

# Control-Oriented Modeling Approach for Open Channel Irrigation Systems<sup>\*</sup>

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**Abstract:** In irrigation, most of the water is transported by networks of open-channel irrigation systems (OCIS). In most cases, the OCIS are manually operated showing low efficiency. Then the incorporation of control strategies is one of the most practical ways to increase the efficiency of these systems. However, in order to design an appropriate control strategy, an accurate control-oriented model that can be used for analyses, simulation, design, and test of the OCIS under realistic scenarios is necessary. In OCIS, obtaining a control-oriented model is a challenge that has aroused interest in the related literature. In spite of the multiple research in modeling of OCIS, the development of control-oriented modeling approaches that describe the dynamic and nonlinear behaviors of OCIS with gate regulation structures, remains an open problem. In this paper, a modeling approach that describes the nonlinear and dynamical behaviors of OCIS using a mass and energy balance by channel is proposed, which is compared with two modeling approaches. The comparison has been performed with a test case proposed in the literature. The results show that the proposed modeling approach is better describing the nonlinear behavior of the OCIS and presents a simpler structure that can be used in the design and test of control, prediction, and estimation strategies for these type of systems.

*Keywords:* Modeling; Hydroinformatics; Water and food security; Water quality and quantity management.

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## 1. INTRODUCTION

Water is a priority because it is a limited resource that needs to be protected. Nearly 70% of the water consumed in the world is used for irrigation, and most of the water is transported through open channels. In this process, the water is taken from a natural source and is transported by networks of open channels; these systems are called open channel irrigation systems (OCIS). In most countries, these systems are manually operated, and large amount of losses are present. The incorporation of control systems to OCIS is considered one of the most reliable ways to reduce losses (Zheng et al., 2019), and in order to reach the most favorable behavior of the controlled system, the existence of accurate models for simulation and control design is essential. The Saint-Venant equations (SVE) offers a rigorous analytical description of the OCIS dynamics. However, the SVE are two nonlinear partial differential equations and the direct use of these equations for control systems design is impractical (Cantoni et al., 2007). For this reason,

in the literature, there is reported the use of different types of modeling approaches (or strategies) that approximate the OCIS dynamics. These modeling approaches can be divided in two fields: i) models that come from simplifications of the SVE (simplified models), such as the use of explicit and implicit finite-difference schemes (e.g., Cen et al. (2017); Bonet et al. (2017); Lacasta et al. (2018)), the transformations, and partial solutions of the linearized SVE (e.g., Litrico and Fromion (2004a); Qiao and Yang (2010); Clemmens et al. (2017)); and ii) models that come from basic physical principles, observations and empirical knowledge (approximated models), such as the Muskingum model (e.g., Bolea et al. (2014b); Horváth et al. (2014)), the Integrator Delay Model (e.g., Van Overloop et al. (2014); Horváth et al. (2015b,a); Zheng et al. (2019)), the use of black-box models or experimental models, which can only be obtained with measurements from a real system (e.g., Rivas Perez et al. (2007); Diamantis et al. (2011); Romera et al. (2013); Herrera et al. (2013); Bolea et al. (2014a); Van Overloop et al. (2014)), and grey-box models, which have a structure based on physical knowledge, and their parameters are identified from measured data (e.g., Mareels et al. (2005); Eurén and Weyer (2007); Ooi and Weyer (2011); Bedjaoui and Weyer (2011)). It has been shown that there are multiple options to obtain simplified

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and approximate modeling strategies for OCIS. However, according to the authors knowledge, there are few reports of simplified or approximated control-oriented modeling approaches for OCIS that can be used to: i) describe most of the dynamic behavior of the real system, even adverse conditions, disturbances, noise, parameter variations, etc.; ii) obtain design models, which are frequently linear models used for control systems design and analyses of performance indices at an operation region; iii) test the designed controllers in presence of external disturbances and realistic scenarios; and iv) describe the nonlinear dynamic behavior of open channels that are interconnected with regulation structures of the type *gates in submerged-flow*, where the flow depends on the upstream depth and downstream depth of the regulation structure.

Therefore, in this work, the analyses of three modeling approaches that can accomplish with the interpose requirements are performed. First, it is analyzed a simplified modeling approach proposed by Litrico and Fromion (2004a), which is obtained by linearization and Laplace transformation of the SVE. However, in this study, the nonlinear hydraulic relations of the regulation structures are kept. Second, it is analyzed a modeling approach proposed by Conde et al. (2019), which describes the dynamical behavior of the system using a mass balance and the hydraulic mathematical descriptions of the regulation structures, assuming that after a time-delay and adjusted by a constant, the dynamic behavior at the downstream end of the channel is similar to the dynamic behavior at the upstream end. Third, as a main contribution, it is proposed a new modeling approach that includes the nonlinear hydraulic relation of the regulation structures, and mass and potential energy balances that describes the dynamical behavior of the OCIS. These approaches are evaluated using the test case proposed by Clemmens et al. (1998) that is implemented in EPA-SWMM (reference model), which is a specialized simulation tool developed by the Environmental Protection Agency of the United States that gives the numerical solution of the SVE of hydraulic systems (Lewis, 2017).

The remainder of the paper is organized as follows. Section 2 starts with a short description and explanation of the OCIS, then the description of the three modeling approaches is performed. In Section 3, the test case and the used operational conditions are presented. Next, in Section 4 the comparisons between the reference model and the modeling approaches are drawn, and the results are discussed. Finally, in Section 5, some conclusions are presented.

## 2. MODELING APPROACHES

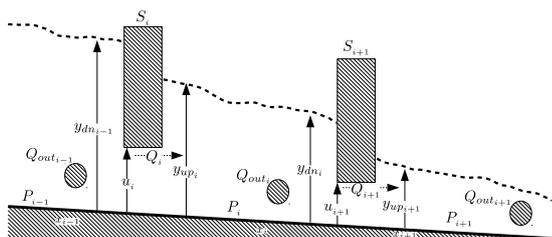


Fig. 1. Proposed representation for OCIS.

Table 1. Notation

$P_i$	Section between two consecutive cross structures (pool, channel, canal).
$i$	Stage number (1 = first channel)
$Q_i$	$P_i$ inflow ( $\text{m}^3/\text{s}$ )
$Q_{i+1}$	$P_{i+1}$ inflow and $P_i$ outflow ( $\text{m}^3/\text{s}$ )
$S_i$	Cross structure that regulate the flow $Q_i$
$u_i$	regulation structure position (m)
$x_i$	Downstream distance from $S_i$ (m)
$y_{x_i}$	Depth at point $x_i$ (m)
$y_{up_i}$	Depth at the upstream end of $P_i$ (m)
$y_{dn_i}$	Depth at the downstream end of $P_i$ (m)
$Q_{out_i}$	Outlet flow ( $\text{m}^3/\text{s}$ )
$W_i$	Channel width (m)
$L_i$	Channel length (m)

In order to explain the objective modeling approaches, first it is introduced a short description and explanation of the OCIS, which are structures used to transport water. In Figure 1, the proposed representation for OCIS is shown, and in Table 1 the associated variables are summarized. In this case,  $P_i$  is fed by the flow  $Q_i$  that comes from the upstream canal  $P_{i-1}$ .  $x_i$  is a position inside  $P_i$ , from the upstream end of the pool, and  $y_{x_i}$  represents the depth at the  $x_i$  position. For control purposes, the most important output variables are the upstream and downstream depths of a pool, denoted by  $y_{up_i}$  and  $y_{dn_i}$ , respectively. From the pool  $P_i$  there could be multiple outflows to other channels. In the representation (Figure 1), the outflows are simplified into the outlet flow  $Q_{out_i}$ , and the flow that feeds the downstream channel  $Q_{i+1}$ . The most notorious dynamic feature is that, in steady-state, the volume in a pool increases when the inflow increases, and decreases when the outflow increases. The flow  $Q_i$  has a hydraulic relationship with the regulation structure that could be divided into gates (Figure 2) and weirs (Figure 3) that regulate in free-flow or submerged-flow (Litrico and Fromion, 2009). In Table 2, the mathematical relationships for the discharge through each type of structure is presented, where  $w$  is the width of the regulation structure,  $g$  is the gravity constant, and  $Cd$  is the discharge coefficient. Furthermore, it is important to highlight that in OCIS, the control variables are the regulation structure positions  $u_i$  or, by the inclusion of a master-slave configuration,  $Q_i$ .

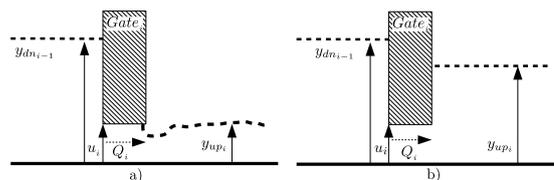


Fig. 2. Flow relation for: a) Gate in free-flow. b) Gate in submerged-flow.

Now, using the proposed notation, in the following subsections each modeling approach is described.

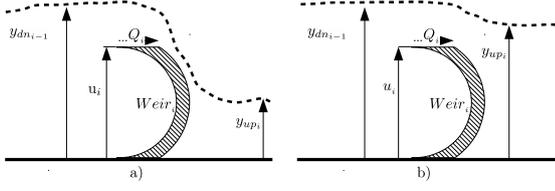


Fig. 3. Flow relation for: a) Weir in free-flow. b) Weir in submerged-flow.

Table 2. Flow relation for different categories of regulation structures

	Free-flow	Submerged-flow
Gate	$Q_i = Cd_i w_i u_i \sqrt{2g} \sqrt{y_{dn_{i-1}} - 0.5u_i}$	$Q_i = Cd_i w_i u_i \sqrt{2g} \sqrt{y_{dn_{i-1}} - y_{up_i}}$
Weir	$Q_i = Cd_i w_i \sqrt{2g} (y_{dn_{i-1}} - u_i)^{3/2}$	$Q_i = Cd_i w_i \sqrt{2g} (y_{dn_{i-1}} - y_{up_i})^{3/2}$

### 2.1 Integrator Delay Zero (IDZ) Modeling Approach

The IDZ modeling approach has been proposed by Litrico and Fromion (2004a) showing that for an open channel, the linearized Laplace transform of the SVE are spatial linear ordinary differential equations that are solved obtaining a transfer function matrix with  $y_{up}$  and  $y_{dn}$  as outputs, and  $Q_i$  and  $Q_{i+1}$  as inputs, where the parameters of the matrix can be obtained from the physical parameters of the system. For low frequencies, this modeling approach describes an open channel with two differential equations given by

$$\begin{aligned} A_{up_i} \dot{y}_{up_i}(t) &= Q_i(t) - Q_{i+1}(t - \tau_i) - Q_{out_i}(t - \tau_i) \\ A_{dn_i} \dot{y}_{dn_i}(t) &= Q_i(t - \tau_i) - Q_{i+1}(t) - Q_{out_i}(t). \end{aligned} \quad (1)$$

In this approach, the system can be analyzed as two storage units. In the first storage unit, with area  $A_{up_i}$ , the channel inflow enters at a time  $t$ , and the channel outflow leaves the storage unit after a time delay  $\tau_i$ . In the second storage unit, with area  $A_{dn_i}$ , the channel inflow enters at a time  $t - \tau_i$ , and the channel outflow leaves the storage unit at a time  $t$ . In the original form of the IDZ the hydraulic relations of the regulation structures are linearized in order to obtain a model for control design. In this work, using the information presented in Table 2, the nonlinear relations are conserved in order to obtain a nonlinear description of the OCIS and validate the IDZ as a control-oriented modeling strategy.

### 2.2 Modeling Approach 1

This modeling approach has been proposed by Conde et al. (2019) using a mass balance per channel. In this model, it is assumed that the upper part of the channel  $i$  has a depth  $y_{up_i}$  and the lower part of the channel has a depth  $l_i y_{up_i}(t - \tau_i)$ . That means, after a time delay, the level at the lower part of the channel is similar to the level at the upper part but attenuated by a constant  $l_i$  associated with a difference of potential along the channel. The mass balance is described as follows:

$$\begin{aligned} A_i \dot{y}_{up_i} &= \overbrace{w_{S_i} u_i \sqrt{2g} Cd \sqrt{l_{i-1} y_{up_{i-1}}(t - \tau_{i-1}) - y_{up_i}}}^{Q_i} \\ &- \overbrace{w_{S_{i+1}} u_{i+1} \sqrt{2g} Cd \sqrt{l_i y_{up_i}(t - \tau_i) - y_{up_{i+1}}}}^{Q_{i+1}} \\ &- \overbrace{w_{out_i} u_{out_i} \sqrt{2g} Cd \sqrt{l_i y_{up_i}(t - \tau_i) - 0.5 u_{out_i}}}^{Q_{out_i}}. \end{aligned} \quad (2)$$

It is important to mention that parameters such as area, discharge structure width, and the discharge coefficient could be obtained by structural channel information. Other parameters such as the time delay and the potential energy difference could be obtained from experimental data as is presented in Conde et al. (2019).

### 2.3 Proposed Modeling Approach

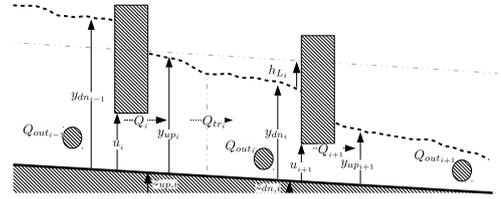


Fig. 4. Graphical description for the proposed energy and mass balances.

In this modeling approach (Approach 2), the inclusion of potential energy balances in the description of the dynamical behavior of the OCIS is proposed. In this approximation, the modeling approach assumes mass balances for two storage units per channel and a transition flow between each storage unit. The transition flow is obtained from a simplification of the concept of energy conservation along each channel. Figure 4 shows a representation of the modeling approach, where the mass balance is given by

$$\begin{aligned} A_{up_i} \dot{y}_{up_i}(t) &= Q_i(t) - Q_{tr_i}(t) \\ A_{dn_i} \dot{y}_{dn_i}(t) &= Q_{tr_i}(t) - Q_{out_i}(t) - Q_{i+1}(t), \end{aligned} \quad (3)$$

with the flows  $Q_i(t)$ ,  $Q_{i+1}(t)$ , and  $Q_{out_i}(t)$  obtained from the flows associated with each discharge structure. The flow transition ( $Q_{tr_i}(t)$ ) is obtained from an energy balance given by

$$z_{up_i} + y_{up_i} + \frac{v_{up_i}^2}{2g} = z_{dn_i} + y_{dn_i} + \frac{v_{dn_i}^2}{2g} + h_{L_i}, \quad (4)$$

where the difference between  $z_{up_i}$  and  $z_{dn_i}$  is the potential energy related to the channel inclination,  $v_{up_i}$  and  $v_{dn_i}$  are the upstream and downstream mean flow velocity,  $\frac{v_{up_i}^2}{2g}$  and  $\frac{v_{dn_i}^2}{2g}$  are the kinetic energy at the upper and lower part of the channel. Besides,  $h_{L_i}$  is known as the head loss due to friction, which can be described by the Darcy-Weisbach equation (U. S. Department of the Interior, 2001) by

$$h_{L_i} = f_i \frac{L_i v_i^2}{D_i 2g}, \quad (5)$$

where  $f_i$  is the friction factor,  $L_i$  is the channel length and  $D_i$  is the hydraulic diameter. In this model, an equal mean flow velocity along the channel is assumed, therefore the mean flow velocity could be approximated by  $v_{up_i} = v_{dn_i} = \frac{Q_{tr_i}}{W_i y_{up_i}}$ . On the other hand,  $f_i$  is a function of the Reynolds number, which is a relation between the viscous and inertial forces in a fluid Chaudhry (2008). Therefore, the strongest assumption is to describe the parameters  $f_i, L_i, D_i, g, W_i$  with a unique transition constant  $K_{tr_i}$  that could be obtained from experimental tests. Then, assuming that  $h_{L_i} \approx \frac{Q_{tr_i}^2}{K_{tr_i}^2 y_{up_i}^2}$ , and performing the energy balance (4), the flow transition is given by

$$Q_{tr_i} = K_{tr_i} y_{up_i} \sqrt{y_{up_i} - y_{dn_i} + z_{up_i} - z_{dn_i}}. \quad (6)$$

The transition constant  $K_{tr_i}$  can be obtained analysing the system in steady state, where  $Q_i = Q_{tr_i}$ . On the other hand, the values of  $A_{up_i}$  and  $A_{dn_i}$  can be obtained using data fitting techniques as shown in Ljung (2010).

### 3. TEST CASE AND EXPERIMENT SETTINGS

In order to obtain a reference model and analyze the behavior of the modeling approaches, the test case proposed by Clemmens et al. (1998) is implemented in EPA-SWMM. This test case is based on the Corning canal in California and has been proposed by the ASCE Task Committee on Canal Automation Algorithms as a standardized test case on canals with well-studied and realistic properties, where the variations in the pool lengths of the channels presents a challenge in modeling and control. The test case is composed by eight channels with cross regulation structures of the type undershoot gates in submerged flow, and outlet regulation structures of the same type. The simulation diagram of the system is shown in Figure 5, where the hydraulic dimensions and operational conditions are given in Table 3.

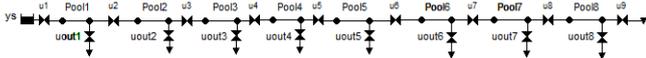


Fig. 5. Simulation diagram of the reference model.

On the other hand, in order to validate the behavior of the presented modeling approaches in a broad operation region, a variation routine for cross and outlet regulation structures is proposed, where from the operational conditions of the system, at a time of 100 hours, the first cross regulation structure ( $u_1$ ) is closed in a 30%, and each 100 hours each regulation structure is changed in a 50%. From the test, it is observed that the average constant time of the channels is  $50 \times 10^3$ s. Therefore, the reference system is sampled with a sampled time ( $\tau_s$ ) of  $1 \times 10^3$ s. The obtained data are used to adjust the parameters of the modeling approaches 1 and 2. In the first approach, the areas ( $A_i$ ) are assumed to be the physical area of each channel. To obtain the  $l_i$  constant, the system is analyzed at the steady-state operation, where  $y_{dn_i} = l_i y_{up_i}$ . For each channel,  $y_{dn_i}$  and  $y_{up_i}$  are obtained from Table 3. The delays  $\tau_i$  are obtained from the dynamic behavior of the reference system as is described by Conde et al. (2019). In the second approach, the transition constants ( $k_{tr_i}$ ) are

Table 3. Hydraulic Characteristics (HC) and Operational Conditions (OC) of the test case.

		$L_1$ 7000 (m)				$L_1$ 4000 (m)	
		$W_1$				$W_1$	
$P_1$	HC	$z_{up1}$	4,4 (m)	HC	$P_5$	$z_{up1}$	1,73 (m)
		$z_{dn1}$	3,29 (m)			$z_{dn1}$	1,1 (m)
		$w_{S1}$	7 (m)			$w_{S1}$	6 (m)
		$w_{out1}$	1 (m)			$w_{out1}$	1 (m)
		$y_{up1}$	2,38 (m)			$y_{up1}$	2,83 (m)
$P_2$	OC	$y_{dn1}$	2,84 (m)	OC	$P_6$	$y_{dn1}$	3,31 (m)
		$u_1$	0,35 (m)			$u_1$	0,78 (m)
		$u_{out1}$	0,22 (m)			$u_{out1}$	0,2 (m)
		$L_1$	3000 (m)			$L_1$	3000 (m)
$P_3$	HC	$W_1$	7 (m)	HC	$P_7$	$W_1$	5 (m)
		$z_{up1}$	3,29 (m)			$z_{up1}$	1,1 (m)
		$z_{dn1}$	2,83 (m)			$z_{dn1}$	0,63 (m)
		$w_{S1}$	7 (m)			$w_{S1}$	5 (m)
		$w_{out1}$	1 (m)			$w_{out1}$	1 (m)
$P_4$	OC	$y_{up1}$	2,65 (m)	OC	$P_8$	$y_{up1}$	2,78 (m)
		$y_{dn1}$	2,93 (m)			$y_{dn1}$	3,11 (m)
		$u_1$	1,18 (m)			$u_1$	0,59 (m)
		$u_{out1}$	0,22 (m)			$u_{out1}$	0,2 (m)
$P_5$	HC	$L_1$	3000 (m)	HC	$P_7$	$L_1$	2000 (m)
		$W_1$	7 (m)			$W_1$	5 (m)
		$z_{up1}$	2,83 (m)			$z_{up1}$	0,63 (m)
		$z_{dn1}$	2,36 (m)			$z_{dn1}$	0,31 (m)
		$w_{S1}$	7 (m)			$w_{S1}$	5 (m)
$P_6$	OC	$w_{out1}$	1 (m)	OC	$P_8$	$w_{out1}$	1 (m)
		$y_{up1}$	2,7 (m)			$y_{up1}$	2,66 (m)
		$y_{dn1}$	3,02 (m)			$y_{dn1}$	2,89 (m)
		$u_1$	0,95 (m)			$u_1$	0,53 (m)
$P_7$	OC	$u_{out1}$	0,21 (m)	OC	$P_8$	$u_{out1}$	0,21 (m)
		$L_1$	4000 (m)			$L_1$	2000 (m)
		$W_1$	6 (m)			$W_1$	5 (m)
		$z_{up1}$	2,36 (m)			$z_{up1}$	0,31 (m)
		$z_{dn1}$	1,73 (m)			$z_{dn1}$	0 (m)
$P_8$	HC	$w_{S1}$	6 (m)	HC	$P_8$	$w_{S1}$	5 (m)
		$w_{out1}$	1 (m)			$w_{out1}$	1 (m)
		$y_{up1}$	2,7 (m)			$y_{up1}$	2,42 (m)
		$y_{dn1}$	3,12 (m)			$y_{dn1}$	2,67 (m)
		$u_1$	0,85 (m)			$u_1$	0,42 (m)
$P_9$	OC	$u_{out1}$	0,21 (m)	OC	$P_8$	$u_{out1}$	0,22 (m)
		$L_1$	4000 (m)			$L_1$	2000 (m)
		$W_1$	6 (m)			$W_1$	5 (m)
		$z_{up1}$	2,36 (m)			$z_{up1}$	0,31 (m)
$P_10$	HC	$z_{dn1}$	1,73 (m)	HC	$P_8$	$z_{dn1}$	0 (m)
		$w_{S1}$	6 (m)			$w_{S1}$	5 (m)
		$w_{out1}$	1 (m)			$w_{out1}$	1 (m)
		$y_{up1}$	2,7 (m)			$y_{up1}$	2,42 (m)
		$y_{dn1}$	3,12 (m)			$y_{dn1}$	2,67 (m)
$P_11$	OC	$u_1$	0,85 (m)	OC	$P_8$	$u_1$	0,42 (m)
		$u_{out1}$	0,21 (m)			$u_{out1}$	0,22 (m)
		$L_1$	4000 (m)			$L_1$	2000 (m)
		$W_1$	6 (m)			$W_1$	5 (m)
$P_12$	HC	$z_{up1}$	2,36 (m)	HC	$P_8$	$z_{up1}$	0,31 (m)
		$z_{dn1}$	1,73 (m)			$z_{dn1}$	0 (m)
		$w_{S1}$	6 (m)			$w_{S1}$	5 (m)
		$w_{out1}$	1 (m)			$w_{out1}$	1 (m)
		$y_{up1}$	2,7 (m)			$y_{up1}$	2,42 (m)
$P_13$	OC	$y_{dn1}$	3,12 (m)	OC	$P_8$	$y_{dn1}$	2,67 (m)
		$u_1$	0,85 (m)			$u_1$	0,42 (m)
		$u_{out1}$	0,21 (m)			$u_{out1}$	0,22 (m)
		$L_1$	4000 (m)			$L_1$	2000 (m)
$P_14$	HC	$W_1$	6 (m)	HC	$P_8$	$W_1$	5 (m)
		$z_{up1}$	2,36 (m)			$z_{up1}$	0,31 (m)
		$z_{dn1}$	1,73 (m)			$z_{dn1}$	0 (m)
		$w_{S1}$	6 (m)			$w_{S1}$	5 (m)
		$w_{out1}$	1 (m)			$w_{out1}$	1 (m)
$P_15$	OC	$y_{up1}$	2,7 (m)	OC	$P_8$	$y_{up1}$	2,42 (m)
		$y_{dn1}$	3,12 (m)			$y_{dn1}$	2,67 (m)
		$u_1$	0,85 (m)			$u_1$	0,42 (m)
		$u_{out1}$	0,21 (m)			$u_{out1}$	0,22 (m)
$P_16$	HC	$L_1$	4000 (m)	HC	$P_8$	$L_1$	2000 (m)
		$W_1$	6 (m)			$W_1$	5 (m)
		$z_{up1}$	2,36 (m)			$z_{up1}$	0,31 (m)
		$z_{dn1}$	1,73 (m)			$z_{dn1}$	0 (m)
		$w_{S1}$	6 (m)			$w_{S1}$	5 (m)
$P_17$	OC	$w_{out1}$	1 (m)	OC	$P_8$	$w_{out1}$	1 (m)
		$y_{up1}$	2,7 (m)			$y_{up1}$	2,42 (m)
		$y_{dn1}$	3,12 (m)			$y_{dn1}$	2,67 (m)
		$u_1$	0,85 (m)			$u_1$	0,42 (m)
$P_18$	OC	$u_{out1}$	0,21 (m)	OC	$P_8$	$u_{out1}$	0,22 (m)
		$L_1$	4000 (m)			$L_1$	2000 (m)
		$W_1$	6 (m)			$W_1$	5 (m)
		$z_{up1}$	2,36 (m)			$z_{up1}$	0,31 (m)
		$z_{dn1}$	1,73 (m)			$z_{dn1}$	0 (m)
$P_19$	HC	$w_{S1}$	6 (m)	HC	$P_8$	$w_{S1}$	5 (m)
		$w_{out1}$	1 (m)			$w_{out1}$	1 (m)
		$y_{up1}$	2,7 (m)			$y_{up1}$	2,42 (m)
		$y_{dn1}$	3,12 (m)			$y_{dn1}$	2,67 (m)
		$u_1$	0,85 (m)			$u_1$	0,42 (m)
$P_20$	OC	$u_{out1}$	0,21 (m)	OC	$P_8$	$u_{out1}$	0,22 (m)
		$L_1$	4000 (m)			$L_1$	2000 (m)
		$W_1$	6 (m)			$W_1$	5 (m)
		$z_{up1}$	2,36 (m)			$z_{up1}$	0,31 (m)
$P_21$	HC	$z_{dn1}$	1,73 (m)	HC	$P_8$	$z_{dn1}$	0 (m)
		$w_{S1}$	6 (m)			$w_{S1}$	5 (m)
		$w_{out1}$	1 (m)			$w_{out1}$	1 (m)
		$y_{up1}$	2,7 (m)			$y_{up1}$	2,42 (m)
		$y_{dn1}$	3,12 (m)			$y_{dn1}$	2,67 (m)
$P_22$	OC	$u_1$	0,85 (m)	OC	$P_8$	$u_1$	0,42 (m)
		$u_{out1}$	0,21 (m)			$u_{out1}$	0,22 (m)
		$L_1$	4000 (m)			$L_1$	2000 (m)
		$W_1$	6 (m)			$W_1$	5 (m)
$P_23$	HC	$z_{up1}$	2,36 (m)	HC	$P_8$	$z_{up1}$	0,31 (m)
		$z_{dn1}$	1,73 (m)			$z_{dn1}$	0 (m)
		$w_{S1}$	6 (m)			$w_{S1}$	5 (m)
		$w_{out1}$	1 (m)			$w_{out1}$	1 (m)
		$y_{up1}$	2,7 (m)			$y_{up1}$	2,42 (m)
$P_24$	OC	$y_{dn1}$	3,12 (m)	OC	$P_8$	$y_{dn1}$	2,67 (m)
		$u_1$	0,85 (m)			$u_1$	0,42 (m)
		$u_{out1}$	0,21 (m)			$u_{out1}$	0,22 (m)
		$L_1$	4000 (m)			$L_1$	2000 (m)
$P_25$	HC	$W_1$	6 (m)	HC	$P_8$	$W_1$	5 (m)
		$z_{up1}$	2,36 (m)			$z_{up1}$	0,31 (m)
		$z_{dn1}$	1,73 (m)			$z_{dn1}$	0 (m)
		$w_{S1}$	6 (m)			$w_{S1}$	5 (m)
		$w_{out1}$	1 (m)			$w_{out1}$	1 (m)
$P_26$	OC	$y_{up1}$	2,7 (m)	OC	$P_8$	$y_{up1}$	2,42 (m)
		$y_{dn1}$	3,12 (m)			$y_{dn1}$	2,67 (m)
		$u_1$	0,85 (m)			$u_1$	0,42 (m)
		$u_{out1}$	0,21 (m)			$u_{out1}$	0,22 (m)
$P_27$	OC	$L_1$	4000 (m)	OC	$P_8$	$L_1$	2000 (m)
		$W_1$	6 (m)			$W_1$	5 (m)
		$z_{up1}$	2,36 (m)			$z_{up1}$	0,31 (m)
		$z_{dn1}$	1,73 (m)			$z_{dn1}$	0 (m)
		$w_{S1}$	6 (m)			$w_{S1}$	5 (m)
$P_28$	HC	$w_{out1}$	1 (m)	HC	$P_8$	$w_{out1}$	1 (m)
		$y_{up1}$	2,7 (m)			$y_{up1}$	2,42 (m)
		$y_{dn1}$	3,12 (m)			$y_{dn1}$	2,67 (m)
		$u_1$	0,85 (m)			$u_1$	0,42 (m)
$P_29$	OC	$u_{out1}$	0,21 (m)	OC	$P_8$	$u_{out1}$	0,22 (m)
		$L_1$	4000 (m)			$L_1$	2000 (m)
		$W_1$	6 (m)			$W_1$	5 (m)
		$z_{up1}$	2,36 (m)			$z_{up1}$	0,31 (m)
		$z_{dn1}$	1,73 (m)			$z_{dn1}$	0 (m)
$P_30$	HC	$w_{S1}$	6 (m)	HC	$P_8$	$w_{S1}$	5 (m)
		$w_{out1}$	1 (m)			$w_{out1}$	1 (m)
		$y_{up1}$	2,7 (m)			$y_{up1}$	2,42 (m)
		$y_{dn1}$	3,12 (m)			$y_{dn1}$	2,67 (m)
		$u_1$	0,85 (m)			$u_1$	0,42 (m)
$P_31$	OC	$u_{out1}$	0,21 (m)	OC	$P_8$	$u_{out1}$	0,22 (m)
		$L_1$	4000 (m)			$L_1$	2000 (m)
		$W_1$	6 (m)			$W_1$	5 (m)
		$z_{up1}$	2,36 (m)			$z_{up1}$	0,31 (m)
$P_32$	HC	$z_{dn1}$	1,73 (m)	HC	$P_8$	$z_{dn1}$	0 (m)
		$w_{S1}$	6 (m)			$w_{S1}$	5 (m)
		$w_{out1}$	1 (m)			$w_{out1}$	1 (m)
		$y_{up1}$	2,7 (m)			$y_{up1}$	2,42 (m)
		$y_{dn1}$	3,12 (m)			$y_{dn1}$	2,67 (m)
$P_33$	OC	$u_1$	0,85 (m)	OC	$P_8$	$u_1$	0,42 (m)
		$u_{out1}$	0,21 (m)			$u_{out1}$	0,22 (m)
		$L_1$	4000 (m)			$L_1$	2000 (m)
		$W_1$	6 (m)			$W_1$	5 (m)
$P_34$	HC	$z_{up1}$	2,36 (m)	HC	$P_8$	$z_{up1}$	0,31 (m)
		$z_{dn1}$	1,73 (m)			$z_{dn1}$	0 (m)
		$w_{S1}$	6 (m)			$w_{S1}$	5 (m)
		$w_{out1}$	1 (m)			$w_{out1}$	1 (m)
		$y_{up1}$	2,7 (m)			$y_{up1}$	2,42 (m)
$P_35$	OC	$y_{dn1}$	3,12 (m)	OC	$P_8$	$y_{dn1}$	2,67 (m)
		$u_1$	0,85 (m)			$u_1$	0,42 (m)
		$u_{out1}$	0,21 (m)			$u_{out1}$	0,22 (m)
		$L_1$	4000 (m)			$L_1$	2000 (m)
$P_36$	HC	$W_1$	6 (m)	HC	$P_8$	$W_1$	5 (m)
		$z_{up1}$	2,36 (m)			$z_{up1}$	0,31 (m)
		$z_{dn1}$	1,73 (m)			$z_{dn1}$	0 (m)
		$w_{S1}$	6 (m)			$w_{S1}$	5 (m)
		$w_{out1}$					

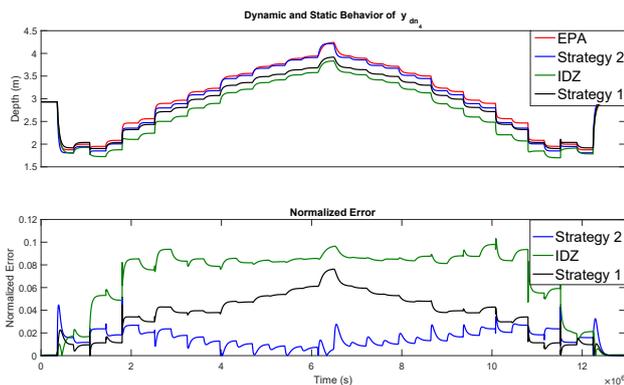


Fig. 6. Comparison of the behavior of the simulated systems at the downstream level of the fourth channel.

at an intermediate part of the system. i.e., at the downstream level of the fourth channel. Therefore, a comparison of the behavior of the simulated systems is shown in Figure 6. Additionally, in this figure, a comparison of the normalized error between the reference model and each modeling approach is shown. From these comparisons, it is possible to establish that the evaluated modeling approaches offer an accurate description of the reference model, with normalized errors lower than a 10% of the overall variation of the system. Also, it is highlighted that the second modeling approach describes a more accurate behavior than the first approach and than the IDZ approach. In the first approach and in the IDZ approach the error is increased at higher depths. However, it is observed that instead of this error, the behavior of the modeling approaches has a high dynamical relation with the reference model. Therefore, in order to only compare the dynamical behavior of the modeling approaches, in Figure 7 a comparison between  $\dot{y}_{dn_4}$  of the reference model and the modeling approaches is presented, where the derivative is approximated using a difference relation. Additionally, in Figure 7, the normalized absolute error between  $\dot{y}_{dn_4}$  of the reference model and the modeling approaches is presented. From these comparisons, it is observed a high dynamic relation between the reference model and the modeling approaches. Furthermore, it is observed that the first modeling approach presents a more accurate behavior than the other two approaches. On the other hand, at some times, the IDZ presents a contrary direction of the dynamics; in other words, the IDZ shows a water level increase when the reference model is showing a water level decreases.

In Figure 8, the normalized mean absolute errors between the reference model and the modeling approaches for all  $y_{up}$ ,  $y_{dn}$ , and the approximations of  $\dot{y}_{up}$ , and  $\dot{y}_{dn}$  are presented. From the comparison of the level errors, it is observed that, for the eight channels, the second approach describes with almost three times lowest error than the other models, the behavior of the reference model. On the other hand, for the eight channels, the three approaches show extremely low error describing the dynamic behavior of the reference model, and the first approach presents the high dynamic relation.

The results of the comparisons show that with the use of the simplified and approximated modeling approaches the dynamical behavior of the OCIS can be described. It is

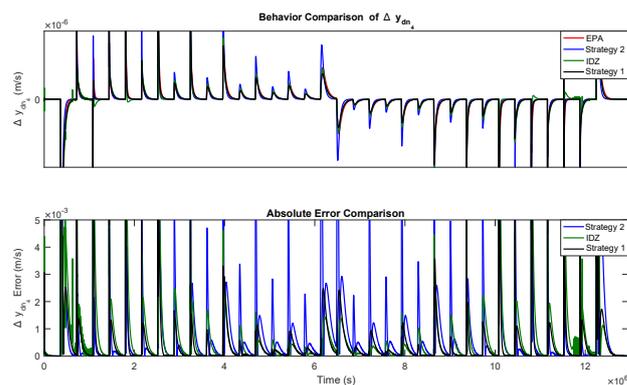


Fig. 7. comparison between  $\dot{y}_{dn_4}$  of the reference model and the modeling approaches.

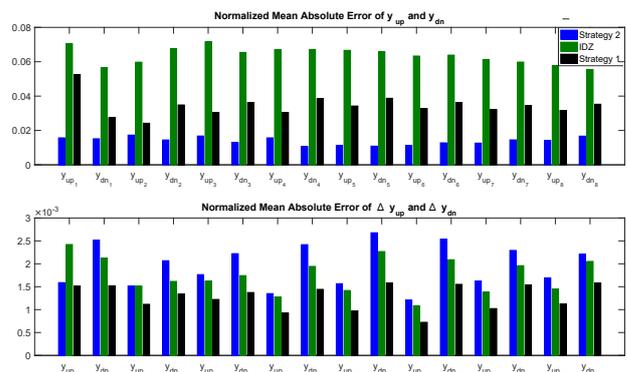


Fig. 8. Normalized mean absolute errors between the reference model and the modeling approaches for the eight pools.

evident that describing the nonlinear behavior of the OCIS the nonlinear behavior of the OCIS, the second approach presents better performance than the other approaches. That is because the IDZ and the first modeling approach lost the nonlinear relationship that exists between the difference of potential and the flow along a channel. On the other hand, the description of the dynamics is performed better by the IDZ and first modeling approaches. This can be because the second modeling approach lacks time delays to describe the time that a wave spends traveling along the channels. However, the performance of the second approach is close to the performance of the other two approaches. Finally, the first modeling approach could be presented as the simplest one, because it describes the dynamics of an open channel using only one differential equation. However, the first approach, and the IDZ approach include delays that increase the complexity of the simulation and control design strategies.

## 5. CONCLUSIONS

An approximated modeling approach for OCIS has been proposed and tested with a reference model and compared with two modeling approaches. The proposed approach has shown better performance describing the nonlinear behavior of the OCIS. Additionally, this approach, which describes the OCIS using a mass and energy balance by channel, presents a simpler structure that can be used in modeling the nonlinear dynamical behavior of OCIS

with multiple configurations and types of regulation structures. Furthermore, it is important to mention that the configuration of the modeling approach is useful to test the behavior of OCIS in presence of physical disturbances (e.g., leaks, overflows, evaporation, and even obstructions), which can be either added or modified into the mass balance equations that describe the dynamical behavior of the system. The proposed approach also can be used for design and test of control, estimation, and prediction strategies. Finally, the modeling approach has been adjusted with high accuracy using structural data and systematic procedures. However, more tests and evaluations with real data are needed.

## REFERENCES

- Bedjaoui, N. and Weyer, E. (2011). Algorithms for leak detection, estimation, isolation and localization in open water channels. *Control Engineering Practice*, 19(6), 564–573.
- Bolea, Y., Puig, V., and Blesa, J. (2014a). Linear parameter varying modeling and identification for real-time control of open-flow irrigation canals. *Environmental Modelling and Software*, 53, 87–97.
- Bolea, Y., Puig, V., and Grau, A. (2014b). Discussion on Muskingum versus integrator-delay models for control objectives. *Journal of Applied Mathematics*, 2014.
- Bonet, E., Gómez, M., Yubero, M., and Fernández-Francos, J. (2017). GoRoSoBo: An overall control diagram to improve the efficiency of water transport systems in real time. *Journal of Hydroinformatics*, 19(3), 364–384.
- Cantoni, M., Weyer, E., Li, Y., Ooi, S., Mareels, I., and Ryan, M. (2007). Control of large-scale irrigation networks. *Proceedings of the IEEE*, 95(1), 75–91.
- Cen, L., Wu, Z., Chen, X., Zou, Y., and Zhang, S. (2017). On Modeling and constrained model predictive control of open irrigation canals. *Journal of Control Science and Engineering*, 2017.
- Chaudhry, M. (2008). *Open-Channel Flow*. Springer Science+Business Media, Columbia, second edition.
- Clemmens, A., ASCE, M., Tian, X., van Overloop, P.J., and Litrico, X. (2017). Integrator delay zero model for design of upstream water-level controllers. *Journal of Irrigation and Drainage Engineering*, 143(3).
- Clemmens, A., Kacerek, T., Grawitz, B., and Schuurmans, W. (1998). Test Cases for Canal Control Algorithms. *Journal of Irrigation and Drainage Engineering*, 124(February), 23–30.
- Conde, G., Quijano, N., and Ocampo-Martinez, C. (2019). Modeling and Control of Interacting Irrigation Channels. In *IEEE 4th Colombian Conference on Automatic Control (CCAC)*, 1–6. Medellín.
- Diamantis, M., Papageorgiou, M., Kosmatopoulos, E., and Chamithoris, G. (2011). Identification and Adaptive Control for Open Channel Water Flow Systems. *Computer-Aided Civil and Infrastructure Engineering*, 26(6), 464–480.
- Eurén, K. and Weyer, E. (2007). System identification of open water channels with undershot and overshot gates. *Control Engineering Practice*, 15(7), 813–824.
- Herrera, J., Ibeas, A., and De La Sen, M. (2013). Identification and control of integrative MIMO systems using pattern search algorithms: An application to irrigation channels. *Engineering Applications of Artificial Intelligence*, 26(1), 334–346.
- Horváth, K., Galvis, E., Rodellar, J., and Valentín, M. (2014). Experimental comparison of canal models for control purposes using simulation and laboratory experiments. *Journal of Hydroinformatics*, 16(6), 1390–1408.
- Horváth, K., Galvis, E., Valentín, M., and Benedé, J. (2015a). Is it better to use gate opening as control variable than discharge to control irrigation canals? *Journal of Irrigation and Drainage Engineering*, 141(3).
- Horváth, K., Galvis, E., Valentín, M., Rodellar, J., Gómez, M., and Rodellar, J. (2015b). New offset-free method for model predictive control of open channels. *Control Engineering Practice*, 41, 13–25.
- Lacasta, A., Morales-Hernández, M., Brufau, P., and García-Navarro, P. (2018). Application of an adjoint-based optimization procedure for the optimal control of internal boundary conditions in the shallow water equations. *Journal of Hydraulic Research*, 56(1), 111–123.
- Lewis, A. (2017). *Storm Water Management Model Reference Manual Volume II Hydraulics*. U.S. Environmental Protection Agency, Cincinnati.
- Litrico, X. and Fromion, V. (2004a). Analytical approximation of open-channel flow for controller design. *Applied Mathematical Modelling*, 28(7), 677–695.
- Litrico, X. and Fromion, V. (2004b). Simplified Modeling of Irrigation Canals for Controller Design. *Journal of Irrigation and Drainage Engineering*, Volume 130(Issue 5), Pages 373–383.
- Litrico, X. and Fromion, V. (2009). *Modeling and Control of Hydrosystems*. Springer London, London.
- Ljung, L. (2010). Perspectives on system identification. *Annual Reviews in Control*, 34(1), 1–12.
- Mareels, I., Weyer, E., Ooi, S., Cantoni, M., Li, Y., and Nair, G. (2005). Systems engineering for irrigation systems: Successes and challenges. *Annual Reviews in Control*, 29(2), 191–204.
- Ooi, S. and Weyer, E. (2011). Detection of oscillations in control loops in irrigation channels. *Control Engineering Practice*, 19(3), 311–319.
- Qiao, Q.S. and Yang, K.L. (2010). Modeling unsteady open-channel flow for controller design. *Journal of Irrigation and Drainage Engineering*, 136(6), 383–391.
- Rivas Perez, R., Feliu Batlle, V., and Sanchez Rodriguez, L. (2007). Robust system identification of an irrigation main canal. *Advances in Water Resources*, 30(8), 1785–1796.
- Romera, J., Ocampo-Martinez, C., Puig, V., and Quevedo, J. (2013). Flooding Management using Hybrid Model Predictive Control: Application to the Spanish Ebro River. *Journal of Hydroinformatics*, 15(2), 366.
- U. S. Department of the Interior (2001). *Water Measurement Manual*. U.S. Government Printing Office, Washington, DC.
- Van Overloop, P., Horvath, K., and Ekin, B. (2014). Model predictive control based on an integrator resonance model applied to an open water channel. *Control Engineering Practice*, 27(1), 54–60.
- Zheng, Z., Wang, Z., Zhao, J., and Zheng, H. (2019). Constrained model predictive control algorithm for cascaded irrigation canals. *Journal of Irrigation and Drainage Engineering*, 145(6).