Multiple Integrator Zero Delay model for flat canals impacted by resonance phenomena

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***Abstract*—Inland navigation networks are composed of interconnected navigation reaches. Some of them are flat and thus especially subject to resonance phenomena: lock operations create waves that can travel back and forth during several hours before their attenuation. The objective of this paper is to propose a model dedicated to this phenomenon, which is called *multiple Integrator Delay Zero* (mIDZ) model. It is based on the well known IDZ model, which is particularly suitable to reproduce the dynamics of free-surface water systems. However, since it is a first order model, it can only reproduce the effect of the first peak and not the effect of the subsequent, attenuated peaks that are observed in the response. The proposed model exploits the performance of the IDZ model in order to reproduce the effect of the following peaks. Its design requires the classical IDZ parameter identification followed by a frequency study to determine the attenuation and the periodicity of the signal. This thorough study aims at obtaining an accurate model that can be used for fault diagnosis and/or fault-tolerant control purposes. The proposed approach is illustrated by considering the Cuinchy-Fontinettes navigation reach located in the north of France.**

***Index Terms*—Free-surface water systems, resonance, modeling, multi-model, frequency analysis.**

1 INTRODUCTION

Inland navigation networks are composed of artificial reaches, some of which are characterized by no slope. These reaches are particularly impacted by resonance phenomena. When a lock operation is performed, waves are created and travel from the upstream to the downstream end (resp. from downstream to upstream). Once they reach one end of the reach, they reflect back in the other direction. This phenomenon may take several hours before it attenuates completely. A correct understanding and modeling of this behavior is required to facilitate the design of control or fault-tolerant control strategies. Hence, models of canals without slope have to be proposed. The partial derivative equations of Saint-Venant [1] remain to this day as the most accurate model for these systems. However, the use of these equations for the design of control strategies is difficult. This is the reason why alternative solutions based on a simplification and a linearization of the Saint-Venant equations around an operating point have been proposed in the literature. The ID (Integrator Delay) model was proposed by [6]. This model is suitable to reproduce the low-frequency dynamics by taking into account time delays. It is therefore not suitable for water systems that are characterized by waves and resonance phenomena, *i.e.* high-frequency dynamics. The IDZ (Integrator Delay Zero) model [3] improved the ID model by considering an additional zero in the model. Thus, it allowed reproducing high-frequency dynamics, but not the resonance phenomena. The IR (Integrator Resonance) model was conceived to design control laws for free-surface water systems whose dynamics are characterized by the resonance phenomena [8]. However, it is a frequency-based model that cannot be easily used to design continuous-time model. Finally, data-based models have been proposed to deal with water systems for which the physical parameters are not well known, such as gray-box [2] and black-box [9] models. These approaches require accurate data that characterize all the operating range of the water systems.

The objective of this work is to propose a model based on physical parameters that can reproduce the high-frequency dynamics and the resonance phenomena. The measured water levels show that the peak that characterizes the high-frequency dynamics is repeated some time after, but attenuated. The first peak can be captured by the original IDZ model, but not the subsequent peaks. Therefore, the multiple IDZ (mIDZ) model is conceived as several IDZ models in cascade, each of them reproducing one of the peaks. The mIDZ model takes advantage of the well-known design methodology of the IDZ model, and extends it in order to be able to predict multiple peaks.

This paper is structured as follows: the description of the IDZ model is given in Section II. Section III is dedicated to the presentation of the mIDZ model and its tuning step. Section IV describes the Cuinchy-Fontinettes reach (north of France), which is used to design and test the proposed approach. The numerical expressions of the IDZ and the mIDZ models are determined according to the physical parameters and then tuned by means of single lock operation scenarios in a 24-hour period. Section V presents the testing of the tuned mIDZ models by considering a low navigation demand scenario (3 lock operations in a 24-hour period) and a high navigation demand scenario (13 lock operations in a 24-hour period). The advantages and limits of the mIDZ models compared to the original IDZ model are discussed. The conclusions and perspectives of this work are given in Section VI.

2 INTEGRATOR DELAY ZERO MODEL

The general IDZ input-output expression that links the discharges and the water depths at the boundaries of a reach can be expressed by means of:

$$\begin{bmatrix} y(0,s) \\ y(L,s) \end{bmatrix} = \begin{bmatrix} p_{11}(s) & p_{12}(s) \\ p_{21}(s) & p_{22}(s) \end{bmatrix} \begin{bmatrix} q(0,s) \\ q(L,s) \end{bmatrix}, \tag{1}$$

where 0 and L are the abscissas for the initial and final ends of the canal; $y(0,s)$ and $y(L,s)$, the upstream and downstream water levels; $q(0,s)$ and $q(L,s)$, the upstream inflow and downstream outflow; and $p_{ij}(s)$, the different terms of the IDZ model. These terms consist of an integrator, a delay and a zero. The two first terms reproduce the low frequencies behavior, while the zero accounts for the high frequencies. Its structure is as follows:

$$p_{ij}(s) = \frac{z_{ij} \cdot s + 1}{\mathcal{A}_{ij} \cdot s} e^{-\tau_{ij} \cdot s}, \tag{2}$$

where z represents the inverse of the zero, $\mathcal{A}$ the integrator gain and τ the propagation time delay. The integrator gain represents the change in volume according to the variation of the water level. The time delay constitutes the required time for a wave to travel from its origin to the measurement points (therefore $\tau_{11} = \tau_{22} = 0$). Finally, the zero approximates through a constant gain the oscillatory phenomena that occurs in high frequencies.

It can be considered that the dynamics of a canal are composed of a uniform part and a backwater part. It is necessary to compute these parameters for each part, and then merge them to obtain the equivalent parameters, which describe the whole pool. However, when the bed slope is equal to zero (flat canal), there is no uniform flow [1]. Therefore, the parameters must only be computed for the backwater part (using the total length of the reach), and no merging formulas are needed afterward [7].

3 MULTIPLE INTEGRATOR DELAY ZERO MODEL

3.1 Structure

The multiple IDZ (mIDZ) model consists in considering the nominal IDZ model and m attenuated and delayed replicas of this nominal model. Its structure is defined as follows:

$$\begin{bmatrix} y(0,s) \\ y(L,s) \end{bmatrix} = \underbrace{\begin{bmatrix} P_{11}(s) & P_{12}(s) \\ P_{21}(s) & P_{22}(s) \end{bmatrix}}_{\boldsymbol{P}(s)} \begin{bmatrix} q(0,s) \\ q(L,s) \end{bmatrix}, \tag{3}$$

where

$$P_{ij}(s) = \sum_{k=0}^{m-1} p_{ij}^k(s) \tag{4}$$

and

$$p_{ij}^k(s) = \frac{m \cdot \xi_k \cdot z_{ij} \cdot s + 1}{m \cdot \mathcal{A}_{ij} \cdot s} e^{-(\tau_{ij} + 2k \cdot \tau_{ij} + \delta)s}, \forall k \in \{0, m-1\} \tag{5}$$

with m the number of considered multimodels, k the index of the k^{th} multimodel, ξ_k the damping coefficient and δ a delay calibration coefficient. For $m = 1$, it is immediate that $P_{ij}(s) = p_{ij}^0(s) = p_{ij}(s)$. The periodicity of the wave reflection is determined according to the delays of the IDZ model. However, it is necessary to determine the damping coefficients ξ_k as well as a calibration factor δ that accounts for the calibration of the period. The first possibility is to use a frequency analysis.

3.2 Frequency analysis

A frequency analysis of open-surface water systems is proposed in [4]. This analysis is based on the Saint-Venant equations by considering the physical characteristics of the water systems. Moreover, it allows to draw Bode plots and to determine the number of peaks, their periodicity and the gain differences between peaks. The multiple IDZ model can be tuned according to these coefficients.

3.3 Parameter optimization

The frequency analysis gives some information about the resonance phenomenon. However, it is often still necessary to calibrate the parameters of each transfer function $p_{ij}^k(s)$. One possibility consists in simulating a step response; in the case of inland navigation reaches, it corresponds to an upstream lock operation to tune the transfer functions $p_{11}^k(s)$ and $p_{21}^k(s)$, and one downstream lock operation to tune $p_{12}^k(s)$ and $p_{22}^k(s)$.

4 CASE STUDY: THE CUINCHY-FONTINETTES REACH

4.1 Description of the system

The Cuinchy-Fontinettes reach (CFr) belongs to the inland navigation network in the north of France. It is one of the longest reaches of this network, with an approximate length of 42300 m. Its geometry is complex, with more than 600 section profiles. However, it can be accurately approximated by considering a rectangular cross section of dimensions 52 m (top width) by 3.8 m (average water depth). It is equipped with a lock and a controlled gate at the upstream end of Cuinchy, and with a lock at the downstream end of Fontinettes. This reach is characterized by a bed slope equal to 0 (flat canal), which is the reason why it is particularly impacted by resonance phenomena, due mainly to the operations of the Fontinettes lock. Indeed, each lock operation in Fontinettes creates a wave of more than 15 cm of magnitude that travels back and forth during several hours. The lock operations lead to water volume exchanges (with the adjacent reaches) of 3000 m^3 (lock of Cuinchy, input volume) and 25000 m^3 (lock of Fontinettes, output volume). Each lock operation lasts 15 min in average. In addition, in order to determine the parameters of the IDZ and mIDZ models, the values of the Manning coefficient ($n = 0.035$) and the daily average discharge ($q = 0.6$ m^3/s) are required.

4.2 Numerical results

4.2.1 IDZ model: The computation of the IDZ model based on the procedure described in [3] yields the following numerical results:

$$\hat{p}_{11} = \frac{6928s+1}{2.2\cdot 10^6 s} \qquad \hat{p}_{12} = \frac{-9544s-1}{2.2\cdot 10^6 s}e^{-6930s}$$
$$\hat{p}_{21} = \frac{9544s+1}{2.2\cdot 10^6 s}e^{-6920s} \qquad \hat{p}_{22} = \frac{-6928s-1}{2.2\cdot 10^6 s} \tag{6}$$

4.2.2 Multiple IDZ model: The mIDZ model is based on the identified IDZ model given in (6). Taking it as a starting point, a frequency analysis is performed. It leads to the Bode plot depicted in Fig. 1 for $p_{12}(s)$ and $p_{22}(s)$. It is possible to determine, according to this plot, the gain of each peak, which leads to the damping coefficients. As an example, Table I summarizes the results for the case of an mIDZ model composed of 4 models, *i.e.* $m = 4$. This table allows to compute the P_{ij} expressions given in (4) and (5), which are not given here due to lack of space. With respect to the distance between two consecutive peaks, it is assumed that it is equal to $2\tau_{ij}$. The Bode plots provide the same values.

The damping coefficients of the mIDZ models should be modified according to the step response that corresponds to a Fontinettes lock operation (*see* Fig. 2), such as those given in Table I. In addition, the resonance period is calibrated by considering δ = 700 s more than the initial period (relative error smaller than 10%). The same calibration step is performed for $p_{11}(s)$ and $p_{21}(s)$ (but not depicted in this paper) by means of a lock operation in Cuinchy. Finally, according to the calibration step, the accuracy of the mIDZ models can be improved.

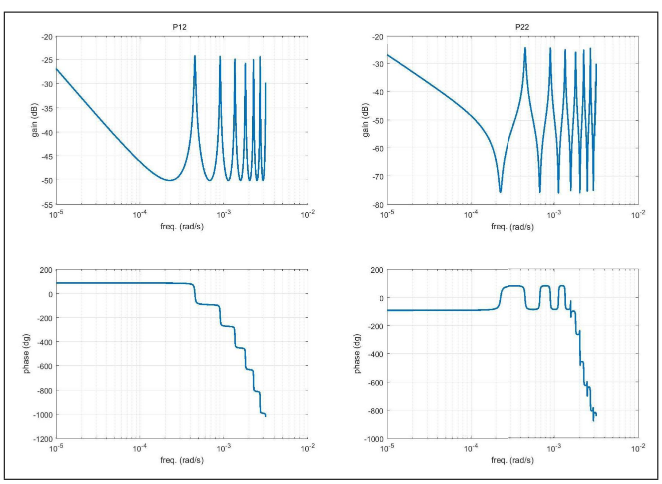

Fig. 1. Bode plot of $p_{12}(s)$ and $p_{22}(s)$.

5 Simulation results

In order to test the proposed approach, several scenarios are considered. The first step involves simulating individual lock operations, first in Cuinchy and after in Fontinettes. In general, linearized models like the IDZ offer an acceptable performance, but they still need to be fitted with respect to the measured reference. Once the predictive power of the model has been improved, a 24-hour realistic profile of the navigation condition in the CFr is simulated. Four different models are tested, the original IDZ model and three mIDZ models with m={2, 4, 6}. For the sake of brevity, they will be referred to as IDZ, 2IDZ, 4IDZ and 6IDZ hereinafter.

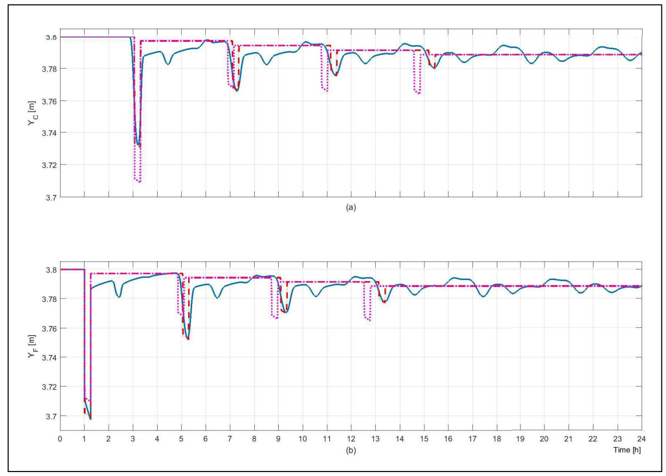

Fig. 2. (a) Cuinchy water level. (b) Fontinettes water level. Reference water level: blue solid line; mIDZ model (m=4) from frequency analysis: magenta dotted line; mIDZ model (m=4) from optimization: red dashed line.

TABLE I
Calibration of the damping coefficients

		ξ_0	ξ_1	ξ_2	ξ_3
P_{12}	*Frequency analysis*	1	0.3	0.29	0.28
	Optimization	0.75	0.32	0.18	0.1
P_{22}	*Frequency analysis*	1	0.3	0.29	0.27
	Optimization	1.12	0.48	0.25	0.13

In order to ensure a more quantitative comparison, the following fit coefficients are computed:

- *Pearson product-moment correlation coefficient*: linear dependence measure between two variables and is defined in the following way:

$$r = \frac{\sum_{t=1}^{T}\left(Y_o(t)-\overline{Y_o}\right)\left(Y_m(t)-\overline{Y_m}\right)}{\sqrt{\sum_{t=1}^{T}\left(Y_o(t)-\overline{Y_o}\right)^2}\sqrt{\sum_{t=1}^{T}\left(Y_m(t)-\overline{Y_m}\right)^2}} \tag{7}$$

  with T the horizon for which the data have been acquired, $Y_o(t)$ the observed water depth at time *t*, $Y_m(t)$ the predicted water depth at time *t* and $\overline{Y_o}$ and $\overline{Y_m}$ the mean value of observed and modeled water depths, respectively. The bounds of this coefficient are +1 (total positive linear correlation) and -1 (total negative linear correlation), and 0 implies no linear correlation.
- *Nash-Sutcliffe model efficiency coefficient*: assesses the predictive power of hydrological models as follows [5]:

$$E = 1 - \frac{\sum_{t=1}^{T} \left(Y_o(t) - Y_m(t)\right)^2}{\sum_{t=1}^{T} \left(Y_o(t) - \overline{Y_o}\right)^2} \quad (8)$$

E can range from 1 to $-\infty$, where 1 indicates a perfect match of modeled and observed values, 0 corresponds to the case in which the model predictions are as accurate as the mean of observed data and $E < 0$ means that the model predictions are less accurate than the mean of observed data.

- *Maximum difference between the modeled and observed data* as a measure of the magnitude of the maximum error. It is computed as:

$$\Delta = \max_{1 \leq t \leq T} |Y_o(t) - Y_m(t)| \quad (9)$$

5.1 Individual lock operations

5.1.1 Cuinchy lock operation: The operation of the lock of Cuinchy at $t = 1\ h$ is simulated. Below, the reference water level (blue solid line) and the predicted water levels by using IDZ, 2IDZ, 4IDZ and 6IDZ models (red dotted line), respectively, are depicted in Fig. 3 and 4. All models are calibrated according to the previous tuning description.

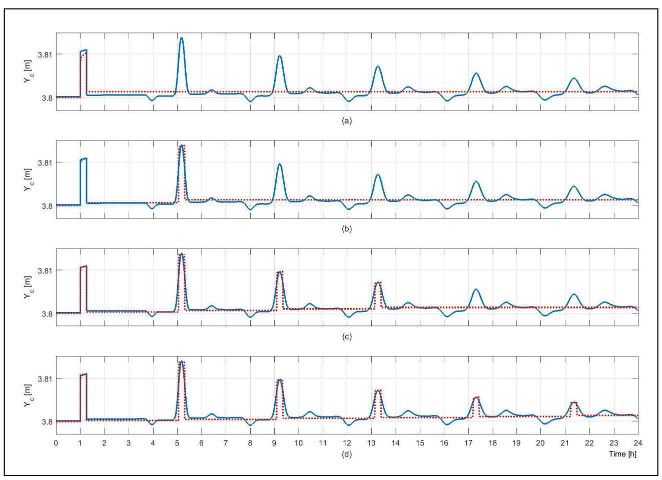

Fig. 3. Water levels in Cuinchy for a lock operation in Cuinchy. (a) IDZ model. (b) 2IDZ model. (c) 4IDZ model. (d) 6IDZ model.

TABLE II
FIT COEFFICIENTS FOR A CUINCHY LOCK OPERATION

		IDZ	2IDZ	4IDZ	6IDZ
C	E	0.21	0.52	0.72	0.76
	r	0.47	0.72	0.85	0.89
	Δ $[m]$	0.0123	0.0082	0.0077	0.0078
F	E	0.20	0.59	0.72	0.75
	r	0.49	0.77	0.86	0.88
	Δ $[m]$	0.014	0.0078	0.0078	0.0079

This individual lock operation is the only disturbance that affects the water level. Therefore, the wave created by the lock operation is immediately measured upstream (since the

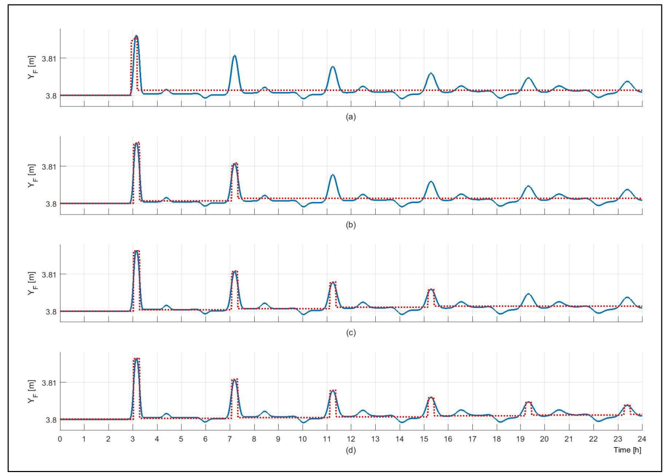

Fig. 4. Water levels in Fontinettes for a lock operation in Cuinchy. (a) IDZ model. (b) 2IDZ model. (c) 4IDZ model. (d) 6IDZ model.

lock and the water level sensor can be considered to be in the same spatial point). Then, the wave is measured downstream after the corresponding time delay has elapsed from the lock operation, and finally it travels back to the upstream end, where it is measured again with an attenuated magnitude. This behavior is repeated in time until a complete attenuation of this phenomenon.

Table II summarizes the fit coefficients for this scenario. Of course, as the number of used IDZ models increases, so do the Nash and the correlation coefficients, because the number of predicted peaks increases. However, it is interesting to note that these indicators grow faster in the beginning (when switching from IDZ to 2IDZ) and more steadily after. This is due to the fact that the magnitude of the peaks attenuates. Therefore, the relative improvement in the prediction accuracy of two consecutive models will be larger in the beginning. The same trend is observed in the case of the maximum absolute difference: this value is reduced much more in the beginning, but it soon reaches a minimum value that can no longer be improved.

5.1.2 Fontinettes lock operation: A similar simulation is presented for the case of a Fontinettes lock operation, which also takes place at $t = 1\ h$. Figures 5 and 6 and Table III illustrate the obtained results.

TABLE III
FIT COEFFICIENTS FOR A FONTINETTES LOCK OPERATION

		IDZ	2IDZ	4IDZ	6IDZ
C	E	-0.13	0.67	0.57	0.41
	r	0.53	0.83	0.81	0.81
	Δ $[m]$	0.084	0.040	0.040	0.040
F	E	0.71	0.81	0.77	0.67
	r	0.85	0.90	0.90	0.90
	Δ $[m]$	0.035	0.032	0.037	0.039

In contrast to the previous scenario, the model that offers the best performance is the 2IDZ model, according to the fit coefficients in Table III. A thorough analysis of Fig. 5 and 6 reveals that after the second peak ($t \geq 5\ h$) of the reference

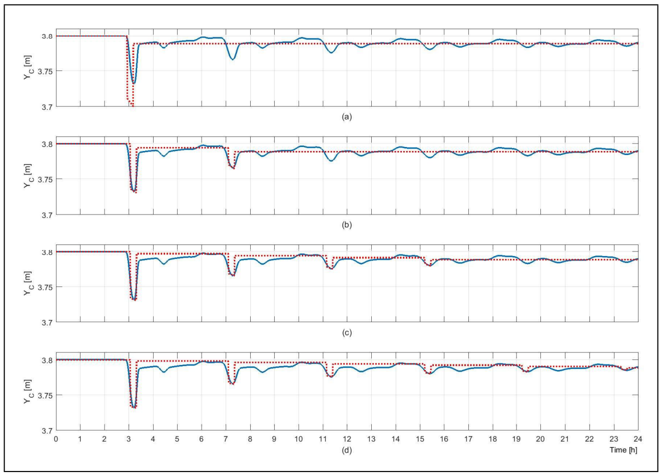

Fig. 5. Water levels in Cuinchy for a lock operation in Fontinettes. (a) IDZ model. (b) 2IDZ model. (c) 4IDZ model. (d) 6IDZ model.

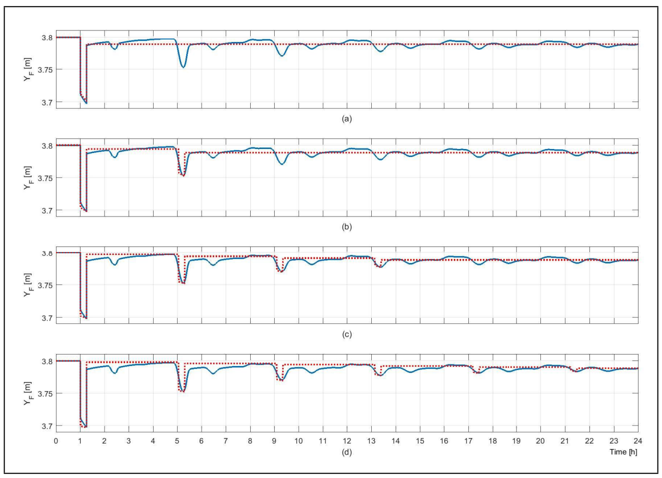

Fig. 6. Water levels in Fontinettes for a lock operation in Fontinettes. (a) IDZ model. (b) 2IDZ model. (c) 4IDZ model. (d) 6IDZ model.

(blue solid line), the 2IDZ model (red dotted line) converges faster to the reference than 4IDZ and 6IDZ. It is true that the two last models are able to predict more peaks than 2IDZ, but their magnitude is such that their prediction does not yield better fit coefficients.

5.2 *Realistic scenarios*

After the simulation of individual lock operation scenarios, all the considered models are calibrated with respect to the reference. Two realistic scenarios are simulated, one with low navigation demand and another one with high navigation demand (*see* Fig. 7). Below, the predicted water levels and the fit coefficients are presented for each scenario.

5.2.1 Low navigation demand: The upstream and downstream water levels corresponding to this scenario (*see* Fig. 7(a) are depicted in Fig. 8 and 9, respectively. The fit coefficients for this scenario are summarized in Table IV.

In this low navigation demand scenario, the 2IDZ model yields the best performance according to Table IV. This model offers a trade-off between settling time (IDZ is the fastest

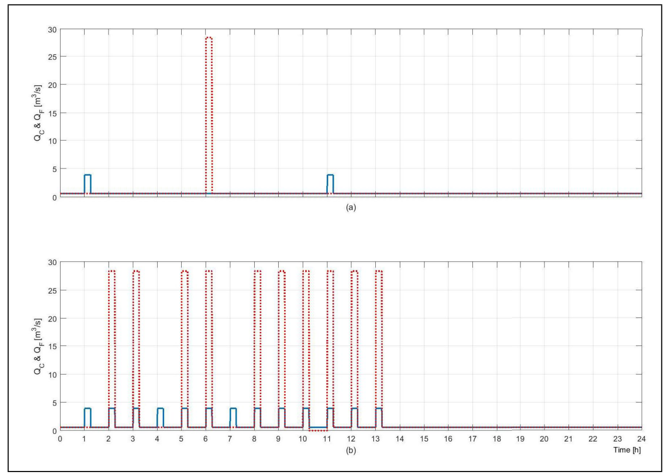

Fig. 7. (a) Low navigation demand scenario. (b) High navigation demand scenario. Upstream discharge: blue solid line; downstream discharge: red dotted line.

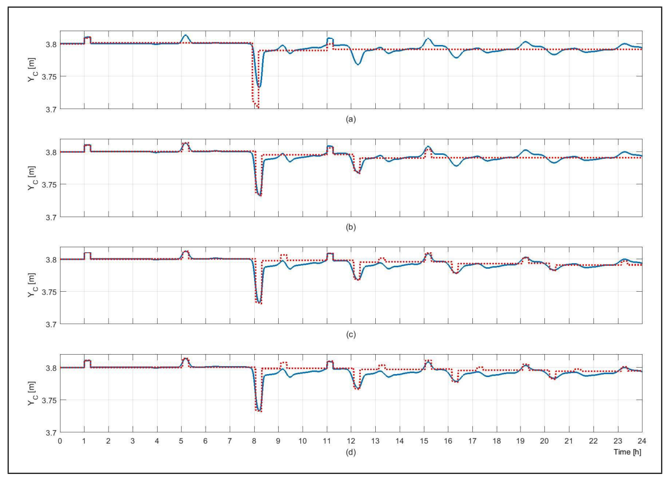

Fig. 8. Water level in Cuinchy for a low navigation demand scenario. (a) IDZ model. (b) 2IDZ model. (c) 4IDZ model. (d) 6IDZ model.

model) and the prediction of multiple peaks (the more models are used, the more peaks can be predicted). Indeed, 4IDZ and 6IDZ overestimate their prediction of the water levels at the beginning and gradually converge to the measured water level. In this way, they need more time to react to changes coming from water volume displacements.

5.2.2 High navigation demand: The upstream and downstream water levels corresponding to this scenario (*see* Fig. 7(b) are depicted in Fig. 10 and 11, respectively. The fit coefficients for this scenario are summarized in Table V.

In this high navigation demand profile, the water levels do not stay close to the initial level, unlike the previous scenario. This is a consequence of the water exchange with other reaches. This scenario considers 13 lock operations in Cuinchy and 10 in Fontinettes (the balance is -211000 m^3). The final water level decrease (after attenuation of the resonant waves) is the result of this emptying.

The 2IDZ model again attains the best performance of the four models, for the same reasons given before.

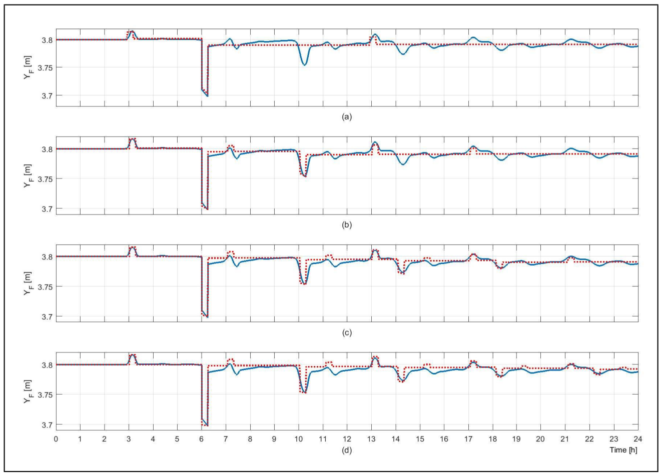

Fig. 9. Water level in Fontinettes for a low navigation demand scenario. (a) IDZ model. (b) 2IDZ model. (c) 4IDZ model. (d) 6IDZ model.

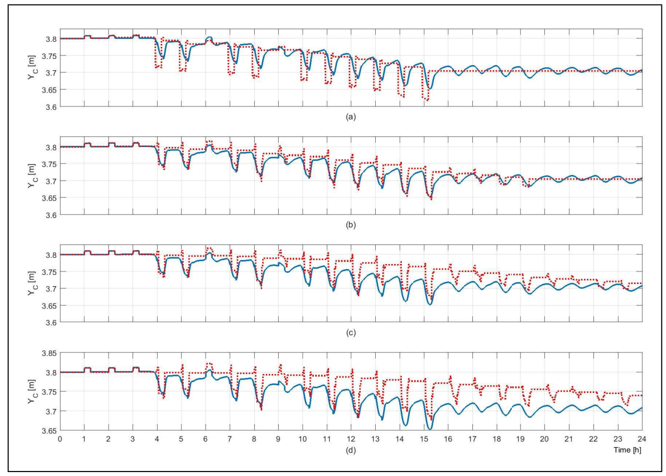

Fig. 10. Water level in Cuinchy for a high navigation demand scenario. (a) IDZ model. (b) 2IDZ model. (c) 4IDZ model. (d) 6IDZ model.

TABLE IV
FIT COEFFICIENTS FOR A LOW NAVIGATION PROFILE SCENARIO

		IDZ	2IDZ	4IDZ	6IDZ
C	E	0.09	0.74	0.69	0.58
	r	0.60	0.86	0.86	0.84
	Δ $[m]$	0.08	0.04	0.04	0.04
F	E	0.75	0.86	0.83	0.76
	r	0.86	0.93	0.93	0.91
	Δ $[m]$	0.030	0.032	0.037	0.039

6 CONCLUSIONS AND FUTURE WORK

This work was dedicated to the development of a multiple Integrator Delay Zero model, which stems from the well known IDZ model. Since it is a first-order model, it is not able to characterize the subsequent peaks that are consequence of resonance phenomena. This behavior is especially critical for flat canals. The mIDZ model makes use of the IDZ model to consider attenuated and delayed replicas and therefore be more precise in the prediction, which is shown in Section V.

The scope of this work was to propose an accurate model that can describe faithfully the dynamics of flat canals. As a future work, with this model it will be easier to detect and diagnose faults that affect these systems, and also to design fault-tolerant control strategies. Therefore, this model has been conceived as a first step towards FDI and FTC of flat reaches.

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TABLE V
FIT COEFFICIENTS FOR A HIGH NAVIGATION PROFILE SCENARIO

		IDZ	2IDZ	4IDZ	6IDZ
C	E	0.60	0.88	0.53	0.14
	r	0.83	0.96	0.90	0.86
	Δ $[m]$	0.098	0.059	0.083	0.095
F	E	0.93	0.93	0.70	0.34
	r	0.97	0.97	0.93	0.88
	Δ $[m]$	0.03	0.04	0.07	0.09

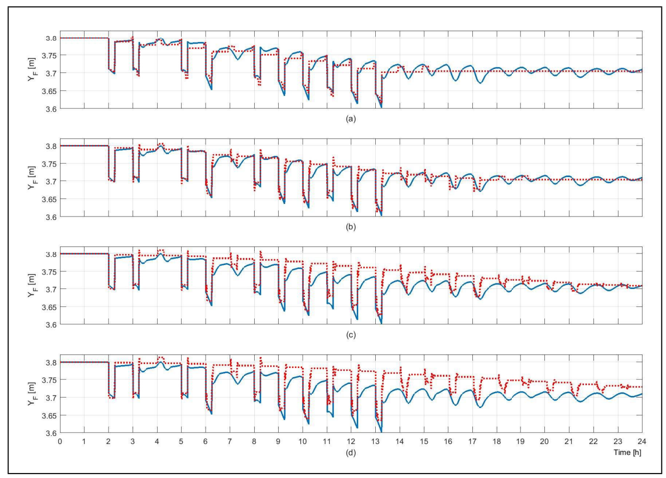

Fig. 11. Water level in Fontinettes for a high navigation demand profile. (a) IDZ model. (b) 2IDZ model. (c) 4IDZ model. (d) 6IDZ model.

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