\mathcal{X}_T(\mathbf{w}_T, \mathbf{p}_T)$ is asymptotically stable for all feasible initial state $x_0 \in \mathcal{X}_N(0)$ for the distributed closed-loop system.*

Proof: The result follows directly from Theorem 3.3 in addition to the convergence and optimality properties discussed before. □

8.4 Numerical Results

Similarly to previous chapters, the effectiveness of the cooperative economic MPC approach proposed in § 8.3 is assessed in this section by means of the case study described in § 2.4. Specifically, the aggregate model of the Barcelona DWN was used (see Figure 2.4) for the simulations discussed in the following. The formulation of the optimisation problems and the closed-loop simulations have been carried out using YALMIP Toolbox, CPLEX solver and Matlab R2012b (64 bits), running in a PC Intel Core E8600 at 3.33GHz with 8GB of RAM.

Figure 8.1 shows the convergence property of Algorithm 5 when solving at $k = 0$ for a prediction horizon of $N = 24$ hours. It can be seen that, even when the magnitude of the cost difference is in practice negligible for this simulation, the property of decreasing open-loop cost holds and it shows a convergent behaviour when more iterations are allowed.

Regarding the closed-loop performance, Table 8.1 shows a comparison between the proposed cooperative distributed economic MPC controller using $p \in \{1, 5, 10\}$, the centralised standard economic MPC with terminal equality constraint described in Chapter 3, the hierarchical-like decentralised MPC proposed in [128] and the multi-layer distributed economic MPC described in Chapter 7. Such controllers are labelled CDEMPC, EMPC, DMPC, and ML-DMPC, respectively. The performance is assessed for a simulation horizon of 96 hours and compared in terms of computational burden and of economic cost as a global management performance indicator. For each MPC approach,

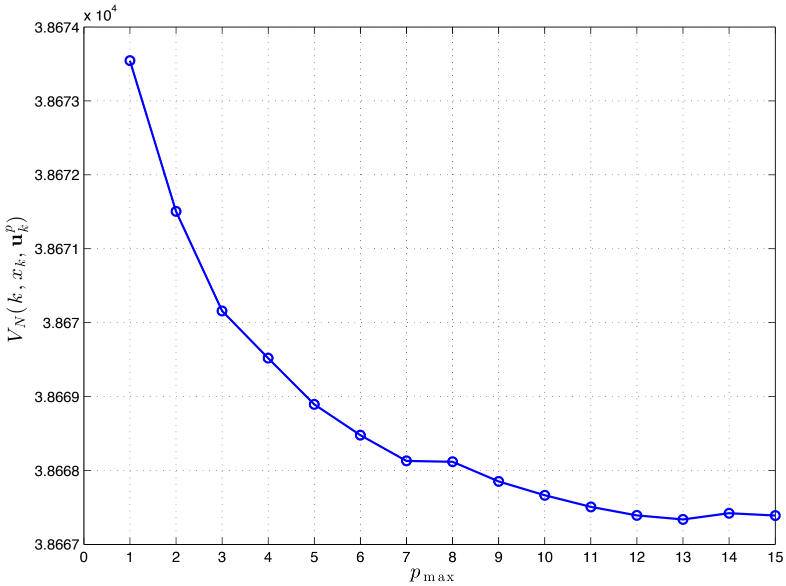


Figure 8.1: Open-loop cost decreasing of V_N as function of the number of Gauss-Jacobi iterations

the average computation time (in seconds) required to solve the optimisation problems and the water, electric and total costs, in economic units (e.u.), are detailed. As can be expected, the DMPC approach outperforms all other controllers in terms of average computational time but at the expense of having the largest sub-optimality (when using the performance of the EMPC approach as a baseline) due to the loss of economic information that comes along with the decentralisation process. In contrast, the ML-DMPC approach outperforms the results of the DMPC approach by using an upper coordination layer, which modifies (non-iteratively) the cost of the shared resources in such a way that each local controller has an approximated global economic information. Regarding the CDEMPC approach, it can be seen that incrementing the intermediate Gauss-Jacobi iterates improves the overall closed-loop performance but without converging to the results of the EMPC approach. In fact, for the selected case study, the rate of improvement is slow. Using $p = 5$ or $p = 10$ does not decrease considerably the overall cost of the CDEMPC approach, but it does increase notably the computation time. Therefore, a trade-off between the level of sub-optimality and the number of

Table 8.1: Performance comparisons

Controller	Water Cost	Electric Cost	Total Cost	CPU time
EMPC	96.468	86.317	182.785	17.493
DMPC	171.136	63.940	235.076	5.787
ML-DMPC	110.720	86.659	197.379	5.819
CDEMPC$_{p=1}$	106.455	82.900	189.355	19.910
CDEMPC$_{p=5}$	106.443	82.907	189.350	100.588
CDEMPC$_{p=10}$	106.444	82.905	189.349	201.218

e.u.: economic units

p-iterations has to be done. The CDEMPC approach outperforms the ML-DMPC approach in terms of economic cost due to the use of a centralised model within each local controller of the CDEMPC approach (no coordination layer is required). Besides, the ML-DMPC approach is non-iterative and its recursive feasibility is enforced by using appropriate robustness constraints that detriment the economic performance. Nevertheless, the ML-DMPC approach reduces considerably the average computation time compared to the CDEMPC approach. The low complexity of the ML-DMPC approach, its level of sub-optimality, and its computation time make such controller appealing for larger problems.

8.5 Summary

This chapter addresses an iterative distributed economic MPC formulation for its application of periodically time-varying generalised flow-based networks that are subject to convex constraints and strictly convex economic cost functions. The distributed algorithm is based on the cooperation of local controllers. Such controllers use the centralised model and objective function of the system but optimise only their corresponding control actions. The communication strategy between subsystems is all-to-all. Therefore, a reliable communication network is required. The main advantage of this approach is that it can tackle easily the interactions related to both the dynamic nodes and the static nodes of the network, without complicating the feasibility analysis after decomposing the control authority. Differently to the ML-DMPC approach of Chapter 7, recursive feasibility of the cooperative algorithm does not rely on robustness constraints. Instead, it relies on convex combinations of the local solutions, which follow the suboptimal MPC philosophy, allowing the early termination of the distributed optimisation before convergence. The scheme uses a periodically time-varying terminal penalty/region that forces the state to lie within the optimal nominal periodic orbit at the end of the prediction horizon. This allows to obtain a priori bounds of the average performance of the closed-loop system. Specifically, the system outperforms in average the best periodic orbit. Additionally, stability to the optimal periodic orbit can be guaranteed and if the algorithm converges, a Nash equilibrium is achieved. As drawbacks of the approach, it can be mentioned that the rate of convergence to the Nash equilibrium could be slow. Hence, a large number of intermediate iterations could be required and that translates in higher computational times.

Part IV

Concluding Remarks

Chapter 9

Concluding Remarks

This chapter concludes this thesis by highlighting its main contributions and by giving an outlook to promising future research perspectives.

9.1 Contributions

Throughout this thesis, each chapter has already presented important conclusions about the performance, advantages and drawbacks of the proposed control strategies. Nonetheless, the main contributions are summarised below.

- Modelling principles of dynamic network flow problems were introduced and a control-oriented model based on the discrete-time state-space framework was formulated.

- A baseline certainty-equivalent MPC strategy was proposed for the integration of the economic scheduling and the real-time control of network flows. Additionally, necessary and sufficient conditions to guarantee the existence of strongly feasible control laws of such baseline controller were derived based on both max-min controllability and set invariance notions.

- Different economic MPC controllers were designed to address the special case of operating under periodically time-varying demands and a multi-objective cost function. Among such controllers, it is important to highlight a single-layer economic MPC controller that copes with changes in the economic parameters of the cost function, and a periodically time-varying terminal cost/region based formulation that enlarges the domain of attraction of the standard economic MPC approach and improves the average performance. For this latter controller, conditions to guarantee recursive feasibility and Lyapunov asymptotic stability were derived for more general periodic linear systems.

- Robust MPC strategies capable to compute dynamic safety stocks and managing the health of actuators in generalised flow-based networks were designed. The strategies optimise the storage at nodes without the expense of excessive stocks due to the exploitation of the quality of information, i.e., the inventories are increased when uncertainty is large and decreased when measurements and forecasts are more accurate. Moreover, the actuators are controlled in a way that their availability is assured for a desired maintenance horizon. These strategies are: (i) a reliability-based MPC controller that combines supply-chain theory and reliability engineering under the MPC framework to guarantee a desired service level in networks with constant uncertainty, and (ii) a tractable chance-constrained MPC controller that assumes networks as stochastic and probabilistic constrained systems and handles constraints by computing a dynamic back-off according to the time-dependent uncertainty of non-stationary disturbances.

- A scenario tree-based MPC strategy was designed and assessed for its application to large-scale networks with uncertain disturbances. The approach does not need to know the probability distribution of the disturbances but a forecast scenario tree of their future evolution. The controller is able to cope naturally with both inequality and equality uncertain constraints. The formulation of the associated optimisation problem requires the introduction of non-anticipativity constraints and involves a large number of decision variables that make the solution computationally demanding for low levels of risk acceptability and a large number of disturbances. Therefore, the approach is attractive but only for uncertain systems whose associated tree-based MPC optimisation problem results in a tractable size. Such size is related to the number of decision variables and constraints that the available processing capability can handle.

- A dynamic tuning for the control move suppression based on the current reliability of the actuators was proposed to distribute the global control effort between redundant actuators that share a demand load. The proposed method avoids the accelerated wear that some equipment may have when no health management is considered.

- A multi-layer MPC controller with self-tuning capabilities for the efficient management of generalised flow-based networks was proposed based on soft-computing techniques (specifically, neural networks and fuzzy logic). The selected parameters to be tuned in the MPC problem were the prediction horizon and the weighting matrices of the multi-objective cost function. The main advantage of the proposed learning-based tuning method is that it is able to tune every element of the network independently, which is a difficult task in analytical approaches due to their lack of intuitiveness for large-scale systems. Although the tuning method was assessed with a centralised problem, it can be similarly applied in each local controller of a distributed MPC scheme, considering the neighbour flow requests as disturbances.

- A non-iterative two-layer economic distributed MPC controller was designed based on temporal, functional and spatial decomposition of the original problem and system. The approach guarantees recursive feasibility by introducing a first-step robustness constraint in the local optimisation problem of each distributed controller and by using a hierarchical-like solution process. The communication strategy is neighbour-to-neighbour. The overall performance is influenced by a coordination layer that projects, by means of a flow-path centralised problem, global economic information related to the control inputs that interconnect subsystems, which are obtained by a graph-based decomposition of the network.

- An iterative cooperative distributed economic MPC strategy that guarantees recursive feasibility and convergence to a Nash equilibrium was designed for linear periodic generalised flow-based networks subject to convex constraints and strictly convex economic cost functions. The approach relies on distributing the optimisation of the centralised problem. In such approach, each distributed controller solves a suboptimal centralised MPC problem with periodic terminal cost/region but optimising only the control inputs that have been assigned to it in a previous decomposition of the input vector. The plant-wide applied control action is obtained by means of a convex combination of the distributed solutions. The proposed cooperative economic MPC controller guarantees asymptotic stability of the economically optimal periodic trajectory of the system.

- A real case study, i.e., the Barcelona drinking water network, was used to test and demonstrate the flexibility and properties of each proposed MPC strategy.

9.2 Directions for Future Research

There are several open problems and interesting avenues of research arising from the results presented in this thesis. In the following, some ideas of possible future lines of research are outlined.

- Incorporate parametric uncertainty and unmeasured disturbances in the model and implement closed-loop predictions by parametrisation of the control actions based on affine disturbance feedback, in order to reduce the conservatism of open-loop predictions.

- Develop algorithms for on-line optimisation of risk allocation in the CC-MPC approach.

- Develop distributed versions of the proposed centralised stochastic MPC controllers. Particularly, the demanding optimisation problem of the tree-based MPC approach could be distributed in parallel sub-problems, where each of them addresses a single disturbance scenario. Moreover, it is of interest to derive bounds for the number of required clustered scenarios in the disturbance tree in order to achieve a desired probability of constraint satisfaction.

- Design economic MPC controllers capable to consider different replenishment cycles at each storage node, demand pooling risks and actuator maintenance quality and cost in the model and stage cost function.

- Improve the coordination mechanism of the ML-DMPC approach in order to avoid the need of an upper optimisation layer that requires plant-wide information. A possible way to address this goal is by designing flow price tracing algorithms based on a proportional sharing assumption for the allocation of costs that are dependent on the flow value. Such tracing methodologies can be rationalised using cooperative game theory and information theory, specifically the Shapley value and the maximum entropy principle.

- Develop system partitioning algorithms to obtain possibly dynamic graphs based directly on the economic cost function and the network reliability. The algorithms should guarantee the existence of non-empty decentralised robust control invariant sets.

- Develop flow-based models considering both time-varying gains and time-varying transit times in the arcs of the network graph to incorporate non-linearities and phenomena of real applications, e.g., delays, pressure relations in water/gas networks, energy loss in electric networks, among others, and analyse how such time-varying properties affect the feasibility, stability and optimality of the economic MPC controllers.

- Design distributed economic MPC controllers without terminal constraints and studying the method of cutting-plane consensus but considering dynamic state, input and output couplings between subsystems.

- Develop distributed observers and distributed fault-tolerant control schemes.

Part V

Appendices

Appendix A

Demand Modelling

A.1 Water Demand Characterisation

Regular forecasting of a vast number of univariate time series is an essential task to develop proper controllers for the operational management of the drinking water network (DWN). The open-loop feed-forward uncertainty in the DWN can be modelled by the relationship between predicted states and predicted disturbances, see (C.2) and (C.3). In the dynamic model (2.10) of the DWN, randomness is directly described by the uncertainty of customer demands, which can be estimated from historical data. Figure A.1 shows the histogram of a specific water demand node in the Barcelona DWN for the same time instant in different days during year 2007. It can be seen, in the envelope of the histogram, that the uncertain demand obeys a probabilistic distribution close to a Gaussian distribution (red curve). In addition, the last two plots of Figure A.2 show that the demand pattern presents two seasonal cycles, one with a daily period and the other one with a weekly period.

In order to compute the forecast of future disturbances and its stochastic properties, this thesis follows the modified exponential smoothing state-space framework developed in [47] for automatic forecasting of complex seasonal time series, such as the ones related to water demands in the Barcelona DWN. This framework extends traditional exponential smoothing models to accommodate multiple seasonality. The forecasting model is named BATS, which is an acronym for time-series models with Box-Cox transformation, ARMA errors, Trend and Seasonality components. Taking the time series of the p demands involved in the Barcelona DWN case studies, and applying the BATS fore-

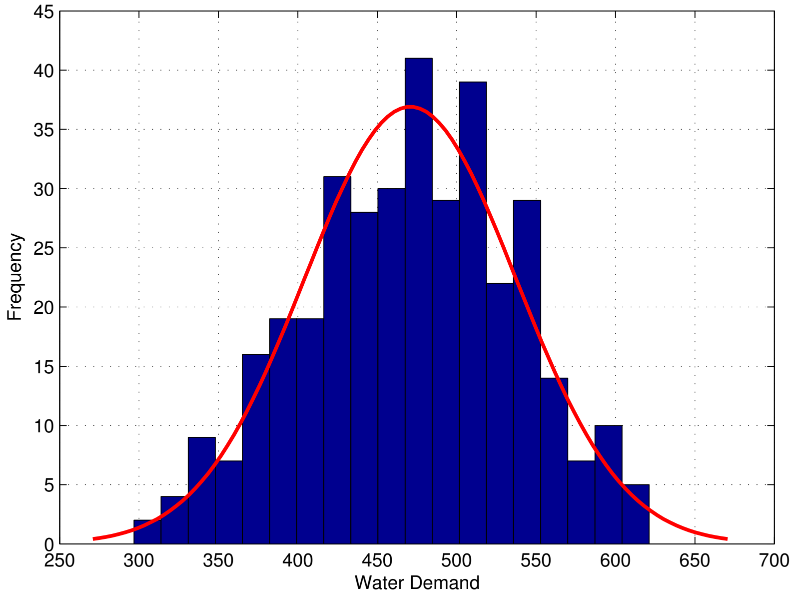


Figure A.1: Histogram of demand $c176BARsud$ in the Barcelona DWN

casting method presented in [47], it is possible to define the elements that complete the deterministic equivalent CC-MPC approach, i.e., the predicted mean and covariance. The other non-stochastic MPC approaches presented in this thesis use only the mean forecast demands, assuming they lie in compact sets that can be inferred from historical data or decision makers' experience. In some networks the bounds of demands might be explicitly stipulated in supply contracts. Figure A.3 shows an open-loop forecast of a demand along a prediction horizon of four days, highlighting the effect of the propagated uncertainty. The thick line is the expectation of future demand, while the thin lines are the upper and lower bounds of the prediction interval for different confidence levels. For more details about the prediction of time-series uncertainty description, the reader is referred to the aforementioned reference.

A.2 BATS Modelling of Water Demand

Demand time series model: Let d_k, $k \in \mathbb{Z}_+$, denote an observed time series of any water demand, and $d_k^{(\omega)}$ the Box-Cox transformed observed value at time k with the parameter ω. The transformed series $d_k^{(\omega)}$, is then decomposed into an irregular

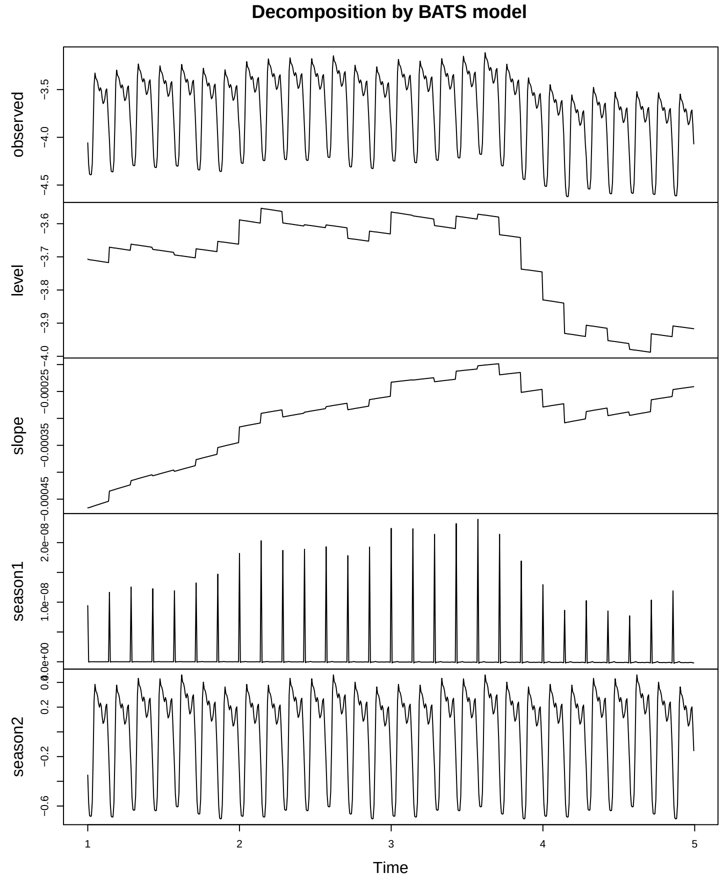


Figure A.2: Time series decomposition by the BATS model for water demand forecasting in the Barcelona DWN

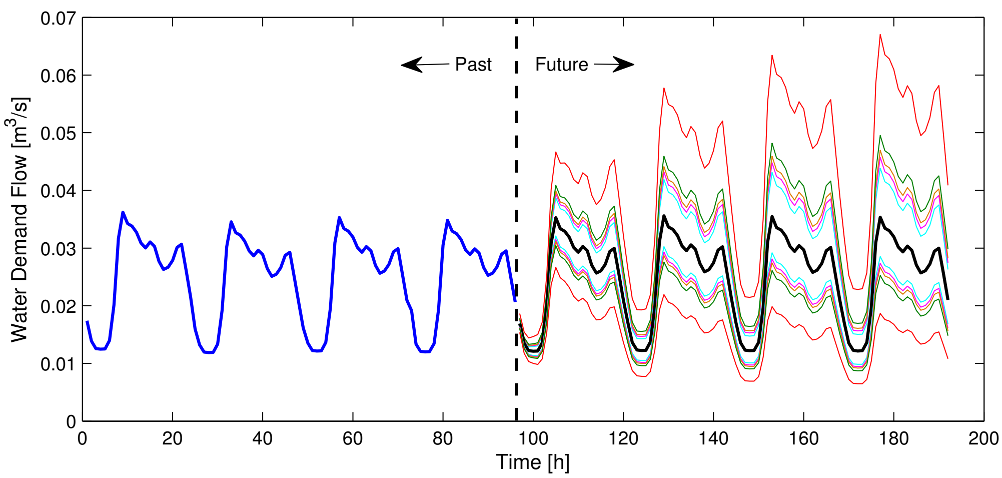


Figure A.3: Forecasting of water demand using a BATS model component h_k, a level component l_k, a growth component b_k and possible seasonal components $s_k^{(i)}$ with seasonal frequencies m_i, for $i = 1, \ldots, P$, where P is the total number of seasonal patterns in the series. In order to allow for possible dampening of the trend, a damping parameter ϕ is included. The irregular component h_k is described by an ARMA(p, q) process with parameters φ_i for $i = 1, \ldots, p$ and θ_i for $i = 1, \ldots, q$, and an error term ε_k which is assumed to be a Gaussian white noise process with zero mean and constant variance σ^2. The smoothing parameters, given by α_d, β_d, $\gamma_{d,i}$ for $i = 1, \ldots, P$, determine the extent of the effect of the irregular component on the states l_k, b_k, $s_k^{(i)}$ respectively.

The equations for the models are

$$d_k^{(\omega)} = \begin{cases} \frac{d_k^{(\omega)} - 1}{\omega}, & \omega \neq 0, \\ \log\left(d_k\right), & \omega = 0, \end{cases} \tag{A.1}$$

$$d_k^{(\omega)} = l_{k-1} + \phi b_{k-1} + \textstyle\sum_{i=1}^{P} s_{k-m_i}^{(i)} + h_k, \tag{A.2}$$

$$l_k = l_{k-1} + \phi b_{k-1} + \alpha_d h_k, \tag{A.3}$$

$$b_k = \phi b_{k-1} + \beta_d h_k, \tag{A.4}$$

$$s_k^{(i)} = s_{k-m_i}^{(i)} + \gamma_{d,i} h_k, \tag{A.5}$$

$$h_k = \textstyle\sum_{i=1}^{p} \varphi_i h_{k-i} + \sum_{i=1}^{q} \theta_i \varepsilon_{k-i} + \varepsilon_k. \tag{A.6}$$

The above model receives the notation BATS$(p, q, m_1, m_2, \ldots, m_P)$ and it can be expressed in the following state-space form:

$$d_k^{(\omega)} = w^\top x_{\mathrm{d},k-1} + \varepsilon_k, \tag{A.7}$$

$$x_{\mathrm{d},k} = F x_{d,k-1} + g\varepsilon_k, \tag{A.8}$$

where $w^\top$ is a row vector, g is a column vector, F is a square matrix and $x_{\mathrm{d},k}$ is the unobserved demand state vector at time k. The details on how these vectors and matrices are defined can be found in [47].

Demand uncertainty propagation: Let ϑ be a vector of all parameters to be estimated in the model (A.1-A.6), including the smoothing parameters and the Box-Cox parameter. Moreover, let k be the actual length of a water demand time series, n be the length of the desired demand forecast horizon, and $d_{k+n|k} \triangleq \{d_{k+n} | x_{d,k}, \vartheta\}$ be a random variable denoting future values of a demand series given the model, its calibrated parameters and the demand state vector at the last observation $x_{\mathrm{d},k}$. A Gaussian assumption for the errors implies that $d^{(\omega)}_{k+n|k}$ is also normally distributed, with mean $\bar{d}^{(\omega)}_{k+n|k}$ and variance $\Sigma^{(\omega)}_{d,k+n|k}$ given by

$$\bar{d}^{(\omega)}_{k+n|k} = w^\top F^{n-1} x_{\mathrm{d},k}, \tag{A.9}$$

$$\Sigma^{(\omega)}_{d,k+n|k} = \begin{cases} \sigma^2 & \text{if } n = 1, \\ \sigma^2 \left[1 + \sum_{j=1}^{n-1} (w^\top F^{j-1} g)^2\right] & \text{if } n \geq 2. \end{cases} \tag{A.10}$$

As demonstrated in [47], point forecasts and forecast intervals are obtained using the inverse Box-Cox transformation.

Taking the n_d time series of demands in the Barcelona DWN, and computing the inverse transformation of (A.9) and (A.10) applied to each of them, it is possible to define the elements that complete the stochastic properties of the DWN model for a prediction horizon $N \in \mathbb{Z}_{\geq 1}$ as follows:

$$\bar{d}_{k+n|k} \triangleq \left[\bar{d}_{1,k+n|k}, \ldots, \bar{d}_{n_d,k+n|k}\right]^\top, \tag{A.11}$$

$$\Sigma_{d,k+n|k} \triangleq \operatorname{diag}\left(\Sigma_{d(1),k+n|k}, \ldots, \Sigma_{d(n_d),k+n|k}\right), \quad \forall n \in \mathbb{Z}_{[1,N]}. \tag{A.12}$$

Appendix B

Reduction of Flow Variables

The reduction of the model is based on the following assumption.

Assumption B.1. *There are more variables than algebraic equations, i.e., $q < m$. The matrix E_u in (2.10b) has maximal rank, i.e.* $\operatorname{rank}(E_u) = q$, *and it can be expressed in a reduced staggered form by using Gauss-Jordan elimination.*

Consider (2.10b) in the following form:

$$\begin{bmatrix} E_u & E_d \end{bmatrix} \begin{bmatrix} u \\ d \end{bmatrix} = 0. \tag{B.1}$$

Assumption B.1 guarantees that there exists a permutation $\tilde{P}$ of the first m variables in (B.1) such that

$$E_u \tilde{P} = \begin{bmatrix} I_q & M_1 \end{bmatrix}, \quad M_1 \in \mathbb{R}^{q \times (m-q)}, \tag{B.2}$$

and

$$\begin{bmatrix} E_u & E_d \end{bmatrix} P = \begin{bmatrix} I_q & M_1 & E_d \end{bmatrix}, \quad E_d \in \mathbb{R}^{q \times p}, \tag{B.3}$$

where

$$P = \begin{bmatrix} \tilde{P} & 0 \\ 0 & I_p \end{bmatrix}. \tag{B.4}$$

Then, it is possible to state that

$$\begin{bmatrix} E_u & E_d \end{bmatrix} \begin{bmatrix} u \\ d \end{bmatrix} = 0 \iff \begin{bmatrix} E_u & E_d \end{bmatrix} P P^\top \begin{bmatrix} u \\ d \end{bmatrix} = 0. \tag{B.5}$$

Defining

$$\begin{bmatrix} v \\ d \end{bmatrix} = P^\top \begin{bmatrix} u \\ d \end{bmatrix} = \begin{bmatrix} \tilde{P}^\top u \\ d \end{bmatrix}, \tag{B.6}$$

where

$$v = \begin{bmatrix} \bar{u} \\ \tilde{u} \end{bmatrix}, \quad \bar{u} \in \mathbb{R}^q, \quad \tilde{u} \in \mathbb{R}^{(m-q)}, \tag{B.7}$$

then it holds

$$\begin{bmatrix} I_q & M_1 & E_d \end{bmatrix} \begin{bmatrix} \bar{u} \\ \tilde{u} \\ d \end{bmatrix} = 0 \implies \bar{u} = -M_1\,\tilde{u} - E_d\,d, \tag{B.8}$$

and

$$\begin{aligned} v = \begin{bmatrix} \bar{u} \\ \tilde{u} \end{bmatrix} &= \begin{bmatrix} -M_1\,\tilde{u} - E_d\,d \\ \tilde{u} \end{bmatrix} \\ &= \begin{bmatrix} -M_1 \\ I_{(m-q)} \end{bmatrix} \tilde{u} + \begin{bmatrix} -E_d \\ 0_p \end{bmatrix} d. \end{aligned} \tag{B.9}$$

Finally, being P a permutation matrix and therefore an orthogonal matrix, i.e., $P^{-1} = P^\top$, and from $\tilde{P}^\top u = v$, the control parametrisation is as follows:

$$u = \underbrace{\tilde{P} \begin{bmatrix} -M_1 \\ I_{(m-q)} \end{bmatrix}}_{\tilde{M}_1} \tilde{u} + \underbrace{\tilde{P} \begin{bmatrix} -E_d \\ 0_p \end{bmatrix}}_{\tilde{M}_2} d. \tag{B.10}$$

Replacing (B.10) in (2.10), the system can be then modelled with the following difference equation:

$$x_{k+1} = Ax_k + \tilde{B}\,\tilde{u}_k + \tilde{B}_d\,d_k, \tag{B.11}$$

where

$$\tilde{B} = B\,\tilde{P}\,\tilde{M}_1, \quad \tilde{B}_d = B\,\tilde{P}\,\tilde{M}_2 + B_d. \tag{B.12}$$

In the same way, constraint (5.1b) is transformed taking into account the control parametrisation in (B.10). Therefore, the set of restricted input constraints is defined as

$$\tilde{\mathbb{U}}_k \triangleq \{\tilde{u} \in \mathbb{R}^{m-q} \,|\, H\tilde{P}\tilde{M}_1\tilde{u} \leq h - F\tilde{P}\tilde{M}_2 d_k\} \quad \forall k. \tag{B.13}$$

Appendix C

Convex Approximation of DWN Chance Constraints

Below are derived the deterministic equivalents of the individual chance constraints that approximate the joint chance constraints in (5.24).

Lower Bound of States: The robust counterpart of the set of individual chance constraints that approximates the joint constraint (5.24c) is derived as follows:

$$
\begin{aligned}
&\forall i \in \mathbb{Z}_{[0,N-1]} \wedge \forall j \in \mathbb{Z}_{[1,n]}, \\
&\qquad \mathbb{P}\left[x_{(j),k+i+1|k} \geq x_{\min(j)}\right] \geq 1 - \frac{\delta_x}{2nN} \\
&\Leftrightarrow \quad \mathbb{P}\left[x_{(j),k+i+1|k} < x_{\min(j)}\right] \leq \frac{\delta_x}{2nN} \\
&\Leftrightarrow \quad \mathbb{P}\left[\frac{x_{(j),k+i+1|k} - \bar{x}_{(j),k+i+1|k}}{\Sigma^{1/2}_{x_{(j)},k+i+1|k}} < \frac{x_{\min(j)} - \bar{x}_{(j),k+i+1|k}}{\Sigma^{1/2}_{x_{(j)},k+i+1|k}}\right] \leq \frac{\delta_x}{2nN} \\
&\Leftrightarrow \quad \Phi\left(\frac{x_{\min(j)} - \bar{x}_{(j),k+i+1|k}}{\Sigma^{1/2}_{x_{(j)},k+i+1|k}}\right) \leq \frac{\delta_x}{2nN}, \\
&\Leftrightarrow \quad \bar{x}_{(j),k+i+1|k} \geq x_{\min(j)} - \Phi^{-1}\left(\frac{\delta_x}{2nN}\right)\Sigma^{1/2}_{x_{(j)},k+i+1|k}.
\end{aligned} \tag{C.1}
$$

The mean and variance of x are computed over the random variable d, as follows:

$$\bar{x}_{(j),k+i+1|k} = A_{(j)}\bar{x}_{k+i|k} + \tilde{B}_{(j)}\tilde{u}_{k+i|k} + \tilde{B}_{d(j)}\bar{d}_{k+i|k}, \tag{C.2}$$

$$\Sigma_{x_{(j)},k+i+1|k} = A_{(j)}\Sigma_{x,k+i|k}A^{\top}_{(j)} + \tilde{B}_{d(j)}\Sigma_{d,k+i|k}\tilde{B}^{\top}_{d(j)},. \tag{C.3}$$

The symmetry of the normal distribution allows to consider the equality $-\Phi^{-1}(p) = \Phi(1-p)$ for any probability level $p \in (0,1)$. In this way, the equivalent of (5.24c) can be finally expressed, $\forall\, i \in \mathbb{Z}_{[0,N-1]}$ and $\forall j \in \mathbb{Z}_{[1,n]}$, by the following single constraints:

$$\bar{x}_{(j),k+i+1|k} \geq x_{\min(j)} + \Phi^{-1}\left(1 - \frac{\delta_x}{2nN}\right)\Sigma^{1/2}_{x_{(j)},k+i+1|k}. \tag{C.4}$$

Upper Bound of States: The same procedure used to derive the lower bound of states yields the robust counterpart of (5.24d), which is expressed, $\forall i \in \mathbb{Z}_{[0,N-1]}$ and $\forall j \in \mathbb{Z}_{[1,n]}$, as

$$\bar{x}_{(j),k+i+1|k} \leq x_{\max(j)} - \Phi^{-1}\left(1 - \frac{\delta_x}{2nN}\right)\Sigma^{1/2}_{x_{(j)},k+i+1|k}. \tag{C.5}$$

Safety Constraint of States: In the operational constraint (5.24e), both sides of the inequality $x_{k+i+1|k} \geq d_{\text{net},k+i+1|k}$ contain random variables. This fact could complicate the definition of a linear deterministic equivalent for the probabilistic constraint, unless appropriate assumptions are made. As it can be seen in (5.24b) and (5.24g), the uncertainty in variables $x_{k+i+1|k}$ and $d_{\text{net},k+i+1|k}$ is directly associated with the stochastic variable in common: the forecasted demand d, which appears in the definition of both sides of the aforementioned inequality, but with a difference of one time instant between each side. Therefore, taking into account that a disturbance prediction model (e.g., time-series model) may allow to estimate the cumulative uncertainty for a multiple-step forecasting process, it can be assumed that the uncertainty of $d_{k+i|k}$ is already considered in the uncertainty of $d_{k+i+1|k}$. This assumption avoids overestimation of uncertainty and aims to reduce conservatism in the controller performance.

The deterministic equivalent is derived as follows:

$$\begin{aligned}
&\forall i \in \mathbb{Z}_{[0,N-1]} \;\wedge\; \forall j \in \mathbb{Z}_{[1,n]}, \quad \mathbb{P}\left[x_{(j),k+i+1|k} \geq d_{\text{net}(j),k+i+1|k}\right] \geq 1 - \frac{\delta_s}{nN} \\
\Leftrightarrow\quad & \mathbb{P}\left[d_{\text{net}(j),k+i+1|k} - x_{(j),k+i+1|k} \leq 0\right] \geq 1 - \frac{\delta_s}{nN} \\
\Leftrightarrow\quad & \Phi\left(\frac{-\mathbb{E}\left[d_{\text{net}(j),k+i+1|k} - x_{(j),k+i+1|k}\right]}{\Sigma^{1/2}_{d_{\text{net}(j)},k+i+1|k}}\right) \geq 1 - \frac{\delta_s}{nN} \\
\Leftrightarrow\quad & \bar{x}_{(j),k+i+1|k} \geq \bar{d}_{\text{net}(j),k+i+1|k} + \Phi^{-1}\left(1 - \frac{\delta_s}{nN}\right)\Sigma^{1/2}_{d_{\text{net}(j)},k+i+1|k}.
\end{aligned} \tag{C.6}$$

Appendix D

Background on LMIs

Lemma D.1 (S-procedure for quadratic functions)**.** *Let $F_0, F_1, \ldots, F_p$ be quadratic functions of the variable $\xi \in \mathbb{R}^n$, i.e.,*

$$F_i(\xi) \coloneqq \xi^\top M_i \xi + 2y_i^\top \xi + z_i, \quad i \in \mathbb{Z}_{[1,p]}$$

where $y_i \in \mathbb{R}^n$, $M_i^\top = M_i \in \mathbb{R}^{n\times n}$ and $z_i \in \mathbb{R}$. If there exists a scalar $\zeta_i > 0$, for all $i \in \mathbb{Z}_{[1,p]}$, such that for all ξ

$$F_0(\xi) - \sum_{i=1}^{p} \zeta_i F_i(\xi) \leq 0,$$

then, $F_0(\xi) \leq 0$ for all ξ such that $F_i(\xi) \leq 0$ for all $i \in \mathbb{Z}_{[1,p]}$ (cf. [28]).

Lemma D.2 (Schur complement)**.** *Given $Q = Q^\top$ and $R = R^\top$, the condition*

$$\begin{bmatrix} Q & S \\ S^\top & R \end{bmatrix} \succeq 0$$

is equivalent to

$$R \succeq 0, \quad Q - SR^\dagger S^\top \succeq 0, \quad S(I - RR^\dagger) = 0,$$

where $R^\dagger$ denotes the Moore-Penrose inverse of R (cf. [28]).

Lemma D.3. *(cf. [193]) Let $M \in \mathbb{R}^{n\times n}$, $y \in \mathbb{R}^n$ and $z \in \mathbb{R}$. The inequality*

$$x^\top M x + 2y^\top x + c \leq 0$$

is satisfied for all $x \in \mathbb{R}^n$ if and only if

$$\begin{bmatrix} M & y \\ y^\top & z \end{bmatrix} \preceq 0.$$

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