Model Based Fault Detection and Isolation for a PEMFC System

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Yet to be decided
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Abstract

Fuel cell systems are considered as clean and efficient power sources, which are under development by manufacturers for both stationary and mobile applications. Polymer Electrolyte Membrane (PEM) fuel cells are considered to have the highest energy density due to the nature of the reaction and the quickest start up time ($\leq 1$ sec). These are the main reasons for being used in applications such as automotive engines, portable and backup power applications. Recent years have seen the proliferation of PEM fuel cell system (PEMFCs) optimization and control applications, where the aim is to obtain a better process performance. Nowadays, increases in safety, reliability and uptime process operation are requiring the inclusion of fault diagnosis algorithms. Because of the lack of space for physical redundancy and cost reduction in automotive applications, the automotive industry is pushing and facing better techniques for fault diagnosis that makes its products compatible in the markets offering to the final customer not only the best quality but also a reliable product. Here an alternative technique to hardware redundancy is the analytical redundancy which uses a mathematical model with input and output measurements as a monitored system signals to generate a fault diagnosis. Analytical redundancy could also allow increasing the fault tolerance, as described in [13], using the recent methods of Fault Tolerant Control (FTC).

The problem of robust fault diagnosis in a PEMFCs is addressed in this Thesis for parametric and additive faults. The model-based fault diagnosis is used to compute robust fault detection and isolation. For fault detection is based on a LPV interval observer, which can face the variation of parameter respect to the operating point schedule.
variable while preserving the linear structure of the model. Parametric uncertainty effects are propagated to the observer output using a zonotope representation to approximate and propagate the set of possible states.

At the same time, the approaches for fault diagnosis in this thesis, addresses the robustness problem against modeling uncertainty included as unknown parameters whose value is bounded by intervals. An important characteristic which allows, isolate faults which otherwise are not able to isolate them is the effects that make the fault in the residuals, called fault sensitivity. Sensitivity offers information, which could be essential to isolate faults where just binary information does not allow them, that why in this thesis use a methodology related to sensitivity analysis.

Fault isolation is based on a set of structured residuals that are analyzed using a relative fault sensitivity analysis approach. The proposed methodology of fault diagnosis is applied to a PEM Fuel Cell system.

Moreover, this thesis provides a control-oriented state space model for a PEM fuel cell system. The model describes the effects of a transient behavior over operating point change such as material and energy balance. The model uses a set of parameters which can be identified base on linear least-squares problem solving and using a set of lab data. The model parameters have been identified for a commercial PEMFC prototype installed in the lab, the model was used to test the fault diagnosis algorithm that is discussed along this thesis.
# Contents

Contents

List of Figures vii

List of Tables ix

Nomenclature ix

1 Introduction 1

1.1 Motivation. 2

1.2 Contributions. 4

1.3 Thesis Outline. 4

2 State of art in Fault Diagnosis. 6

2.1 Introduction 6

2.2 Scope of fault detection and fault isolation. 11

2.2.1 Model-free methods. 12

2.2.2 Model-based methods. 13

2.3 Fault detection using system models. 13

2.4 Robust Fault Detection 17

2.5 Fault isolation using system models 18

2.5.1 Models applied to fault isolation. 19

2.5.2 Models mapping the space of binary fault signals into the space of faults. 20

2.5.3 Models mapping the space of multi-value fault signals into the space of faults. 22
2.5.3.1 Fault information system: FIS. 22
2.5.3.2 Models mapping the space of continuous fault signals into the space of faults. 23
2.5.4 Model-based fault isolation techniques. 24
2.6 General approaches to fault isolation. 25
2.6.1 Fault isolation based on the binary diagnostic matrix. 25
2.6.1.1 Rules of parallel diagnostic inference on the assumption of a single fault. 26
2.6.1.2 Rules of series diagnostic inference on the assumption of a single fault. 28
2.6.2 Diagnosing based on the information system. 29
2.6.3 Analytical model-based fault isolation techniques. 29
2.6.4 Fault isolation based on interval observers. 33
2.7 Summary. 33

3 PEM Fuel Cell System Model. 35
3.1 Introduction. 35
3.1.1 Fuel Cell Classification. 36
3.1.2 PEM Fuel Cell System operation. 38
3.2 PEMFC Non-linear Dynamic Model. 41
3.2.1 Auxiliary Components modelling. 42
3.2.1.1 Air supply system model. 42
3.2.1.2 Supply and return Manifold. 48
3.2.1.3 Humidifier model. 51
3.2.1.4 Cooling System Model. 53
3.2.2 Fuel cell stack modelling. 55
3.2.2.1 Anode mass flow model. 56
3.2.2.2 Cathode mass flow model. 56
3.2.2.3 Membrane. 57
3.2.2.4 Stack voltage. 60
3.2.3 Thermal modeling. 64
3.2.3.1 Heat produced by reaction ($Q_{gen}$). 65
3.2.3.2 Electrical power ($P_{elec}$). 66
3.2.3.3 Heat Removal ($Q_{dis}$) .......................... 66
3.3 Summary ........................................... 71

4 Fault detection for a PEMFC system. .................. 73
  4.1 Introduction. ...................................... 73
  4.2 LPV Modeling in PEM Fuel Cells. .................. 74
  4.3 Fault detection using LPV modeling. ............... 74
      4.3.1 Problem formulation. .......................... 75
      4.3.1.1 Linear Parameter Varying Model (LPVM). .... 76
      4.3.1.2 Linear Parameter Varying Observer (LPVO). .. 78
      4.3.1.3 Adaptive threshold using zonotopes. .......... 80
      4.3.1.4 Robust Fault Detection using Linear Parameter Varying Interval Observer (LPVIO). .......... 87
  4.4 Summary. ......................................... 98

References ............................................. 103
List of Figures

2.1 Place of faults in a Plant. ........................................ 7
2.2 Hardware and Analytical redundancy architecture. .......... 8
2.3 Framework for fault diagnosis. ................................. 9
2.4 Scheme for model-based fault detection. ....................... 10
2.5 Architecture of model-based fault detection using residuals. . 14
2.6 Fault isolation model mapping (fault signals and faults). ... 19
2.7 Binary fault detection and fault isolation architecture. ....... 30

3.1 Diagram of a single cell of a PEMFC stack. .................... 37
3.2 Schematic of capillary flow of liquid water through the fuel cell GDL. 39
3.3 Fuel cell system. .................................................. 42
3.4 Block diagram of the blower model with corresponding inputs and outputs. .................................................. 44
3.5 Supply and return manifold process diagram. ................... 49
3.6 Humidity Exchanger diagram. .................................... 51
3.7 Cooling ambient air flow diagram. ................................. 54
3.8 Stack mass flow. ..................................................... 56
3.9 Example of a polarization curve showing the different loss regions. 62
3.10 Areas associated with a rectangular surface. .................... 70

4.1 Residual generation using the LPV observer approach. ....... 75
4.2 Zonotope architecture. .............................................. 81
4.3 Simulation performance for Reduced and Full PEMFCS model. . 99
4.4 Simulation performance in a fault free case using LPVO and adaptive threshold. .............................................. 100
4.5 Simulation performance in $f_1$ case ($FSM_{i,1} = [(−1), 0, 0]^T$) . . . . 101
4.6 Simulation performance in $f_2$ case ($FSM_{i,2} = [(−1), 0, 0]^T$) . . . . 102
### List of Tables

<table>
<thead>
<tr>
<th>Table</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1</td>
<td>Fuel Cell Advantage and disadvantage</td>
<td>36</td>
</tr>
<tr>
<td>3.2</td>
<td>Classification of fuel cells.</td>
<td>37</td>
</tr>
<tr>
<td>3.3</td>
<td>The electrochemical reaction for the anode and cathode.</td>
<td>38</td>
</tr>
<tr>
<td>3.4</td>
<td>Input and Output of the non-linear model, see Appendix ??</td>
<td>43</td>
</tr>
<tr>
<td>3.5</td>
<td>Corrected parameter correlation.</td>
<td>46</td>
</tr>
<tr>
<td>3.6</td>
<td>Compressor parameter</td>
<td>46</td>
</tr>
<tr>
<td>3.7</td>
<td>Specific heat value</td>
<td>53</td>
</tr>
<tr>
<td>3.8</td>
<td>Anode flow properties</td>
<td>57</td>
</tr>
<tr>
<td>3.9</td>
<td>Cathode flow properties</td>
<td>58</td>
</tr>
<tr>
<td>4.1</td>
<td>Description of the fault benchmark.</td>
<td>89</td>
</tr>
<tr>
<td>4.2</td>
<td>Eigenvalues, eigenvector of PEMFC linear model at $I_{st} = 120[A]$</td>
<td>93</td>
</tr>
<tr>
<td></td>
<td>and observability.</td>
<td></td>
</tr>
<tr>
<td>4.3</td>
<td>Binary diagnostic matrix ($FSM$) for the proposed fault benchmark.</td>
<td>97</td>
</tr>
</tbody>
</table>
Chapter 1

Introduction

In last few years, major efforts to reduce greenhouse gas emissions have increased the demand for pollution-free energy sources. Fuel cells have been pointed out as a promising alternative way for an energy source in the future, this is because fuel cell not only running on pure hydrogen, which is considered as a zero-emission power source, but also very quiet, which reduces noise pollution. Their energy density makes them specially suitable for embedded generation systems in transport applications. However, fuel cell technology is still too expensive to be accessible to a mass market, and there are still considerable obstacles to overcome. One of main drawbacks of fuel cell systems is related to the stack lifetime and reliability when working at strongly changing charge conditions, that could be usually found them in transport applications. To overcome these drawbacks, an adequate fault diagnosis and control strategy are needed to avoid failure modes which could compromise stack reliability, degradation processes or even compromise stack’s integrity.

An adequate real time system diagnosis can provide valuable information either to optimize control and fault diagnosis design strategies or to suggest preventive maintenance actions to prolong the lifetime of the system and thereby avoid future damage in the equipment. Most classic fault diagnosis tools need certain amount of apriori knowledge of the system and fault which wants to be modeled and identified respectably; however, fuel cell systems are complex and strongly non-linear in which constantly changing parameters are difficult to identify, even for static operation modes. To obtain models with a reasonable
computing complexity, many simplifying hypotheses have to be introduced to model, limiting their generalization capabilities to certain previously chosen operation modes under some restrictive conditions. Different authors have chosen different approaches for fuel cell system modeling. Electric equivalent models [69], state space models [42; 99] and bond graph representation models [106] are the most representative approaches. Even if these models can represent fuel cell behavior under certain operating conditions, their application to diagnosis is not so straightforward and most of them are related just in the stack and not in the overall system.

Available variable measurements in embedded systems such as fuel cell stack are relatively limited due to reduced space, technological instrumentation limitations and to the cost that will represent the sensors needed for obtaining the wanted or needed measurements, for fault diagnosis as an immediate result, available information is frequently insufficient to deduce failure origin.

Due to the system nature, some important constraints must be respected when developing a diagnosis strategy. First, computation time must be small since stack’s electrical dynamics are fast, compared to system ancillary components, and thus adequate control strategies have to be taken rapidly to avoid an overall system performance degradation. Secondly, diagnosis should perform well in a wide operation range, including system normal degradation due to ageing avoiding false alarms. Finally, diagnosis strategy should minimize hardware costs if it is intended to serve mass production markets.

1.1 Motivation.

As was said before, the energy generation systems based on fuel cells are complex since they involve thermal, fluidity and electrochemical phenomena that must keep in control for safety and efficiency reasons. That is the reason why, PEMFC need a set of auxiliary elements (valves, compressor, sensors, regulators, etc.) to make the fuel cell working at the pre-established operating point with optimal conditions. Therefore, as many auxiliary equipment are needed as the system is vulnerable to faults that can cause an emergency shut down or a permanent damage of the fuel cell. To guarantee a safe operation of the fuel cell systems,
it is necessary to use systematic techniques, like the recent methods of Fault Tolerant Control (FTC) [13], which allow increasing the fault tolerance in fuel cell systems. The first task to achieve active tolerant control in real-time is based on the inclusion of a fault diagnosis system. The diagnosis system should not only be able to detect and isolate faults but also estimate the fault magnitude.

Although the complexity of the fuel cell systems, in this thesis, a LPV model-based fault diagnosis technic is proposed as a way to diagnose faults in fuel cell systems. The model-based fault diagnosis is based on comparing on-line the real behavior of the monitored system obtained using sensors with a predicted behavior obtained using a mathematical model, where here a LPV model with a Luenberger interval observer scheme is used. In case of a significant discrepancy (residual) is detected between both the model outputs and the measurement signals obtained by the sensors, the existence of a fault is assumed. If a set of measurements is available, it is possible to generate a set of residuals (indicators) that present a different sensitivity to the set of possible faults. The fundamental purpose of a Fault detection in the FDI scheme is to generate an alarm (promptly) when a fault is presented in the system. Since a fuel cell dynamic is fast, a fault undetected, could have catastrophic consequences not only in safety but also in financial losses and process up-time reduction.

In senses of fault effect in a system study, sometimes where the plant does not allow to apply a physical fault, for safety and costly reasons, an alternately way to explore the effects caused by a fault in the system is to simulate the faultless and faulty systems model. Consequently the faulty system must be modeled as much detail as accurately for fault diagnosis performance is decided. One approach is to model sensor faults as additive perturbation to an input/output model of the plant [30] and other kind of fault is the multiplicative fault, which is modeled as parameter change. Such a representation in an appropriate way to model subtle drifts in measuring devices, abrupt sudden failures of sensors, or actuation devices becoming struck or failing to respond to a reference command signal. A degradation or malfunction of any part of the plant, this could be inferred as multiplicative or parametric fault.
1.2 Contributions.

The main contributions related to this research work could be separated in system modeling and fault diagnosis. For plant modeling has been built a model which considered not only the fluidics phenomena in the overall system but also the energy balance across the system. Another important characteristic in modeling, is the fact that the model scheme presents the capability to be calibrated for a commercial PEMFC prototype such as Ballard©, Nexa 1,2 KW. To test the fault diagnosis algorithm proposed here a fault benchmark is implemented in the plant, over the assumption of single fault.

For fault diagnosis, the problem of robust fault detection is complex systems, such as PEMFC System (PEMFCS) is addressed based on internal observer to generate adaptive thresholds, in other to isolate faults which are not able to isolate them with conventional technics, such as fault signature matrix, here is proposed the structural residual analysis bases on sensitivity presented in the residual over a fault.

1.3 Thesis Outline.

Chapter 2: State of Art in Fault Diagnosis. This chapter aim to bring the main ideas about some important basic concepts and review the state of the art in fault diagnosis. First part introduces concepts and definitions such as FDI approach, type and fault classification. In the second part, some methods for fault detection and isolation are briefly described and finally Robustness. The state of the art is mainly focused on model-based technics which is the core of this Thesis in the senses to develop the proposed fault diagnosis.

Chapter 3: PEM Fuel Cell System Model. This chapter introduction to PEMFC system technology, based on the different types of PEM as well as the king of their classification. Here is introduced on system modeling and concludes with a theoretical model which can be calibrated by adjusting parameters for a specific PEMFC system prototype. The parameter identification process uses a $lsq$-non linear fitting approach which minimize the error between estimated model output and measurement at each $k$ sampling time for a specific vector of
parameters.

Chapter 4: Fault Detection for PEMFC System. In this chapter address the complex problem for power sources system fault detection such as PEM-FCS is. The problem of robust fault detection with an interval observer using zonotope where measurement noise and modeling uncertainty is considered unknown but bounded by intervals. Their effect is addressed using an interval state observation method applied in PEMFCS as example in a Fault benchmark.

Chapter 5: Fault Isolation for PEMFC System. In this chapter a new methodology is introduced bases on sensitivity analysis to improve fault diagnosis in PEMFC systems. Some classical technics such as fault signature matrix, analytical redundancy, are used and compared with the new methodology based on relativity fault sensitivity for parametric and additive faults with theoretical Examples. Another important contribution, a fault benchmark is proposed as case study.

Chapter 6: Fault Diagnosis Case Study: BALLARD®, NEXA 1,2 KW. It is in this chapter which address the issue of robust fault diagnosis using the theoretical model already developed in Chapter 3 and calibrated results are shown here, additionally of the modeling for a trading PEMFC system, such as Ballard ©, Nexa, the methodology developed in Chapters 4 and ?? is used to carried out fault diagnosis in a realistic case.

Chapter 7: Conclusions and Future Work. Finally in this part of the thesis are discussed the results and offers conclusion of the research work. Also additional open issues are mentioned for further research works.
Chapter 2

State of art in Fault Diagnosis.

2.1 Introduction

Since the beginning in 1970’s, research in fault diagnosis has been gained more interest in world-wide in both theory and application [34; 87]. This development is still mainly stimulated by the trend of automation towards more complexity and the growing demand to reach safe control systems. However, a strong interest also comes from the side of modern control theory that has brought powerful techniques such as; mathematical modeling, state estimation and parameter identification. Staring from this point, fault diagnosis is still an open issue for automotive area, which uses technics from modern control.

Before the start, it is necessary to introduce some basics principles and definitions, such as fault, type of fault, fault diagnosis system. In the following part, these ideas are discussed. A fault is deemed to occur when the system experiences an abnormal condition, such as a component malfunction, it may not represent physical failure or breakdown. Fault is commonly denoted in the literature as $f$. The fault acting upon a system (see Figure 2.1) can be divided into three types of faults.

- **Sensor** faults: This type of faults occurring with sensors, such as scaling errors, drift, dead zones, and so on.
- **Actuator** faults: This king of faults associated with the actuator, such as damage in the bearings, deficiencies in force or momentum and so on.
- Component (system) faults: These faults happening in the framework of the process, such as a broken element of the plant.

Faults also could be categorized into additive and multiplicative faults, in terms of how the faults influence the system variable. Additive faults influence the variables additively, such as offsets of sensors; while multiplicative faults usually affect the system parameters by a product factor. Another way of categorizing faults is time-based, which is called as abrupt or incipient faults. An abrupt fault represents an undesired and sudden change, while an incipient fault is a fault that causes undesirable drifts away from healthy operating values and gradually grows with time.

![Figure 2.1: Place of faults in a Plant.](image)

Because a fault disturbs the normal operation of an automatic system, thus causing an unacceptable degradation of the system performance or even leading the system into a long shut down. A monitoring system which is commonly used as an expert tool for fault diagnosis and identification with as much information as possible (location and significance in a system), this application is commonly called a fault diagnosis system and uses a physical or analytical redundancy architecture, in order to pictured out the last concept we can see Figure 2.2.

A traditional approach to fault diagnosis in a wider application context is based on physical or hardware redundancy methods which use multiple lanes of sensors, actuators, computers and software to measure and/or control a particular variable.
One of the main advantage of this approach is the confidence of the redundancy is totally related to the quality of the hardware (sensor or actuator) and does not need expert analysis, just an engineering design. By other hand, the major problems of this approach are commonly inferred to the fix cost (back up or warehouse stocks) and variable cost (maintenance) increases, in some cases the size or volume of the plant does not allow the use of extra equipment. The analytical redundancy approach, which is related to mathematical modeling, represents a suitable alternative to face this kind of problems.

Fault detection. simply involves a decision based on the monitored data as to whether there is a fault or the system is running under normal conditions. For Fault isolation infers an execution to identify the type and location of a fault after the fault detection has triggered an alarm so that corrective action can be made. These two process together is known as fault detection and isolation (FDI). Fault identification in some authors refers to identify the fault magnitude and the time of occurrence. Another frequently used therm, fault diagnosis, is generally refereed to as the combination of fault detection, isolation and identification. Figure 2.3 is a general framework for fault diagnosis system

The effectiveness of the FDI method is also affected by some factors other than faults ($f_a$, $f_u$, $f_y$), such as system uncertainties, noise ($n$), perturbation ($d$) and
Symptoms evaluation

Fault isolation

Fault type

Fault magnitude and time

Plant

Yes

No

Figure 2.3: Framework for fault diagnosis.

model mismatch. These might not influence the system operation under normal conditions, but can be very problematic to FDI, where a fast model dynamic is expected to, then a quick fault diagnosis is needed to avoid any kind of system damage.

Therefore there must be a compromise between robustness and quick detection and isolation. For what is considered as quick detection; the ability of FDI system to be able to detect and isolate faults quickly enough to avoid damage during abnormal conditions, isolability is the capability of distinguishing between different faults based on the monitored plant data, and robustness. The robustness is the ability of the system to make a correct decision within the tolerance of system uncertainty and noise, later in this chapter will be addressing the issue of robustness.

How to detect and isolate faults quickly and correctly while being insensitive to uncertainties and noise is an important challenge to FDI approaches. In additions, incipient faults tend to be hidden by disturbances, where an incipient fault do not necessarily cause immediate damage.

Model-based FDI approaches are based on a mathematical model representation of the system. They are also associated with analytical redundancy, where
the available input or measurements and a system model are used to cross-check the signal information to detect faults. Figure 2.4 depicts the general architecture of model-based FDI, a system model is used to estimate the system behavior under normal operation, the deviation between model outputs and real system measurements generate the residuals. The residual would be zero or small enough to cross any alarm when the system is fault-free but noticeable when faults occur. The residual is analyzed and evaluated for fault detection and isolation.

\[
\begin{align*}
\text{Model} & \quad \hat{y} \\
\text{Residual generation} & \quad r \\
\text{Residual evaluation} & \quad r \rightarrow \phi \\
\end{align*}
\]

Figure 2.4: Scheme for model-based fault detection.

As pointed out previously, model-based FDI methods make use of the mathematical model of the monitored system. However, an exact and complete model of the monitored system is never available because the presence of model uncertainty, unknown model parameter values, disturbances and finally noises. Hence, there is always a mismatch between the actual process and its mathematical model even if there are no process faults, the model must be suitable enough to fault diagnosis avoiding or missing false alarms.

The effect of modeling uncertainty is therefore the most crucial point in the model-based FDI concept. To increase the confidence of FDI a robust FDI is necessary. A robust FDI architecture consists on a FDI scheme designed to provide sensitivity to faults avoiding noise and model uncertainty effects. Several FDI methods have been proposed to solve this problem, for example, the unknown input observer [38; 46], eigenstructure assignment [83; 84], optimally robust parity relation methods [17; 79].
2.2 Scope of fault detection and fault isolation.

Fault detection is the process of generating fault signals ($\phi$) by residual analysis on the fields of the process variables ($x$) to detect faults. Thereby, detection algorithms should be able to generate fault signals which ought to contain information about faults. The mapping of the space of process variables into the space of fault signals as well as the evaluation of these signals to detect the fault during the detection stage.

Fault isolation is carried out mainly on fault signals generated by the detection module. The result of isolation process is a diagnosis of the overall system showing the type of fault and actual state of the system. The knowledge of the relationship between the fault signals and the technical states of the system is necessary to perform the fault isolation. Thus, a completely reliable and unequivocal presentation of the fault isolation result is not always possible to perform straightforward, due to lack of the system knowledge, limited fault distinguishability or states, uncertainty of fault signals, etc.

In general, fault diagnosis methods could be classified into two major groups [116]: those which do not use the model of the plant to express the knowledge about its physics and those which do. Regarding the last group, there are two mainly approaches: the analytical one where the process or plant understanding is expressed in terms of mathematical functional relationships between the inputs and outputs of the system and the qualitative one where these relationships are expressed in terms of qualitative functions.

The analytical approach is mainly used by the well known FDI methodology which is based on control theory and statistics. On the other hand, another important methodology is the known as DX which is based on the fields of computer science and artificial intelligence applying qualitative models.

The techniques analyzed in this section are just classified according to its application (fault detection or fault isolation). Although this thesis is devoted to the model-based methods, the model-free techniques will be briefly reviewed. There are many ways to categorize the different diagnosis schemes described in literature, but here it is divided then into two groups, Model-free and Model-based methods.
2.2.1 Model-free methods.

In this approach, the fault detection and isolation methods do not use the mathematical model of the monitored system, then they are related to hardware redundancy, limit checking, frequency spectrum, statistical analysis and logical reasoning techniques.

- **Physical redundancy.** In this approach, multiple sensors are installed to measure the same physical variable. Any serious discrepancy between the measurements shows a sensor fault. With only two parallel sensors, fault isolation is not possible. With three sensors, a voting scheme can be built which allows to isolate the faulty sensor. Physical redundancy involves extra hardware cost and extra weight, the latter representing a serious concern, for example, in aerospace or automotive applications.

- **Limit checking.** In this approach, the system variable measurements are compared to preset limits by a computer, in some applications are called as alarms and they have different levels. Exceeding the threshold shows a fault situation.

- **Spectrum analysis of plant measurements.** Most plant variables exhibit a typical frequency spectrum under normal operating conditions; any mismatch from this is an indication of abnormality. Certain types of faults may even have their characteristic signature in the spectrum, making fault isolation a simple task.

- **Logic reasoning.** These techniques form a broad class which is complementary to the methods outlined above, in that they are aimed at evaluating the fault signals obtained by the detection hardware or software. The simplest techniques consist of logical trees of if-then rules: ”**IF** fault signal $\phi_i$ - **AND** fault signal $\phi_j$-**THEN** conclusion”. Each conclusion can, in turn, serves as a fault signal in the next rule, until the final conclusion is reached.
2.2.2 Model-based methods.

The general idea of fault diagnosis model based methods is to compare the available measurement vector of the monitored system \( (y) \) with their corresponding prediction vector \( (\hat{y}) \) obtained using a system model, either analytical or qualitative [116]. The analytical redundancy, incorporates the concept of residuals \( (r) \), where under ideal conditions the residual vector in discrete time is computed as:

\[
r_k = y_k - \hat{y}_k \tag{2.1}
\]

If an accurate enough model is available and their associated residual values are significant to cross an alarm, then it may be determined that a fault has occurred. Nonetheless, the problem with this approach is that a precise description of a system that takes the effects of faults into account is usually impossible, and even if such a description exists, the dependence that characterizes particular faults cannot be defined on the grounds of it. Therefore, different kinds of simplified models are used in FDI.

When classifying models applied to the diagnostics of processes (systems), it is possible to distinguish models of systems applied to fault detection and models used for fault isolation. Models used for fault detection describe relationships existing within the system between the input \( (u) \) and output \( (y) \) signals, and allow detecting changes (fault signals \( (\phi) \)) caused by faults. Models used for fault isolation define the existing relationship between fault signals \( (\phi) \) and faults \( (f) \), which will be represented by \( \phi \rightarrow f \).

2.3 Fault detection using system models.

In general, model-based fault detection methods use system (process) models to generate residuals [40; 44]. Thus, Figure 2.5 illustrates the general and conceptual structure of a model-based fault detection module which consists in two main parts: the residual generation stage and the residual value stage. This two-stage structure was first suggested by [22]. Nowadays this is widely accepted by the fault diagnosis community. The main purposes of these two main stages are described as follow:
Figure 2.5: Architecture of model-based fault detection using residuals.

- **Residual Generation**: Its purpose is to generate a fault-showing based on signal-residual, using available input and output information from the monitored system. This auxiliary signals is designed to reflect the onset of a possible fault in the analyzed system. This is regard to the residual analysis which is characteristically independent of system inputs and outputs, in ideal conditions. The algorithm used to generate residuals is called a residual generator. Residual generation is thus a procedure for extracting fault signals $\phi$ from the system, with the fault signal represented by the residual signal $(r)$. The residual should ideally carry only fault information and to ensure reliable fault detection, the loss of fault information in residual generation should be the minimum as possible.

- **Residual value evaluation**: Every fault detection algorithm that makes use of an analytical, fuzzy or neural model contains the decision-making part, in which the evaluation of the residual value takes place. In this stage, the decision about the existence of a fault is made together with a possible indication of this event generating the corresponding fault signal. This signal should carry out information about the effect of the fault on the residual set, then the fault isolation module can isolate the fault.

A decision process may consist in a simple threshold test on the instantaneous values or moving averages of the residuals [92; 98], or it may consist of methods of statistical decision theory, e.g., generalized likeli-
hood ratio (GLR) testing or sequential probability ratio testing (SPRT) [9; 11; 114; 118]. On the other hand, this residual evaluation can also be carried out with the use of fuzzy or neural logic [56] or qualitative methods [62; 66].

Model-based fault detection is mainly concert to On-line fault diagnosis and Robustness. Where on-line fault diagnosis is carried out during a normal system operation, this is because the system input-output information required by model-based fault detection module is only available when the system is in operation.

Finally, robustness is related to disturbances dismiss, noise and modeling uncertainty that arises from the lack of knowledge and understanding of the monitored processes. Consequently, the residual generated to indicate faults may also react to the presence of noise, disturbances and model errors. Decoupling the residuals from these sources is the most important aspect in the design of the FDI algorithm. In particular; residual filtered, disturbance decoupling, robustness in the face of error modeling.

Because of the presence of noise and model errors, the residuals are never zero, even if there is no fault. Therefore, the residual evaluation requires testing the residuals against thresholds, obtained empirically or by theoretical considerations. As it was already mentioned before, another approach to achieve robust residual evaluation would be the use of a fuzzy or neural logic. On the other hand, to perform easily fault isolation, the residual generators are usually designed (enhanced residuals) to exhibit structural or directional properties. The isolation decisions then can be obtained in a structural (Boolean) or directional (geometric) framework, with or without the inclusion of statistical elements. Concerning residual generation methods in FDI model-based, four main approaches can be considered within the group of analytical methods applied to fault detection.

- Diagnostic Observers. The basic idea behind the diagnostic observer approaches is to estimate the outputs of the system from the measurements (or a subset of measurements) by using either Luenberger observers in a deterministic setting [12; 55] or Kalman filters in a stochastic setting [10; 70; 118]. Then, the weighted output estimation error is used as a residual. Thereby, while the flexibility in selecting the observer gains is used to minimize the
noise effect on the fault detection result in the Kalman approach, this freedom is applied to enhance the residual fault detection and isolation properties. As a result, the dynamics of the fault response can be controlled, within certain limits, by placing the poles of the observer. A long line followed this trend of researchers, including [18; 33; 35; 82; 97; 117].

- **Parity (consistency) relations.** Parity relations are rearranged direct to input-output model equations, subjected to a linear dynamic transformation. The transformed residuals are used for detection and isolation processes. The residual sequence is colored, just like for observers. The design freedom provided by the transformation can be used for disturbance decoupling and fault isolation enhancement. Also, the dynamics of the response can be assigned, within the limits posed by the requirements of casualty and stability. The parity relation approaches to generate the residual, based on consistency checking system input and output data over a time window, was originally proposed in [71; 72]. The approach was later proposed in [22], and has been expressed in several different versions [20; 40]. The latest development regarding parity relation approaches can be found in [18; 39; 40; 91].

- **Parameter estimation.** Another FDI approach is the use of parameter estimation which is based directly on system identification techniques. In [50] is illustrated in a survey paper that the fault detection and isolation processes can be achieved using the estimation of non-measurable process parameters and/or state variables. Parameter estimation is a typical approach for parametric (multiplicative) faults detection and isolation. A model of the monitored plant is obtained by first identifying the plant in fault-free situation. Thus, a deviation from the reference model serves as a basis for detection and isolation. Parameter estimation may be more reliable than the analytical redundancy methods, but it requires a high computational effort and is also more demanding in terms of on-line computation and input excitation requirements and consequently, this approach will not be considered in the thesis. The latest development and applications can be found in [48; 51; 52].
2.4 Robust Fault Detection

When building a model of a complex system to monitor its behavior, there is always a mismatch between the modeled and real behavior since some effects are neglected, some parameters have tolerance, some errors in parameters or in the structure of the model are introduced in the calibration or linearization process, etc. Many times, these modeling errors could be bounded and included in the fault detection model. There are several ways of modeling the uncertainty associated with the model.

For instance, the interval model (or worst case) approach provides a nominal model plus the uncertainty on every parameter value bounded by intervals, this technic is later address in this thesis. In the FDI and automatic control community, this type of uncertainty is called structured because it is assumed that the structure of the model is known but not the model parameter values, in opposition to a more general type of uncertainty which considers the structure of the model is not completely known but only bounded.

In FDI community a fault detection algorithm able to handle uncertainty is called robust. The robustness of fault detection algorithm is the degree of sensitivity to faults compared to the degree of sensitivity to uncertainty [86]. In the last years, research in robust fault detection methods has been very active in the FDI community [19]. Active approach, is based on generating residuals which are insensitive to uncertainty, while at the same time sensitive to faults, some techniques are: unknown input observers, robust parity equations, $H_{\infty}$, etc. In [19; 40] there is an excellent survey of the active approach. On the other hand, there is a second approach, called passive, which enhances the robustness of the fault detection system at the residual value evaluation stage. This approach is still under research. Several techniques have been used, but most of them are based on using an adaptive threshold at the residual value evaluation stage. According to [41], there is no algorithm which is robust under arbitrary model error conditions. To design an algorithm for robustness, rather more detailed information is necessary about the nature of errors and uncertainties, and such information is seldom available. However even if it is, what can be achieved is limited.
Adaptive thresholding techniques were first proposed in [23], who suggests an empirical relation between the operation point and the corresponding detection threshold. Further approaches by [31], who develops a theoretical relation between the operation point, the model uncertainty and the detection threshold. This approach is based on $H_{\infty}$ techniques and it was further explored by [28; 36]. Another approach for adaptive threshold generation was proposed by [45], this is based on a dynamical optimization assuming parametric uncertainty.

The advantage of passive over the active approach is the fact that it can achieve robustness in the detection process despite the number of uncertain parameters in the model, and without using any approximation based on the simplification of the underlying parameter representation [41]. The passive approach based on adaptive thresholds is not only based on avoiding the effect of uncertainty in the residual through perfect decoupling, but also in propagating the parameter uncertainty to the residual, and then bounding the residual uncertainty using an interval.

2.5 Fault isolation using system models

According to [116], model-based fault isolation has been approached over the last two decades from two different scientific communities: Artificial Intelligence, also known as the DX approach [43; 105] and Automatic Control, also known as FDI approach [13; 40; 85]. The DX approach relies on a well-founded and logically based theory for diagnosis of static systems. From a logical point of view, fault detection is performed through a consistency-check and organized around the conflict concept (fault signal).

In DX approach, fault localization or isolation is automatically derived from the conflict detection stage, which usually relies on some kind of dependency recording. On the other hand, the FDI approach considers fault diagnosis in two separate tasks: fault detection and fault isolation based on generating and evaluating a set of analytical redundancy relations obtained off-line from elementary component models of the physical systems. Fault detection has traditionally been more deeply investigated in the FDI community using a broad set of techniques (parity equations, observers and parameter estimation) and looking at
the nuisance effects of noise, perturbations and model uncertainty (\textit{robust fault detection}) [18].

Conversely, fault isolation has been more deeply investigated in the DX community regard to the logical diagnosis theory developed by Reiter [105]. However, when this theory has been applied to dynamical systems, some problems have appeared, which does not allow to use it straightforward. In fact, the theoretical formalization for fault diagnosis applied to dynamic systems is still nowadays an open issue. The last situation has motivated further research on analyzing off-line the set of dependencies, which could become conflicts (fault signals) as in the FDI approach [102], based on the common framework provided in [25].

### 2.5.1 Models applied to fault isolation.

Models applied to fault isolation or system state recognition should therefore, map the space of fault signals ($\phi$) into the discrete space of faults $f$ ($\phi \rightarrow f$). Figure 2.6 shows the conceptual idea for model isolation process.

![Fault isolation model mapping](image)

Figure 2.6: Fault isolation model mapping (fault signals and faults).

According to [57], in fault isolation, it is possible to point out the following kind of models:

- models that map the space of binary fault signals into the space of faults,
- models that map the space of multi-value fault signal into the space of faults,
- models that map the space of continuous fault signals into space of faults.

The above models can be defined using different techniques, such as training, knowledge about the hardware redundant structure and modeling the influence
of faults on residual values or the expert’s knowledge. Training data for the state of complete efficiency and for all the states with faults, or at least a definition of the fault states, are necessary for applying the training procedure. However, such data are difficult and often impossible to obtain for the industrial process diagnostics. Conversely, it is relatively easy to define the relationship that exists between fault signal values when the hardware redundant structure in the diagnosed system is used. Nevertheless, such a solution is very rarely applied due to its high costs.

If equations for the generation of residuals that contain the effect of faults are known, then it is possible to define residual value ranges as well as fault signal values that correspond to residuals for the state without faults and for the states with faults. Thereby, sets of fault signal values are obtained for particular faults and for the state of complete efficiency. These sets define specific regions in the space of fault signals. This method, as well requires a mathematical description of the system which must allow deriving the effect of faults on the residuals or fault signals. Another method consists in using an expert’s knowledge. The expert should define fault signal values that correspond to particular faults. As a result, the fault signal space regions that correspond to states with a single fault and to the non-faulty state are arbitrarily defined.

2.5.2 Models mapping the space of binary fault signals into the space of faults.

Binary fault signals $\phi_i \in \{0, 1\}$ are obtained as a consequence of a two-value evaluation of residuals or process variable value features. They are also generated as a result of implementing tests, which consist in controlling limits or examining the heuristic relationships existing among process variables. In this model group, it is possible to list out the following approaches:

*Binary diagnostic matrix*: The model most often applied is a relation defined on the Cartesian product of the sets of faults $f = \{f_j : j = 1, 2, \ldots, n_f\}$ and fault signals $\phi = \{\phi_i : 1, 2, \ldots, n_\phi\} : FSM \subset \phi \times f$, where $FSM$ is the binary diagnostic matrix [25; 40] which is also well known in the FDI community as the theoretical fault signature matrix [40]. The matrix stores the binary influence of
a given fault $f_j$ (column of $FSM$) on a given fault signal $\phi_i$ (row of $FSM$). Thereby, if the element $FSM_{ij}$ of this matrix is equal to 1, it means the fault $f_j$ causes the occurrence of the fault signal $\phi_i$. Otherwise, when the fault $f_j$ has no effect on the fault signal $\phi_i$, the element $FSM_{ij}$ is equal to 0. Such as the $FSM$ matrix is defined, its $j^{th}$-column contains the binary effect of fault $f_j$ on the fault signal set $\phi_i$. In consequence, this column is known as the theoretical fault signature of $f_j$. This binary diagnostic matrix could be defined on the grounds of the residual equations which take into account the effect of faults.

**Diagnostic trees and graphs:** The relationship that exists between faults and fault signals can be presented as a binary tree that defines the method of diagnostic inference [115]. The tree vertices correspond to fault signals (tests). Out of each of the vertices come out two branches corresponding to two values of the fault signal: the true and the false result of the test. A fault signal having a value that is analyzed as the first one is the root of the tree. Such a tree can be defined using the theoretical fault signature matrix $FSM$ [13], or directly because of an expert’s knowledge.

**Rules and logic functions:** According to [57], the relationship existing between faults and binary fault signals can be defined as the following types:

**Algorithm 2.5.2.a**

1: if $\phi_1 = 0$ and ... and $\phi_j = 1$ ... and $\phi_{n_\phi} = 1$ then
2: fault $\leftarrow f_k$
3: end if

**Algorithm 2.5.2.b**

1: if $\phi_1 = 1$ then
2: fault $\leftarrow f_\alpha$ or ... or $f_k$ or ... or $f_n$
3: end if

Every column of the theoretical fault signature matrix $FSM$, allows to obtain a rule of the type given by Algorithm 2.5.2.a. This type of rules is also well known as parallel reasoning, in general they are used in the FDI approaches. Conversely, the rows of $FSM$ allows defining rules of the type given by Algorithm 2.5.2.b which are known as series’s reasoning, and usually they are used in DX approaches.

The logic function is the simplest possible relationship that exists between fault signals ($\phi$) and faults ($f$). Binary fault signals are considered as input
for fault isolation process, and the result of this function shows the state of a particular fault, i.e., its existence or absence. In a general case, such a function takes the following form:

\[ z_{f_k} = [\phi_a \lor \cdots \lor \phi_b] \land [\phi_i \lor \cdots \lor \phi_j] \land \cdots \land [\phi_m \lor \cdots \lor \phi_n] \] (2.2)

where \( z_{f_k} \) is a the system binary state which indicates the occurrence of fault \( f_k \): its value is 1 when \( f_k \) is isolated and 0 when it is not.

### 2.5.3 Models mapping the space of multi-value fault signals into the space of faults.

Multi-value fault signals appear as a result of residual value or signal feature quantization. They can also result from a process variable limit control with the application of several boundary values. It is assumed that a different set of values \( w_i \) could correspond to each one of the faults signals \( \phi_i \). Thereby, models necessary for fault isolation using multi-value fault signals realize the mapping [57] as

\[ \phi \in w_i \times \cdots \times w_i \times \cdots \times w_{n_f} \Rightarrow f \in \{0, 1\}_{n_f} \] (2.3)

Then, it is possible to say that the following models belong with the groups: *fault information systems, diagnostic trees, graphs, and if-then rules.*

#### 2.5.3.1 Fault information system: FIS.

Then, it is possible to say that the following models belong with the groups: *fault information systems, diagnostic trees, graphs, and if-then rules.* The fault information system models (FIS), which were introduced in [59], they can be seen as an extension of the binary theoretical fault signature matrix (FSM), where the values of its non-null elements are not a binary one but they belong to the set of values \( (w_{ij}) \), the fault signal \( \phi_i \) and the fault \( (f_j) \). This fact allows adding fault distinguishability when the fault isolation process is a complicated task.
According to [59], a fault information system $FIS$ is defined as follows:

$$FIS = \langle f, \phi, w_\phi, \mu \rangle \quad (2.4)$$

where $w_\phi$ is the set that contains all the values of $w_{ij}$ associated with every couple $\langle \phi, f_j \rangle$.

$$w_\phi = \bigcup_{i=1}^{n_\phi} \bigcup_{j=1}^{n_f} \quad (2.5)$$

Regarding $\mu$, this is a function defined as

$$\mu : \phi \times f \rightarrow w_\phi \quad (2.6)$$

that determines for every couple $\langle \phi_i, f_j \rangle$, the corresponding set of values related to $\phi_i$:

$$\mu (\phi_i, f_j) = w_{ij} \quad (2.7)$$

Therefore, $FIS$ is a table that defines the fault signal pattern values for particular faults. This table has the following characteristics regarding the binary FSM:

- Individual set of fault signal values can exist for each one of the fault signals.
- The set of the $i^{th}$-fault signal values can be a multi-value one.
- Any element of the $FIS$ can contain either one fault signal value or a subset of values.

2.5.3.2 Models mapping the space of continuous fault signals into the space of faults.

Residuals obtained using system models generate continuous fault signals. Here, models applied to fault isolation consider the mapping:

$$\phi \in \mathbb{R}^{n_\phi} \Rightarrow f \in \{0, 1\}^{n_f} \quad (2.8)$$

Thereby, considering the space of continuous fault signals, the effect of a fault $f_j$ on the fault signal set $\phi$ is defined by a region which should be a characteristic
of each fault so that a fault can be isolated. Some methods used to model the mapping defined by Eq.(2.8) are: classic methods of pattern recognition, neural networks, and neural fuzzy networks.

**Pattern pictures:** The construction of a model for fault isolation consists in defining regions in the space of fault signals, which constitute pattern pictures of faults. These regions can be defined in different ways. Thus, it is possible to picture out geometrical, polynomial and statistical classifiers [112]. Pattern recognition could be obtained from the data training process. To achieve this, it is necessary to have available the training data set for all faults. However, this is extremely difficult, and for many industrial systems even impossible or unrealistic. To mitigate or solve this problem, the data set can be obtained by fault simulation with the use of the system analytical model that takes the effect of faults into account.

**Fuzzy neural networks:** Fuzzy neural networks applied to fault isolation take into account the fuzzy evaluation of residual values as well as diagnostic inference [111]. The structure of a fuzzy neural network applied to fault isolation different from the structures used for system modeling. It contains no layer in which defuzzification is carried out. The number of network outputs is equal to the number of distinguished faults or system states.

### 2.5.4 Model-based fault isolation techniques.

There are plenty of isolation methods, the work developed by [53] distinguishes two basic groups: classification and automatic concluding methods. According to [60], the fault isolation methods can be classified depending on how the knowledge about the relation between fault signals and faults is obtained. Based on the last criterion, signals out the following methods.

- Methods in which the diagnostic relationship results from the structure of mathematical or qualitative models used for fault detection.

- Methods that require defining the relation between fault signals and faults during the training phase.
• Methods based on expert’s knowledge.

• Methods in which the relation between fault signals and faults results from a redundant hardware structure. Concerning the relation between fault signals and faults, the following model-based fault isolation methods can be highlighted
  – Graphs, [13; 90; 113].
  – Binary diagnostic matrix, [40].
  – Information system, [59].
  – Fault-attributed regions in the space of fault signals, [40; 54].
  – Neural networks, [32].
  – Fuzzy neural networks, [1].

2.6 General approaches to fault isolation.

In this section, general fault isolation methods using the fault isolation models already introduced in Section 2.5 will be described.

2.6.1 Fault isolation based on the binary diagnostic matrix.

The binary diagnostic matrix $FSM$ presents the relation between the values of binary fault signals $\phi$ and faults $f$. It could be designed using system equations and taking the effect of faults into account or based on expert knowledge. Fault isolation inference is carried out by the binary diagnostic matrix which can be realized with the use of classical [40] or fuzzy logic [59] methods. One great advantage of the latter approach is the fact that allows taking fault signal uncertainty into consideration.

The binary diagnostic matrix is used for fault isolation together with different fault detection methods under the specification that the fault signals, which are the outputs of the fault detection algorithms, which have to be a binary format. Fault isolation methods using the binary fault isolation matrix can isolate faults
comparing the value of the observed fault signals with the information stored in
the FSM. Thereby, there are two main groups for fault isolation methods using the
binary FSM depending on how they carry out the comparison between parallel
and series inference approaches.

2.6.1.1 Rules of parallel diagnostic inference on the assumption of a
single fault.

Parallel diagnostic inference, based on the binary diagnostic matrix, consists in
formulating a fault isolation result comparing the observed binary fault signals
\( \phi = \{\phi_i : i = 1, 2, \ldots, n_\phi\} \), where \( \phi_i \in \{0, 1\} \), with the theoretical fault sig-
nature associated with all considered fault hypotheses determined by the set
\( f = \{f_j : j = 1, 2, \ldots, n_f\} [40; 58] \). The theoretical fault signature of the fault hypothesis \( f_j \) is stored in the \( j^{th} \)–column of the matrix FSM where each element
of this matrix \( FSM_{ij} \in \{0, 1\} \).

Assuming that only a single fault exists, fault isolation is carried out using
the set of observed fault signals \( \phi \). Thereby, the inference procedure consists in
comparing the binary fault signals \( (\phi_i) \) computed at every time instant \( (k) \) by the
fault detection module with the theoretical binary fault signature related to every
fault hypothesis \( f_j \) of the set faults \( (f) \) which are stored in the \( j^{th} \)–column of
the binary matrix FSM. If all fault signals are zero-valued, the fault isolation
module shows a lack of any fault:

\[
\forall \phi_i \in \phi : \phi_i = 0 \Rightarrow DGN = 0 \tag{2.9}
\]

where \( DGN \) is the set of fault hypothesis \( f_j \) which are consistent with the ob-
served fault signals. When some fault signal values equal to one, according to the
residual evaluation carried out by the fault detection module, the result of the
fault isolation algorithm is a subset of the fault hypothesis set \( f \), whose signatures
are consistent with the observations:

\[
DGN = \{f_j \in f : \phi_i = FSM_{ij}, \forall \phi_i \in \phi\} \tag{2.10}
\]

In case, where a robust fault detection approach based on interval models is
used, the binary value of every fault signal is computed as it follows

\[ \phi_i = \begin{cases} 0 & y_{ik} \in [\hat{y}_{ik}] \\ 1 & y_{ik} \notin [\hat{y}_{ik}] \end{cases} \] (2.11)

Then, a general approach to compute the comparison between the observed fault signature \( \phi_i \) and the theoretical fault signature related to every fault hypothesis is computed the distance between both vectors: \( \phi_i \) and the \( j^{th} \)-column of matrix \( FSM \) for the hypothesis \( f_j \), e.g. using the Hamming distance measurement. Due to this comparison, a distance measurement \( d_{jk} \) is obtained for every fault hypothesis \( f_j \), being \( d_k \) the vector of all the computed distances at the time instant \( k \): \( d_k = [d_{1k}, d_{2k}, \ldots, d_{nfk}] \). If the Hamming distance approach is applied, then

\[ d_{jk} = \sum_{i=l}^{n_f} (FSM_{ij} \oplus \phi_{ik}) \] (2.12)

where \( \oplus \) is the XOR logic operator. Then, the shortest distance from the fault hypotheses set to the current observed fault signature \( \phi \) is considered as the fault isolation result:

\[ DGN = \left\{ f_j \in f : \forall_j \text{ where } d_{jk} = \min_{v \in \{1, \ldots, n_f\}} [d_{jk}] \right\} \] (2.13)

This approach gives a simple idea of the fault isolation problem, but it has many drawbacks, which can lead towards a wrong diagnosis result: e.g., this algorithm always provides a diagnosis result, even when a no fault hypothesis exactly matches the current observed fault signature vector. Consequently, this can cause the diagnosis to jump from a fault hypothesis to another fault hypothesis, every time instant when a new symptom appears. Mostly, this fault isolation methods using the binary theoretical fault signature matrix and based on a parallel inference are used by the FDI community which uses analytical models for monitoring system and is well known as the fault isolation column view approach.

In general, the assumption of a single fault is not always justified. In that situation, the states of multiple faults should be taken into account in the diagnostic inference. It is usually enough to widen the set of the analyzed states of
the system behavior with multiple faults. This can be done by increasing the number of columns of the theoretical FSM or increasing the fault set, \( f \), grouping single faults. Thereby, there will be a new column for each of the new considered multiple fault states where each of these columns shows the theoretical effect of the related multiple fault state on the residuals.

### 2.6.1.2 Rules of series diagnostic inference on the assumption of a single fault.

Thereby, this subset \( DGN_1 \) of \( f \) is the set of possible fault hypotheses, which must be considered when the next fault signal will be observed. In consequence, the performance of this method allows to narrow the set of possible fault hypotheses every time a new fault signal occurs. In general, once the series diagnostic inference has been applied \( p - 1 \) times and a new fault signal \( \phi_{ipk} \) has been observed, the result of the new series of diagnostic inference can be written as it follows:

\[
DGN_1 = \left\{ f_j : \forall_{j,f_j \in f} \left[ FSM_{i \mid j} = 1 \right] \right\} \tag{2.14}
\]

Thereby, this subset \( DGN_1 \) of \( f \) is the set of possible fault hypotheses, which must be considered when the next fault signal will be observed. In consequence, the performance of this method allows to narrow the set of possible fault hypotheses every time a new fault signal occurs. In general, once the series diagnostic inference has been applied \( p - 1 \) times and a new fault signal \( \phi_{ipk} \) has been observed, the result of the new series of diagnostic inference can be written as it follows:

\[
DGN_p = DGN_{p-1} \cap \left\{ f_j : \forall_{j,f_j \in f} \left[ FSM_{i \mid p} = 1 \right] \right\} \tag{2.15}
\]

In general, the series of diagnostic inference only considers those fault hypotheses which can explain all the observed fault signals. If the theoretical fault signature related to a given fault hypothesis \( f_j \) contains a null value (\( FSM_{ij} = 0 \)), where a fault signal has been observed, \( \phi_{ik} = 1 \), this fault hypothesis will be automatically rejected by this method. Regarding the parallel diagnostic inference method presented previously, this series reasoning avoids the flickering of the fault diagnosis result computed in the parallel diagnostic approach, since this series inference
method is based on an incremental reasoning and not on an absolute reasoning such as it is used by the column view approach. Therefore, this method is more suitable for those dynamic systems with time delays. On the other hand, the drawback of this incremental reasoning is that every inference process leads to a set of possible faults requiring the observation of all affected fault signals in order to give the final fault isolation result but conversely, the time required is unknown in this method because temporal or dynamics aspects of the fault signals are not considered.

2.6.2 Diagnosing based on the information system.

The information system is a generalization of the binary diagnostic matrix, FSM, allows applying a multi-value evaluation of residuals, carried out individually for each fault signal. Thereby, a generalization of the binary parallel and series inference methods can be distinguished in this approach. Moreover, although this approach is defined in the single fault assumption, it can also deal with multiple fault hypotheses increasing the fault set of fault hypotheses.

2.6.3 Analytical model-based fault isolation techniques.

Traditionally, applying this analytical approach to fault isolation, the relationship between fault signals \( \phi \) and fault set \( f \) is inferred using the analytical model of the system. In general, this relationship is obtained using the residual equations, described in Eq. (2.1) or the analytical redundant relations (ARR’s) built on the grounds of the considered system model.

Although using accurate system models, the most of the FDI fault isolation approaches just consider a binary interface between fault detection and fault isolation modules. These approaches are based, in general, on the binary diagnostic matrix FSM. This architecture can be described in Figure 2.7.

Pursuant to this poor interface, these methods are affected by certain drawbacks, which can lead to a wrong or inefficient fault diagnosis:

- The presence of noise produces chattering using the binary evaluation of the residual.
All faults signal $\phi_{ik}$ affected by a certain fault $f_{jk}$ according to the structure of the matrix $FSM$ should be activated at the same time instant, they should be persistently observed during the whole fault isolation process. Otherwise, a wrong fault diagnosis result could be given.

The relation between fault signals is restricted to a binary one causes a loss of useful information that can add fault distinguishability and accurateness to the fault isolation algorithm avoiding possible wrong fault diagnosis results. The occurrence of a fault causes the apparition of a certain subset of fault signals, such as each of them has dynamic properties, which give important information that its uses can improve the performance of the fault isolation process.

In general, restricting the interface between fault detection and fault isolation to binary one causes the loss of crucial information, which can enhance the whole fault diagnosis process. Significant fault signal properties that should be considered are:

- The sign of the fault signal.
- The sensitivity of the fault signal regarding to each fault hypothesis.
- The order of the fault signal occurrence.
- The time required for a fault signal to be observed once the fault occurs.
Some of the most important non-binary fault isolation methods that try to
tackle some of the drawbacks mentioned previously are:

- **DMP-Diagnostic Model Processor** [89]. The binary fault indication test in
  Eq. (2.11) is replaced by a fuzzy reasoning based on the Kramer function
  preventing the chattering of the test result. Besides, instead of using the
  FSM, a matrix with the same structure is applied but each element is
  related to the steady-state value of the fault residual sensitivity property
  associated with this element [40]. Regarding the fault isolation, this is still
  based on a kind of parallel diagnostic inference method, in spite of its men-
  tioned weaknesses: e.g., DMP-method is blind to unexpected fault signals
  and thus, a wrong fault diagnosis can be obtained

- **Finite State Automatons** [67]. Here every state represents a partial or com-
  plete diagnosis. During the reasoning process, the automaton switches from
  an initial state to partial-diagnosis states within the end to a final diagno-
  sis. Every transition is connected to a condition that depends on time and
  the upcoming fault signals. Thus, time windows can be defined for every
  transition and the time dependent fault pattern can be codified. Although
  this method considers time aspects related to the fault signals, it is still a
  pure binary method being affected by the already mentioned drawbacks of
  the binary approaches.

- **DTS-Dynamic Table of States** [58]. One great advantage of this method, is
  the capability to establish the detection time based on fault signal, which
  describes the maximum time between the occurrence of a certain fault and
  the observation of a certain fault signal. Those times can be derived from
  the monitored system model, or they are known from the system expert
  knowledge. However, as a disadvantage is that this algorithm belongs to
  a parallel diagnosis inference approach and therefore, it suffers from the
  drawbacks mentioned previously.

- **BM- Behavioural Modes** [80]. Here the monitored system is divided in sev-
  eral components. Each of them has a different Behavioural Mode (BM):
fault-free mode and different faulty ones. In contrast to many other methods, the BM method uses explicit models of every behavioural mode of the components. Thus, this approach compares the real system to those system models. An advantage of this approach is that it is possible to isolate multiple faults because of the negative reasoning. Additionally, any fault type can be modelled and many different ones for every component. Besides, the whole dynamic behaviour of a fault is modelled. On the other hand, the size of the fault has to be known in order to get model-based estimations of the faulty system behaviour that exactly correspond to the behaviour of the real system. Thereby, this method requires a big effort in order to model all the behavioural modes of the system components and to identify the fault is affecting the system.

Considering the drawbacks of all the approaches presented previously, this thesis is focused on the interface between fault detection and fault isolation proposed by [48]. This method can be viewed as an extension of the binary diagnostic matrix proposed by [40] in the sense that considers one diagnostic matrix FSM for each fault signal property: binary, sign, fault residual sensitivity, fault signal occurrence order and fault signal occurrence time. Thereby, this method, once the fault signal appears, registers all these properties in order to compare their values with the theoretical information stored in several diagnostic matrices. Moreover, instead of using the binary fault detection test given by Eq. (2.11), this method uses a fuzzy test based on the method proposed by the DMP-method. As illustrated in [48] the interface has an appealing performance when used in dynamical systems. Nonetheless, it is affected by some weaknesses, which are derived from some simplifications:

- Although every fault signal has its own dynamics and consequently, its properties evolve with the time, this method just considers the steady-state value of them.

- This method does not consider interval models. It uses a fixed fault detection threshold instead of an adaptive one.

- This method just considers a simulation model of the monitored system.
Regarding to the fault isolation interface, one of the goals of this thesis will be to tackle the previous weaknesses of this interface using an interval observer model of the system. On the other hand, concerning the fault isolation algorithm, this thesis deals with the fact that a fault affecting the monitored system will generate a unique temporal signature of fault signals (events) where every fault signal evolves according to its own dynamics.

2.6.4 Fault isolation based on interval observers.

The main idea of using an extension of the fault detection and fault isolation interface proposed by [48] is that these two processes cannot be considered separately when a fault diagnosis process is carried out. The reason is that the result of the fault detection module, a temporal sequence of fault signals, has a crucial influence of the whole fault isolation process since:

- When a permanent fault occurs, a fault signal might not be observed permanently due to the fault following effect of the used system model.

- Certain expected fault signals cannot be observed because the size of the fault is not big enough so that the related fault detection test (see Eq. (2.11)) can indicate the fault according to the corresponding adaptive threshold.

- In general, fault isolation requires the observation of a subset of fault signals in order to determine a result. In consequence, the fact that every fault signal has its own dynamics and some of them cannot be observed persistently or simply, they are not observed, can lead to a wrong fault isolation result if the whole fault diagnosis process does not take into account these circumstances.

2.7 Summary.

This chapter has addressed the fundamental bases which are counted for fault detection and isolation in a dynamic process, taking into account different tech-
niques according to the statement of the problem and the characteristics of the system to treat Inherency.

The fault detection systems for non linear process, such as PEMFC, are under a process reinforced being based on these fundamental principles discussed in this Chapter, this Thesis bases its studies in robust fault detection and isolation for complex systems as the PEMFC which is addressed in depth latter in Chapter 3.
Chapter 3

PEM Fuel Cell System Model.

3.1 Introduction.

Thomas Grubb and Leonard Niedrach invented and developed the first Polymer electrolyte membrane (PEM) fuel cell\cite{121}. It was initially developed under a program with the US Navy’s Bureau of Ships and U.S. Army Signal Corps to supply portable power for personnel in the field. In the early 1960’s, General Electric (GE) made a significant breakthrough in fuel cell technology. The power generation systems based on fuel cell technology convert the electrochemical energy released in a chemical reaction directly into electricity. Because fuel cells running on pure hydrogen are considered as a zero-emission power source, also are very quiet, which reduces noise pollution. Fuel cells do not need to be recharged or replaced as is the case for a standard battery since the fuel and oxidant are supplied from outside the cell, which is a great advantage. Some stationary fuel cells use natural gas or hydrocarbons as a hydrogen feedstock, but even those produce far fewer emissions than conventional power plants.

There are several types of fuel cells currently under development for practical applications today. Polymer Electrolyte Membrane Fuel Cells (PEMFC), due to the relatively quick startup times and high power density compared with another type of fuel cells, they are the primary type of fuel of interest for the automotive industry. For further details, about PEMFC applications, see \cite{5; 47}.

PEMFC potential applications include stationary power generation, distributed
power generation, transport and portable applications. Furthermore, PEMFCs are expected to replace the internal combustion engine (ICE) in motor vehicles in the next years. PEMFCs also have many other fuel options, which range from hydrogen to ethanol and biomass-derived materials. These fuels can either be directly feeding into the fuel cell, or sent to a reformer to extract pure hydrogen, which is then directly feeding to the fuel cell.

Some advantages and disadvantages of PEMFC as power source application are listed in Table 3.1.

<table>
<thead>
<tr>
<th>Advantages</th>
<th>Disadvantages</th>
</tr>
</thead>
<tbody>
<tr>
<td>High operating efficiency</td>
<td>FC are currently costly.</td>
</tr>
<tr>
<td>There are many types of fuel sources, and methods of supplying fuel to a fuel cell</td>
<td>Fuel Reformation Technology can be costly and heavy and need power in order to run.</td>
</tr>
<tr>
<td>Fuel cells have a highly scalable design</td>
<td>Performance decreases over time due to catalyst degradation</td>
</tr>
<tr>
<td>Fuel cells produce no pollutants</td>
<td></td>
</tr>
<tr>
<td>Fuel cells are low maintenance</td>
<td></td>
</tr>
<tr>
<td>Fuel cells do not need to be recharged as battery</td>
<td></td>
</tr>
</tbody>
</table>

A common cell of a PEMFC architecture consists of a proton exchange membrane (PEM), catalyst and gas diffusion layers, flow field plates, gaskets and end plates. The actual fuel cell layers are the PEM, gas diffusion and catalyst layers. These layers are sandwiched together, and are called as the membrane electrode assembly (MEA), see Figure 3.1.

3.1.1 Fuel Cell Classification.

Since the invention of fuel cells by Sir William Grove in 1839, several types of fuel cells have been developed in the nineteenth and the twentieth centuries. Nearly as many classifications have appeared in the literature [2; 14; 76] because there is a vast number of variable among the fuel cell systems, such as type of fuel and electrolyte, operating temperature, primary and regeneration systems, and direct or indirect systems.

The common for a fuel cell is based on that to generate electricity from an electrochemical reaction between oxygen and hydrogen, creating water and heat.
as byproducts. Moreover, most fuel cells are able to use oxygen from air, then a fuel cell must be designed to use either pure hydrogen or reformate (hydrogen derived from hydrocarbon or other fuels).

Table 3.2 lists the various types of fuel cells along with electrolyte used, operating temperature, and electrode reactions.

Table 3.2: Classification of fuel cells.

<table>
<thead>
<tr>
<th>Fuel cell type</th>
<th>Electrolyte used</th>
<th>Operating temperature (°C)</th>
<th>Electrode Reactions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Polymer electrolyte</td>
<td>Polymer membrane</td>
<td>60-140</td>
<td>Anode $H_2 \rightarrow 2H^+ + 2e^-$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Cathode $1/2O_2 + 2H^+ + 2e^- \rightarrow H_2O$</td>
</tr>
<tr>
<td>Direct methanol</td>
<td>Polymer Membrane</td>
<td>30-80</td>
<td>Anode $CH_3OH + H_2O + CO_2 + 6H^+ + 6e^-$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Cathode $3/2O_2 + 6H^+ + 6e^- \rightarrow 3H_2O$</td>
</tr>
<tr>
<td>Alkaline</td>
<td>Potassium hydroxide</td>
<td>150-200</td>
<td>Anode $H_2 + 2OH^- \rightarrow H_2O + 2e^-$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Cathode $1/2O_2 + H_2O + 2e^- \rightarrow 2OH^-$</td>
</tr>
<tr>
<td>Phosphoric acid</td>
<td>Phosphoric acid</td>
<td>180-200</td>
<td>Anode $H_2 \rightarrow 2H^+ + 2e^-$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Cathode $1/2O_2 + 2H^+ + 2e^- \rightarrow H_2O$</td>
</tr>
<tr>
<td>Molten carbonate</td>
<td>Lithium/potassium carbonate</td>
<td>650</td>
<td>Anode $H_2 + CO_2^- \rightarrow H_2O + CO + 2e^-$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Cathode $1/2O_2 + CO_2 + 2e^- \rightarrow CO_2^-$</td>
</tr>
<tr>
<td>Solid oxide</td>
<td>Yttria stabilized zirconia</td>
<td>1000</td>
<td>Anode $H_2 + O_2^- \rightarrow H_2O + 2e^-$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Cathode $1/2O_2 + 2e^- \rightarrow O_2^-$</td>
</tr>
</tbody>
</table>

This research is focused on fault diagnosis model-based for a PEMFC feed with pure hydrogen store in tanks, as a case study, also this type of fuel cells is known as Proton Exchange Membrane Fuel Cell because of the type of reaction that happens into the stack.

To perform a fault diagnosis approach base on models, here it is essential to have a fuel cell model available, that why a theoretical non-linear model is developed and described in detail in this Thesis in the following sections. The
PEMFC is not only the most popular type of fuel cell but also this is growing an interest in some research areas such as modeling, control and automation [27; 101].

### 3.1.2 PEM Fuel Cell System operation.

The overall electrochemical reaction in a PEMFC occurs in two stages at two different locations inside the cell; the hydrogen pass from the anode site to membrane, and oxygen pass from the cathode to membrane. The last is pictured out in Figure 3.2.

The first half of the electrochemical reaction occurs at the anode. Here, hydrogen is brought into the cell through the flow channels, and it diffuses through the GDL to the catalyst layer on the electrolyte membrane. Then, the catalyst breaks the hydrogen molecules into hydrogen ions (protons) and electrons. The hydrogen ions pass through the electrolyte membrane which is conductive to protons but not electrons. Since the electrons cannot pass through the membrane, they are carried away from the reaction site through the electrically conductive GDL, into the anode collector plate and then through an electric circuit, this part of reaction is shown in Table 3.3 at the anode section.

<table>
<thead>
<tr>
<th>Section</th>
<th>Reaction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Anode</td>
<td>$2H_2 \rightarrow 4H^+ + 4e^-$</td>
</tr>
<tr>
<td>Cathode</td>
<td>$O_2 + 4H^+ + 4e^- \rightarrow 2H_2O$</td>
</tr>
<tr>
<td>Overall</td>
<td>$2H_2 + O_2 \rightarrow 2H_2O$</td>
</tr>
</tbody>
</table>

The only co-product of the reaction is water, which is far better for the environment than the harmful products of combustion in a gasoline or diesel ICE.

Mathematical modeling provides a key benefit to the fuel cell designing. It does not only allow a proper physical components analysis but also optimization and safety algorithms design. The work proposed in this thesis, presents a realistic PEMFCS model, where well known physical equations are used and some empirical relationships are used as well a model calibration processing, is taking into account to mimic a real PEMFCS dynamic over known operating conditions, therefore this model is considered as a *pseudo-empirical* model class.
Figure 3.2: Schematic of capillary flow of liquid water through the fuel cell GDL.

Mathematical models are also useful when limited hardware is available and when PEMFC designers need to have a better understanding about expected to the system when runs under limit conditions that sometimes, in real PEMFC these conditions infer a dangerous scenario.

The model proposed here, basically can be split in three parts; the stack, auxiliary components and thermal modeling. The stack model considered important features such as the mass transport descriptions of the anode, cathode, and electrolyte. Several mass transport models are commonly used as Simple Fick diffusion models and effective Fick diffusion models typically use experimentally determined effective transport coefficients instead of Fick diffusiveness, and do not account for convective flow contributions. Therefore, many models use Nernst-Planck mass transport expressions that combine Fick’s diffusion with a convective flow. The convective flow is typically calculated from Darcy’s law using different

---

1Fick’s laws of diffusion describe diffusion and can be used to solve for the diffusion coefficient, D. They were derived by Adolf Fick in the year 1855.

2In fluid dynamics and hydrology, Darcy’s law is a phenomenologically derived constitutive equation that describes the flow of a fluid through a porous medium.
formulations of the hydraulic permeability coefficient. Some models use Schlogl’s formulations for convective flow instead of Darcy’s law, which also accounts for electroosmotic flow, and can be used for mass transport inside the PEM.

A very simple method of incorporating electroosmotic flow in the membrane is by applying the drag coefficient model, which assumes a proportion of water and fuel flow to proton flow. Another popular type of mass transport description is the Maxwell-Stefan formulation for multicomponent mixtures, for modelling simplification this type of model is considered in this Thesis. This model type has been used for gas-phase transport, because it is considered that the mixture for liquid-vapor-phase mass transport is presented in the stack.

A very simple method of incorporating electroosmotic flow in the membrane is by applying the drag coefficient model, which assumes a proportion of water and fuel flow to proton flow. Another popular type of mass transport description is the Maxwell-Stefan formulation for multicomponent mixtures, for modelling simplification this type of model is considered in this thesis. This model type has been used for gas-phase transport, because it is considered that the mixture for liquid-vapor-phase mass transport is presented in the stack.

Mass transport models that use effective transport coefficients and drag coefficients usually only yield a good approximation to experimental data under a limited range of operating conditions.

A model is only as accurate as its assumptions allow it to be, the assumptions need to be well understood to known the model’s limitations and accurately interpret its results. The common assumptions used for voltage stack by some author in [81; 99; 108] for fuel cell modeling are:

- Isothermal operation is also assumed. The cell is treated as one which all

---

1Scholgl’s model is the canonical example of a chemical reaction system that exhibits bistability.

2The Maxwell-Stefan Diffusion (or Stefan-Maxwell-Diffusion) is a model for describing diffusion in multicomponent systems. The equations that describe these transport processes have been developed independently and in parallel by James Clerk Maxwell for diluted gases and Josef Stefan for fluids.

3The Maxwell-Stefan Diffusion (or Stefan-Maxwell-Diffusion) is a model for describing diffusion in multicomponent systems. The equations that describe these transport processes have been developed independently and in parallel by James Clerk Maxwell for diluted gases and Josef Stefan for fluids.
the cell has the same temperature along the stack.

- Single phase assumption hold, and water is assumed to exist solely in the vapor phase.
- Ideal gas behaviour of all gas components and mixtures is assumed.
- Mass and energy transport is modeled from a macro-perspective using volume-averaged conservation equations.
- Laminar flow.

Commonly, a PEMFCS needs auxiliary components to keep the cell running at preestablish optimal conditions such as the compressor, humidifier, cooling system, etc. For auxiliary component modelling some thermodynamic and mechanical equations are used. The model details vary not only somewhat from method to the method, but also for the final use of the model. Taking into account on this principle, in this Thesis uses a model suitable to apply a fault in simulation in any model component of the system.

3.2 PEMFC Non-linear Dynamic Model.

The model proposed in this section, is a control-oriented, which its structure is suitable for experimental parameter calibration to a real PEMFC prototype using an optimization processing (lsq-non linear fitting data). In some part is refereed to Nexa® as particular PEMFC as a case study.

The overall FC system could be partitioned into subsystems is shown in Figure 3.4. The auxiliary components are compressor, supply and return manifold, hydrogen supply, cooling system and humidifier. For fuel cell stack: stack voltage, anode and cathode flow, membrane hydration models belong to the fuel cell stack subsystem.

The PEMFC non-linear model consists of teen states, including blower speed, mass transport for hydrogen, oxygen, nitrogen and vapor flow, pressure and mass transport at the supply and return manifold, finally stack temperature change. The estimated outputs are the blower speed, Hydrogen consumption ratio, stack
temperature, stack voltage and supply manifold pressure. The inputs and outputs of each section of the non-linear model are shown in Table 3.5.

3.2.1 Auxiliary Components modelling.

3.2.1.1 Air supply system model.

In general the common PEMFCS use as the air supply system a compressor (for high pressure systems) or blower (for low pressure systems). For air feeding is commonly used a controller to regulate the amount of air required based on the current or voltage needed.

For simplification reasons a static feed-forward or look-up table is commonly used. In this case, the system is considered as a low pressure system, then the input model of the air supply system is the blower motor voltage, $v_{bl}$, and the outlet is the compressor outlet air mass flow ($W_{bl}$).

where $k$ is the parameter set to be estimated using non linear fitting data, their values are reported later in Section ??.
Table 3.4: Input and Output of the non-linear model, see Appendix ??

<table>
<thead>
<tr>
<th>Inputs Parameter</th>
<th>Model Outputs</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_{st}$</td>
<td>$k$</td>
</tr>
<tr>
<td>$v_{bl}$, $(p, T, \phi)_{atm}$</td>
<td>On board controller</td>
</tr>
<tr>
<td>$p_{sm}$, $\omega_{EP}$</td>
<td>(blower)</td>
</tr>
<tr>
<td>$I_{st}$, $m_{H_2}$, $T_{st}$,</td>
<td>$k$</td>
</tr>
<tr>
<td>$m_{an}$, $\eta_{bl}$</td>
<td>Blower dynamic</td>
</tr>
<tr>
<td>$I_{st}$, $m_{O_2}$, $T_{st}$,</td>
<td>$m_{H_2}$, $p_{H_2}$</td>
</tr>
<tr>
<td>$m_{N_2}$, $\eta_{bl}$</td>
<td>$(p, \phi)_{an}$</td>
</tr>
<tr>
<td>$I_{st}$, $\phi_{an}$, $\phi_{ca}$, $T_{st}$, $A_{fc}$, $\phi_{fc}$</td>
<td>Anode $m_{H_2}$, $p_{H_2}$, $(p, \phi)_{an}$</td>
</tr>
<tr>
<td>$I_{st}$, $\phi_{an}$, $\phi_{ca}$, $T_{st}$, $A_{fc}$, $\phi_{fc}$</td>
<td>Cathode $p_{ca}$, $p_{O_2}$, $p_{N_2}$, $m_{N_2}$, $\eta_{bl}$ $\phi_{O_2}$, $m_{H_2}O_2$</td>
</tr>
<tr>
<td>$I_{st}$, $\phi_{an}$, $\phi_{ca}$, $T_{st}$, $A_{fc}$, $\phi_{fc}$</td>
<td>Membrane $W_{vme}$, $\lambda_{me}$</td>
</tr>
<tr>
<td>$(T, \phi)<em>{st}$, $(m, p)</em>{sm}$, $P_{sm}$, $k$</td>
<td>Stack Voltage $(v, i)_{st}$</td>
</tr>
<tr>
<td>$(m, p, W, \phi)<em>{sm}$, $RH M$, $\eta</em>{hm}$</td>
<td>Supply Manifold $(W, T, \phi)_{sm}$</td>
</tr>
<tr>
<td>$(W, T, p, \phi)<em>{hm}$, $\eta</em>{O_2}$</td>
<td>Humidifier (Supply section) $(T, p, W, W_{inj}, \phi)_{hm}$</td>
</tr>
<tr>
<td>$p_{ca}, T_{st}, y_{O_2}$, $p_{sm}, \phi_{ca}$</td>
<td>Inlet flow properties at cathode $(m_{Q_2}, m_{N_2}, W_{vca})_{i}$</td>
</tr>
<tr>
<td>$p_{sm}, (m, p)<em>{rm}$, $T</em>{st}$, $p_{amb}, \phi_{ca}$, $k$</td>
<td>Outlet flow properties at cathode $(m_{Q_2}, m_{N_2}, W_{vca})<em>{o}$, $W</em>{ca}$</td>
</tr>
<tr>
<td>$(m_{O_2}, m_{Q_2}, W_{vca}, y_{O_2})<em>{i}$, $(p, W, \phi)</em>{rm}$, $T_{st}$, $(W, T)<em>{sm}$, $(T, W</em>{inj})_{hm}$</td>
<td>Humidifier (Return section) $(T, W, p, \phi)_{rm}$</td>
</tr>
<tr>
<td>$W_{H_2}$, $H_2$ Feed</td>
<td>$p_{ca}$, $P_{sm}$</td>
</tr>
<tr>
<td>$W_{H_2}$, $\phi_{H_2}$, $p_{an}$</td>
<td>Inlet flow properties at anode $(m_{H_2}, W_{vca})_{i}$</td>
</tr>
<tr>
<td>$T_{st}$, $(\phi, p)_{an}$</td>
<td>Outlet flow properties at anode $(m_{H_2}, W_{vca})_{o}$</td>
</tr>
<tr>
<td>$v_{st}$, $n_{fc}$, $I_{st}$</td>
<td>$k$</td>
</tr>
</tbody>
</table>

Blower dynamic. The air supply system consists of a blower with a supply manifold that provides a mass flow of pressurized air to the fuel cell stack at cathode side using an electro-motor that drives the blower. Typical blower motors used in fuel cell applications are three-phase brushless DC-motors due to their high dynamics and high efficiency. The dynamical behavior of such motors is
very similar to brushed DC-motors.

The air supply system consists of a blower with a supply manifold section that provides a mass flow of pressurized air to the fuel cell stack at the cathode side. Typical blower motors used in fuel cell applications are three-phase brushless DC-motors due to their high dynamics and high efficiency. The dynamical behavior of such motors is very similar to brushed DC-motors.

In this part, the blower model is explained in detail, as is shown in Figure 3.4 the proposed dynamic model is separated into two parts; bower map and blower motor.

![Blower model diagram](image)

Figure 3.4: Block diagram of the blower model with corresponding inputs and outputs.

In [100] the motor driving the blower is simplified to its static behavior because the fuel cell system is a large system with related large time constants. Therefore, the motor time behavior can be assumed as static. For smaller fuel cell systems (like the 1.2 kW system used here) the time constants are smaller and the bandwidth is higher, so the need for a fast controller is larger. By calculating the dynamical behavior of the motor, the accuracy of the model increases and a better controller can be developed. The dynamical behavior of the blower speed is modeled by Eq. (3.1).

\[
\frac{d\omega_{bl}}{dt} = \frac{\tau_{cm} - \tau_f - \tau_{bl}}{J_{bl}}
\]  

(3.1)

where \( J_{bl} \) denotes the rotary inertia of the blower and \( \tau_{cm} \) denotes the blower motor torque, \( \tau_{bl} \) the external load on the motor, and finally \( \tau_f \) denotes the
torque loss in the motor due to damping and friction. The blower motor torque is calculated using the motor current \((i_{cm})\) and motor constant factor \((k_{cm})\) by

\[ \tau_{cm} = k_{cm} \cdot i_{cm} \] (3.2)

The torque loss \((\tau_f)\) is assumed proportional to the blower speed by a constant.

\[ \tau_f = b_{cm} \cdot \omega_{bl} \] (3.3)

The blower motor torque \((\tau_{bl})\) is defined using thermodynamic principals in the blower map section.

**Blower map.** The blower map as depicted in Figure 3.4, the outputs are the air mass flow rate \([\text{kg/sec}]\) from the blower \((W_{bl,o})\), the temperature of the air leaving the blower \((T_{bl})\) and the blower torque, which are determined using static equations.

The inputs to the blower map are the inlet \((p_{amb})\) and outlet \((p_{sm})\) air pressure, the blower speed \((\omega_{bl})\), as well as the air temperature entering the blower \((T_{amb})\). A typical approach to determine the blower map is realized by including measurements in look-up-tables are neither continuous nor differentiable as sometimes discontinuities appear, which can slow down simulations [26].

Therefore, look-up tables are not well suited for usual control-oriented dynamic models. Instead, the measurement data of the blower performance are represented by continuous functions, determined using nonlinear curve fitting methods. As described by [99], the mass flow rate from the blower \((W_{cp})\) can be expressed as a function of the pressure ratio \((p_r)\) and turbine shaft speed \((U_{bl})\).

To calibrate the model with lab data the Jensen and Kristensen method, described in [75], is used. To reflect variations in the inlet condition of the compressor (temperature and pressure), the corrected\(^1\) value of mass flow rate and compressor speed are used in the blower map, see Table 3.5.

The normalized blower flow rate \((\Phi)\) is correlated with a dimensionless head

\(^1\)The corrected variables correspond to the values which would be measured at ambient conditions on a standard day at sea level, i.e. a temperature of 15 °C and a pressure of 101325Pa.
Table 3.5: Corrected parameter correlation.

<table>
<thead>
<tr>
<th>Corrected parameter</th>
<th>Units</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corrected Compressor speed</td>
<td>[rpm]</td>
<td>$\omega_{cr}^{*} = \omega_{bl}/\sqrt{\theta}$</td>
</tr>
<tr>
<td>Mas flow</td>
<td>[kg/s]</td>
<td>$W_{cr} = W_{bl}\sqrt{\theta}/\delta$</td>
</tr>
<tr>
<td>Temperature</td>
<td>[K]</td>
<td>$\theta = T_{bl,i}/288$</td>
</tr>
<tr>
<td>Pressure</td>
<td>[atm]</td>
<td>$\delta = p_{sm}/p_{atm}$</td>
</tr>
</tbody>
</table>

parameter ($\Psi$) and mach number ($M$) then

$$\Phi = \frac{W_{bl}}{4\pi d_{c}^{2}\rho_{air}U_{bl}^{4}}$$

(3.4)

The dimensionless head parameter ($\Psi$) is defined by

$$\Psi = \frac{C_{p}T_{amb}}{1/2U_{bl}^{2}} \left[ \frac{\gamma}{p_{r}} - 1 \right]$$

(3.5)

where $U_{bl}$ is the blower blade tip speed

$$U_{bl} = \frac{\pi}{60} \cdot d_{c} \cdot \omega_{bl}^{\gamma}$$

(3.6)

and $C_{p}$ denotes the specific heat capacity, $\gamma$ the heat capacity ratio of air [75] and the inlet mach number is introduced as

$$M = \frac{U_{bl}}{\sqrt{\gamma R_{a}T_{bl,i}}}$$

(3.7)

The constants are shown in Table 3.6 to solve the Eqs. (3.5) and (3.6) are taken from thermodynamic properties [109] and the manufacturer data sheets [7].

Table 3.6: Compressor parameter

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Units</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_{a}$</td>
<td>[J/(kg·K)]</td>
<td>287</td>
</tr>
<tr>
<td>$\rho_{a}$</td>
<td>[kg/m$^3$]</td>
<td>1.23</td>
</tr>
<tr>
<td>$d_{c}$</td>
<td>[m]</td>
<td>0.07</td>
</tr>
</tbody>
</table>
For a mathematical representation of the compressor flow characteristics, there are two options. I) The flow through the compressor can be expressed as a function of the pressure ratio \( p_r = \frac{p_{sm}}{p_{amb}} \) and turbine shaft speed \( \omega_{bl} \).

II) The pressure ratio across the compressor can be modeled as a function of compressor flow \( W_{bl} \) and turbine speed \( \omega_{bl} \).

A problem could be appeared with these approach, because two options cannot be easily incorporated into a dynamic model that exhibits surge behavior \[88\]. In other to solve this problem some methods are available in literature such as Mueller method, zero slope line and Jensen & Kristensen methods, because in this case it is assumed that a blower is a low pressure system, a linear function is achieved to compute the normalized compressor flow rate \( \phi \), such as:

\[
\Phi = f \{ p_r, M, z \} \tag{3.8}
\]

where \( z \) is the parameter set to be estimated using non linear fitting data, their values are reported later in Section ??.

The temperature of the air leaving the blower is given by

\[
T_{bl,o} = T_{amb} + \Delta T_{bl} \tag{3.9}
\]

where \( \Delta T_{bl} \) is the temperature change across the bower, which is calculated as

\[
\Delta T_{bl} = \frac{\Delta h_{s,bl}}{C_p a \eta_{bl}} \tag{3.10}
\]

where \( \Delta h_{s,bl} \) denotes the isentropic enthalpy change across the blower and is calculated as

\[
\Delta h_{s,bl} = \frac{\gamma a}{\gamma a - 1} R a T_{amb} \left( \frac{p_r^{(\gamma a - 1)/\gamma a}}{p_{amb}^{(\gamma a - 1)/\gamma a}} - 1 \right) \tag{3.11}
\]

According to Eqs (3.9) to (3.11), it follows that the temperature at outlet flow of the blower \( T_{bl,o} \) results to

\[
T_{bl,o} = T_{amb} \left( 1 + \frac{\gamma a}{(\gamma a - 1) C_p a} \frac{R a}{\eta_{bl}} \left( \frac{p_r^{(\gamma a - 1)/\gamma a}}{p_{amb}^{(\gamma a - 1)/\gamma a}} - 1 \right) \right) \tag{3.12}
\]
The relation between the heat capacity ratio ($\gamma_a$) and the gas constant of air ($R_a$) is given by

$$\frac{R_a}{Cp_a} = \frac{\gamma_a - 1}{\gamma_a}$$  \hspace{1cm} (3.13)

Using the last relation in Eq. (3.18), gives finally

$$T_{bl,o} = T_{amb} \left(1 + \frac{p_r^{\frac{\gamma_a - 1}{\gamma_a}} - 1}{\eta_{bl}}\right)$$  \hspace{1cm} (3.14)

Another output of the blower map, which is used as well as an input to the blower motor, is the blower torque ($\tau_{bl}$), which is calculated as

$$\tau_{bl} = \frac{P_{bl}}{\omega_{bl}}$$  \hspace{1cm} (3.15)

where the compressor power ($P_{bl}$) is defined by

$$P_{bl} = Cp_a W_{bl,o} T_{amb} \left(\frac{p_r^{\frac{\gamma_a - 1}{\gamma_a}} - 1}{\eta_{bl}}\right)$$  \hspace{1cm} (3.16)

Finally, it flows from Eq. (3.20) to (3.21), then

$$\tau_{bl} = \frac{Cp_a W_{bl,o} T_{amb}}{\omega_{bl}} \left(\frac{p_r^{\frac{\gamma_a - 1}{\gamma_a}} - 1}{\eta_{bl}}\right)$$  \hspace{1cm} (3.17)

3.2.1.2 Supply and return Manifold.

In some fuel cell systems, which their capacity is not too high (as Nexa© Ballard, 1.2 Kw), generally the humidifier section belongs to the manifold. As it is shown in Figure 3.5, the supply manifold belongs to a humidifier section which is called as the supply section, which cared out the air from the compressor to the cathode site at stack, the return section, which takes the remain air, that did not react into the stack, to exhaust. The last is important to keep into account to compute the mass and energy balance.

The manifold behavior is governed by the mass and energy conservation equa-
tions, linearized nozzle flow equation, and ideal gas law. In a general form the material balance is described as

\[
\frac{dm}{dt} = W_i - W_o
\]  

(3.18)

where \(m\) is the mass of the gas accumulated in the manifold volume, \(W_i\) and \(W_o\) are the inlet and outlet flow rates in the manifold section respectively. If the air temperature is expected to change in the manifold, the pressure dynamic equation from the energy conservation and the ideal gas law is expressed as

\[
\frac{dp}{dt} = \frac{R}{V} (W_i T_i - W_o T_o)
\]  

(3.19)

**Supply Manifold section.** Since the rate of flow is a function of the upstream pressure \((p_{bh})\) and downstream pressure \((p_{sm})\) and if the difference between the manifold and the downstream volume is small, the flow rate can be calculated by a linearized form of the sub-critical nozzle flow. Following the last descriptions
in the supply manifold section, the inlet outlet flow rate is

\[ W_{sm,o} = k_{sm,o} (p_{sm} - p_d) \]  \(\text{(3.20)}\)

where \(k_{sm,o}\) is the linear constant for supply manifold outlet flow. Because a change in the temperature occurs in the supply manifold because of the thermal effect by humidification and using Eqs. (3.19) and (3.20), the change in pressure is expressed as

\[ \frac{dp_{sm}}{dt} = \frac{\gamma R_a}{V_{sm}} (W_{bl} T_{bl} - W_{sm,o} T_{sm}) \]  \(\text{(3.21)}\)

**Return Manifold section.** The temperature of the air leaving the stack \((T_{ext})\) is relatively low compared to stack temperature \((T_{st})\). Therefore, the return manifold behavior is governed by the ideal gas law through isothermal assumption and the mass conservation.

Since the pressure drop between the return manifold and the ambient pressure is relatively small, the equation of return manifold exit flow is close to the supply manifold equation, then

\[ W_{rm,o} = k_{rm,o} (p_{rm} - p_{ca}) \]  \(\text{(3.22)}\)

where \(k_{rm,o}\) is the linear constant for return manifold outlet flow, since the differential in temperature occurs in the supply manifold because of the thermal effect related to humidification and taking into account on the Eqs. (3.18) and (3.19) with \(T_{ca} = T_{st}\), then

\[ \frac{dp_{rm}}{dt} = \frac{\gamma R_a}{V_{rm}} (W_{ca} T_{ca} - W_{rm,o} T_{rm}) \]

\[ = \frac{\gamma R_a T_{st}}{V_{rm}} (W_{ca} - W_{rm,o}) \]  \(\text{(3.23)}\)

where \(v_{rm}\) is the return manifold volume section, which is part of the humidifier volume and \(p_{rm}\) and \(T_{rm}\) are the pressure and temperature at the return manifold respectively.
3.2.1.3 Humidifier model.

PEMFC membranes are extremely sensitive to changes in water content and incoming reactant gas humidity. The humidification of incoming gases is critical to fuel cell performance and reliability. If the membrane becomes too dry, proton transport in the membrane will be reduced and the oxygen reduction reaction at the cathode then a decrease in fuel is presented producing a poor fuel cell performance or failure.

For simplification reasons, the humidifier model proposed here is considered as a simple humidity exchanger to transfer water vapor from the cathode exhaust stream into the cathode reactant air stream. Where the humidifier is a static heat-and-moisture exchanger core comprises an enclosed shell structure having a plurality of heat-and-moisture transfer members statically disposed there in superposed and spaced apart relationship defining between each two adjacent member a static flow path. The conceptual idea is pictured out in Figure 3.6.

Humidifier Mass Balance. Air flow from the blower is humidified before entering the stack by injecting water into the humidifier from return manifold air flow
Since the maximum amount of water vapor that the air can hold depends on the temperature and pressure of the air. Thus the relative humidity (\( \phi \)) is used, which is defined as the ratio of the mole fraction of the water vapor in the mixture to the mole fraction of vapor in a saturated mixture at the same temperature and pressure. If it is assumed that \( T_{hm} \approx T_{st} \) into the humidifier in the steady state, then it is possible to determine the vapor pressure at the humidifier. A full humidified is achieved when \( p_v = p_{sat} \), where \( p_v \) is the partial pressure of the water and \( p_{sat} \) is the saturated vapor pressure of the water.

\[
p_{v,sm} = \phi_{sm} \cdot p_{sat_{atm}}
\]  

(3.24)

If the partial pressure is then calculated as

\[
p_{a,sm} = p_{sm} - p_{v,sm}
\]

(3.25)

the humidity ratio, can be calculated as

\[
\bar{\omega}_{sm} = \frac{M_v \cdot p_{v,sm}}{M_a \cdot p_{a,sm}}
\]

(3.26)

and finally, the mass flow rate of dry air is

\[
W_{a,sm} = \frac{W_{sm}}{1 + \bar{\omega}_{sm}}
\]

(3.27)

then, the mass flow vapor is computed as

\[
W_{v,sm} = W_{sm} - W_{a,sm}
\]

(3.28)

To know the desired stack inlet humidity (\( \phi_{des} \)), it's assumed to be constant and not controlled. Some work as [103] and [27] assumes that a properly humidified process is done, then here is assumed that \( \phi_{des} \approx 1 \). With the last assumption, we can know the required amount of water injected to the flow rate as.

\[
W_{v,inj} = \frac{M_v \cdot \phi_{des} \cdot p_{sat_{st}} \cdot W_{a,hm}}{M_a \cdot p_{a,hm}} - W_{v,hm}
\]

(3.29)
The inlet flow rate and pressure to the cathode are calculated as

\[ W_{ca,i} = W_{sm} + W_{v,inj} \]  
(3.30)

and

\[ p_{ca,i} = p_{a,sm} + \phi_{des} \cdot p_{sat,sl} \]  
(3.31)

**Humidifier Head Exchange Balance.** Assuming as a heat exchanger in the humidifier, then the energy balance is described in Eq. (3.32) where \( \varepsilon_{\text{heat}} \approx 0.6 \) is the value for sensible heat transfer factor [107].

\[ T_{sl} - T_{ex} = \varepsilon_{\text{heat}} \cdot (T_{hm} - T_{sm}) = \varepsilon_{\text{heat}} \cdot \Delta T_{e} \]  
(3.32)

Using Eq. (3.32), the energy balance yields to

\[ (T_{sl} - T_{ex}) = \frac{\dot{m}_{air,ca} \cdot C_{p,air} \cdot (T_{air,ca} - T_{hm})}{(\dot{m}_{O2,o} \cdot C_{p,O2} + \dot{m}_{N2,o} \cdot C_{p,N2} + \dot{m}_{H2O(v)} \cdot C_{p,H2O(v)})} \]  
(3.33)

Solving to stack exhaust temperature taking into account the thermodynamic parameter shown in Table 3.7, then

<table>
<thead>
<tr>
<th>Table 3.7: Specific heat value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter</td>
</tr>
<tr>
<td>( C_{p,O2} )</td>
</tr>
<tr>
<td>( C_{p,N2} )</td>
</tr>
<tr>
<td>( C_{p,H2O} )</td>
</tr>
<tr>
<td>( C_{p,air} )</td>
</tr>
</tbody>
</table>

**3.2.1.4 Cooling System Model.**

To maintain the desired range of temperature inside the cells, the heat generated as a by-product in the electrochemical reactions must be taken away from the stack, cooling with a coolant flowing between the cells. Coolant may be deionized water, antifreeze coolant, or air. Cooling may be arranged between
each cell, between the pair of cells, for small stack, which is the case considered in this Thesis. The common cooling flow used is ambient air flow.

**Active cooling area.** From the heat removal the point of view, a fuel cell stack may be considered as a heat exchanger with internal heat generation, see Figure 3.7(a).

![Cooling system diagram](image1)

![Cooling channel diagram](image2)

**(a) Cooling system diagram**

**(b) Cooling channel diagram**

Figure 3.7: Cooling ambient air flow diagram.

With the last assumption, the amount of heat removed by the cooling air flow is refereed by cooling contact area, considered a square configuration for the stack and vertical cooling channel configuration, see Figure. 3.7(b).

The cooling channel perimeter is then calculated as

\[ P = 2 \cdot (l + w) \]  
(3.34)

where \( l \) and \( w \) are the length and width respectively of a cooling channel. Then the area of a single channel cross section \( (A_c) \) is

\[ A_c = l \cdot w \]  
(3.35)

Hydraulic diameter of rectangular ducts \( (D_H) \) can be calculated as a ratio of a
single channel cross section area divided by its perimeter

\[ D_H = \frac{4 \cdot A_c}{P}; \]  

(3.36)

The amount of air cooling flow \( W_{cool} \) that cross through a section area in [Kg/sec] is expressed in Eq. (3.37). The section inside the stack is calculated by the volume of the total cell channel and cells \( (n_{ch} \times n_{fc}) \), where the average velocity of the flow inside in a channel \( (U_{cool}) \) is estimated as

\[ W_{cool} = U_{cool} \cdot A_c \cdot \rho_{air} \cdot n_{fc} \cdot n_{ch} \]  

(3.37)

where

\[ A_c \]  
\text{Active cooling area. [m}^2\text{]} \]

\[ \rho_{air} \]  
\text{Density of the air at } T_{amb}. [\text{kg/m}^3]\]

\[ n_{fc} \]  
\text{Cell number. [\text{]} \]

\[ n_{ch} \]  
\text{Channel number. [\text{]} \]

\subsection*{3.2.2 Fuel cell stack modelling.}

The fuel cell stack is the core component of any fuel cell system. The electrochemical reaction described in Table 3.3, occurs inside the stack, where the reactants (hydrogen and oxygen) are fed to the anode and cathode side of the fuel cell stack respectively.

At the catalyst, the hydrogen splits in protons \( (H^+) \), which can travel through the polymer membrane, and in electrons \( (e^-) \), which have to travel through the metal electrodes. At the cathode side, oxygen \( (O_2) \) reacts under the influence of the catalyst with the protons and electrons to form water \( (H_2O) \). The electrons traveling in an external circuit are doing \textit{electrical work}. The model proposed in this Thesis, takes into an account the material mass balance in the anode, membrane and cathode side. Figure 3.8 depicts the flow of the raw, produced and purge materials in a cell.

A fuel cell stack model usually consists of four sub-models: \textit{the stack voltage model}, \textit{the anode mass flow model}, \textit{the cathode mass flow model}, and \textit{the membrane hydration model}. 

55
3.2.2.1 Anode mass flow model.

The hydrogen flow supplied to fuel cell is assumed feed instantaneously regards to the high pressure vessel where the hydrogen is stored. The hydrogen feeding flow is related to change in pressure of the stack ($P_{ca}$ and $P_{sm}$). Assuming that a homogenous temperature at the stack and the temperature at the anode as the stack temperature ($T_{an} = T_{st}$), finally that all hydrogen supplied is consumed by the chemical reaction in the cell, then $m_{H_2,o} = 0$.

The hydrogen partial pressure ($P_{H_2}$) and anode flow humidity are determined by material balance between hydrogen and water in both inlet and outlet side of the anode, in Table 3.8 we can see the thermodynamic correlations related to anode mass flow for each part of the anode.

3.2.2.2 Cathode mass flow model.

In the cathode mass flow model is computed the air flow behavior that occurs into the cathode side. This model uses the mass conservation principle and thermodynamic properties of mass mixtures. This section of the model uses the air flow from the humidifier in the return manifold section. Some assumptions are taken into account, such as; all gases behave like an ideal gas, the temperature of the fuel cell stack is constant along all stack, but it can suffer change as stack current change as well and finally the gas composition, humidity and composition are assumed to be the same as the variables inside the cathode flow channel. The inlet section of the cathode takes the thermodynamic conditions from the return manifold, using the principal of mass conservation. For the outlet flow section

![Figure 3.8: Stack mass flow.](image)
Table 3.8: Anode flow properties

<table>
<thead>
<tr>
<th>Section</th>
<th>Flowproperty</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inlet</td>
<td>Vapor partial pressure</td>
<td>$p_{v_{an,i}} = p_{sat_{T=st}} \cdot \phi_{an}$</td>
</tr>
<tr>
<td>Inlet</td>
<td>Humidity ratio</td>
<td>$\varpi_{O_{an,i}} = \frac{M_v}{M_{H_2}} \cdot \frac{p_{an,i} - p_{v_{an,i}}}{W_{an,i}}$</td>
</tr>
<tr>
<td>Inlet</td>
<td>Hydrogen mass flow</td>
<td>$W_{H_2,i} = \frac{W_{an,i}}{1 + \varpi_{O_{an,i}}}$</td>
</tr>
<tr>
<td>Inlet</td>
<td>Vapor mass flow</td>
<td>$W_{v_{an,i}} = W_{an,i} - W_{H_2,i}$</td>
</tr>
<tr>
<td>Outlet</td>
<td>Vapor partial pressure</td>
<td>$p_{v_{an,o}} = p_{sat_{T=st}} \cdot \phi_{an}$</td>
</tr>
<tr>
<td>Outlet</td>
<td>Hydrogen partial pressure</td>
<td>$p_{H_2_{an,o}} = p_{an} - p_{v_{an,o}}$</td>
</tr>
<tr>
<td>Outlet</td>
<td>Humidity ratio</td>
<td>$\varpi_{O_{an,o}} = \frac{M_v}{M_{H_2}} \cdot \frac{p_{v_{an,o}} - p_{v_{an,o}}}{W_{H_2,o}}$</td>
</tr>
<tr>
<td>Outlet</td>
<td>Hydrogen mass flow</td>
<td>$W_{H_2,o} = 0$</td>
</tr>
<tr>
<td>Outlet</td>
<td>Vapor mass flow</td>
<td>$W_{v_{an,o}} = W_{an,o} - W_{H_2,o}$</td>
</tr>
<tr>
<td>Consumption</td>
<td>Hydrogen reacted</td>
<td>$m_{H_2r} = \frac{\eta_f}{\mathcal{E}} \cdot M_{H_2} \cdot I_{st}$</td>
</tr>
</tbody>
</table>

of the cathode with the knowledge of the total flow rate at the cathode using the consideration that the flow channel keeps constant across the cathode it is possible to estimate the outlet conditions of the gases. In Table 3.9 are shown the equation which are related to each cathode section.

### 3.2.2.3 Membrane.

The main function of membrane is to conduct hydrogen ions (protons) but not electrons like a short circuit effect in the fuel cell. The membrane must also not allow either gas to pass to the other side of the cell, a problem known as gas crossover. Finally, the membrane must be resistant enough to the reducing environment at the cathode as well as the harsh oxidative environment at the anode.

**Electro-osmotic drag coefficient.** The water diffusivity ($\lambda_D$), the proton conductivity in the membrane ($\mathcal{D}_w$), and the electro-osmotic drag coefficient for water ($\lambda_j$) are all functions of the water content of the membrane ($\lambda_m$) which was
expressed as a function of the water activity, by \([110]\) as follows

\[
\lambda_j = \begin{cases} 
14 + 1.4 \cdot (\phi_j - 1) & \phi_j \geq 1 \\
0.043 + 17.81 \cdot \phi_j - 39.85 \cdot \phi_j^2 + 36 \cdot \phi_j^3 & 1 > \phi_j
\end{cases} \tag{3.38}
\]

where \(j\) is related to the cathode \((ca)\), anode \((an)\) and membrane \((m)\). For membrane \(\phi_m = \frac{\phi_{ca} + \phi_{an}}{2}\). Then the electro-osmotic drag coefficient can be ex-
pressed as a polynomial function [110] as

\[ n_d = 0.0029 \lambda_m^2 + 0.05 \lambda_m - 3.4E - 19 \] (3.39)

**Water concentration.** The equivalent water concentration in the membrane is defined in Eq. (3.40), where \( j \) is related to cathode (ca) and anode (an)

\[ c_j = \frac{\delta_{\text{mem}}}{\rho_{\text{mem}}} \cdot \lambda_j \] (3.40)

**Water diffusion coefficient.** The proton conductivity in the membrane and the electroosmotic drag coefficient of water implemented in the present model follow the empirical correlations described in [29].

\[ D_w = D_{\lambda} \cdot \exp \left( 2416 \left( \frac{1}{303} - \frac{1}{T_{st}} \right) \right) \] (3.41)

where

\[
D_{\lambda} = \begin{cases} 
1.25E - 6 & \lambda_m \geq 4.5 \\
(3 - 1.67 \cdot (\lambda_m - 3)) & \lambda_m \geq 3 \\
(1 + 2 \cdot (\lambda_m - 2)) & 3 > \lambda_m \\
1E - 6 & 2 > \lambda_m \\
0 & 1 > \lambda_m
\end{cases}
\] (3.42)

**Membrane water flow.** The membrane water hydration represents the water content in the membrane and the rate of water mass flow that crosses the membrane. If it is assumed that there is a uniform layer of water in overall membrane layer, then membrane water content and the rate of water mass flow in the membrane is a function of current [29], where the water transport across the membrane is related to the *electro-osmotic* and *back diffusion of water* phenomena.

- **Electro-osmotic:** The electro-osmotic drag coefficient \( (n_d) \) is defined as the number of water molecules carried by each proton, then *net water transfer* coefficient per proton is expressed as

\[ N_{v,os} = n_d \cdot \frac{i_{st}}{F} \] (3.43)
where
\[ i_{st} = \frac{I_{st}}{A_{fc}} \]  \hspace{1cm} (3.44)

- Back-diffusion of water from cathode to anode: This is a gradient of water concentration across the membrane, that is caused by the difference in humidity in anode and cathode flow, this phenomena is expressed as
\[ N_{v,dis} = D_w \frac{dc_v}{dy} \]  \hspace{1cm} (3.45)

Taking into account a constant distribution of water layer in the membrane and the phenomena described by Eqs. (3.43), (3.45). The total stack mass flow rate across the membrane, can be calculated from
\[ W_{v_{mem}} = M_v \cdot A_{fc} \cdot n_{fc} \cdot n_d \cdot \left( \frac{i_{st}}{F} - D_w \cdot \frac{(c_{vc} - c_{va})}{n_d \cdot l_m} \right) \]  \hspace{1cm} (3.46)

where \( A_{fc} \) is the active area [m²].

### 3.2.2.4 Stack voltage.

A fuel cell stack is a series connection of various single cells to increase the output voltage. There is no limitation in the number of single cells placed in series. However, it is also possible to stack single cells in parallel, to increase the output current. This Thesis is focused on a series cell array, then the total stack voltage is given by the single voltage times the number of cells of the stack, then the stack total voltage is given by
\[ v_{st} = n_{fc} \cdot v_{cell} \]  \hspace{1cm} (3.47)

where \( n_{fc} \) denotes the number of cells and \( v_{fc} \) represents the single cell voltage, which is calculated as
\[ v_{cell} = E - v_{loss} \]  \hspace{1cm} (3.48)

where \( E \) denotes the open circuit voltage and \( v_{loss} \) the total voltage losses, which will be discussed in the next section. The voltage \( E \) can be calculated according
to the Nernst equation [16; 63] by

\[
E = -\frac{\Delta g_f}{2 \cdot F} = E^\circ + \frac{R \cdot T}{2 \cdot F} \ln \left( \frac{p_{H_2} \cdot p_{O_2}^{1/2}}{p_{H_2O}} \right) \tag{3.49}
\]

where

- \( E^\circ \) Reversible cell voltage.
- \( R \) Gas constant. 8.3144 \([\text{J/mol K}^{-1}]\)
- \( F \) Faraday’s constant. 96,495 \([\text{C/mol}]\)
- \( T \) Temperature. \([\text{K}]\)
- \( p_i \) Partial pressure of \( i \)-element. \([\text{bar}]\)

and (\( \Delta S \) and \( \Delta H \)) are the differences in entropy and enthalpy respectively. If is considered the therm \( \Delta H/2 \cdot F \) at standard state (25°C and 1 atm) according to [4], the value of \( E^\circ \) is 1.229, then

\[
E^\circ = 1.229 + (T_{st} - T_0) \cdot \left( \frac{\Delta S}{2 \cdot F} \right) \tag{3.50}
\]

where

- \( \Delta S \) Entropy change. \([\text{J/K}]\)
- \( T_{st} \) Stack temperature. \([\text{K}]\)
- \( T_0 \) Standard state temperature. 298.15 \([\text{K}]\)
- \( p_i \) Partial pressure of \( i \)-element. \([\text{bar}]\)

Using Eqs. 3.49 and 3.50 according to [99] it is possible to express the open circuit voltage as

\[
E = 1.229 - 8.5E - 4 \cdot (T_{st} - T_0) + 4.308E - 5 \cdot \left( \ln \left( \frac{p_{H_2}}{1.01325} \right) \right) + 0.5 \cdot \ln \left( \frac{p_{O_2}}{1.01325} \right) \tag{3.51}
\]

**Losses.** Irreversible losses that occur under operating conditions make the voltage of a fuel cell stack less than open circuit voltage (OCV). The main voltage
drops are, the activation voltage drop \( v_{act} \), the ohmic voltage drop \( v_{ohm} \), and the concentration voltage drop \( v_{con} \). Voltage drops in a fuel cell can also occur either for cathode flooding, when the water produced by the chemical reaction is not effectively removed or for membrane dehydration. Mathematically, the total voltage loss is represented by

\[
v_{loss} = v_{act} - v_{oh} - v_{con}
\]  \hspace{1cm} (3.52)

Figure 3.9 shows a typical representation of a \( I_{st} - v_{st} \) curve for a single cell.

![Figure 3.9: Example of a polarization curve showing the different loss regions.](image)

**Activation.** This loss is caused by slow kinetics of the electrochemical reaction at the electrodes. The total activation voltage loss was parameterized according to Tafel [64]. This loses is considered as an initial dramatic voltage loss in low temperature fuel cells.

\[
v_{act} = a \cdot \ln \left( \frac{i}{i_0} \right)
\]  \hspace{1cm} (3.53)

where
In [99], the Tafel is approximate by a exponential function as

\[
v_{\text{act}} = v_0 (p_{O_2}, p_{H_2}, T_{st}, h_i) \cdot (1 - \exp (h_i \cdot i_{st})) \tag{3.54}
\]

where

- \(v_0\): Voltage drop at zero current density. [Volt]
- \(v_a\): Voltage activation slope. [Volts]
- \(h_i\): Parameter \(i = 1...h\).

Here, the parameters \(v_0\), \(v_a\), and \(c_i\) keep a linear relationship to the temperature and partial pressure of each component in the cathode and parameter \(h_i\) to be identified, these parameters are introduced in [99].

**Concentration.** This loss occurs due to mass transport/concentration problems, which are those directly related to pressure [104]. Using Eq.(3.49) the voltage drop can be determined if \(P_{O_2}\) and \(P_{H_2O}\) are constant, and \(p_{H_2}\) changes as \(\Delta p = p_2 - p_1\), then solving to the hydrogen term Eq. (3.49) gives

\[
\Delta v = \frac{R \cdot T}{2 \cdot F} \ln (\Delta p) \tag{3.55}
\]

With the last equation, it is possible to see the relationship between system temperature and pressure over voltage changes. To adapt this equation, we assume a limiting current density \(i_m\) at which the fuel is used up at a rate equal to maximum supply speed [64]. The maximum current density acts as a current ceiling, since there will be no more fuel to advance the density, then it is to say the pressure of excess hydrogen will be zero.

\[
v_{\text{con}} = i_{st} \cdot \left( \frac{c_2 \cdot i_{st}}{i_m} \right)^{a_1} \tag{3.56}
\]
In \cite{99} proposes an equation to compute $c_2$, which switches between two functions depending on the oxygen partial pressure and the water saturation pressure. However, while here a low-pressure fuel cell system is considered, the variable $c_2$ is described as

$$c_2 = (a_2 \cdot T_{st} + a_3) \cdot \left( \frac{p_{o2}}{a_4} + p_{sat,T_{st}} \right) + a_5 \cdot T_{st} + a_6$$  \hspace{1cm} (3.57)

**Ohmic.** The ohmic loss in a fuel cell is attributed to the ionic, electronic, and contact resistances of components in the fuel cell \cite{37}. The ohmic the membrane conductivity controlled voltage loss as well as the contact and bulk electrical resistance of the conductive materials. Because of the ohmic loss is proportional to the current density, then

$$v_{ohm} = i_{st} \cdot R_i$$

$$= i_{st} \cdot \frac{t_m}{\sigma_m}$$  \hspace{1cm} (3.58)

where

- $t_m$ membrane thickness. [m$^2$]
- $\sigma_m$ membrane conductivity factor. [Ω$^{-1}$·m$^{-2}$]

The membrane conductivity $\sigma_m$ can be calculated as

$$\sigma_m = b_1 \cdot \exp \left( b_2 \cdot \left( \frac{1}{303} - \frac{1}{T_{st}} \right) \right)$$  \hspace{1cm} (3.59)

where

$$b_1 = (b_{11} \cdot t_m + b_{12})$$

$$b_2 = b_{21}$$

**3.2.3 Thermal modeling.**

The thermal model can be obtained by an energy balance across the stack, defining the energy produced in the chemical reaction ($Q_{gen}$), the electric power sup-
plied \((P_{\text{elec}})\), and the total heat removed \((Q_{\text{dis}})\) by natural convective and radial, forced or active heat removed by the cooling system.

The rate of heat transferred by conduction in the \(x\) direction through a finite cross-sectional area is, according to the Fourier Law of Conduction, proportional to the temperature difference, in order to simplify the thermal model, its distribution is considered as uniform with a unique temperature profile along the stack volume. The heat removed by the cooling system is defined by \(Q_{\text{dis}}\), then the total energy balance is expressed as:

\[
\frac{dT_{\text{st}}}{dT} = \frac{1}{m_{\text{st}} \cdot C_{\text{st}}} (Q_{\text{gen}} - P_{\text{elec}} - Q_{\text{dis}}) \tag{3.60}
\]

where \(m_{\text{st}}\) and \(C_{\text{st}}\) are the mass of the fuel cell stack and the heat capacity respectively. These parameter values are expressed as a polynomial expression based on stack temperature as

\[
m_{\text{st}} = u_1 \tag{3.61}
\]

\[
C_{\text{st}} = u_2 \cdot T_{\text{st}}^2 + u_3 \cdot T_{\text{st}} + u_4 \tag{3.62}
\]

where the constants \(u_i\) for \(i = 1, \ldots, 4\) are parameters determined through curve fitting data. The identification of those parameters is shown later in section ??.

### 3.2.3.1 Heat produced by reaction \((Q_{\text{gen}})\).

In general, energy of fuel (higher heating value) is converted into either electricity or heat:

\[
\text{Energy of fuel reacted} = \text{Heat generated} + \text{Electricity generated}
\]

or

\[
\frac{1}{2F} H_{HHV} n_{\text{cell}} = Q_{\text{reac}} + I_{\text{st}} V_{\text{st}} \tag{3.63}
\]

Heat generated in reaction, is then

\[
Q_{\text{reac}} = (k_{t,1} \cdot n_{\text{cell}} - V_{\text{st}}) I_{\text{st}} \tag{3.64}
\]
If the water leaves the stack as liquid at 25 °C, which could be happened if the inlet is fully saturated at the stack operating temperature, the upper value of $k_{t,1}$ is given as 1.482 otherwise, If all the product water leaves the stack as vapor, then the lower value of $k_{t,1}$ is 1.254 [64].

### 3.2.3.2 Electrical power ($P_{elec}$).

The electric power is the amount of electrical energy, which has been converted to power, it is considered as energy removed from the system, and it is expressed as

$$P_{elec} = I_{st} \cdot v_{st}$$

### 3.2.3.3 Heat Removal ($Q_{dis}$).

Fuel cell is a device where a chemical reaction is produced, as coproduct of this reaction is not only water but also heat. In order to keep the desired range of temperature inside the cell, the heat generated as the electrochemical reaction must be removed from the stack. Based on the size of the physical application, the cooling system is designed, for stack over 50 kW, is used coolant such as deionized water, antifreeze coolant but in the case of under a 5 kW, the air is commonly used as coolant flow. In this Thesis, we focused on cooling system by air. The terms related to removed heat are: the active heat removed by cooling system and natural heat removed by convective and the radiation phenomena.

$$Q_{dis} = Q_{act} + Q_{nat}$$

**Natural convection.** The natural convection ($Q_{nat}$) phenomena is presented when the heat is transferred by the circulation of fluids due to buoyancy from the density changes induced by heating itself. Maximum heat that the stack may lose through natural convection and radiation to the surrounding is defined by

$$Q_{nat} = \frac{T_{st} - T_{amb}}{R_{th}}$$
where

\[ T_{st} \] Stack surface temperature. [K]
\[ T_{amb} \] Ambient temperature. [K]
\[ R_{th} \] Thermal resistance. [ ]

The thermal resistance is defined as

\[ R_{th} = \frac{1}{1/R_c + 1/R_R} \] (3.68)

where the convective thermal resistance \((R_c)\) and the radiative thermal resistance \((R_R)\) are

\[ R_c = \frac{1}{h \cdot A_s} \] (3.69)
\[ R_R = \frac{1}{\sigma \cdot F_k \cdot A_s \cdot (T_{st} + T_{amb}) \cdot (T_{st}^2 + T_{amb}^2)} \] (3.70)

where

\[ \sigma \] Stefan-Boltzman constant. 5.67E-8 [Wm\(^2\) K]
\[ F_k \] Shape factor. [ ]
\[ A_s \] Stack exposed surface area. [m\(^2\)]

The heat transfer coefficient \((h)\) in Eq. (3.69) is a function of the Nusselt number \((Nu_L)\), as

\[ h = \frac{k}{L} Nu_L \] (3.71)

For vertical plates and natural convection, the Nusselt number is some empirical function of Prandtl \((Pr)\) and Rayleigh \((Ra_L)\) numbers, see[109], such as

\[ Nu_L = \left\{ 0.825 + \frac{0.387 \cdot Ra_L^{1/6}}{\left[ 1 + \frac{0.59/16}{Pr} \right]^{9/16}} \right\}^2 \] (3.72)
where

\[
Ra_L = \frac{g \cdot \beta \cdot (T_{st} - T_{amb}) \cdot L^3}{\alpha^2 \cdot Pr}
\]  

and

\[
g \quad \text{Gravity acceleration.} \quad 9.81 \, [\text{ms}^{-2}]
\]

\[
\beta \quad \text{Thermal expansion coefficient.} \quad (\beta = 1/T_{amb}) \, [\text{K}^{-1}]
\]

\[
l \quad \text{Length of travel of the fluid in the boundary layer.} \quad [\text{m}]
\]

\[
\nu \quad \text{Kinematic viscosity.} \quad [\text{m}^{-2}\text{s}^{-1}]
\]

\[
\alpha \quad \text{Thermal diffusivity.} \quad [\text{m}^{-2}\text{s}^{-1}]
\]

**Active Heat Removal.** In terms of heat removal, a fuel cell stack it is considered as a heat exchanger with internal heat generation. In order to simplify the model, the walls of the fuel cell cooling channels are considered as constant temperature and have the constant heat flux. The heat to be removed by active \((Q_{act})\) depends basically by the active cooling area \((A_c)\), the conditions of the cooling flow, \(f = (W_{cool}, T_{amb})\) and the average velocity of the air \((U_{cool})\) in the cooling area.

The modeled cooling system, describe in section 3.2.1.4, is based on rectangular channels running vertically through the bipolar plate (see Figure 3.7(b)), this leaves an array of symmetrical cooling channels. Since the channel divider material is relatively thick, then its span is relatively small, and the material that most bi-polar plates are made from high electrical and thermal conductivity, it is assumed that the channel walls are at a uniform temperature equal to the stack temperature \((T_{st})\). The heat transferred to the cooling fluid is represented as

\[
\frac{dQ_{act}}{dA_c} = h_f (T_{st} - T_c)
\]  

(3.74)

where, \(T_{st}\) and \(T_c\) are the fuel stack and cooling fluid temperature respectively, and \(A_c\) is called as the heat exchange or surface area of the cooling channels. Considering the cooling area as a heat exchanger in Ec. (3.74) can be expressed as
\[ Q_{act} = UA_c LMTD \]
\[ = (h_f \cdot P \cdot B \cdot LMTD) \cdot n_{fc} \cdot n_{ch} \]

where

- \( h_f \): Local heat transfer coefficient. \([\text{W}(\text{m}^2\text{K})^{-1}]\)
- \( U \): Overall heat transfer coefficient. \([\text{W}(\text{m}^2\text{K})^{-1}]\)
- \( B \): Base of cooling area. \([\text{m}]\)
- \( P \): Perimeter of cooling area. \([\text{m}]\)
- \( n_{fc} \): Number of cells. \([\text{m}]\)
- \( n_{ch} \): Number of channel per cell. \([\text{m}]\)
- \( LMTD \): Logarithmic mean temperature difference. \([\text{K}]\)

The Heat transfer coefficient is calculated as

\[ h_f = \frac{Nu \cdot k_{air}}{D_H} \]

where

- \( Nu \): Nusselt Number for uniform wall temperature
- \( k_{air} \): Thermal conductivity of the air. \([\text{W} \ (\text{m}^2\text{K})^{-1}]\)

The LMTD is a logarithmic average of the temperature difference between the hot and cold streams at each end of the exchanger. The larger the LMTD, the more heat is transferred. The use of the LMTD arises straightforward from the analysis of a heat exchanger with constant flow rate and fluid thermal properties.

\[ LMTD = \frac{(T_{st} - T_{out}) - (T_{st} - T_{amb})}{\ln \left( \frac{T_{st} - T_{out}}{T_{st} - T_{amb}} \right)}; \]

Using the fin efficiency and mechanisms of heat exchange formulation \[94\], fin is used to enhance convective heat transfer in a wide range of engineering applications, and offers a practical means for achieving a large total heat transfer surface area without the use of an excessive amount of primary surface area. With
the last principle, it is possible to estimate the outlet temperature at the outlet
surface.

\[ T_{out} = T_{st} - (T_{st} - T_{amb}) \cdot \exp \left( \frac{-2 \cdot n_0 \cdot h_f \cdot A_t}{W_{cool} \cdot C_{p_{air}}} \right) \quad (3.79) \]

where

- \( n_0 \) Overall fin surface efficiency. [%]
- \( A_t \) Total area including base area na total finned surface. [m²]
- \( A_f \) Surface area of a single fin. [m²]
- \( W_{cool} \) Overall heat transfer coefficient. [kg/seg]
- \( C_{p_{air}} \) Heat capacity of the air. [J/kg]

The overall fin effectiveness depends on the fin density\(^1\) as well as the effectiveness of the individual fins, see Figure 3.10.

\[ n_0 = 1 - \frac{n_{ch} \cdot A_f}{A_t} \cdot (1 - n_f) \quad (3.80) \]

Using the diagram shown in Figure 3.10, the surface area of the fin is calcu-

\(^1\)number of fins per unit length (\( n_{ch} \)).
lated by

\[ A_f = 2 \cdot w \cdot l + w \cdot t_i \]
\[ \approx 2 \cdot w \cdot l \]  
(3.81)

Defining the fin efficiency as

\[ n_f = \frac{\text{tanh} \left( \frac{m \cdot l}{2} \right)}{m \cdot \frac{l}{2}} \]  
(3.82)

where the ratio of temperature change \( (m) \) is a function of the thermal conductivity factor \( (k_o) \) of the cell material, then \( m \) is expressed as

\[ m = \left( \frac{h_f \cdot P_f}{k_o \cdot A_{cf}} \right)^{1/2} \]  
(3.83)

and the perimeter \( (P_f) \) and area \( (A_{cf}) \) of a fin plate are estimated as

\[ P_f = 2 \cdot (t_i + B) \]  
(3.84)
\[ A_{cf} = t_i \cdot B \]  
(3.85)

where \( t_i \) and \( B \) are the thickness and base of the fin. The total convective cooling area is denoted as

\[ A_t = n_{ch} \cdot B \cdot l + n_{ch} \cdot A_f \]
\[ = n_{ch} \cdot l \cdot (B + w) \]  
(3.86)

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### 3.3 Summary

This Chapter has addressed the issue of modeling a power generation sources system based on PEMFCS. The proposed model is intended to be used in a fault
detection system, which is based on models. This model has the characteristic that can be calibrated by identification of parameters to mimic a conventional stack, which is later discussed in Chapter ???. The model has ten states \((m_{O_2}, m_{H_2}, m_{N_2}, \omega_{bl}, p_{sm}, m_{sm}, m_{w;an}, p_{rm}, m_{rm}, T_{st})\) and one input \((I_{st})\).
Chapter 4

Fault detection for a PEMFC system.

4.1 Introduction.

The methodology of fault diagnosis, which is used in this Thesis, is mainly based on classic theory of model-based diagnosis described in [41; 52; 96]. Model based diagnosis, as was discussed before in Chapter 2, can be divided in two subtasks: fault detection and fault isolation.

The principle of model-based fault detection is to check the consistency of observed behaviour using the relationship in Eq. (4.1) while fault isolation tries to identify the component that is in fault. In this case where the residual vector \( r \) is computed considering the estimated output from the mathematic model.

\[
\phi_{ik} = \begin{cases} 
0 & \text{if } |r_i| \leq \tau_i \\
1 & \text{if } |r_i| > \tau_i 
\end{cases}
\]  

(4.1)

where \( \tau_i \) is the threshold associated to the residual \( r_i \).

When building a model for fault detection from a complex system, such as PEMFC, to monitor its behaviour, there is always a mismatch between the estimated and real behaviour since some effects are neglected, some parameters have tolerance, some errors in parameters or in the structure of the model are introduced during the calibration process, etc. In order to avoid problems such
as false alarms robust model design must be carefully analyzed.

4.2 LPV Modeling in PEM Fuel Cells.

As was discussed before, Fuel cells are devices that enable the direct conversion of chemical energy into electrical energy. The need to maintain signal constraints during operation, combined with importance of unmeasured variables, indicated the need for control-oriented models can be used to estimate the model outputs for model predicted control or any FDI based on models. Like was shown in Chapter 3 and farther more in [15; 99; 120], PEMFCS non lineal model is considered as a complex model, where there is no way to obtain an explicit analytical redundancy relations (ARRs) using parity space, i.e. where there is not mathematically way to express the estimated sensor signal with the available theoretical model without uses a numerical manipulation, such as model linearization o estimation. This problem was addressed in [78].

In other to solve this problem two alternative are possible; one of those is related to placing more sensors to generate more degrees of freedom in the parity space, but in the case of PEMFCS, there is not faceable because of the physical size does not allow the sensor placed, the second one is related to using the observer approaches, where in this case to solve the mathematical constrain referred to computational model is solved using the LPV approach, where is possible to use the LTI tools. In the following sections, we discuss the development of a control oriented dynamic model of a PEMFC. Using a detailed physical model as a starting point, we demonstrate the utility of a LPV model structure as a mechanism for residual generation in robust fault diagnosis.

4.3 Fault detection using LPV modeling.

Quantitative model-based FDI methods rely on the comparison of a system’s available measurements, with a priori information represented by the system’s mathematical model. There are two main trends of this approach: analytical redundancy or residual generation methods and parameter estimation [93]. In the present work, the FDI methodology is developed based on residual generation.
Since quantitative model-based methods for FDI are developed based on some fundamental understanding of the physics of the process, FDI using analytical redundancy is a viable implementation for systems where \textit{a priori} knowledge is available in terms of mathematical relationships between the inputs and outputs of the system. While there are several methods for residual generation, this Thesis is focused on residual generation using \textit{Linear Parameter Varying Observer} (LPVO).

In some real applications, where there is not a theoretical model of the plant available, to deal with this problem it is possible to identify the plant or system based on model parameter estimation approach around an operating point using just input and output signals (black box). One great advantage using this approach is the fact that the time consumption for model computing is considerably reduced, but in another hand, the model contains uncertainty because of a model parameter identification process.

The required architecture is considered as a conservative one, because it uses just a Luenberger observer structure, this architecture is shown in Figure 4.1.

\begin{align*}
L \cdot r &= f\{A(\vartheta), C(\vartheta)\} \cdot r \\
\dot{x}_{k+1} &= A(\varphi) \dot{x}_k + B(\varphi) u_k + w_k + H_k \\
\hat{y}_k &= C(\varphi) \hat{x}_k + D(\varphi) u_k + v_k
\end{align*}

Figure 4.1: Residual generation using the LPV observer approach.

\subsection{Problem formulation.}

To proceed in the analysis of residual generation using a LPVO approach, we need an analytical model or Input-Output signals from a plant. In this Thesis, a
state-representation of the PEMFCS model is used as

\[
\dot{x}_k = f(x_k, u_k) \\
y_k = h(x_k, u_k)
\]  (4.2)

4.3.1.1 Linear Parameter Varying Model (LPVM).

Some methods use nonlinear equation of the system to derive a LPVM such as state transformation, substitution of functions and methods using the well known Jacobian linearization. Another kind of method uses multi-model identification that consists in two different steps. First a set of LTI model is identified at different equilibrium points by classical methods (on-line or off-line), as second part of this methodology is getting a multi-model by interpolation law that allows to commute among a local LTI model at each operating point [8; 77].

Let us consider the nonlinear system in Eq. (4.2) can be described by its LPV representation as follows

\[
x_{k+1} = \tilde{A}(\vartheta_k)x_k + \tilde{B}(\vartheta_k)u_k + w_k \\
y_k = \tilde{C}(\vartheta_k)x_k + \tilde{D}(\vartheta_k)u_k + v_k
\]  (4.3)

where \(x_k \in \mathbb{R}^{n_x}, u_k \in \mathbb{R}^{n_u}\) and \(y_k \in \mathbb{R}^{n_y}\) are, respectively, the state, input, and output vectors. The process and measurement noises \(w_k \in \mathbb{R}^{n_x}\) and \(v_k \in \mathbb{R}^{n_y}\) are respectively; that are considered unknown but bounded as \(v_k \in \mathbb{V}\) and \(w_k \in \mathbb{W}\) which are interval boxes.

The vector of time-varying parameters \(\vartheta_k \in \mathbb{R}^{n_{\vartheta}}\) that changes with the operating point scheduled by some measured system variables \((p_k)\) that can be estimated using some known function \((\vartheta_k = f(p_k))\). However, some uncertainty in the estimated parameter values is considered to be bounded by the set \(\Theta\), which represents the uncertainty about the exact knowledge of the real system parameters \(\vartheta_k\).

\[
\Theta_k = \{\vartheta_k \in \mathbb{R}^{n_{\vartheta}} \mid \underline{\vartheta}_k \leq \vartheta_k \leq \overline{\vartheta}_k\}
\]  (4.4)
The system in Eq. (4.3) describes a model parametrized by a scheduling variable denoted by \( p_k \). In this section, the kind of LPV system considered are those parameters whose vary affinely in a polytope \([6]\). In particular the state-space matrices range in a polytope of matrices defined as the convex hull of a finite number \((N)\) of matrices. That is,

\[
\begin{pmatrix}
A_j(\vartheta_j) & B_j(\vartheta_j) \\
C_j(\vartheta_j) & D_j(\vartheta_j)
\end{pmatrix}
\in \left\{ \begin{pmatrix} A_j(\vartheta^j) & B_j(\vartheta^j) \\ C_j(\vartheta^j) & D_j(\vartheta^j) \end{pmatrix} \right\} \\
:= \sum_{j=1}^{N} \alpha_j(p_k) \begin{pmatrix} A_j(\vartheta^j) & B_j(\vartheta^j) \\ C_j(\vartheta^j) & D_j(\vartheta^j) \end{pmatrix}
\]

(4.5)

with \( \alpha_j(p_k) \geq 0 \), \( \sum_{j=1}^{N} \alpha_j(p_k) = 1 \) and \( \vartheta^j = f(p^j) \) is the vector of uncertain parameters of \( j^{th} \) model where each \( j^{th} \) model is called vertex system, and it is assumed according to Eq. (4.5) that \( \vartheta^j \in [\vartheta^\ell, \vartheta^\ell] \) (here \( \vartheta^\ell \) and \( \vartheta^\ell \) represent the maximum and minimum values of \( \vartheta^j \), respectively), when the varying parameter \((\vartheta_k)\) evolves in a polytopic domain \( \Theta \) of vertices (where the vertices are the extreme values of the parameter \( \vartheta \)).

Consequently, the LPV system, represented in Eq. (4.3), can be expressed as follows

\[
x_{k+1} = \sum_{j=1}^{N} \alpha_j(p_k) \left[ A_j(\vartheta^j) x_k + B_j(\vartheta^j) u_k \right] \\
y_k = \sum_{j=1}^{N} \alpha_j(p_k) \left[ C_j(\vartheta^j) x_k + D_j(\vartheta^j) u_k \right]
\]

(4.6)

Here \( A_j, B_j, C_j \) and \( D_j \) are the state space matrices defined for \( j^{th} \) model. Notice that, the state space matrices of the system in Eq. (4.3) is equivalent to the interpolation between LTI models, for example; \( \tilde{A}(\vartheta_k) = \sum_{j=1}^{N} \alpha_j(p_k) A_j(\vartheta^j) \).

The polytopic system is scheduled through functions \( \alpha_j(p_k), \forall j \in [1, \ldots, N], \)
that lie in a convex set

\[ \Psi = \left\{ \alpha^j(p_k) \in \mathbb{R}^N, \alpha^j(p_k) = [\alpha^1(p_k), \ldots, \alpha^N(p_k)]^\top, \alpha^j(p_k) \geq 0, \forall j, \sum_{j=1}^{N} \alpha^j(p_k) = 1 \right\} \]

(4.7)

There are several ways of implementing the system in Eq. (4.5) depending on how \( \alpha^j(p_k) \) functions are defined [77], here the approach used in [8] is expressed as

\[
\left( \begin{array}{c}
\hat{A}(\vartheta_k) \\
\hat{C}(\vartheta_k)
\end{array} \right) 
\begin{pmatrix}
\frac{2}{3} \sum_{i=1}^{v} \prod_{m=1}^{\mu_{m,i}} (p_{m}^i) \\
\frac{2}{3} \sum_{i=1}^{v} \prod_{m=1}^{\mu_{m,i}} (p_{m}^i)
\end{pmatrix}
\left( \begin{array}{c}
\alpha^j(p_k)
\end{array} \right)
\]

(4.8)

with \( \mu_{m,1} = \frac{(p_{m}^k - p_{m}^l)}{p_{m}^l - p_{m}^r} \) and \( \mu_{m,2} = 1 - \mu_{m,1} \) where \( p_{m}^l \) and \( p_{m}^r \) represent the upper and lower bounds of \( p_m \) respectively and \( v \) is the number of scheduling variables.

4.3.1.2 Linear Parameter Varying Observer (LPVO).

A LPVO with Luenberger structure for the state estimation of the system described in Eq. (4.3) is given by

\[
\begin{align*}
\hat{x}_{k+1} &= \tilde{A}(\vartheta_k)\hat{x}_k + \tilde{B}(\vartheta_k)u_k + \tilde{L}(\vartheta_k)(y_k - \hat{y}_k) + w_k \\
\hat{y}_k &= \tilde{C}(\vartheta_k)\hat{x}_k + \tilde{D}(\vartheta_k)u_k + v_k
\end{align*}
\]

(4.9)

Since a set of matrices that describe the linear dynamic system around an operating point defined by Eq. (4.8) and the definition of state estimation by Eq. (4.9), then it can conclude with the design of an observer state estimation in interpolating form as
\[
\dot{x}_{k+1} = \sum_{j=1}^{N} \alpha_j(p_k) \begin{bmatrix} A_j^\varphi(\varphi^j) & B_j(\varphi^j) \end{bmatrix} \cdot \begin{bmatrix} \hat{x}_k \\ u_k \end{bmatrix} \\
\hat{y}_k = \sum_{j=1}^{N} \alpha_j(p_k) \begin{bmatrix} C_j(\varphi^j) & D_j(\varphi^j) \end{bmatrix} \cdot \begin{bmatrix} \hat{x}_k \\ u_k \end{bmatrix}
\] (4.10)

where \( A_j^\varphi = A_j(\varphi^j) - L_j(\varphi^j) \cdot C_j(\varphi^j) \), \( u_k \) is the measured system input vector, \( \hat{x}_{k+1} \) is the estimated system state vector, \( \hat{y}_k \) is the estimated system output vector and \( L_j \) is the observer gain that has to be designed to stabilize the observer given by Eq. (4.10) for all \( \varphi^j \in \left[ \varphi^j, \varphi^j \right] \).

**Definition 4.1** Consider the state estimator given by Eq. (4.10), an initial compact set \( X_0 \) and a sequence of measured inputs \((u_j)_{k=0}^{k-1}\) and outputs \((y_j)_{k=0}^{k}\). The **exact uncertain estimated state set** at time \( k \) is expressed by

\[
X_k = \left\{ \hat{x}_k : (\hat{x}_j = \hat{A}(\varphi_{j-1})\hat{x}_{j-1} + \hat{B}(\varphi_{j-1})u_{j-1} + w_{j-1} + \hat{L}(y_{j-1} - y_{j-1}))_{j=1}^{k}, \\
(\hat{y}_{j-1} = \hat{C}(\varphi_{j-1})\hat{x}_{j-1} + \hat{D}(\varphi_{j-1})u_{j-1} + v_{j-1})_{j=1}^{k} \\
x_0, \hat{x}_0 \in X_0, (\varphi_{j-1}) \in \Theta, w_{j-1} \in W_{j-1}, v_{j-1} \in V_{j-1})_{j=1}^{k} \right\}
\] (4.11)

where \( \hat{L} = \sum_{j=1}^{N} \alpha_j(p_k) L_j \).

The uncertain state set described in **Definition 4.1** at time \( k \) can be computed approximately by admitting the split of the existing relations between variables of consecutive time instants\(^1\). This makes possible to compute an approximation of this set from the approximate uncertain state set at the time \( k - 1 \).

**Definition 4.2** Consider a system given by Eq. (4.10), the set of uncertain states at time \( k-1 \), \( X_{k-1} \) and the input/output values \((u_{k-1}, y_{k-1}, y_k)\). Then, the

\(\footnote{However, the problem of uncertainty propagation (wrapping effect) could appear when this set is approximated in this way because of the accumulation of overestimation along the time and deriving in an explosion of uncertainty.}

1
approximated set of estimated states at time \( k \) based on the measurements up to time \( k - 1 \) is defined as

\[
X_e^k = \{ \hat{x}_k : \tilde{A}(\vartheta_{j-1}) \hat{x}_{k-1} + \tilde{B}(\vartheta_{j-1}) u_{k-1} + \tilde{L}(y_{k-1} - \hat{y}_{k-1}) + w_{k-1}, \\
\hat{y}_{k-1} = \tilde{C}(\vartheta_{j-1}) \hat{x}_{k-1} + \tilde{D}(\vartheta_{j-1}) \bar{u}_k + v_{k-1} \}
\]  

(4.12)

Analogously, considering measurement in Eq. (4.10) the approximated set of estimated outputs \( Y_e^k \) can be determined. Since the set of estimated states \( X_e^k \) is difficult to compute, one way is to bound it using some geometric regions easy to compute for example: a box (interval hull).

Using Definition 4.2, the set of estimated states (or outputs) introduced in Definition 4.1 will be approximated interactively using zonotopes. From these zonotopes, an interval for each state variable can also be obtained by computing the interval hull of the zonotope. The sequence of interval hulls \( \Box X_e^k \) with \( k \in [0, N] \) will be called the interval observer estimation of the system given by Eq. (4.10). Analogously, the sequence of interval hulls \( \Box Y_e^k \) can be obtained.

**4.3.1.3 Adaptive threshold using zonotopes.**

In this section, adaptive threshold generation for robust fault detection of a non-linear system, such as PEMFCS, described by Linear Parameter Varying (LPV) model is analyzed. Uncertainties due to parameter variations are considered unknown but bounded by intervals. Their effect is addressed using an interval LPV observer, descriptive in previous Section 4.3.1.2, incorporating the zonotopes algorithm. The design procedure of this observer is carried out via pole placement using LMI. The optimal advanced threshold is generated by evaluating the worst-case residual’s energy evolution in the time domain. When the advanced adaptive threshold is used, the minimum detectable fault is characterized.

**Zonotope architecture.** In this method, zonotopes are used for guaranteed state estimation in the case of non-linear discrete systems with bounded uncertain parameters, where each sample time, a guaranteed bound (zonotope) of the uncertain trajectory of the system parameters is calculated using interval arith-
metic [68; 74]. A zonotope is an affine transformation of a unitary hypercube. Zonotompe are proposed by [95] and [24] to build a state bounding observer. Let us introduce zonotopes using the following definitions.

**Definition 4.3** The Minkowski sum of two sets $X$ and $Y$ is defined by $X \oplus Y = \{ x + y : x \in X, y \in Y \}$.

**Definition 4.4** Given a center vector $p \in \mathbb{R}^n$ and a matrix $H \in \mathbb{R}^{n \times m}$ the Minkowski sum of the segments defined by the columns of matrix $H$, is called a zonotope of order $m$, this set is represented as:

$$X = p \oplus H\beta^m = \{ p + Hz : z \in \beta^m \}$$

where $\beta^m$ is a unitary box, composed by $m$ unitary intervals.

(a) Zonotope of order $m = 3$. (b) Zonotope of order $m = 9$. (c) Zonotope of order $m = 14$.

Figure 4.2: Zonotope architecture.

**Definition 4.5** The interval hull $\Box X$ of a closed set $X$ is the smallest interval box that contains $X$. Given a zonotope such as $X = p \oplus H\beta^m$, its interval hull can be easily computed by evaluating $p \oplus H\beta^m$, for all $i = 1, \ldots, n : \Box X = \{ x : |x_i - p_i| \leq \|H_i\|_1 \}$ where $H_i$ is $i^{th}$-row of $H$, and $x_i$ and $p_i$ are $i^{th}$ component of $x$ and $p$, respectively.

**Implementation of LPVO using zonotopes.** To implement LPVO using zonotopes, it should be noticed that using Eq. (4.9) as the expression of a model
estimation as a discrete-time system with one input that can be reorganized as

\[ \hat{x}_{k+1} = A_0 \hat{x}_k + B_0 u_0^k \]  

(4.14)

where

\[ A_0 = \tilde{A}(\vartheta_k) - \tilde{L}(\vartheta_k) \tilde{C}(\vartheta_k) \]
\[ B_0 = \begin{bmatrix} \tilde{B}(\vartheta_k) - \tilde{D}(\vartheta_k)/\tilde{L}(\vartheta_k) & I & \tilde{L}(\vartheta_k) & \tilde{L}(\vartheta_k) \end{bmatrix} \]
\[ u_0^k = \begin{bmatrix} u_k & y_{k+1} & w_k & y_k & v_k \end{bmatrix}^\top \]  

(4.15)

Then, the problem of interval estimation can be formulated as a problem of interval simulation, which requires a characterization of the set \( \mathcal{X}_e_k \). This set can be viewed as a \emph{direct image evaluation} of Eq. (4.14) and can be implemented using zonotopes.

**Implementation of prediction set step.** The predicted set step \( \mathcal{X}_e_{k+1} \), which is the direct image evaluation of Eq. (4.14), should be bounded. There are different algorithms to bound such an image using ellipsoids [68] or zonotopes [61]. To bound such image using zonotopes the following result is used.

**Theorem 4.1 (Zonotope Inclusion [3].)** Consider a family of zonotopes represented by \( \mathcal{X} = p \oplus MB^m \) where \( p \in \mathbb{R}^n \) is a real vector and \( M \in \mathbb{I}^{n \times m} \) is an interval matrix. A zonotope inclusion \( \odot(\mathcal{X}) \) is defined by

\[ \odot(\mathcal{X}) = p \oplus \left[ \text{mid}(M, G) \right] \begin{bmatrix} B^m \\ B^n \end{bmatrix} = p \oplus J B^{n+m} \]  

(4.16)

where \( G \in \mathbb{R}^{n \times n} \) is a diagonal matrix that satisfies: \( G_{ii} = \sum_{j=1}^{m} \frac{\text{diam}(M_{ij})}{2}, i = 1, 2 \ldots n \). with \( \text{mid} \) and \( \text{diam} \) denote respectively the center and diameter of the interval according to [74], under this definition \( \mathcal{X} \subseteq \odot(\mathcal{X}) \).

The prediction step aims to compute the zonotope \( \mathcal{X}_{e_{k+1}} \) that bounds the trajectory of the system at instant \( k+1 \), from the previous approximating zonotope.
\(X_k\) at the instant \(k\). The last is possible, if is used the natural interval extension in Eq. (4.14) as suggested by [74] and the zonotope inclusion operator, as a generalization of Kühn’s method [61] as follow

\[
X_{k+1}^x = p_{k+1} \oplus H_{k+1}^x B^x
\]

(4.17)

where

\[
p_{k+1} = \text{mid}(A_o)p_k + \text{mid}(B_o)u_k^o
\]

and

\[
H_{k+1}^x = \begin{bmatrix}
J_1^x & J_2^x & J_3^x
\end{bmatrix}
\]

\[
J_1^x = \bigodot(A_o H_{k+1}^x)
\]

\[
J_2^x = p_k(\text{diam}(A_o)/2)
\]

\[
J_3^x = u_k^o(\text{diam}(B_o - \tilde{L}(\vartheta_k) \cdot \tilde{D}(\vartheta_k))/2 \cdot \text{diam}(\tilde{L}(\vartheta_k))/2)
\]

The term \(J_1^x\) is calculated using the zonotope inclusion operator. It is important to notice that the set of estimated states has an increasing number of segments generating the zonotope \(X_{k+1}^x\) using this method. Analogically, it is possible to compute the estimate set of output value \(Y_k^x\).

\[
Y_{k}^x = p_{k} \oplus H_{k}^x B^x
\]

(4.18)

where

\[
H_{k}^y = \begin{bmatrix}
J_1^y & J_2^y & J_3^y
\end{bmatrix}
\]

\[
J_1^y = \bigodot(C(\vartheta_k)H_k^x)
\]

\[
J_2^y = p_k(\text{diam}(C(\vartheta_k))/2)
\]
where the operating point boundary is denoted by upper boundary operating point model \( (y_{up}^i) \) and lower boundary point model \( (y_{lo}^i) \) from \( y_{op}(\vartheta_k) \).

\[
y_{op}(\vartheta_k) = y_{lo}(\vartheta_k) + \begin{bmatrix} y_{up}^i \\ y_{lo}^i \end{bmatrix}
\]

(4.19)

**Checking for intersection emptiness.** Compute the set of estimated output \( Y^e_k \), requires to check if the intersection of \( [y_k] \cap Y^e_k \), is not the empty set, before introducing such operation, an additional definition is introduced.

**Definition 4.6** \( Y^e_k = p \oplus HB^r \), the strip \( [y_k] = \{ x \in \mathbb{R}^n \mid c^T x - d \leq \sigma \} \), a hyperplane \( S = \{ x : c^T x = q \} \) is a supporting hyperplane of a zonotope \( Y^e_k \). If either \( c^T x \leq q, \forall x \in Y^e_k \) or else \( c^T x \geq q, \forall x \in Y^e_k \) with equality occurring for \( x \in Y^e_k \).

The two constants \( q_u \) and \( q_d \) characterizing the supporting hyperplanes are easily calculated as

\[
q_u = c^T p + \| H^T c \|_1
\]

(4.20)

\[
q_d = c^T p - \| H^T c \|_1
\]

(4.21)

where \( \| \cdot \|_1 \) is the 1-norm of a vector. Then the intersection check is very easy to perform just considering that each new measurement, which is defined as a set of consistent states as

\[
F_k = \{ x_k \in \mathbb{R}^n : -\sigma \leq y_k - C x_k \leq \sigma \}
\]

(4.22)

where \( F_k \) is the region between two hyperplanes and the output \( y_k \) is considered component-wise. The normalized form of this strip is written as

\[
\tilde{F}_k = \{ x_k \in \mathbb{R}^n : \frac{y_k}{\sigma} - c^T x_k \leq 1 \}
\]

(4.23)
Calculating the supporting hyperplane constant $q_u$ and $q_d$, the intersection is empty if and only if:

$$q_u < \frac{y_k}{\sigma} - 1 \text{ or } q_d > \frac{y_k}{\sigma} + 1$$

(4.24)

**Observer Gain design.** The design of the LPV observer gain ($\hat{L}$), in Eq. (4.9) can be solved with the LMI pole placement technique [21], that allows to locate the poles of the observer in a subregion of the left half-plane using a LMI region.

Consider a given $2d \times 2d$ Hermitian matrix defined as

$$R = \begin{bmatrix} R_{00} & R_{10} \\ R_{10}^* & R_{11} \end{bmatrix} \in \mathbb{C}^{2d \times 2d}, R_{11} \in \mathbb{C}^{2d \times 2d} \geq 0$$

(4.25)

and the feasibility set of an associated LMI defined as

$$\mathcal{D} = \left\{ s \in \mathbb{C} : R_{00} + (R_{10}s)^H + R_{11}s^*s < 0 \right\}$$

(4.26)

where $(R_{10}s)^H$ denotes the Hermitian transpose of $R_{10}s$. The sets defined according to Eqs. (4.25) and (4.26) are called $\mathcal{D}$-regions, allowing to characterize some multiple temporal specifications. For instance, the vertical left half-plane characterized by $x < \lambda$, the associated matrix $R$ is

$$R = \begin{bmatrix} -2\lambda & 1 \\ 1 & 0 \end{bmatrix}$$

(4.27)

while, for an open disk with a center $c = c_1 + c_2i$ and radius $r$ is

$$R = \begin{bmatrix} c_1^2 + c_2^2 - r^2 & -c_1 + c_2i \\ -c_1 - c_2i & 1 \end{bmatrix}$$

(4.28)

Using these formulations, it is easy to verify that the classical stability regions for continuous-time (left half-plane) and discrete time (origin-centered unitary
disk) systems are associated to the matrices

\[
R_{ct} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad R_{dt} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}
\] (4.29)

In particular, let consider a disk LMI region called \( D \) defined by center \( c \) (in this case \( c = c_1 \) and \( c_2 = 0 \)) and radius \( r \) such that \((c + r) < 1\). Here, the two scalars \( c \) and \( r \) are used to determine a specific region included in the unit circle where the observer eigenvalues will be placed. Therefore, this circular region puts a lower bound on both the exponential decay rate and the damping ratio of the closed-loop response. The design of the interval LPV observer (4.10) such that the observer poles are placed in this LMI region requires to find for each vertex \( j^{th} \) (with \( j \in [1, \ldots, N] \)) the observer gain \( (L_j) \) and unknown symmetric matrix \( X_j = X_j^\top > 0 \) that’s satisfies the following LMI

\[
\begin{bmatrix}
-rX_j & cX_j + \left( A_o^j (\vartheta^j)^\top X_j \right)^\top \\
\left( c + A_o^j (\vartheta^j)^\top \right) X_j & -rX_j
\end{bmatrix} < 0
\] (4.30)

for \( \vartheta^j \in [\vartheta^j, \vartheta^j] \), that corresponds to Eq. (4.25) in with matrix \( A \) being transpose of the observer matrix \( A_o^j \) [21].

Note that Eq. (4.30) is a Bilinear Matrix Inequality (BMI) which cannot be solved with LMI classical tools. However, substituting \( W_j = L_j^\top X_j \), it is possible to transform it into

\[
\begin{bmatrix}
-rX_j & \cdots & cX_j + X_j^\top A_j (\vartheta^j) - W_j^\top C_j (\vartheta^j) \\
\left( c + A_j (\vartheta^j)^\top \right) X_j - C_j (\vartheta^j)^\top W_j & \cdots & -rX_j
\end{bmatrix} < 0
\] (4.31)

Then, the design procedure boils down to solve the LMI in Eq. (4.31) and determining \( L_j = (W_j X_j^{-1})^\top \). Finally, the observer gains \( L_j \) will be interpolated to obtain the interval LPVO in Eq. (4.10).

**Observability analysis.** Observability defined as a measurement property of the number of states that can be inferred or estimated based on the outputs. For
the study of the degree of observability of a system at each operating point \((\vartheta)\) is done using the procedure \(\text{rank}\) in (4.32). The analysis of a dynamic system with respect to the degree of observability of a system makes use of the procedure \(\text{cond}\) in (4.32).

\[
\text{rank}
\begin{bmatrix}
\lambda I - A \\
C
\end{bmatrix}
\quad \text{cond}
\begin{bmatrix}
\lambda I - A \\
C
\end{bmatrix}
\quad (4.32)
\]

### 4.3.1.4 Robust Fault Detection using Linear Parameter Varying Interval Observer (LPVIO).

The consistency check is based on computing residuals \(r_k\). The residuals are obtained from the discrepancy of measured signal input \((u_k)\) and outputs \((y_k)\) at operating point \((\vartheta_k)\) using the set of sensors installed in the process with an analytical relationship which is obtained by the system modeling. The last dependency can be expressed as

\[
r_k = \psi (y_k, u_k, \vartheta_k)
\quad (4.33)
\]

where \(\psi\) is the residuals generator function that depends on the type of detection strategy used (parity equation [40] or observer [18]. At each time instance, \(k\), the residual is compared with a threshold value (zero in ideal case or almost zero in real case).

In the case where a LPVIO is used for fault detection where an adaptive threshold is used to generate the signal map \((\phi)\). The robustness of a FDI system should be able to avoid false alarms, in order to face this issue a robust fault detection procedure is presented in Algorithm 1.

**Example 4.1** Considered the well known PEMFC system non-linear model available in the literature and developed by [99]. In this case, the model is considered as nine states. In another hand but like the water vapor mass in the cathode \((m_{w,ca})\) always predicts excessive water flow from the anode to the cathode which results in fully humidified cathode gas under all nominal conditions. Additionally, the effects of liquid condensations also known as flooding, is not included in the model but considered on the fuel cell voltage response. Hence, in this case, we can
Algorithm 1 Robust Fault Detection Process Using LPVIO.
1: \( \text{fault} \leftarrow \text{FALSE} \)
2: \( k \leftarrow 0 \)
3: \( \mathcal{X}_k^e \leftarrow \mathcal{X}_0 \)
4: while \( \text{fault} = \text{FALSE} \) do
5: Obtain input-output data \( \{u_k, y_k\} \)
6: Compute the approximated set of estimated states, \( \mathcal{X}_k^e \)
7: Compute the approximated set of estimated outputs, \( \mathcal{Y}_k^e \)
8: Compute the interval hull of the approximated set of estimated states, \( \mathcal{X}_k^e = [\underline{x}_k, \overline{x}_k] \)
9: Compute the interval hull of the approximated set of estimated outputs, \( \mathcal{Y}_k^e = [\underline{y}_k, \overline{y}_k] \)
10: if \( \{y_k\} \cap \mathcal{Y}_k^e = \emptyset \) and \( u_k = u_{k-1} \) then
11: \( \text{fault} \leftarrow \text{TRUE} \)
12: end if
13: \( k \leftarrow k + 1 \)
14: end while

reduce the original model from nine to eight state variables without \( m_{w,ca} \).

Problem formulation:

The dynamic model with eight states is expressed as

\[
\begin{align*}
\dot{m}_{O_2} &= W_{O_2,i} - W_{O_2,o} - W_{O_2,r} \\
\dot{m}_{H_2} &= W_{H_2,i} - W_{H_2,o} - W_{H_2,r} \\
\dot{m}_{N_2} &= W_{N_2,i} - W_{N_2,o} \\
\dot{\omega}_{cp} &= \frac{1}{J_{cp} \cdot \omega_{cp}} \cdot (P_{cm} - P_{cp}) \\
\dot{p}_{sm} &= \frac{\gamma \cdot R_a}{V_{sm}} (W_{cp} \cdot T_{cp} - W_{sm,o} \cdot T_{sm}) \\
\dot{m}_{sm} &= W_{cp} - W_{sm,o} \\
\dot{m}_{w,an} &= W_{van,i} - W_{van,o} - W_{v_{mbr}} \\
\dot{p}_{rm} &= \frac{R_a \cdot T_{rm}}{V_{rm}} (W_{ca,o} - W_{rm,o}) 
\end{align*}
\]

where
States: \( x = [m_O_2, m_H_2, m_N_2, \omega_{cp}, p_{sm}, m_{sm}, m_{w,an}, p_{cm}]^\top \).

Outputs: \( y = [W_{cp}, p_{sm}, v_{st}]^\top \).

Input: \( u = [I_{st}]^\top \).

Fault benchmark:

In order to test the proposed methodology in the PEMFC system model described in Eq. 4.34, a set of common possible fault scenarios was considered and implemented in simulation. Table 4.1 describes the set of faults, which was considered.

Table 4.1: Description of the fault benchmark.

<table>
<thead>
<tr>
<th>ID</th>
<th>Fault Description</th>
<th>Type</th>
<th>Magnitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f_1 )</td>
<td>There is a suddenly increase of friction in the mechanical part of the compressor.</td>
<td>Parametric &amp; Abrupt</td>
<td>( \Delta R_{cm} = 60% )</td>
</tr>
<tr>
<td>( f_2 )</td>
<td>Degradation in an electrical component of the compressor</td>
<td>Parametric &amp; Abrupt</td>
<td>( \Delta k_v = 40% )</td>
</tr>
<tr>
<td>( f_3 )</td>
<td>Degradation in the cells at stack is presented because of contact-sensitivity reactions against to a reaction killer.</td>
<td>Parametric &amp; Abrupt</td>
<td>( \Delta A_{fc} = 20% )</td>
</tr>
<tr>
<td>( f_4 )</td>
<td>An increase of the fluidic resistance is presented, because of over flooding diffusion layer.</td>
<td>Parametric &amp; Abrupt</td>
<td>50%</td>
</tr>
<tr>
<td>( f_5 )</td>
<td>A suddenly leak of air is presented at the inlet section of humidifier.</td>
<td>Parametric &amp; Abrupt</td>
<td>30%</td>
</tr>
<tr>
<td>( f_6 )</td>
<td>Hydrogen leak in the anode is presented because of seal degradation.</td>
<td>Parametric &amp; Abrupt</td>
<td>400%</td>
</tr>
</tbody>
</table>

Fault benchmark: implementation in simulation:

This section, as part of this example, describes how faults in Table 4.1 were included in the model. One of the main assumptions for fault simulation, is the fact that just single fault plays in the system.

**Fault 1 and Fault 2:** These faults are simulated with an increment, \( \Delta R_{cm} \) and \( \Delta K_v \), respectively. For \( f_1 \) is presented in the compressor motor resistance \( (R_{cm}) \) parameter, the fault effect is translated in a change in the compressor torque \( (\tau_{cm}) \). In \( f_2 \) a degradation in the electrical part of the compressor is presented.
Note that both faults are directly related to the compressor speed ($\omega_{cp}$). The Eq. (4.35) describes the faults’ implementation, where: $\eta_{cm}$ is the motor mechanical efficiency, $k_t$ is a motor constant.

$$\tau_{cm} = \frac{\eta_{cm} k_t}{(R_{cm} + \Delta R_{cm})} (v_{cm} - (k_v + \Delta k_v) \omega_{cm})$$  \hfill (4.35)

Furthermore, considering the fault interaction overall system dynamic, the parameter $\tau_{cm}$ is associated to $\omega_{cp}$ which is straightforward related to the amount of air feed to the system, the $f_1$ and $f_2$ are related to the state of $p_{sm}$ because its dynamic is governed by the compressor inlet air flow

$$W_{ca,i} = k_{sm} (p_{sm} - p_{cp})$$  \hfill (4.36)

where $p_{cp}$, $p_{sm}$ are the compressor and the supply manifold pressure and $k_{sm}$ is the supply manifold mass flow constant. Because of a change in the air mass flow feed to the system is affected by the faults, the total mass ($m_{O_2}, m_{N_2}$) balance across the FC changes, with the last it is possible to know that overall partial pressure for each raw material component changes as well producing an effect over the stack voltage.

**Fault 3:**

This fault is presented as a contamination into the stack, reducing the chemical reaction efficiency by abating of catalysis active area. The current density ($i$) is defined as cell current, which equals stack current $I_{st}$ (A), per cell active area ($A_{fc}$ [cm$^2$]).

$$i = \frac{I_{st}}{A_{fc} \cdot \Delta A_{fc}}$$  \hfill (4.37)

Because of the majors voltage drops are associated with current density for non-linear relations [64], current density is an important issue for total stack voltage, then producing a change by $f_3$ in the active a change in the stack voltage is expected, where in Eq. (4.37) $\Delta A_{fc}$ is the active area contaminated.

**Fault 4:**

This fault is implemented at the return manifold outlet section, which implies a reduction of outlet flow. The change produced by $\Delta k_{rm}$ is related to material
balance and system pressure. The fault implementation is shown as follows

\[ W_{rm,o} = (k_{rm} - \Delta k_{rm}) \cdot (p_{sm} - p_{cp}) \]  

(4.38)

**Fault 5:** This fault appears at the cathode inlet flow. Because this fault is considered as a leak, it is introduced in the mass balance

\[ W_{hm,f} = W_{hm,i} - W_{hm,i} \cdot f_5 \]  

(4.39)

Note that the amount of air that does not enter the system will not only be created an abrupt change in the total mass balance, where the state \((m_{O_2}, m_{N_2})\) are mainly involved, but also a system pressure change.

**Fault 6:** The term \(W_{H_2,nl}\) introduced in [49] represents the natural leak from the anode of the fuel cell stack. This leak is always present due to the physical stack sealing design. It is assumed that the natural leak is governed by a standard orifice relation through an effective area \((A_{nl})\). This parameter has been obtained in [49] for some particular stacks. To simulate a degradation in the seal a change in \(A_{nl}\) is used as \(A_{nl,f} = A_{nl} \cdot f_6\).

\[ W_{H_2,nl} = \frac{A_{nl,f} \cdot p_{an}}{\sqrt{R_{an} \cdot \phi_{an}}} \cdot p_r^{(1/\gamma)} \cdot \left( \frac{2\gamma}{\gamma - 1} \left[ 1 - p_r^{\frac{\gamma - 1}{\gamma}} \right] \right)^{1/2} \]  

(4.40)

where \(p_r = p_{an}/p_{atm}\) is the pressure ratio across the assumed leak and the anode gas constant \((R_{an})\) is calculated through the universal gas constant \((R)\) as follows

\[ R_{an} = R/(y_{H_2} \cdot M_{H_2} + (1 - y_{H_2}) \cdot M_{H_2O}) \]  

(4.41)

where the molar fraction of hydrogen in the anode is given by

\[ y_{H_2} = \frac{p_{an} - \phi_{an} \cdot p_{sat,T_{an}}}{p_{an}} \]  

(4.42)

**Residual generation:**

The implementation of the model-based fault diagnosis system following the approach proposed in this section has been done using MATLAB/SIMULINK environment. The first step of model-based FDI is to generate signals called residuals that reflect the consistency between actual data and the model.
The residuals, using Eq. (2.1) and considering the available measurements, are computed as follows

\[ r_1 = W_{cp} - \hat{W}_{cp} \]  
(4.43)

\[ r_2 = p_{sm} - \hat{p}_{sm} \]  
(4.44)

\[ r_3 = v_{st} - \hat{v}_{st} \]  
(4.45)

LPVM problem formulation:
The non-linear model (shown in Eq. (4.34)) can be transformed into a LPVM in state space representation (Eq. (4.3))

\[
A(\vartheta_k) = \begin{bmatrix}
  a_{11} & 0 & a_{13} & 0 & a_{15} & 0 & 0 & a_{18} \\
  0 & a_{22} & 0 & 0 & a_{25} & 0 & a_{27} & 0 \\
  a_{31} & 0 & a_{33} & 0 & a_{35} & 0 & 0 & a_{38} \\
  0 & 0 & 0 & a_{44} & a_{45} & 0 & 0 & 0 \\
  a_{51} & 0 & a_{53} & a_{54} & a_{55} & a_{56} & 0 & 0 \\
  a_{61} & 0 & a_{63} & a_{64} & a_{65} & 0 & 0 & 0 \\
  0 & a_{72} & 0 & 0 & a_{75} & 0 & a_{77} & 0 \\
  a_{81} & 0 & a_{83} & 0 & 0 & 0 & 0 & a_{88}
\end{bmatrix} ;
B(\vartheta_k) = \begin{bmatrix}
  b_{11} \\
  b_{21} \\
  0 \\
  b_{41} \\
  0 \\
  0 \\
  b_{71} \\
  0
\end{bmatrix}
\]

\[
C(\vartheta_k) = \begin{bmatrix}
  0 & 0 & 0 & c_{14} & c_{15} & 0 & 0 & 0 \\
  0 & 0 & 0 & c_{24} & c_{25} & 0 & 0 & 0 \\
  c_{31} & c_{32} & c_{33} & 0 & 0 & 0 & 0 & 0 \end{bmatrix} ;
D(\vartheta_k) = \begin{bmatrix}
  0 \\
  0 \\
  d_{31}
\end{bmatrix}
\]

(4.46)

where \( x = [m_{O_2} m_{H_2} m_{N_2} \omega_{cp} p_{sm} m_{sm} m_{w,n} p_{rm}]^\top \), \( u = [I_{st}]^\top \) and \( y = [W_{cp} p_{sm} v_{st}]^\top \), the units of states and outputs are compatible magnitudes (kg/s, kPas, kRPM, A, vol).

In the case of scheduling variable, this is the current \( I_{st} \), then Eq. (4.4) is \( \vartheta_k = f(I_{st,k}) \). With the last it is possible to express the system in Eq. (4.4) in computational form with polytope structure as Eq. (4.10). Since the non-linear model, it is possible to express a LPVM as polytope structure, and using Eq. (4.4), then it is possible to compute a LPVO in polytope form as Eq. (4.10).
Considering an operating point of interest as \(I_{st} = 120[A]\), the LTI matrix model is shown as follows

\[
A = \begin{bmatrix}
-12.31 & 0 & -10.85 & 0 & 83.74 & 0 & 0 & 22.61 \\
0 & -268.6 & 0 & 0 & 85.3 & 0 & -30.06 & 0 \\
-36.84 & 0 & -45.22 & 0 & 275.7 & 0 & 0 & 147.1 \\
0 & 0 & 0 & -11.54 & 154.4 & 0 & 0 & 0 \\
2.351 & 0 & 2.686 & 0.297 & -37.28 & 0.07345 & 0 & 0 \\
33.28 & 0 & 38.03 & 4.32 & -518.4 & 0 & 0 & 0 \\
0 & -342.8 & 0 & 0 & 108.4 & 0 & -68.51 & 0 \\
4.045 & 0 & 4.621 & 0 & 0 & 0 & 0 & -55.08
\end{bmatrix},
\]

\[
B = \begin{bmatrix}
-3.16E-2 \\
-9.98E-3 \\
0 \\
2.373 \\
0 \\
0 \\
0 \\
0
\end{bmatrix},
\]

\[
C = \begin{bmatrix}
4.32E-3 & -0.155 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
-0.8183 & 15.01 & 24.75 & 0 & 0 & 0 & 0 & 0
\end{bmatrix},
\]

\[
D = \begin{bmatrix}
-0.32
\end{bmatrix}.
\]

(4.47)

**LPVM observability analysis:**

Using the LTI matrix shown in Eq. (4.46) for state space representation, an observability analysis described in Section 4.3.1.3 is performed to improve the observer gain design, which is normally at least twice as fast as the dominant closed loop eigenvalues (see Table 4.2).

Table 4.2: Eigenvalues, eigenvector of PEMFC linear model at \(I_{st} = 120[A]\) and observability.

<table>
<thead>
<tr>
<th>Eigenvector</th>
<th>(m_{c_2})</th>
<th>(m_{g_2})</th>
<th>(m_{N_2})</th>
<th>(\omega_{sc})</th>
<th>(p_{sm})</th>
<th>(-\omega_{sc})</th>
<th>(-p_{sm})</th>
<th>(m_{w, an})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(m_{c_2})</td>
<td>-6.01E-17</td>
<td>0.178913</td>
<td>0.10627</td>
<td>-8.42E-17</td>
<td>-0.06127</td>
<td>-0.08417</td>
<td>-7.14E-01</td>
<td>-0.0238</td>
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<tr>
<td>(m_{g_2})</td>
<td>0.577627</td>
<td>-0.01596</td>
<td>-0.01681</td>
<td>0.122982</td>
<td>-0.00551</td>
<td>-0.01246</td>
<td>-0.00196</td>
<td>-0.00294</td>
</tr>
<tr>
<td>(m_{N_2})</td>
<td>8.38E-16</td>
<td>0.747169</td>
<td>0.147621</td>
<td>-2.00E-17</td>
<td>-0.17058</td>
<td>-0.29483</td>
<td>6.16E-01</td>
<td>-0.06032</td>
</tr>
<tr>
<td>(\omega_{sc})</td>
<td>1.89E-17</td>
<td>0.083664</td>
<td>0.278987</td>
<td>-1.20E-15</td>
<td>0.959015</td>
<td>-0.75948</td>
<td>-1.07E-01</td>
<td>-0.13479</td>
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<tr>
<td>(p_{sm})</td>
<td>-5.25E-17</td>
<td>-0.04378</td>
<td>-0.06491</td>
<td>-1.05E-17</td>
<td>-0.01421</td>
<td>-0.03759</td>
<td>-5.98E-03</td>
<td>-0.00913</td>
</tr>
<tr>
<td>(-\omega_{sc})</td>
<td>1.00E-15</td>
<td>-0.02211</td>
<td>-0.92714</td>
<td>3.45E-16</td>
<td>-0.21572</td>
<td>-0.57156</td>
<td>-9.81E-02</td>
<td>-0.08868</td>
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<tr>
<td>(-p_{sm})</td>
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<td>-0.06054</td>
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<td>0.006361</td>
<td>0.000025</td>
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<table>
<thead>
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<tbody>
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<td>rank</td>
<td>S</td>
<td>S</td>
<td>S</td>
<td>S</td>
<td>S</td>
<td>S</td>
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<td>S</td>
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<td>1.495874</td>
<td>2.320664</td>
<td>7.497177</td>
<td>10.11705</td>
<td>26.87191</td>
</tr>
</tbody>
</table>

The high condition number for slow eigenvalue could degrade the observer performance. When these eigenvalue are moved, the resulting observer gain is large, and thus producing a large overshoot in observer error. For an implementation point of view, when it is combined with a FDI strategy, a large observer gain can
produce an untruth alarm. The eigenvector associated to eigenvalues reveal that
the unobservable states are the mass of vapor in the cathode, \((m_{v,an})\) and pressure
at the return manifold \((p_{rm})\). Moreover, the high condition number for slow
eigenvalues could degrade the observed performance. The last is possible to see in
simulation, which is pictured out in Figure 4.3, where the full LPVM performance
is unsuitable to be used as LPVO because sensor noise and operating point change
produce a large overshot in the estimation, in other hand these phenomena do not
appear in the reduced model, which its performance offers a state estimation over
an operating point change and sensor noise.

To design a reduced model, considering just the observable part avoiding the
weakly observable part using the Gramian-based input-output balancing of state-
space realizations \([65; 73]\). Consider a continuous LTI state space model, with
controllability and observability gramians \(W_c\) and \(W_o\). The state coordinate trans-
formation \(\bar{x} = Tx\) produces the equivalent model

\[
\begin{align*}
\dot{\bar{x}} &= TAT^{-1}\bar{x} + TBu \\
y &= CT^{-1}\bar{x} + Du
\end{align*}
\]

(4.48)

Computing the Gramian-based input-output balancing, a particular similarity trans-
formation \(T\) is obtained such that

\[
W_c = W_o = diag(G)
\]

(4.49)

In the case of LTI matrix model in Eq. (4.47), the grammian and transfor-
mation matrix are
For stable systems, is an equivalent realization for which the controllability and observability Gramians \( G \) are equal and diagonal, their diagonal entries forming the vector \( G \) of Hankel singular values. Small entries in \( G \) indicate states that can be removed to simplify the model. The nominal or full continuous LTI state space model already balance such as

\[
\begin{bmatrix}
\dot{x} \\
y
\end{bmatrix} =
\begin{bmatrix}
A_b & B_b \\
C_b & Du
\end{bmatrix} \begin{bmatrix}
x \\
u
\end{bmatrix} \tag{4.51}
\]

where \( A_b = T \begin{bmatrix} A_1 & \cdots & A_r \end{bmatrix} T_n^{-1} \), \( B_b = T B \) and \( C_b = C T_n^{-1} \). The full state vector \( x = [x_1; x_2] \) where \( x_2 \) is to be discarded, and the reduced states is set to \( x_r = x_1 + H x_2 \), where \( H \) is chosen to enforce matching DC-gain \(^1\) (steady-state response)\[^{119}\] between reduced model and full model.

In order to determine \( x_1 \) is then first isolate states with a negligible contribution to the I/O response (\( G \) value), then

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2
\end{bmatrix} =
\begin{bmatrix}
A_{11} & A_{12} \\
A_{21} & A_{22}
\end{bmatrix} \begin{bmatrix}
x_1 \\
x_2
\end{bmatrix} +
\begin{bmatrix}
B_1 \\
B_2
\end{bmatrix} u
\]

\[
y =
\begin{bmatrix}
C_1 & C_2
\end{bmatrix} x + Du \tag{4.52}
\]

\(^1\)With the matched DC gain method, \( A_{22} \) must be invertible in continuous time, and \( I - A_{22} \) must be invertible in discrete time.
Next, the derivative of $x_2$ is set to zero and the resulting equation is solved for $x_1$. The reduced-order model is given by

$$
\dot{x}_1 = \left[ A_{11} - A_{12}A_{22}^{-1}A_{21} \right] x_1 + \left[ B_1 - A_{12}A_{22}^{-1}B_2 \right] u
$$

$$
y = \left[ C_1 - C_2A_{22}^{-1}A_{21} \right] x + \left[ D - C_2A_{22}^{-1}B_2 \right] u
$$

(4.53)

Considering the LTI PEMFCS model in Eq. (4.47) that any state such that $2.5E-2 > G$ is a negligible contribution to model response, then $x_2 = [m_{w,an}, p_{rm}]$ with the transformation matrix $T$ in Eq. (4.50), the reduced model in state space is then

$$
A_R = \begin{bmatrix}
-1.888 & -4.76 & 1.326 & 0.005991 & 0.08481 & -1.312 \\
4.76 & -13.93 & 9.422 & 0.04033 & 0.5884 & -9.136 \\
0.000606 & 0.04469 & 9.875 & -0.001725 & -0.06132 & 1.275 \\
-0.1514 & 1.047 & 15.26 & -0.07748 & -30.57 & 827.8 \\
3.983 & -27.55 & -397.8 & 2.06 & 1017 & -2.959e + 004
\end{bmatrix}
$$

; $B_R = \begin{bmatrix}
1.553 \\
-1.656 \\
-0.5473 \\
0.002709 \\
0.06225 \\
-1.638
\end{bmatrix}$

$$
C_R = \begin{bmatrix}
0.0009353 & -0.004203 & 0.00169 & -0.0008263 & -0.01199 & 0.3441 \\
0.01035 & 0.005019 & -0.007979 & 0.0007771 & 0.04992 & -1.504 \\
1.553 & 1.656 & -0.5473 & -0.00246 & -0.0352 & 0.5494
\end{bmatrix}
$$

; $D_R = \begin{bmatrix}
0 \\
0 \\
-0.32
\end{bmatrix}$

(4.54)

Simulation: LPVM performance:

Figure 4.3 shows, the model performance for reduced and full model in a fault free case, here is clear that the reduced model offers a better model performance avoiding to overshoot in operating point change at $k = 30$. The residual vector for each plant measurement, described in Eq. (4.43), is shown in the bottom of each subfigure.

Simulation: Robust fault detection; In order to prove a robust fault detection process is performed, an adaptive threshold is used in this case, the adaptive threshold offers a robustness beside of fix threshold, using zonotopes architecture, which was discussed previously in Section 4.3.1.4. In a fault free case, looking at Figure 4.4, we can see an operating point change is produced at $k = 30$, the LPVO fix the output estimation within the up and low threshold.
To test the robust fault detection, the simulation of faults $f_1$ and $f_2$ is shown in Figures 4.4 and 4.5. In both cases the process suffers an operating point change at $k = 30$ of $\Delta I_{st} = 80 \text{[A]}$. The plant outputs keep within the threshold. The plant suffers the fault a time $k = 50$, looking at Figures 4.4 and 4.5 both presents the same binary signature matrix (FSM) where faults affect the adaptive threshold performance because of observer gain $L$, both faults $f_1$ and $f_2$ affect the same component but the effect in residuals is different.

At this time, we can consider that the model estimation and adaptive threshold offers a robust fault detection because the noises and parameter model uncertain is involved into the adaptive threshold besides fix threshold is commonly fix by statistical analysis, moreover fix threshold does not take into account model uncertainties. Moreover, the effects to the faults are not negligible during the fault detection process If it is just considered the binary information available in Table 4.3, it is impossible to isolate faults, but taking into account the effect of the fault and residual can be used in order to enhance fault isolation, some technics are discussed later in Chapter ??.

<table>
<thead>
<tr>
<th></th>
<th>$f_1$</th>
<th>$f_2$</th>
<th>$f_3$</th>
<th>$f_4$</th>
<th>$f_5$</th>
<th>$f_6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_1$</td>
<td>$(-)1$</td>
<td>$(-)1$</td>
<td>$(-)1$</td>
<td>$(+)1$</td>
<td>$(-)1$</td>
<td>$0$</td>
</tr>
<tr>
<td>$r_2$</td>
<td>$0$</td>
<td>$0$</td>
<td>$0$</td>
<td>$(+)1$</td>
<td>$0$</td>
<td>$0$</td>
</tr>
<tr>
<td>$r_3$</td>
<td>$0$</td>
<td>$0$</td>
<td>$(+)1$</td>
<td>$0$</td>
<td>$(+)1$</td>
<td>$0$</td>
</tr>
</tbody>
</table>

Conclusion:

In this example, we have intruded on a parametric fault benchmark, which as been implemented in simulation using the well known model in the literature [99]. Considering a realistic case, where it is not available a theoretical model a LPVM is an alternative to compute residual for fault diagnosis following the FDI philosophy. Moreover, the case of robust fault detection has been faced using adaptive thresholding with zonotopes architecture. For observer gain design a reduced model is proposed to improve the model estimation performance, additionally the gain design follows the LMI pole placement warranty the system stability placing the poles within the unitary circle.
4.4 Summary.

This chapter has addressed the issue of robust detection using LPV models and interval observer (LPVO) applied to PEMFCs, adaptive threshold is introduced for fault detection. Moreover this chapter address the analysis of observability to avoid model estimation problems.
(a) Compressor air feed (Plant and model (Full and Reduced) performance, models error)

(b) Pressure manifold (Plant and model (Full and Reduced) performance, models error)

(c) Stack voltage (Plant and model (Full and Reduced) performance, models error)

Figure 4.3: Simulation performance for Reduced and Full PEMFCS model.
4. My Third Chapter

(a) Compressor air feed (Model performance with adaptive threshold and model error).

(b) Pressure manifold (Model performance with adaptive threshold and models error).

(c) Stack voltage (Model performance with adaptive threshold and model error).

Figure 4.4: Simulation performance in a fault free case using LPVO and adaptive threshold.
(a) Compressor air feed (Model performance with adaptive threshold and model error).

(b) Pressure manifold (Model performance with adaptive threshold and models error).

(c) Stack voltage (Model performance with adaptive threshold and model error).

Figure 4.5: Simulation performance in $f_1$ case ($FSM_{1,1} = [(-1), 0, 0]^T$).
4. My Third Chapter

(a) Compressor air feed (Model performance with adaptive threshold and model error)

(b) Pressure manifold (Model performance with adaptive threshold and models error)

(c) Stack voltage (Model performance with adaptive threshold and model error)

Figure 4.6: Simulation performance in $f_2$ case ($FSM_{i,2} = [(-1), 0, 0]^\top$).
References


REFERENCES


