Projecte de Tesi

Singularity-Invariant Transformations in Stewart-Gough Platforms: Theory and Applications

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Contents

1 Introduction

The generalized Stewart-Gough platform, as it is understood today, consist of two rigid bodies (the base and the platform), connected trough six actuated extensible legs, each with spherical joints at both ends, or spherical joint at one end and universal joint at the other (see Fig. 2). These joints are usually known as attachments.

The first prototype of a platform manipulator with 6 legs was made by Gough in 1947 (Fig. 1). The manipulator structure known as the Stewart-Gough platform has its origin in the design by Stewart of a 6-DOF mechanism to simulate flight conditions by generating general motion in space [1]. Stewart's mechanism consisted of a triangular platform supported by ball joints over three legs of adjustable lengths and angular altitudes connected to the ground through two-axis joints. Gough suggested the use of six linear actuators all in parallel, similar to the tyre test machine designed by Gough and Whitehall [2], and thereby making the platform manipulator a fully parallel actuated mechanism.

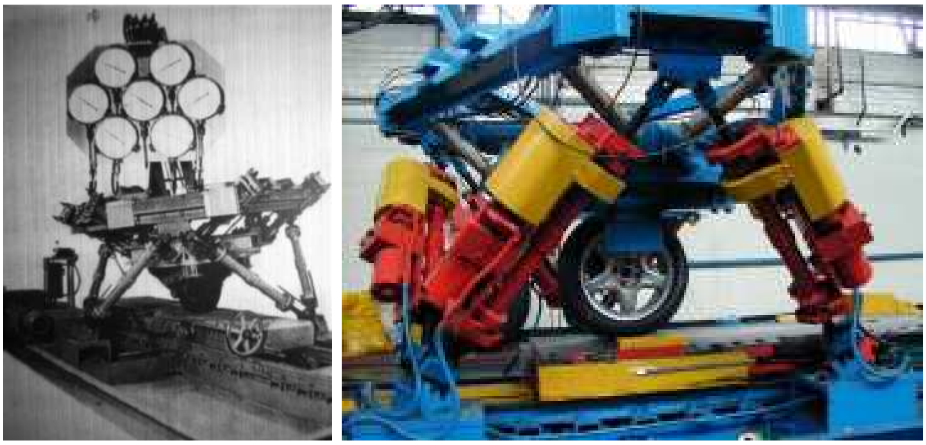


Figure 1: Two different examples of Stewart-Gough platforms.

Since 1990's, there has been a steady increase in research interest in the field of parallel manipulators in general, and the Stewart-Gough platform in particular. Parallel manipulators have greater rigidity and superior positioning capabilities with respect to serial manipulators. In addition, they have much better load/mass ratio because load is distributed on the legs [20, 43].

The analysis of robot singularities is probably the most active research topic in kinematics nowadays. Peter Donelan, from Victoria University of Wellington in New Zealand, maintains a database currently containing over 770 publications on this topic [45]. Those devoted to the singularity analysis of parallel robots represent an steadily increasing number but, unfortunately, most of them tackle the analysis of particular parallel architectures on an ad hoc basis. In this context, the development of new mathematical tools for the singularity analysis of parallel robots, though difficult, is a must.

The main goal of the proposed thesis is the characterization of the geometric transformations on the locations of the attachments of a Stewart-Gough that leave its singularities invariant. It is easy to see that the legs length of the platform, before and after such kind of transformations, must be in one-to-one relation. As a consequence, the forward kinematics of a platform undergoing a singularity-invariant transformation is essentially the same.

This thesis proposal is organized as follows. In Section 2 the most relevant publications related with the thesis topic are reviewed. In Section 3 the PhD candidate defines the main objectives of the thesis, which are detailed into 8 separated modules. Section 4 and 5 detail the contributions that are expected to achieve with the project, detailing the already attained objectives. In Section 6 all the objectives are scheduled through four years of work and finally section 7 list the achieved and submitted publications of the PhD candidate.

2 State of the art

Next, the most relevant contributions on the analysis of Stewart-Gough platforms are reviewed, paying special attention to those topics in which a contribution is expected to be done.

2.1 Kinematics of Stewart-Gough platforms

There is a large number of publications regarding the kinematics of Stewart-Gough platforms, both for remarkable particular configurations of the attachments [31, 8, 37, 30], and for the general case [36, 12].

Contrary to what happens to serial manipulators, the forward kinematics of Stewart-Gough platforms is a very challenging problem, while their inverse kinematics is trivial.

The resolution of the forward kinematics problem is essential for control, on-line simulation and for performance analysis. For a Stewart-Gough platform, it consist in finding the position and orientation (i.e., the pose) of the platform, given its leg lengths.

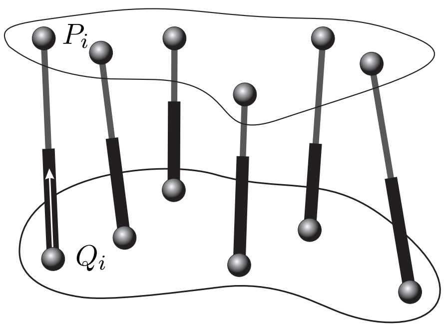


Figure 2: A general Stewart-Gough platform with base attachments Q_i and platform attachments at P_i, $i = 1, ..., 6$.

Geometrically, and following the notation of Fig. 2, it is equivalent to the problem of placing a rigid body such that six given points of the body lie on six given spheres. Mathematically, this can be expressed as:

$$\|P_i - Q_i\| = L_i^2, \qquad i = 1, ...6 \tag{1}$$

In general, the above system of six equations can have several solutions. In other words, there are several poses for which the corresponding leg lengths are the same. Each valid pose is called an assembly mode. The number of assembly modes depends on how the attachments are arranged. The topology of a given arrangement of attachments is called an architecture and each architecture have an associated maximum number of assembly modes. This number ranges from 8 to 40.

For some particular architectures, a closed-form solution for their forward kinematics is known [8, 37]. Closed-form solutions simplify the corresponding error analysis, and provide accurate and fast computations. For these particular architectures, either several attachments merge into multiple spherical joints or some alignment or coplanarity constraints must be satisfied between attachments.

For the general case, it was proved that the maximum number of assembly modes was 40 (Lazard [9], Ronga [10], Raghavan [11] (1992)). In fact, a 40th degree polynomial allowing to calculate (numerically) all possible assembly modes was found by Husty in 1994 [12]. Finally, Dietmaier, in 1998 [17] found a Stewart-Gough platform with 40 real assembly modes for a particular set of leg length using continuation theory.

2.2 Classifying Stewart-Gough platforms

One of the existing classifications of Stewart-Gough platforms is based on treating them as bipartite graphs (see Fig. 3 for several examples using such representation). For example, an architecture with m and n different attachments either on the base or the platform is referred to as an $m - n$ Stewart-Gough platform. Using this nomenclature, the simplest architecture is of type 3-3 and the most general of type 6-6. Using this approach, an incomplete classification appeared in [13]. Later on, Faugère and Lazart made a detailed classification of all $m - n$ classes of Stewart-Gough platforms with all possible combinations of connections between attachments [14]. They enumerated 35 different classes representing all possible Stewart-Gough platforms, giving the maximum number of assembly modes for each of them. Nowadays, this classification is widely used when describing Stewart-Gough architectures. Unfortunately, it does not take into account non-generic cases, that is, architectures with alignment or coplanarity restrictions in their attachments which are very common in many implementations.

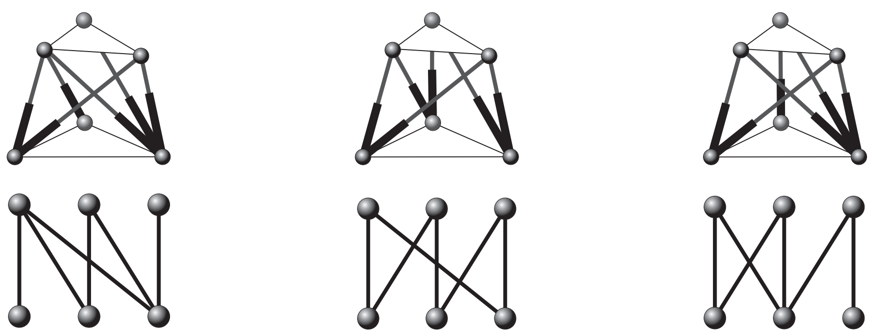


Figure 3: The three possible 3-3 Stewart-Gough platforms (top) and their symbolic representation as bipartite graphs (bottom).

Kong and Gosselin presented a different classification [23]. They defined rigid substructures with less than 6 legs (called components) and then they form manipulators by joining different components. These components are:

PP: Point-Point (a single leg);

PL: Point-Line (two legs sharing a spherical joint);

PB: Point-Body (three legs sharing a spherical joint);

LL: Line-Line (four legs, their endpoints lying on two lines); and

LB: Line-Body (five legs, an endpoint from each lying on a line).

The idea of Kong and Gosselin was to solve the forward kinematics of each component and then, given a platform, to solve its forward kinematics from the solution to the forward kinematics of each of its components. Indeed, the existence of components in a platform greatly simplifies its analysis, but it is not clear how to put together the solution to the forward kinematics of each of its component to form the solution for the entire manipulator.

2.3 Singularities

If we rewrite the system of equation in (1) as a implicit relation between the leg lengths, $\Theta = (l_1, ..., l_6)$, and the pose of the platform, $X = (p_x, p_y, p_z, \alpha_x, \alpha_y, \alpha_z)$, we can rewrite the system in (1) as [43]

$$F(\Theta, X) = 0. \tag{2}$$

When differentiating this expression with respect to time, we obtain

$$A\dot{X} + B\dot{\Theta} = 0 \tag{3}$$

where $A = \frac{\partial F}{\partial X}$ and $B = \frac{\partial F}{\partial \Theta}$.

In 1990, Gosselin and Angeles [6] using this formulation classified the singularities of Stewart-Gough platforms into three categories:

- Type I: (called Serial singularities in [26]): When $|\frac{\partial F}{\partial \Theta}| = 0$. Occurs when the manipulator reaches the boundary of the workspace or internal boundaries limiting different subregions of the workspace. In such singularities, exist nonzero $\dot{\Theta}$ for which $\dot{X} = 0$, so, there are velocities that cannot be reproduced at the output. One can say that, at type I singularities, the manipulator loses degrees of freedom.

- Type II: (called Parallel singularities in [26]): When $|\frac{\partial F}{\partial X}| = 0$. Occurs when the platform is locally movable even when the actuated joints are locked and happens within the workspace. That is, the vector $\dot{\Theta} = 0$ but the corresponding $\dot{X}$ nonzero. One can say that, at type II singularities, the manipulator gains degrees of freedom.

- Type III: (called Architectural singularities in 1991 by Ma and Angeles in [7]). When both $|A| =$ and $|B| = 0$. Such singularities are of different nature, as depend on some conditions at the linkage parameters. It was defined in [7] as a manipulator that is singular in all the points of the workspace. At type III singularities, the manipulator exhibits a self-motion for any set of leg lengths.

It was shown in [6] that Stewart-Gough platforms have very simple Type I singularities, as the matrix B will be singular only when any of the leg lengths is zero, which in practice can never happen due to joint limits. In fact, type I singularities only happen at the limits of the prismatic joints, just because at the limit of the workspace some velocities cannot be reproduced by the platform, but without losing the capability of controlling it.

On the other hand, in [24] it was proved that the type II singularities on Stewart-Gough platforms are much more complicated and much research efforts have been put on the characterization of this type of singularities.

Equation (3) can be reformulated using the Jacobian matrix, that is, the matrix that relates the velocities of the actuated joints with linear and angular velocities of the platform. Equation (3) is written as

$$\dot{\Theta} = J^{-1}\dot{X}. \tag{4}$$

If J^{-1} is singular, there exists $\dot{X} \neq 0$ so that $\dot{\Theta} = 0$. Matrix J^{-1} can be written in terms of matrices A and B as $J^{-1} = -B^{-1}A$, but, as for Stewart-Gough platforms $B = Diag(l_1, ..., l_6)$ [6], normally the study of singularities reduces to the study of matrix A, and making an abuse of language, it is normally called J.

Due to the reciprocity relation between linear velocities (angular velocities) and forces (torques), matrix J in (4) can also be used to study the statics of Stewart-Gough platforms. Actually, the relationship between forces and torques on the actuated joints, τ, with the resultant forces and torques acting on the platform, F, can be expressed as:

$$F = J^{-T}\tau. \tag{5}$$

Thus, forces on the actuated joints may go to infinite at a singularity.

An important property of Stewart-Gough platforms was shown by Merlet in [5]. The jacobian matrix J can be written as

$$J = ((P_i - Q_i), (Q_i \times (P_i - Q_i))^T \tag{6}$$

where leg i goes from the base attachment Q_i to the platform attachment P_i (see Fig. 2). Thus, the entries of column i of the Jacobian matrix are the Plücker coordinates of line P_iQ_i. This characterization of the Jacobian permits to study the singularities of Stewart-Gough platforms as arrangements of lines. Several works deal with singularities of Stewart-Gough platforms by studying the line complexes formed by their legs [25, 35]. Furthermore, Merlet noticed that Grassmann Geometry was a useful mathematical tool for this study [26, 5].

An analytic characterization of singularities was provided in [38], but it does not say much about their nature and the topology of their singularity loci in the configuration space of the platform with respect to the base. The only Stewart-Gough platforms for which a cell decomposition of their singularity loci is available are the flagged parallel manipulators (another 3-3 architecture) [40], [41].

2.4 Architectural singularities

Special attention has been paid to the characterization of type III singularities, or architectural singularities. Such singularities must be avoided in the design process, as architecturally singular manipulators cannot be controlled in any position of its workspace.

The study of architectural singularities in Stewart-Gough Manipulators has been divided into two big categories: manipulators whose platforms are planar and those whose platforms are non-planar. Making an abuse of language, they are referred as planar platforms and non-planar platforms.

Non-planar platforms have been studied in [19], [22] and [44]. M. Husty and A. Karger present a list of non-planar architecturally singular Stewart-Gough platforms. All them have some kind of alignment on the base and/or on the platform, and indeed, they state that any architecturally singular non-planar Stewart-Gough platform has this kind of alignment restriction on the attachments. In particular, they give two algebraic characterizations of the architectural singularities of the Line-Line and the Line-Plane subassemblies, with a vague geometrical interpretation.

The major part of publications on this topic deal with planar-platforms. Working with planar polygonal platforms in [7] , using theory of linear manifolds of correlations in [18], imposing zeros on the jacobian matrix determinant in [32, 34] or imposing some kind of algebraic relation between the platform attachments and the base attachments in [29]. But all those works can be unified with an important theorem that is used in many works to characterize architectural singularities on planar-platform manipulators [16, 28, 27]. The theorem published in 1851 by M. Chasles, states that

Theorem 2.1 (Chasles Theorem) *Given any two conic sections in space between which a projective correspondence p exists, straight lines connecting p-corresponding points on the conics belong to a linear complex.*

Lately in 1896, R. Bricard reformulate it in the context of kinematics, relating the stability of the position of a rigid body supported by 6 bars.

As it was stated by Merlet in [5], singularities of Stewart-Gough manipulators happen when its leg lines form a linear complex. So, architecturally singular platforms are manipulators whose legs are always forming a linear complex. All the works dealing with planar platforms listed in the previous paragraph are particular cases of when such projective correspondence between platform and base attachment exist, and so, the Chasles theorem can be applied: The manipulator is architecturally singular if, and only if, the upper and lower attachments lie on a conic.

On the other hand, when such projectivity doesn't exist, A. Karger studied in [33] the characterization of other kind of architectural singularities that deal with projectivities relating only a subset of the attachments.

Although a lot of work dealing with architectural singularities has been done, it is based in algebraic methods that study the casuistry of the analytical coefficients of the Jacobian matrix determinant. Thus, the existing characterizations are based on very complicated and nonintuitive demonstrations. As a consequence, the geometrical understanding of such singularities is still an open problem.

Finally, an alternative approach to analyze the singularities of a given architecture, and in particular, its possible architectural singularities, is due to Ben-Horin and Shoham based on Grassman-Cayley algebra [42]. This technique has been successful at characterizing the singularities of the octahedral manipulator (a 3-3 architecture) [30], and at simplifying the analysis of platforms with aligned attachments [42].

2.5 Workspace and dexterity

One of the major difficulties in the workspace and dexterity analysis of the Stewart-Gough platforms is the strong coupling between position and orientation. Furthermore, such an analysis should preferably be done in association with the singularity analysis because a workspace segmented by singularity barriers will not be fully usable in practice. In addition, it must be taken into account the crucial role of leg collision in limiting the platform workspace [15].

To define a well-conditioned workspace it is necessary to define indexes representing the "distance" to singularities. As no mathematical distance can be defined because of the simultaneous use of variables representing translations and orientations, several indexes to quantify the dexterity of a Stewart-Gough platform [39] have been proposed instead.

Assuming that the errors on the actuated joints are limited, $\|\Delta\Theta\| \leq 1$, and using the Jacobian matrix definition, the manipulability ellipsoid is defined as

$$\Delta X^T J^{-T} J^{-1} \Delta X \leq 1. \tag{7}$$

When this ellipsoid is a sphere, the platform is said to be isotropic. An isotropic manipulator is optimally well-conditioned, but no Stewart-Gough platform can be constructed to be isotropic all over its workspace.

Using equation (7), in [4] the manipulability index is defined as

$$m(J) = \sqrt{|JJ^T|}. \tag{8}$$

On the other hand, the isotropy condition can be written as

$$\sqrt{\frac{\lambda_{max}}{\lambda_{min}}} = 1, \tag{9}$$

where λ_{min} and λ_{max} are the minimum and maximum eigenvalues of matrix JJ^T. This value is also known as the inverse of the condition number which is another index defined as

$$\kappa(J) = \kappa(J^{-1}) = \|J^{-1}\|\|J\|. \tag{10}$$

This later index is commonly used to indicate the factor of error amplification, define the accuracy/dexerity of the robot, and also the closeness of a pose to a singularity. Moreover, it is a performance criterium for optimal design, and also a criterium to determine the useful workspace [39].

However, due to the complexity of the obtained expressions for these indices, they have only been computed analytically for a limited number of platforms. Furthermore, all these indices are sensitive to the chosen units and the used norm. As a consequence, none of them can be used to represent the positioning accuracy of the platform [39].

In short, determining the geometry of a platform to ensure that a given region of its workspace is free from singularities is a difficult problem that has only been solved for some simple architectures.

A critical issue for parallel robots is the dimensioning in the design process. This dimensioning can be formulated as an multiobjective optimization problem in which it is necessary to take into account aspects such as positioning accuracy, dexterity, repeatability, stiffness, vibration, workspace volume and singularities. Most of the indexes that have been proposed to quantify them rely on the Jacobian matrix or its inverse. Therefore, the characterization of singular-invariant transformation, that is, transformations that are able to alter the value of the determinant of the Jacobian except when its value is zero, is of clear interest to decouple the problem of locating the singularities to that of improving, for example, the accuracy in a given region of the configuration space.

2.6 Conclusions

In conclusion, a significant volume of research has been carried out on the kinematics of Stewart-Gough platforms. So far, as the issues of singularity and workspace analysis are concerned, partial answers to many questions are available, but a complete analysis is yet to be performed.

With respect to the kinematics problems, many of them have been solved, and so, only more clear and intuitive solutions to old problems can be provided.

Regarding the existing classifications, they don't provide meaningful information, as each manipulator must be studied separately independently of the class it belongs, both in the classification appeared in [14] and in [23].

On the other hand, many contributions can be done in the characterization of architectural singularities. Despite the amount of work published in this topic, it is still not clear the geometrical understanding of them.

Finally, the study of the structure of the singularity hypersurface is a difficult problem that has not been solved in general, indeed, the characterization of the singularity-free regions of the workspace is an important problem for the design of useful Stewart-Gough platforms. Contributions on this topic can be provided with our transformations as will be stated in the following sections. There are very few works on the systematic design of Stewart-Gough platforms. As a consequence, an step in this direction is important for the enhancement and realization of its potential.

3 Objectives

The main goal of the proposed thesis is the characterization of the geometric transformations on the locations of the attachments of a Stewart-Gough platform that leave its singularities invariant. In general, substituting one leg of a Stewart-Gough platform by another arbitrary leg modifies the platform singularity locus in a rather unexpected way. Nevertheless, in those cases in which the considered platform contains rigid subassemblies, or *components* [23], legs can be rearranged so that the singularity locus is modified in a controlled way provided that the kinematics of the components are not modified. The complete characterization of these transformations would permit to:

1. Generate families of Stewart-Gough platforms with the same singularity locus without the limitations of previous classification which do not consider the possibility of collinear or coplanar attachments. This technique has already been successfully applied to generate the family of all flagged parallel platforms.

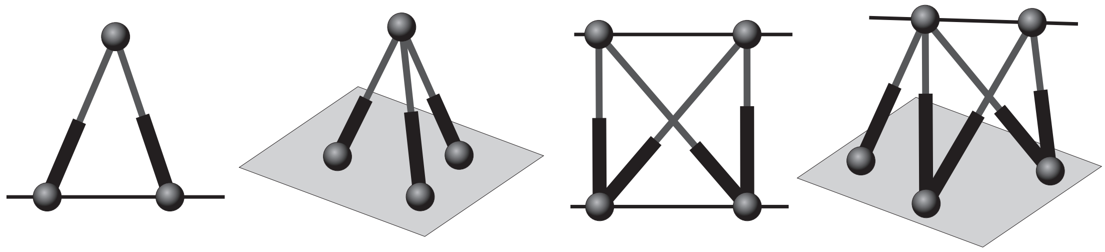
2. Classify and characterize all architectural singularities arising in Steward-Gough platforms. We actually conjecture that all possible singularity-invariant transformation that can be obtained by continuous transformations in the location of the attachments of a Stewart-Gough platform can lead to architectural singularities by properly adjusting the involved geometric parameters and, conversely, all architecturally singular platforms can be obtained by applying a degenerate singularity-invariant transformation to a non singular platform.

3. Decouple the problem of locating the singularities of a Stewart-Gough platform to that of improving, for example, its dexterity in a given region of its configuration space. This would be possible because singularity-invariant transformations permit modifying the value of the platform Jacobian determinant by a constant multiple.

4. Optimize a given parallel platform to improve its dexterity or to avoid possible collisions between its legs, in given region of its configuration space, without altering its singularity locus. This includes the possibility of eliminating multiple spherical joints which are always difficult to implement.

5. Design reconfigurable platforms whose attachments can be modified statically or dynamically to adapt them to different tasks. Since the legs lengths, before and after such kind of transformations, would be in one-to-one relation, the control of such reconfigurable platforms would not be increased notably by the possibility of reconfiguring them.

The classification of Stewart-Gough platforms on the basis of the components they contain was addressed in [23]. Each class consists of all the manipulators obtained by adding to a given component the remaining legs up to 6 in all possible topological configurations. Note that the platforms in a class have neither the same forward kinematics nor the same singularity structure. The proposed thesis, on the contrary, would try to come up with transformations that preserve the platform's singularities,



(a) (b) (c) (d)

Figure 4: Point-line, point-plane, line-line and line-plane components.

thus opening up the possibility of classifying platforms in families sharing the same singularity structure, as mentioned above.

The legs involved in a component fix the relative position between two affine geometric entities (points, lines, or planes). Fig. 4 shows possible configurations of legs in components fixing the location of (a) a point with respect to a line, (b) a point with respect to a plane, (c) a line with respect to another line, and (d) a line with respect to a plane. The case in with a plane is fixed with respect to another plane can be seen as a complete platform with coplanar attachments. At this moment, it is not known how many singularity-invariant transformations exists but preliminary results endorse the idea that no singularity-invariant transformation exists for platforms containing none of these components.

Note that we have formulated two conjectures. Namely,

C1 All singularity-invariant transformations, under degenerate circumstances, can lead to architectural singularities and, conversely, all architecturally singular platforms can be obtained by applying a degenerate singularity-invariant transformation to a non architecturally singular platform.

C2 All singularity-invariant transformations can be classified in five classes according the component they modify, as no singularity-invariant transformations exists for platforms containing no components.

It can be seen that if C1 is proved, C2 would come as a corollary. This is why we call C1 and and C2 the strong and week conjectures on singularity-invariant transformations, respectively. The tasks to be developed in this thesis are designed assuming that the strong conjecture is true. Nevertheless, the proposed tasks will provide, at the end of their completion, the needed mathematical tools to prove or disprove our conjectures. Next, the foreseen tasks are grouped in 8 modules specifying their objectives separately.

3.1 Module 1: Building a background

The PhD candidate started by studying the most relevant literature on the kinematics of Stewart-Gough platforms. This module also included:

1. the attendance to academic courses in the *"Robotització Avançada y Robotica"* doctorate program,

2. the assistance to the EURON winter school *Parallel robots: theory and applications*, organized by Universidad Miguel Hernandez, which included master classes by J-P. Merlet, R. Di Gregorio, S. K. Agrawal, A. Wolf, F. Thomas and others.

3. the attendance to national and international scientific conferences (Congreso Nacional de Mecánica, IEEE International Conference on Robotics and Automation, Advances in Robot Kinematics (ARK 2008), International Workshop on Fundamental Issues and Future Research Directions for Parallel Mechanisms and Manipulators and others).

4. a stay of three months at Ferrara University, under the supervision of Prof. Raffaele Di Gregorio, a well-recognized scientist in the area of parallel robot kinematics.

Of particular relevance has been the stay at Ferrara University where the PhD candidate get used to the algebraic methods developed by Prof. Di Gregorio providing her with an alternative view to the geometric methods she would apply in the development of this proposal.

3.2 Module 2: Study of new mathematical tools for the analysis of singularities

The mathematical tools that are planned to be used during the development of the proposed thesis are of geometric nature in the sense that all obtained algebraic expressions using them should have an immediate geometric interpretation. This is why coordinate-free methods, such as those based on Distance Geometry, will be preferred instead of those based on well-known coordinate-based transformations. Also, the use of Projective Invariants will surely play a fundamental role in the development of the proposed tasks as architectural singularities seem to be always characterizable using this kind of invariants.

Nevertheless, the mathematical tools to be applied will not be limited to those streaming from Distance Geometry or the Theory of Projective Invariants. In this module, the PhD candidate would study the existing literature on Geometric Algebra, which can be seen as an extension of Grassmann-Cayley algebra, with the aim of simplifying the obtained formulations that characterize the singularities of Stewart-Gough platforms, so that they they can be easily factorized.

Obtaining factorizations of the singularity conditions has been devised as a fundamental need in the analysis of possible singularity-invariant transformations. To this end, the PhD candidate has been using Grassmann-Cayley algebra but the obtained formulations get rapidly involved in most cases. It is expected that the proposed stay at Amsterdam University, under the supervision of Prof. Leo Dorst, will allow the PhD candidate to come up with the required simplifications.

3.3 Module 3: Singularity-invariant transformations in point-line components

The study of the point-line component kinematics has already allowed the PhD candidate to develop the first singularity-invariant transformation: the Δ-transform (Fig.5).

Despite its apparent simplicity, when several of this transforms are applied simultaneously or sequentially, it is possible to obtain singularity-invariant transformations for general point-plane and line-line components, and even for some instances of the plane-plane component.

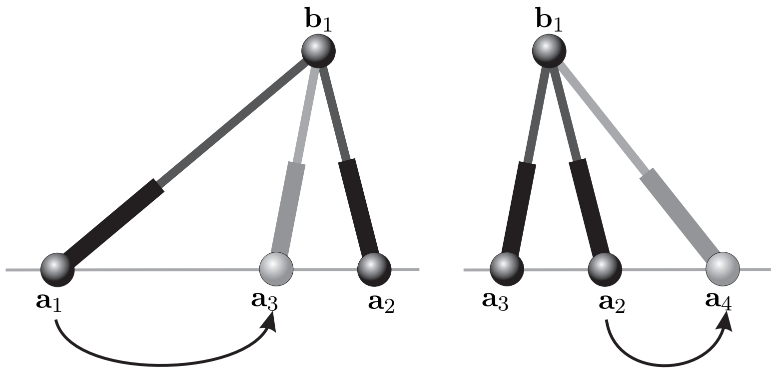


Figure 5: Two Δ-transforms can be applied to move the two attachments, $\mathbf{a}_1$ and $\mathbf{a}_2$, along the line in a point-line component.

The objectives of this module, that have been already achieved using Δ-transforms, are:

1. Characterize the architectural singularities of all manipulators to which the Δ-transform applies (for example, the Zhang-Song and Griffis-Duffy platforms) in terms of projective invariants.

2. Give simple procedures to characterize the self-motions associated with the aforementioned architectural singularities which in the case of the Zhang-Song platform amounts to an atlas.

3. Generate the complete family of flagged parallel platforms.

At this point, this module has been fully and successfully completed. Other unexpected results have been obtained and are listed in Section 5.

3.4 Module 4: Singularity-invariant transformations in line-plane components

In this module, the PhD candidate has concentrated her efforts in characterizing the singularity-invariant transformations for arbitrary line-plane components (Fig. 6). This has already been possible thanks to the definition of a hypersurface in $\Re^3$ that is determined by the coordinates of the attachments of a line-plane components in their local reference frames. A point in this hypersurface determines the attachments for one leg. Thus, five points determine a complete architecture for the line-plane component and, in general, independently on how this five points are chosen, this component will have the same singularity locus.

It has already been proved that, in some degenerate circumstances, the chosen five points will lead to architecturally singular manipulators.

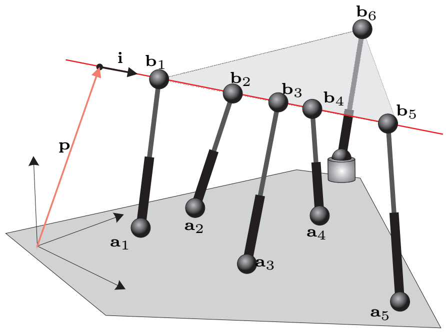


Figure 6: Stewart-Gough platform containing the line-plane rigid subassembly studied in Module 4.

The PhD candidate has already been able to devise two practical applications of the obtained results:

1. A reconfigurable 5 DOF parallel robot, in collaboration with the robotics group of Cassino University, whose singularity locus remains invariant under the allowed reconfiguration so that its control is greatly simplified.

2. A low-cost five-axis milling machine with very favorable kinematic properties. This design is being studied as a prospective patent.

It can be said that the expected theoretical results for this module have already been achieved but important efforts have still to be applied to the two above practical applications.

3.5 Module 5: Singularity-invariant transformations in Stewart-Gough platforms with coplanar attachments

In this module, the PhD candidate will study the most complex component: the plane-plane one. It actually can be seen as a complete Stewart-Gough platform. Therefore, it is a very difficult problem, probably as complex as the general body-body Stewart-Gough platform.

It is expected to obtain the singularity-invariant transformations for this case using a similar method to that used in the previous module but, if this fails, a numerical approach based on continuation would be adopted.

3.6 Module 6: Architectural Singularities and Projective Invariants

In this module, the PhD candidate will contribute to the architectural singularities literature with a geometrically meaningful characterization that can be derived from the singularity-invariant transformations developed in the previous modules.

From the literature analysis on this topic, it is clear the importance of Projective Geometry in the description of this type of singularities. Thus, Theory of Projective Invariants will be used to characterize them.

The objectives of this module are to characterize all the architectural singularities for planar-platforms Stewart-Gough manipulators, and relate each of them with the singularity-invariant transformations.

Until the moment, the PhD candidate has already achieved contributions to this topic, as it will be explained in Section 5, using the Δ-transform defined in Module 3 and the Line-Plane transform defined in Module 4.

3.7 Module 7: Theoretical consequences

After completing the six aforementioned modules, the PhD candidate will be able to prove or disprove the conjectures that, at least in part, motivated this proposal. It would be also possible to classify all possible Steward-Gough platforms in families of platforms having the same singularity locus. This exhaustive classifications will require the design and implementation of some combinatorial algorithms to explore all alternatives.

3.8 Module 8: Compilation and composition of results

The last module of this proposal is assigned to the elaboration of the dissertation and the preparation of its public defense.

The schedule, that spans over four years, for the tasks assigned to the above modules appears in Section 6.

4 Contributions

The achievement of the objectives presented in Section 3 would contribute to a better understanding of the Stewart-Gough platform kinematics by

1. classifying these platforms using more practical criteria than those based on pure topological concepts;
2. increasing the family of platforms with known closed-form solution for their forward kinematics;
3. improving the characterization of their architectural singularities, avoiding the case-by-case analysis that dominates the literature on this kind of singularities;
4. providing new tools for the design of optimal platforms; and
5. opening up the possibility of designing reconfigurable robots with simplified statics analysis.

A complete characterization of the singularity locus of arbitrary Stewart-Gough platforms is an extremely difficult problem that, for the moment, has only been solved for the flagged parallel platforms. Thus, in this context, if is very important to generate families of platforms with the same singularity locus because, once this characterization is obtained for one member of the family, the same is valid for all of its members.

It is also important to stress the fact that all members of a singularity-invariant family of platforms have essentially the same forward kinematics because the sets of leg lengths between any two members of the family are in one-to-one correspondence. Thus, if a closed-form solution for the forward kinematics of one member of the family is obtained, this result can be rapidly extended to all members of the family. This novel approach has already been successfully applied to prove that the forward kinematics of the Zang-Song platform can actually be solved by a sequence of three trilaterations.

Regarding architectural singularities, a great deal of research has been carried out but all obtained results are based on long algebraic developments that lead to polynomial equations of high degree with very difficult geometric interpretations. Singularity-invariant transformations provide a new way to characterize this type of singularities that depend only on the location of the attachments and, as a consequence, intimately linked to the proposed transformations. The techniques developed to characterize these transformations will permit, as a by-product, a better understanding of this kind of singularities.

A complete characterization of all possible singularity-invariant transformations would also provide a new tool for the design process. They provide a set of modifications that can be applied to the location of the attachments so that the designer can optimize aspects of the platform like its dexterity, risk of collisions between legs, etc. with the advantage that the resulting platform will have the same singularities and forward kinematic solution branches as the original one. Furthermore, these transformations can be used to eliminate multiple spherical joints that exist in most platforms with closed-form forward kinematics that make them of little practical interest.

In conclusion, to the best of our knowledge, the use of singularity-invariant transformations is a completely new technique that seems to provide elegant solutions to several well-known problems concerning Stewart-Gough platforms.

5 Achieved contributions

The objectives associated with modules 1 and 3 have completely been accomplished. Most of the objectives concerning module 4 have also been attained and only some aspects concerning two practical applications of the obtained theoretical results, included in module 4, have to be solved. Thus, it can be said that two singularity-invariant transformations have already been identified. These results have led to the publications detailed in Section 7 [1,2,3,5,7,8].

As a consequence, contributions explained in the previous section are achieved with respect to both defined transformations. The advantage of the incremental structure is that we have already obtained almost all objectives, and now, each step, we can only improve them all by adding new transformations.

Referring at the objectives of Module 2, the PhD candidate is working on an improvement of the algorithm based on Grassman-Cayley algebra, used in [42] but with a much more simple approach. The improvements introduce much simpler and easy-interpretable results and allows to give a group structure to the whole family of Stewart-Gough manipulators with planar base and platform. In addition, other contributions on this topic using Geometric Algebra are expected.

Relevant and unexpected contributions attained with the objectives of module 3 are

1. an alternative formulation for the Griffis-Duffy architectural singularity condi-

tions;

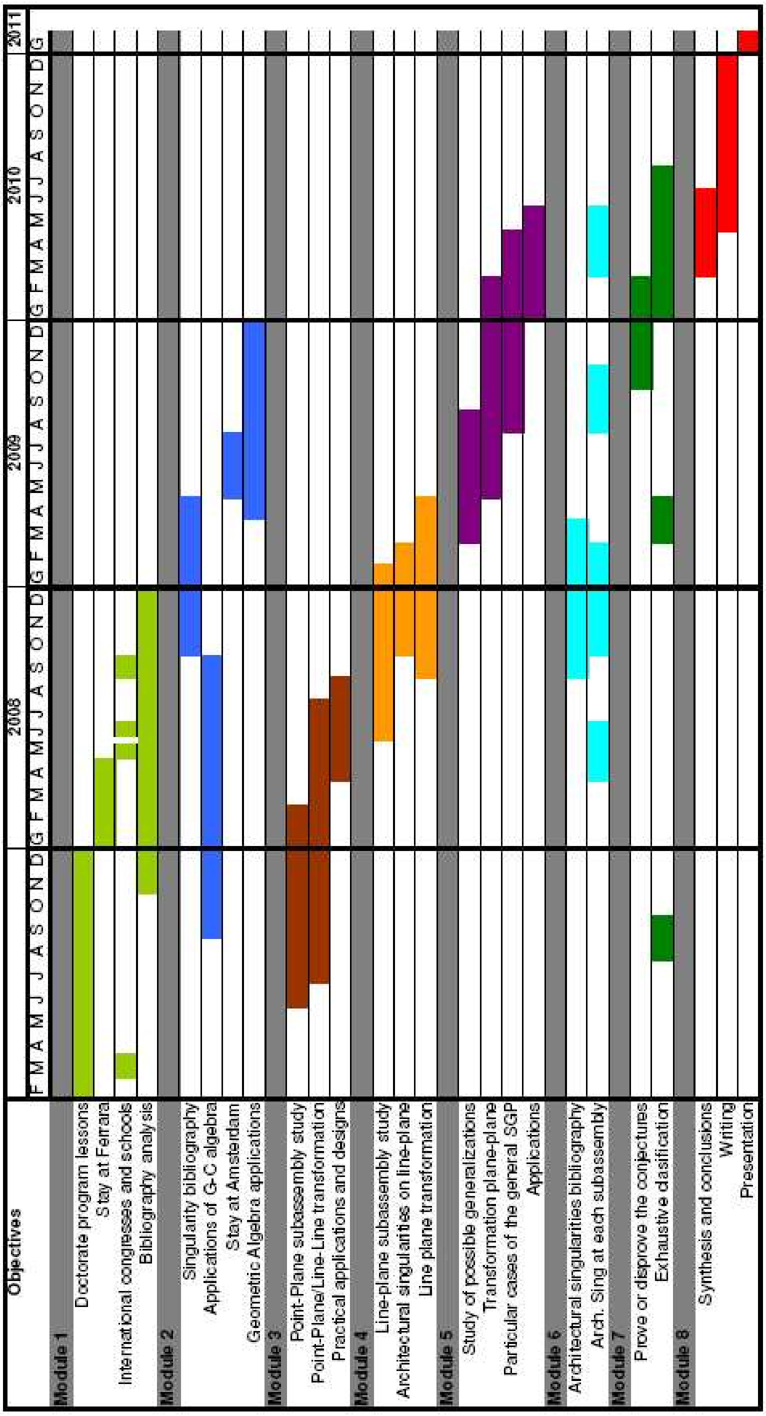
2. the proof that the flagged 3-3 architecture (the first 3-3 manipulator of Fig. 3) is equivalent at the Zhang-Song platform, a quite unexpected result; and

3. a demonstration that the very well known octahedral architecture (the second 3-3 manipulator of Fig. 3) is equivalent at the Griffis-Duffy manipulator (a relation that, to the best of our knowledge, has not appeared in the literature). The large number of publications on the kinematics and singularities of the octahedral manipulator now can be applied to the Griffis-Duffy platform.

Regarding Module 6, the most relevant contribution has been done in the characterization of the architectural singular condition of the Line-Plane subassembly. It has been proved that the previously existing conditions appeared in [22] are not complete, and indeed, the PhD candidate has provided a counterexample. Furthermore, a complete understanding of its geometrical meaning allows the formulation of this characterization into a single condition, while until the moment two long algebraic conditions were needed. This improvement is due to the definition of the Line-Plane transformation attained in Module 4. It is expected to achieve more improvements on the characterization of such singularities, or at least, clarifying its geometrical meaning.

6 Work scheduling

The objectives modules detailed in Section 3 are scheduled here through the four years from 2007 to 2011, both past and future events.



7 Publications

The complete list of accepted and submitted publications during the first two years is:

[1] J. Borràs, F. Thomas, E. Ottaviano and M. Ceccarelli, "A Reconfigurable 5-DOF 5-SPU Parallel Platform," submitted to *IEEE/ASME International Conference on Reconfigurable Mechanisms and Robots.*

[2] J.Borràs and F.Thomas, "Kinematics of the Line-Plane Subassembly in Stewart Platforms," submitted to *IEEE International Conference on Robotics and Automation.*

[3] J.Borràs, F.Thomas and C.Torras, "On Δ-transforms," submitted to the *IEEE Transactions on Robotics* (second revision).

[4] J.Borràs and R. Di Gregorio, "Polynomial Solution To The Position analysis Of Two Assur Kinematic Chains With Four Loops And The Same Topology," *Journal of Mechanisms and Robotics*, to appear in 2009.

[5] J.Borràs, F.Thomas and C.Torras, "Analyzing the Singularities of 6-SPS Parallel Robots Using Virtual Legs," *Proc. II International Workshop on Fundamental Issues and Future Research Directions for Parallel Mechanisms and Manipulators*, pp. 145-150, 2008.

[6] J.Borràs and R. Di Gregorio, "Direct Position Analysis of a Large Family of Spherical and Planar Parallel Manipulators with Four Loops," *Proc. II International Workshop on Fundamental Issues and Future Research Directions for Parallel Mechanisms and Manipulators*, pp. 19-29, 2008.

[7] J.Borràs, F.Thomas and C.Torras, "Architecture Singularities in Flagged Parallel Manipulators," *IEEE Internacional Conference on Robotics and Automation*, pp. 3844-3850, 2008.

[8] J. Borràs, E. Ottaviano, M. Ceccarelli, and F. Thomas, "Optimal Design of a 6-DOF 4-4 Parallel Manipulator with Uncoupled Singularities," *Revista de la Asociación Española de Ingeniería Mecánica*, Año 16, Vol. 2, pp. 1047-1052, 2008.

8 Resources

Except for the short stays at foreign universities, all the work will be developed at the *Institut de Robòtica i Informàtica Industrial-IRI* (UPC-CSIC) from Barcelona, working in the framework of the Geometric Methods Applied to Robotics department, financed by the Spanish Council of Scientific Research (CSIC) under the I+D project DPI2007-60858.

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