A multiobjective-based switching topology for hierarchical model predictive control applied to a hydro-power valley

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Abstract: In a Hierarchical Model Predictive Control (H-MPC) framework, this paper explores suitable time-variant structures for the hierarchies of different local MPC controllers. The idea is to adapt to different operational conditions by changing the importance of the local controllers. This is done by defining the level of the hierarchy they belong to, and solving within each level the local MPC problem using the information provided by the higher levels at the current time step and the predicted information from the lower levels obtained in the previous time step. As selecting a hierarchy results in a combinatorial optimization problem, it is explicitly solved for a limited number of relevant topologies only and then the switching between topologies is defined with a multiobjective optimizer, so as to decide the best H-MPC scheme according to the expected performance. A comparison with fixed-topology H-MPC controllers is done, showing the effectiveness of the proposed approach for the power control of a hydro-power valley.

Keywords: Hierarchical Model Predictive Control, Switching Topologies, Supervisory Control, Multiobjective Optimization, Hydro-Power Valley.

1. INTRODUCTION

Despite of the advantages of Model Predictive Control (MPC) over other control methods, the application of this control technique in large-scale systems resulted to be impractical in many application domains due to the computation time required to solve on-line the corresponding optimization problem. To make the real application of MPC in large-scale systems possible, different hierarchical and distributed MPC (HD-MPC) approaches have been proposed in the literature during the recent years like Rawlings and Stewart (2008); Venkat et al. (2008); Doan et al. (2011); Valencia et al. (2011); Ocampo-Martinez et al. (2012). HD-MPC schemes can deal with the control of large-scale MPC problems since they divide the complex overall problem into several sub-problems, trying to achieve some degree of coordination among subsystems that are solving local simpler MPC problems with locally relevant variables, control goals, and constraints, without solving the centralized MPC problem.

Compared with the centralized MPC solution, the performance of the system with an HD-MPC controller might decrease, but the total computational cost can be reduced by defining proper communication and cooperation strategies, and in some algorithms by negotiation among subsystems. In Alvarado et al. (2011) a comparison of different HD-MPC methods is presented, each of them exhibiting different performance, computational effort, and complexity characteristics. Among the many schemes available in the literature, this paper considers those based on hierarchical MPC (H-MPC), see Scattolini (2009). In H-MPC controllers, the system is divided into different functional layers. The control actions of a layer are then computed considering the information from other local controllers at a higher level of the hierarchy. Although such approaches have been reported to produce suitable results, the communication requirements and the selection of the hierarchies are crucial.

The main contribution of this paper consists in a novel scheme where the H-MPC controller has the flexibility to switch between different hierarchies (solving sequence), but they all have a common cost function. Then, the control action applied to the system will be selected according to the decision of a multiobjective optimization module, comprising not only the global objective, but also some others specifications and management of constraints (due to the hierarchical decomposition). For the particular case study of this paper, the cost functional is a combination of tracking error of the power demanded and the economic benefits. We assume the availability of H-MPC structures that assure stability and feasibility of the controller under the hierarchical decomposition, while the switchings between those H-MPC controllers will be activated just in case it is possible to increase some of the performance indices while keeping the others constant (Pareto dominance criterion).

The remainder of the paper is organized as follows. In Section 2, H-MPC theory is introduced. In Section 3, an H-MPC controller capable of switching between different solving sequences based on a multiobjective optimizer is presented. In Section 4, the hydro-power valley case study is introduced and numerical results are presented. Different schemes considering fixed and non-fixed communication topologies are compared, showing the benefits of the switching scheme proposed in this paper, as different structures fit better when the system is operating in different power reference regions. Finally, the main conclusions and some lines of future research are presented in Section 5.

2. HIERARCHICAL MODEL PREDICTIVE CONTROL (H-MPC)

2.1 Model Predictive Control

Consider the discrete-time non-linear system whose dynamic evolution is described by the following state-space model:

$$x(k+1) = f(x(k), u(k)), \tag{1}$$

with $x(k) \in \mathbb{R}^n$ and $u(k) \in \mathbb{R}^m$ being the state and the input vector respectively, and $f : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$. Often, a quadratic function of the form

$$\begin{aligned} J(\widetilde{x}, \widetilde{u}) = & \sum_{l=1}^{N_\mathrm{p}} x^T(k+l)Qx(k+l) \\ & + \sum_{l=0}^{N_\mathrm{u}-1} u^T(k+l)Ru(k+l) \\ & + \sum_{l=0}^{N_\mathrm{u}-1} \Delta u^T(k+l)S\Delta u(k+l) \end{aligned} \tag{2}$$

is used to measure the performance of the system. Here,

$$\widetilde{x} = [x^T(k+1), \ldots, x^T(k+N_\mathrm{p})]^T, \tag{3a}$$

$$\widetilde{u} = [u^T(k), \ldots, u^T(k+N_\mathrm{u}-1)]^T, \tag{3b}$$

where N_p, N_u are the prediction and control horizon respectively; Q, R, and S are weighting matrices positive definite, and $\Delta u(k+l) \triangleq u(k+l) - u(k+l-1)$. The superscript T denotes the transpose operator.

Assume $x(k) \in \mathbb{X}$, and $u(k) \in \mathbb{U}$ for all $k \in \mathbb{N}$, where $\mathbb{X}$ and $\mathbb{U}$ determine the feasible values of the states and the inputs respectively, and they are given by the physical and operational constraints of the system. Then, the MPC problem can be formulated as a non-linear optimization problem

$$\min_{\widetilde{u}} J(\widetilde{x}, \widetilde{u}) \tag{4a}$$

subject to

$$\begin{aligned} & x(k+1+l) = f(x(k+l), u(k+l)), \quad x(0) = x_0, \\ & x(k+l) \in \mathbb{X}, \quad u(k+l-1) \in \mathbb{U}, \quad l = 1, \ldots, N_\mathrm{p}-1, \\ & u(k+l) = u(k+N_\mathrm{u}-1), \quad l = N_\mathrm{u}, ..., N_\mathrm{p}. \end{aligned} \tag{4b}$$

Typically, such minimization is implemented in a centralized way. For large-scale systems, centralized MPC may become impractical since it might require to optimize a huge amount of variables, and to exchange large amounts of information, which in turn might imply a huge computational burden. Therefore, H-MPC schemes are proposed to deal with large-scale MPC problems given their capabilities to divide a complex problem into several less complex sub-problems.

2.2 Hierarchical Model Predictive Control

Since the goal is to deal with large-scale systems, H-MPC arises as an alternative to overcome the computational problems associated with the implementation of a centralized MPC scheme. Assume M subsystems distributed in L hierarchical levels, where there are P_q subsystems at each level, for $q = 1, \ldots, L$. To place a stronger focus on the case study in the present paper rather than attempting to present a very general framework, we consider the case $L = M$, so we allow just one subsystem in each hierarchy. Therefore, each subsystem may be denoted with just one subindex r as S_r, with $r \in \{1, \ldots, M\}$.

In this paper, we assume no state coupling. This assumption will hold for example in flow networks, whose subsystems are just connected through control actions, or if the partitions are selected in such a way that the manipulated signals are the only common/shared variables, see, e.g., Ocampo-Martinez et al. (2011), or the states affecting subsystems can be written as functions of their own control variables. This assumption can be relaxed by selecting the adequate class of H-MPC controller that can deal with the state coupling. The dynamics of subsystem S_r are given by

$$x_r(k+1) = f_r(x_r(k), u_r(k), u_{\mathrm{h},r}(k), u_{\mathrm{l},r}(k)), \tag{5}$$

where $x_r(k)$ and $u_r(k)$ correspond to the state and the input vectors of S_r, $u_{\mathrm{h},r}(k)$ is the vector of inputs belonging to subsystems at higher hierarchy levels, and $u_{\mathrm{l},r}(k)$ is the vector of inputs belonging subsystems in lower hierarchies. The solving sequence of the optimization problems is given by the predefined hierarchy, where subsystems in a higher level are solved first, communicating their resulting control sequences and state trajectories to the subsystems in a lower level. For the information needed in subsystem S_r that comes from lower hierarchies but it is not yet calculated, predictions coming from their local MPC controller from previous time steps are used.

Sequences $\widetilde{x}_r$, $\widetilde{u}_r$, $\widetilde{u}_{\mathrm{h},r}$ and $\widetilde{u}_{\mathrm{l},r}$ are accordingly defined as in (3), Ω_r is the set of feasible control actions for $\widetilde{u}_r$, and $\widehat{\Xi}_r(\widetilde{u}_r(k), \widetilde{u}_{\mathrm{h},r}(k), \widetilde{u}_{\mathrm{l},r}(k); x_r(k))$ is the feasible set for the states, the initial condition $x_r(k)$, and the prediction model (5). Then, the local optimization problem related to subsystem S_r and arising from the partitioning of the whole large-scale system can be formulated as

$$\min_{\widetilde{u}_r} J_r(\widetilde{x}_r, \widetilde{u}_{\mathrm{h},r}(k), \widetilde{u}_{\mathrm{l},r}(k)), \tag{6a}$$

subject to

$$\widetilde{x}_r \in \widehat{\Xi}_r(\widetilde{u}_r(k), \widetilde{u}_{\mathrm{h},r}(k), \widetilde{u}_{\mathrm{l},r}(k); x_r(k)), \tag{6b}$$

$$\widetilde{u}_r \in \Omega_r, \tag{6c}$$

where $J_r(\cdot)$ denotes the cost function related to subsystem S_r. Hence, this H-MPC approach is decomposed into local optimization problems (6) and the procedures used to guarantee the communication among subsystems. From

(6), local optimization problems are coupled to each other, i.e., the value of the cost function $J_r(\cdot)$ in (6a) and the decision space of S_r depend on the decisions of the remaining subsystems.

However, the way the hierarchies are selected is a crucial step. For example, as shown in Figure 1 (like in the case study with $M = 3$ subsystems), there are in total 6 possible hierarchical structures than can be considered. In this paper, we want to have an extra flexibility in the H-MPC controller, in order to have the opportunity to change the hierarchies. What we expect in general is that the subsystems at the higher levels will have a better local performance than the rest as they will impose their control actions to the other subsystems. However, when the subsystems are coupled, the way the lower level subsystems react will also affect the higher levels. So the question about which is the best hierarchy remains open and, as a possible answer, in the next section we propose a switching method to include extra flexibility in conventional H-MPC controllers, so as to change the structure of the H-MPC according to a better expected performance.

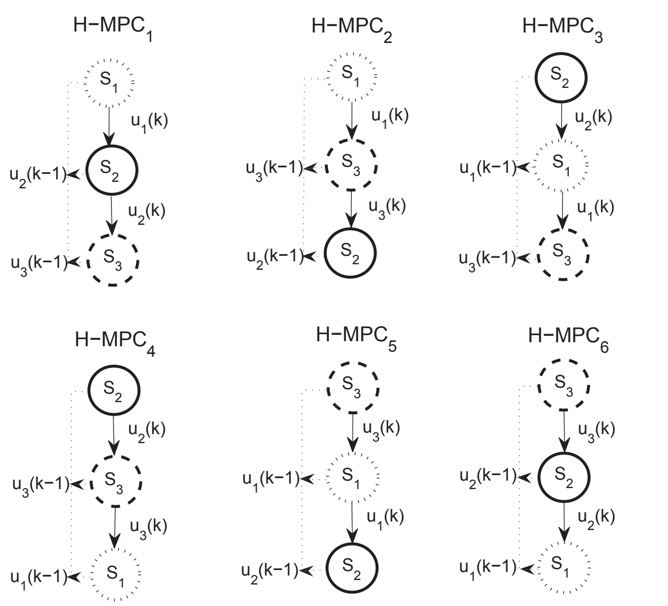


Fig. 1. Different possible hierarchies for the case of $M = 3$ subsystems. In dotted lines the information that is shared at the beginning of the time step k. Then, following the solving sequence from top to bottom, the local MPC controllers calculate their control actions and transmit them (dashed lines) to the controllers in the lower levels

3. SWITCHING METHODOLOGY BASED ON MULTIOBJECTIVE OPTIMIZATION FOR H-MPC

In this section, a methodology to control the way solving sequence of the MPC controllers of the subsystems is presented. The idea is to control the large-scale system by using a set of H-MPC controllers with different hierarchies, to get better control decisions. Once the H-MPC controllers suggest their sequence of control actions, a multiobjective switching algorithm will process the information and will determine the best control action according to multiple performance indices, see, e.g., Bemporad and Muñoz de la Peña (2009) and Núñez et al. (2010). It is important to stress that each local controller may have different cost functions, not necessarily related to the indices in the multiobjective switching algorithm. In Figure 2, a scheme of the operation of the controller is presented.

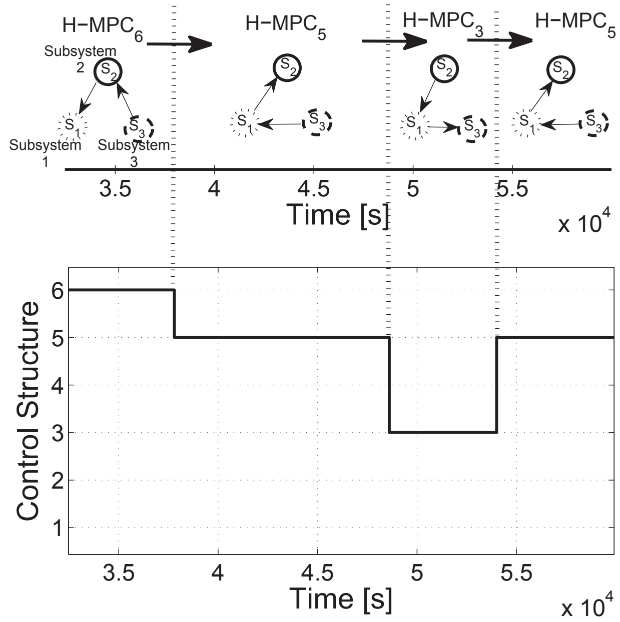


Fig. 2. Dynamic switching topology

According to the criteria, different H-MPC controllers are used at different time steps. For example, in Figure 2, when switching from H-MPC$_6$ to H-MPC$_5$, the subsystem S_3 has the higher priority, but the priorities of subsystem S_1 and S_2 change, giving subsystem S_1 a higher importance. This setup fits in systems where for example, in the morning some specific subsystems are more relevant to be optimized, but then because of the changes over time of the operational conditions, the optimization of other subsystems in the afternoon is more important.

It is assumed that the communication between the M subsystems is always available, but we can control how to use it in an H-MPC structure. Assume we have selected N_{ctrl} different H-MPC controllers, each controller c with a different configuration of hierarchies. So in total, there are N_{ctrl} selected information exchange combinations (hierarchies). The switching strategy between all possible hierarchies is time consuming, especially when no parallel computation is available. To select a proper number of structures, some criteria could be, e.g., the computation time, decay rate of transmission, RMS of the output, distributed balancing of the performance of the different regions, among others. In this paper we just use all the possible combinations (six H-MPC controllers in total), and even they can be run in parallel, a pruning or better selection of just the most used hierarchies can be done, so to reduce the number of combinations.

In Figure 3, a scheme of the overall controller is presented. In total N_{ctrl} H-MPC controllers work in parallel. The switching between the control actions suggested by each controller is decided by a multiobjective decision algorithm, the control action of which is applied based on the performance indices.

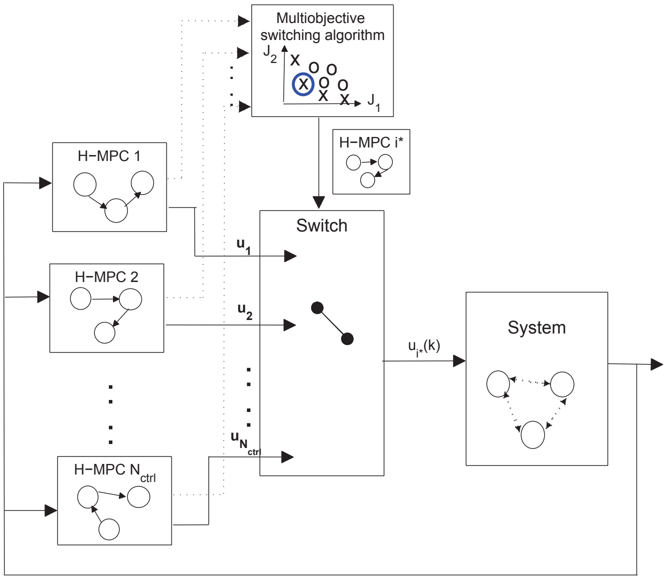


Fig. 3. Overall control scheme

The optimal control action sequences $\mathbf{u}_c$, $c = 1, ..., N_{\mathrm{ctrl}}$ (comprising the information for all the subsystems) coming from each of the H-MPC controllers are used to evaluate in open loop a global model of the process over the prediction horizon. With the information of those N_{ctrl} controllers, the optimal input $\mathbf{u}_{c^*}$ from the controller c^* is selected based on the evaluation of multiple objective functions and this controller will be used during the sampling time defined for the switching algorithm. The switching criteria will depend on a criteria that might change according to multiple factors. Often the objective functions are conflicting, i.e., a solution that optimizes one objective may not optimize others, see, e.g., Bemporad and Muñoz de la Peña (2009); Núñez et al. (2010). Thus, instead of minimizing a single objective function, we consider more performance indices:

$$\min_{\mathbf{u}_c} \{J_1(\mathbf{u}_c, x_0), J_2(\mathbf{u}_c, x_0), ..., J_l(\mathbf{u}_c, x_0)\} \qquad (7)$$

the variables $\mathbf{u}_c$ and $J_\ell(\mathbf{u}_c, x_0)$, $\ell = 1, ..., l$, are the sequence of future control actions and the objective functions to minimize respectively. The solution of multiobjective problems is a set of control action sequences called Pareto-optimal set. In this paper, the Pareto-optimal set is obtained by just evaluations of the given $\mathbf{u}_c$, $c = 1, ..., N_{\mathrm{ctrl}}$. One method to select a solution from the Pareto-optimal set is by using a weighted sum of the different objectives, so the best controller c will be the one that minimizes $\sum_{\ell=1}^{l} w_\ell J_\ell(\mathbf{u}_c, x_0)$, with w_ℓ the weighted factors. From the control sequence of controller c, just the first component $u(k)$ has to be applied to the system.

To assure the feasibility of the controller and the convergence of the iterative process under the hierarchical decompositions, one conservative assumption is to require that each H-MPC can satisfy those conditions. Then, as for the effects of the switching control topologies on the closed-loop stability and performance, the dwell time should be selected in such a way to assure asymptotic stability, see Liberzon (2003). In this paper, the switching is done only if another H-MPC control provides a Pareto dominant solution (thus, some indices will get at least a better performance, while the others are constant), and then, among the Pareto solutions, the one that minimizes the function $\sum_{\ell=1}^{l} w_\ell J_\ell(\mathbf{u}_c, x_0)$. For the implementation in this work, the conditions of the systems were safe enough to not have to face any of the typical problems related with feasibility or convergence (a through analysis of these issues is part of the further research).

In Venkat et al. (2008) and Giovanini (2011), for distributed model predictive controllers, it is shown for linear systems, that it is possible instead of using a switching method to obtain similar results making the weights of the cost function time-varying and allowing the optimization find the optimal solution. This might be interesting to address in a further research, so to have rules (time-varying weights) that indicate in a fast way the best hierarchy based on the conditions of the system, so as to emulate the integer optimization problem that is actually solved in this paper with a complete enumeration of almost all the hierarchies run in parallel (as N_{ctrl} might not be small in general).

4. RESULTS

The Hydro-Power Valley (HPV) benchmark presented in Savorgnan et al. (2011) is used for testing the novel approach, see Figure 4. The system is partitioned into 3 subsystems with the following elements:

- Subsystem 1: Lakes 1 and 2.
- Subsystem 2: Lake 3.
- Subsystem 3: River dams.

In order to verify the superiority of the proposed switching H-MPC, first six regular fixed H-MPC controllers with different structures have been implemented. In Figure 4, the scheme of the partition of the system and the set of N_{ctrl} controllers are shown.

The objective function considers the reference tracking and the reduction of the strong variations in the level of the lakes (state variables). It is expressed as

$$\sum_{t=k+1}^{k+N_{\mathrm{p}}} \lambda |p_{\mathrm{ref}}(t) - p(t)| + (h(t) - h_{\mathrm{ss}})^T Q (h(t) - h_{\mathrm{ss}}), \quad (8)$$

where $p(t)$ is the power generated by the HPV at step t, with $p(t) = \sum_{i=1}^{8} p_i(t)$. Moreover, λ is a weighting factor, $p_{\mathrm{ref}}(t)$ is the reference of the power, $h(t)$ is the vector with the water level of the lakes, and h_{ss} is a reference for the levels. For the local MPC controllers, $N_{\mathrm{p}} = 48$, and $N_{\mathrm{u}} = 32$. The mean square error (MSE) and mean absolute error (MAE) of the tracking error will be used for comparison purposes.

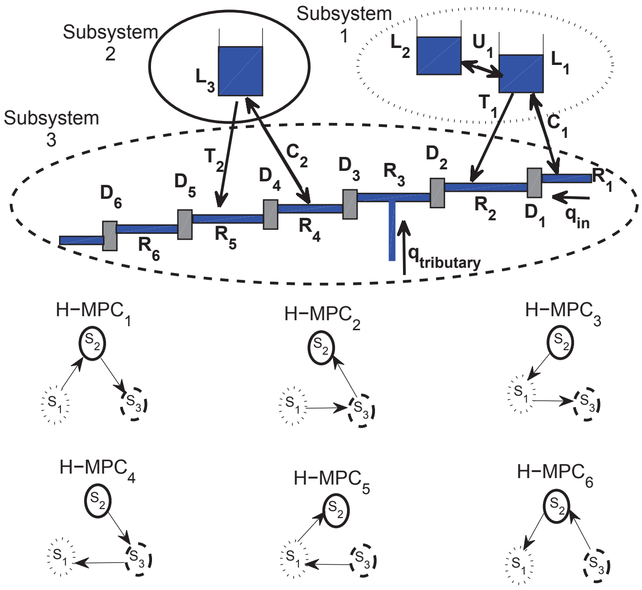


Fig. 4. Proposed control scheme for the benchmark

Figure 5 shows the reference power trajectory and the generated power for the fixed H-MPC schemes. The responses of the controllers are suitable; however, we can see that the performance of some of them is better in some regions than in others. The most notorious examples of tracking error are highlighted by some circles in the figure. This might is due to the lack of flexibility of the hierarchies, at some time steps, it is more relevant to control first the lakes, and then, given their control signal, to control the river dams (or vice-versa).

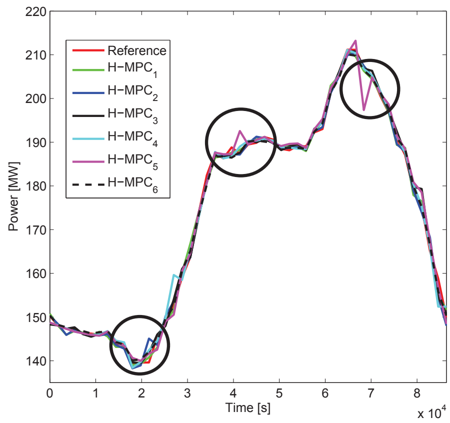


Fig. 5. HPV responses the different controllers

Now, all $N_{\text{ctrl}} = 6$ configurations are used in the multiobjective-based switching topology for H-MPC controllers. Three quantitative indices will be used to trigger the multiobjective switching mechanism:

(1) Mean square tracking error (MSE):

$$J_1(\mathbf{u}_c, x_0) = \sum_{t=k+1}^{k+N_\text{p}} \left(p_{\text{ref}}(t) - p(t)\right)^2 \tag{9}$$

(2) Economic performance reference tracking in Euros (MAE Economic), where $c(t)$ shown in Figure 6 is the cost of the electricity at time t:

$$J_2(\mathbf{u}_c, x_0) = \sum_{t=k+1}^{k+N_\text{p}} c(t) \max\left(p_{\text{ref}}(t) - p(t), 0\right) + 0.5 \sum_{t=k+1}^{k+N_\text{p}} c(t) \max\left(p(t) - p_{\text{ref}}(t), 0\right) \tag{10}$$

(3) Weighted sum, combining the effects of both objectives (normalized):

$$J_3(\mathbf{u}_c, x_0) = 0.25 \frac{J_1(\mathbf{u}_c, x_0)}{J_1^{\text{N}}} + 0.75 \frac{J_2(\mathbf{u}_c, x_0)}{J_2^{\text{N}}}, \tag{11}$$

where J_1^{N} and J_2^{N} are values used to normalize the objective functions (computed using their maximum value among the evaluation of the N_{ctrl} controllers).

To obtain the predictions in each of those indices, the control actions suggested by each H-MPC controller are applied to a global model of the system.

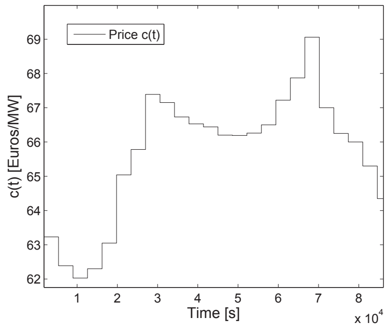
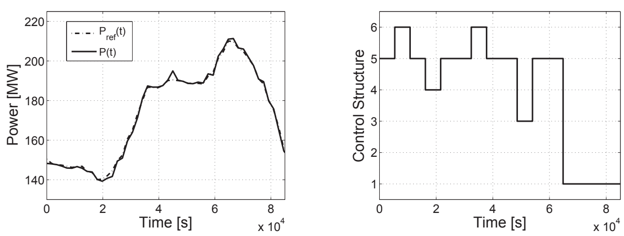


Fig. 6. Prices $c(t)$ used in the economic objective function $J_2(\mathbf{u}_c, x_0)$

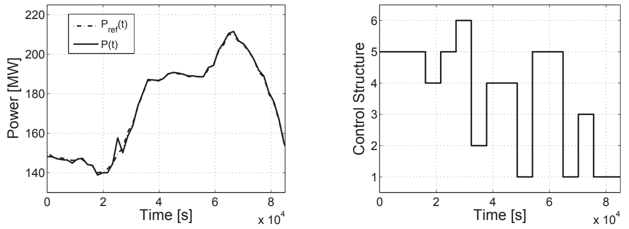
Figures 7, 8, and 9 show the results of power tracking and the selected H-MPC controller during the simulation, for the three cases MSE, MAE Economic, and Weighted sum (MO), respectively. The weighted sum case, which is a combination of the criteria MSE and Economic MAE, provides the best results in terms of both objective functions.



(a) Power performance, MSE (b) Switching, MSE

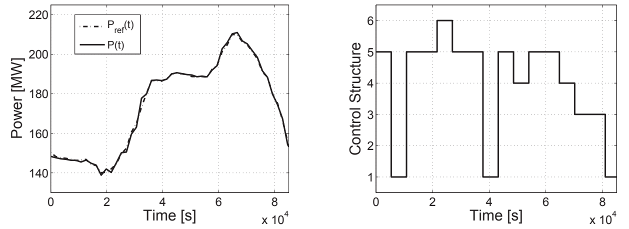
Fig. 7. HPV responses (MSE)

Table 1 presents the MAE and MSE comparison for the three switching configurations. The one using MSE reflects the negative effect of providing less power than the one needed (reference). Using MAE Economic, the structures generating less power than the power reference are penalized, but the tracking error increases. With the combination of both objective functions, we managed to reduce the undesired effects of the other controllers, reaching a better performance in terms of both MSE and MAE indices.



(a) Power performance, MAE Economic (b) Switching, MAE Economic

Fig. 8. HPV responses (MAE)



(a) Power performance, MO (b) Switching, MO

Fig. 9. HPV responses (MO)

Table 1. MAE and MSE of different schemes of selection

Criterion		MAE	MSE
$J_1(\mathbf{u}_c, x_0)$	MSE	0.8044	1.3469
$J_2(\mathbf{u}_c, x_0)$	MAE Economic	0.7996	2.0836
$J_3(\mathbf{u}_c, x_0)$	Multiobjective	0.6020	0.7381

In the example, it is highly surprising that better tracking *and* better economic performance are achieved by minimizing a cost function that penalizes a combination of tracking and economy, rather than one that penalizes tracking error only (or economic performance only). This must be due to the particular structure of the system, and because in the dynamic simulations just the starting point is the same in all the cases (in closed-loop, the performance is dependent of the different decision at every time step, and for the three controllers the inputs are different).

5. CONCLUSIONS AND FUTURE WORK

A hierarchical model predictive control (H-MPC) approach that adapts to different operational conditions by switching between solving sequence topologies using a multiobjective optimizer, is proposed in this paper. The approach was tested in the control of a hydro-power valley, showing its effectiveness in comparison with a fixed-structure H-MPC controller.

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