Linear Parameter Varying Modelling and Identification for Real-time Control of Open-flow Irrigation Canals

Yolanda Bolea, Vicenç Puig, Joaquim Blesa

Abstract— **Irrigation canals are open-flow water hydraulic systems, whose objective is mainly to convey water from its source down to its final users. They are large distributed systems characterized by non-linearity and dynamic behavior that depends on the operating point. Moreover, in canals with multiple reaches dynamic behavior is highly affected by the coupling among them. The physical model for those systems leads to a distributed-parameter model whose description usually requires partial differential equations (PDEs). However, the solution and parameter estimation of those PDE equations can only be obtained numerically and imply quite time-consuming computations that make them not suitable for real-time control purposes. Alternatively, in this paper, it will be shown that open-flow canal systems can be suitably represented for control purposes by using linear parameter-varying (LPV) models. The advantage of this approach compared to the use of PDE equation is that allows simpler models which are suitable for control design and whose parameters can be easily identified from input-output data by means of classical identification techniques. In this paper, the well known control-oriented, model named integral delay zero (IDZ), that is able to represent the canal dynamics around a given operating point by means of a linear time-invariant (LTI) model is extended to multiple operating points by means of an LPV model. The derivation of this LPV model for single-reach open-flow canal systems as well as its extension to multiple-reach open-flow canals is proposed. In particular, the proposed methodology allows deriving the model structure and estimating model parameters using data by means of identification techniques. Thus, a gray-box control model is obtained whose validation is carried out using single-pool and two-pool test canals obtaining satisfactory results.**

I. Introduction

Irrigation canals are open-flow water hydraulic systems, whose objective is mainly to convey water from its source down to its final users. They are large distributed parameter systems described by Saint-Venant's partial-differential equations (Chow, 1959) (Litrico, 2004) which are nonlinear partial derivative hyperbolic equations (distributed model). There is no known analytical solution in real geometry and these equations have to be solved numerically. Then, the hydraulic behavior of

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this canal system can be studied through various numerical methods such as the method of characteristics, several finite difference numerical schemes (either explicit or implicit), the well-known Preissmann implicit scheme among others, (see Cunge, 1980). Because of the complexity and the computational load of this complete distributed model, it presents little advantage for control purposes. It is therefore important to obtain simplified models of open-flow canals for control design. Such models would allow handling the dynamics of the system with few parameters; understanding the impact of physical parameters on the dynamics; and facilitating the development of a systematic design method (Litrico 2004). Then, for control purposes, the used models should be precise but simpler. So far, mainly linear time-invariant (LTI) models have been considered for control neglecting the non-linear behavior and the dependence of the parameters with the operating point. This is the case of the models as the integrator delay (ID) model (Schuurmans. 1999); the Hayami model (Litrico, 1999); the Muskingum model (Gomez, 2002); the integrator delay zero (IDZ) model (Litrico, 2004) that extends the ID model by adding a zero in the high frequencies range leading to a better fit in high frequencies and increasing the accuracy of the time-domain simulations; or black-box models identified using parameter estimation (Weyer, 2001) (Euren and Weyer 2007).

The SISO (single-input, single-output) model for a single pool system proposed in (Schuursmans, 1999) (where the input variables of the model are the reach's inflow and outflow discharges and the water level is obtained at its end) has been also used to generate state-space MIMO (multiple-input, multiple-output) models in (Clemmens, 2004a) (Clemmens, 2004b) (Wahlin, 2004) (Van Overloop, 2005). However, these types of models neither preserve the parameter/delay dependence with the operating point nor the coupling between pools (Litrico, 2009). In order to consider such effects, linear-parameter varying (LPV) models can be used. This type of models was already proposed by (Belforte, 2005) in the context of environmental systems described by parameter distributed models given in a form of partial differential equations. LPV models are based on a linear lumped parameter model whose parameters are not constant but a function of external parameters or/and the system states (operating point) (Rugh, 2000). For the previous reasons, a LPV control oriented model is suitable for the representation of the open-flow system dynamics.

The use of LPV models in open-flow irrigation canals is not new: In (Bolea et al. 2004) and (Bolea et al. 2005), a single-reach LPV model was proposed using empirical parameter estimation combining with some hydraulic rules and applied for control purposes in (Bolea et al. 2011). On the other hand, an LPV Hayami model was proposed for fault detection and control purposes in (Blesa et al., 2010) and (Bolea et al. 2013) with positive results. An equivalent modeling approach, named *state-*

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dependent parameter (SDP), has been used for characterizing flow and solute transport in real river systems (see Young (2011) and the references therein).

In this paper, an approach to obtain an LPV canal model for control is introduced by combining two methodologies:

- Physical modeling. The model structure is obtained by physical laws taking into account that the model parameters vary according the the operating conditions.

- Identification of model parameters at each operating point by means of the method of least squares and the interpolation of the dependence between operating points using some known function.

As a result, a gray-box modeling approach for obtaining an LPV canal control model based on the IDZ model (Litrico, 2004) is introduced. The IDZ model is nowadays a widely accepted model in the canal control community that has been rigorously derived by approximating the Saint-Venant's equations around a given operating point. In this paper, a first-order plus dead-time (FOPDT) LPV model which parameters have a physical meaning is obtained by the application of the IDZ modeling approach. The variation of these parameters can be approximated by polynomial functions and estimated by system identification techniques. The model is first formulated for SISO case and then extended to MIMO case. Finally, the model is validated using test bench canals (single-pool and two-pool open-flow canals).

The paper is organized as follows: In Section II, the control oriented open-flow canal modeling problem is introduced, and a single pool and a multiple pool test bench canals are described in order to verify the proposed modeling approach. In Section III, a physical LPV model based on the IDZ modeling approach is introduced in SISO case and also its MIMO extension. In Section IV, an LPV identification methodology is proposed to estimate the parameters of the proposed model structure. In Section V, the proposed modeling and identification approach is validated on the test bench canals (both, in the single pool canal and multivariable pool canal). Finally, main conclusions are given in Section VI.

II. THE GENERAL MODELLING PROBLEM

An irrigation canal is an open water hydraulic system, whose objective is mainly to convey water from reservoirs down to its final users. Cross structures (mainly hydraulic gates) are operated in order to control the water levels, discharges and/or volumes along this canal (see Figure 1). In this figure, a simplified view of a typical irrigation canal is presented. It receives water from a source and lets the water freely flow by gravity following the slope. The intermediate control gates, represented by vertical lines, regulate through their openings (U_i) the desired discharge, so as to maintain the water depth ($Y_{d\,i}$) in the points, where water is diverted for irrigational purposes (Q_{Li}).

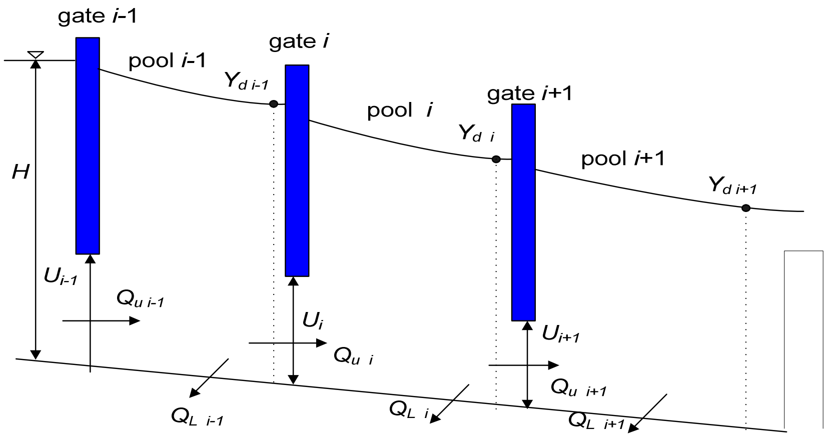

Figure 1. Irrigation canal schematic.

For modeling purposes, a natural way of partitioning a canal is dividing it into reaches (also called pools). A reach is a portion of a canal between two gates. So, a normal canal can have several reaches with different characteristics (length, slope, width, etc.). However, all the reaches share a common structure. Thus, the modeling problem for an irrigation canal consists in finding a suitable model for reaches. In that manner, the problem can be solved by an addition of the same model structure with only different parameter values.

Open flow canals are matter transport systems (distributed parameters) that present two features: a varying dynamics depending on the operating point and transport delays or large dead times that vary with the operating point. In MIMO case, with more than a single pool, an additional problem appears: the pool interactions, one or more outputs depend on more than a single input. This fact makes impossible to control each output independently and does not allow applying the well-known tools used in SISO systems. In this case, it is interesting to break down the MIMO problem into a set of independent SISO designs through decoupling. To decouple a plant is to achieve that each output depends on a single input. Then, the decoupling methods try to diagonalise the system or at least to assure the diagonal dominance. If it is impossible to decouple, it is required to use control techniques that support and treat in a natural manner multivariable systems (e.g. optimal control, predictive control and robust control). This increases the complexity of the solution since multivariable control theory leads to more complex controllers than single loop theory.

Next, the test bench canals features used in the present modeling study will be explained in detail.

A. Single reach (SISO) case

The single reach case consists of a single pool canal equipped with an upstream sluice gate and a downstream spillway (Figure 2). An electromotor is driving the gate position (U) and one sensor located at the end of the canal (in the spillway) is measuring the level Y_d. Upstream of this gate, there is a reservoir of constant depth H = 3.5 m. The total length of the pool is L=2km. The canal width, bottom slope and Manning roughness coefficient are B=2.5m, I_0= 5.10^{-4} and n=0.014. The operating range of the gate is limited to the interval $U \in [0, 0.9]$ m, the gate discharge coefficient and gate width are C_{dg}= 0.6 and b = 2.5m. Finally, the downstream spillway height and coefficient are Y_s = 0.7m and C_{ds}= 2.66, respectively.

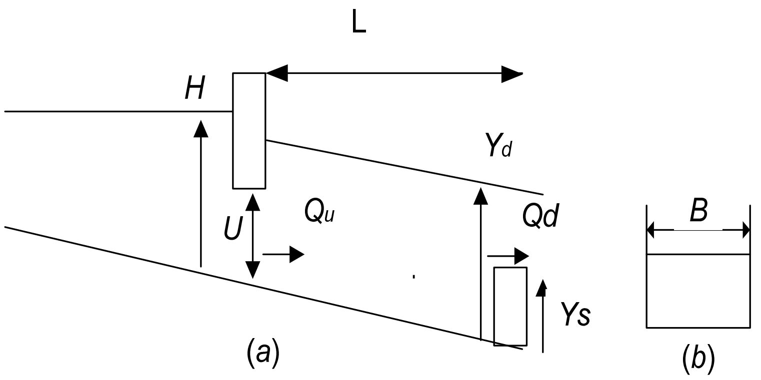

Figure 2. Canal scheme.(a) Longitudinal and (b) cross section.

B. Multiple reach (MIMO) case

The multiple reach case consists of a canal with two pools equipped with two sluice gates and a downstream spillway (see Figure 3). A servomotor is used in each gate to drive the control gate position (U_1 and U_2) and there are two level sensors located at the end of the two canal pools ($Y_d^{(1)}$ and $Y_d^{(2)}$). As in the SISO case, upstream of the first gate there is a reservoir with a constant level H = 3.5 m. The total length of the first pool is L_1 = 2 km while in the second L_2 = 4 km. The canal width, bottom slope and Manning roughness coefficient, operating range of the gate, gate discharge coefficient, gate width, spillway height and coefficient are the same as in the SISO case.

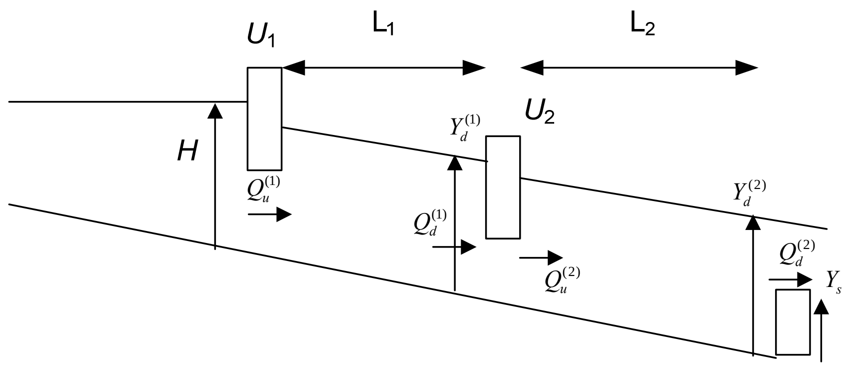

Figure 3. Two-pool canal system

In this paper, the "real" behaviour of the two bench test is accurately reproduced by the simulator developed in (Mantecon, 2002). This simulator solves numerically Saint-Venant's equations (Chow, 1959), which describes the dynamics of these test-bench canals by the conservation of mass and momentum principles in a one-dimensional free surface flow. This pair of partial-differential equations constitutes a non-linear and hyperbolic system that for an arbitrary geometry lacks of analytical solution. Thus, it has to be solved numerically and therefore the simulations are time-consuming for on-line applications.

III. LPV CONTROL MODEL DERIVATION

A. SISO LPVcontrol model derivation

The complete dynamics of the single-pool irrigation canal presented in Figure 2 is classically modeled with the Saint-Venant Equations. However, as discussed in Section I, for control purposes let us consider the IDZ model proposed in (Litrico, 2004). According to this modeling approach, the single canal reach dynamics can be approximated for low frequencies around the stationary values (denoted as Y_{u0} and Y_{d0} for upstream/downstream levels, Q_{u0}, and Q_{d0} for upstream/downstream flows) as follows

$$y_u(s) = P_{11}(s)q_u(s) + P_{12}(s)q_d(s) \tag{1a}$$

$$y_d(s) = P_{21}(s)q_u(s) + P_{22}(s)q_d(s) \tag{1b}$$

where: y_u and y_d for levels and q_d, and, q_u for flows denote upstream/downstream deviations from stationary values, $P_{12}(s) = \frac{-e^{-\tau_u s}}{A_u s}$, $P_{21}(s) = \frac{e^{-\tau_d s}}{A_d s}$ and $P_{22}(s) = \frac{-1}{A_d s}$ with τ_u and τ_d are the upstream and downstream transport delays and A_u and A_d are the upstream and downstream backwater areas, all depending on the operating point.

Remark: In the following, for simplicity the adjective "variation" in lowercase variables will be implicitly assumed and omitted in the text, capital letters in variables will denote absolute values and when subindex "0" is added refer to stationary values. Thus, a relation will be established among them and illustrated in the case of downstream level as follows: $Y_d = Y_{d0} + y_d$

Remark: Because downstream level equation (1b) is the most useful to design distant downstream or local upstream controllers (Litrico, 2006), only transfer functions $P_{21}(s)$ and $P_{22}(s)$ are considered in the reminder of this paper.

Remark: For a canal in uniform flow, the downstream time delay τ_d is equal to $L/(V+C)$ where L is the length of the pool, V is the water velocity and C is the celerity. According to (Litrico, 2004), the theoretical value of the downstream time delay is evaluated by computing the integral:

$$\tau_d = \int_0^L \frac{dx}{V(x) + C(x)} \tag{2}$$

This corresponds to the minimum time required for a perturbation to travel from upstream to the downstream of the pool. We recover the classical value in the uniform case when V and C are constant: $V = \frac{Q}{A}$ and $C = \sqrt{gD}$ where Q is the flow, A is the cross-sectional area, g is the acceleration of gravity and D is the hydraulic depth (cross-sectional area divided by top width). Then $\tau_d = L/(\mathrm{V+C})$ and $\tau_u = L/(\mathrm{C-V})$. Moreover, in the uniform case, the coefficient A_d of the $P_{21}(s)$ and $P_{22}(s)$ transfer functions reflects the way in which the downstream water level varies when the upstream and downstream discharge varies. It can be evaluated by computing the variation of the volume of the pool V_{pool} with respect to the downstream water elevation:

$$A_d = \frac{\partial V_{pool}}{\partial Y_d} \tag{3}$$

Then, it is clear that this coefficient depends on the way the volume changes, which is difficult to account for in a simple way (Litrico, 2004).

A linearised relation between the upstream flow q_u and the gate opening u around their stationary values Q_{u0} and U_0 can be introduced as follows

$$q_u(s) = k_u\, u(s) \tag{4}$$

where $k_u = \frac{\partial f_{k_u}}{\partial u}$ is a parameter varying respect to the operating point with $q_u = f_{k_u}(U)$ being the non-linear relation between the downstream flow q_u and gate opening U. In a similar way, a linearised relation between the downstream flow q_d and level y_d in the spillway can be established

$$q_d(s) = k_d\, y_d(s) \tag{5}$$

where $k_d = \frac{\partial f_{k_d}}{\partial q_d}$ is a parameter varying with respect to the operating point too with $q_d = f_{k_d}(U)$ being the non-linear relation between the downstream flow and level.

Combining equations Eqs.(1b), (4) and (5), the following first order plus time delay (FOPDT) model can be obtained

$$y_d(s) = G(s)u(s) = \frac{k_u P_{21}(s)}{1 - k_d P_{22}(s)} u(s) = \frac{k_u e^{-\tau_d s}}{A_d s + k_d} u(s) = \frac{K}{Ts+1} e^{-\tau_d s} u(s) \tag{6}$$

at each operating point with a gain $K = \frac{k_u}{k_d}$ and a time constant $T = \frac{A_d}{k_d}$. This result is in agreement with some previous works where a first order plus time delay model has been proposed without assuming an IDZ model as a starting point (Bolea, 2004).

Taking into account that the operating point can be characterized by the gate opening u, model (6) can be seen as a FOPDT LPV model (Bolea, 2004)[1]:

$$y_d(s,\theta) = G(s,\theta)u(s) = \frac{K(\theta)}{T(\theta)s+1} e^{-\tau_d(\theta)s} u(s) \tag{7}$$

where the scheduling variable is the gate opening (as proposed in Bolea, 2013), that is, $\theta = U = u + U_0$,. Similar result could be obtained by using as scheduling variable the downstream level (Bolea, 2011)[2]. Model (7) can be used to design a

[1] In the following, for simplicity and with abuse of notation, transfer functions are used for LPV systems, although computations are performed entirely in the time domain using the state space representation.

[2] Notice that using an upstream or a downstream variable could lead to an over optimistic or pessimistic model prediction because some dynamics (delays). This can be clearly seen looking at equations (4) and (5), where: if gate opening (upstream flow) is used as scheduling variable of the gains k_u and

controller that considers the canal dynamics for the whole set of operating points.

Remark: The parameter dependency with the operating point is indicated where it should be made explicit. It is not included everywhere in order to not overload the notation.

Following the method proposed in (Duviella et al., 2010), the operating range of the system has been divided in 4 operating points. Figure 4 presents step responses obtained by simulations at each operating point over the test canal (Bolea, 2005). From these figures, it can be observed that the obtained step response corresponds to the one of a first order system plus time delay. Moreover, evaluating the gain K, the constant time T and delay τ_d at the different operating points (see Table 1 in Section V.A), it can noticed that their values varies with the gate opening. Thus, the FOPDT LPV model (7) is in agreement with what can be observed from simulations using a high-fidelity model based on the Saint-Venant equations.

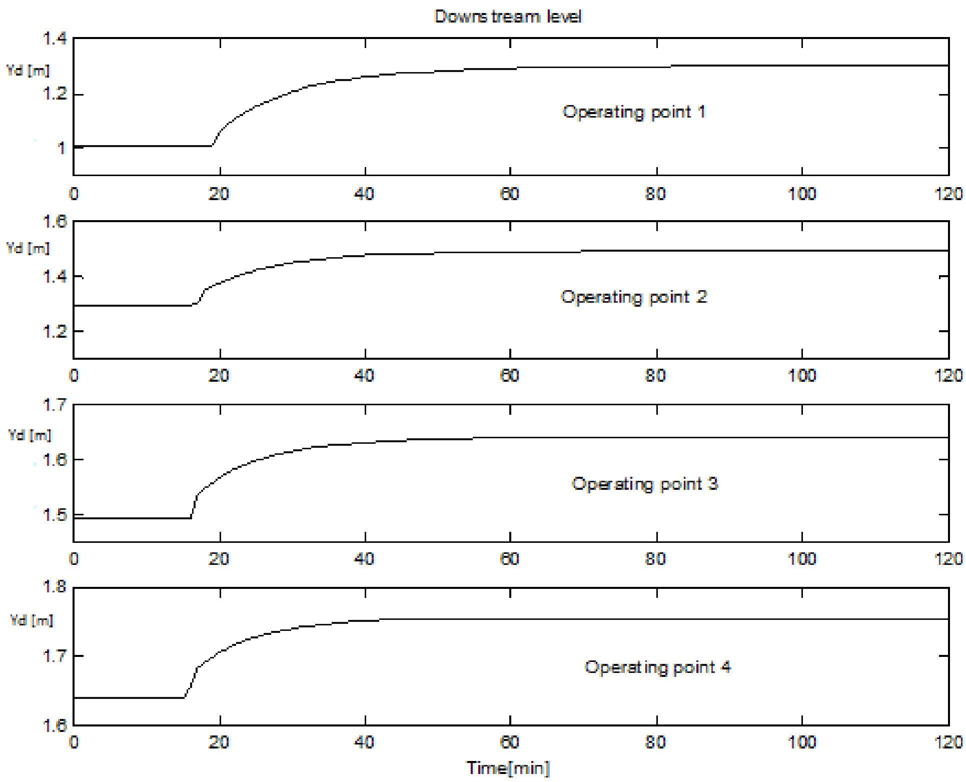

Figure 4. Time response to an opening gate step applied at t=10 min for the four operating points: 1 (θ from 0.1 to 0.3m), 2 (θ from 0.3 to 0.5m), 3 (θ from 0.5 to 0.7m) and 4 (θ from 0.7 to 0.9m) that cover of all system operation range.

Next, it is discussed how LPV parameters ($K(\theta)$, $\tau_d(\theta)$, $T(\theta)$) can be expressed in terms of the operating point θ from

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k_d, the gain k_u is right scheduled while k_d is scheduled with some anticipation. The contrary applies in case the downstream level (flow) is used as scheduling variable. The same applies for the other parameters as for example the case of the transportation delay. This means that selecting an upstream or an downstream variable as scheduling variable is right regarding some parameters but not that accurate for the others.

physical reasoning and experiments.

The dependency of the steady state gain *K* with the operating point θ can be extracted directly from linearizations (4) and (5). On the other hand, the delay $\tau_d(\theta)$ associated to the model (7), as was discussed above, can be derived by physical laws as $\tau_d = \frac{L}{V+C}$, that is equivalent to $\tau_d(\theta) = \frac{k_1}{1+k_2\theta}$, where k_1 and k_2 are functions of the operating point (θ) and the geometry of the canal, see (Bolea, 2005).

Finally the time constant $T(\theta)$ can be approximated considering the physical relation with the delay as was proposed in (Bolea et al., 2004)

$$T(\theta) \approx \kappa \tau_d(\theta) \tag{8}$$

where κ is an empirical constant that depends on the canal geometry

In the same way in (Litrico, 2004), considering that the test-bench canal is rectangular and assuming that this pool is small, the downstream area of the pool is assumed as a storage area. That is, $A_d = \alpha_d B L$ where *B* is the canal width, *L* is the canal length and α_d is a constant that takes into account only the downstream area (0< α_d <1), see Figure 5. Furthermore, considering $q_d = VA_w$ where A_w is the variation of cross-sectional area of the water at the end of the pool that is given by $A_w = By_d$. Then

$$T = \frac{\alpha_d BL}{VA_w / y_d} = \frac{\alpha_d BL}{VBy_d / y_d} = \alpha_d \frac{L}{V} \tag{9}$$

and considering that the estimated delay can be expressed as $\tau_d = \frac{L}{V+C}$, we have that $T = \alpha_d \frac{L}{V} = \alpha_d \frac{(V+C)}{V} \tau_d$. Since v varies according to the operating point θ, the previous equations can be rewritten as:

$$T(\theta) = \alpha_d \frac{L}{V(\theta)} = \alpha_d \frac{V(\theta)+C(\theta)}{V(\theta)} \tau_d(\theta) \tag{10}$$

In (Bolea *et al.,* 2004), the authors observe experimentally that the constant of test-bench canal system has the the form of Eq.(8). Therefore, Eq.(8) is equivalent to Eq.(10) where: $\kappa = \alpha_d \frac{V+C}{V}$. Finally, since V and C vary according to θ, the parameter κ varies in the range: $\kappa \in \left[\alpha_d \frac{V+C}{V}\Big|_{\min}, \alpha_d \frac{V+C}{V}\Big|_{\max} \right]$

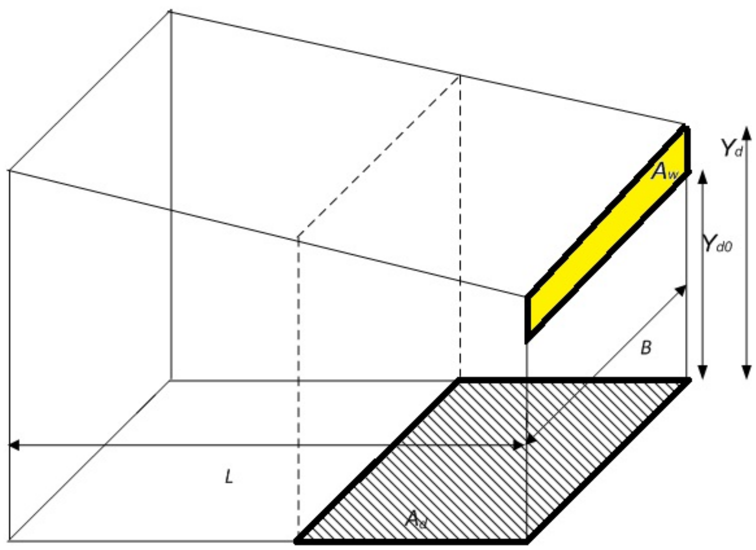


Figure 5. Downstream and wet areas of a small pool or a pool with very small slope.

B. Extension to multiple pool canals: Proposed MIMO LPV control model

So far, an LPV control model has been proposed for a single pool canal (SISO case). In this section, this type of model is extended to the MIMO case, i.e., to an *n*-pool canal system. In this methodology, each pool is modeled around a given operating point using a transfer function matrix (for illustrative purposes, a two-pool irrigation canal is assumed, see Figure 3). This modeling approach applied to the first pool and their hydraulic structures leads to:

- Pool 1:

$$y_d^{(1)}(s) = P_{21}^{(1)}(s) q_u^{(1)}(s) + P_{22}^{(1)} q_d^{(1)}(s) \tag{11}$$

where $y_d^{(1)}$ is the downstream level of the pool 1, $q_u^{(1)}$ is the upstream flow of the gate 1 and $q_d^{(1)}$ is the downstream flow of the pool 1.

- Upstream gate of the pool 1:

$$q_u^{(1)}(s) \approx k_{u_1} u_1(s) \tag{12}$$

- Gate between the pool 1 and the pool 2

The interactions with the flow in the gate between pools 1 and 2 are linearized as proposed in (Litrico, 2006):

$$q_d^{(1)}(s) \approx q_u^{(2)}(s) = k_d^{(1)} y_d^{(1)}(s) + k_{u_2} u_2(s) \tag{13}$$

Replacing (12-13) in (11) leads to:

$$y_d^{(1)}(s) = P_{21}^{(1)}(s) k_{u_1} u_1(s) + P_{22}^{(1)}(s)(k_d y_d^{(1)}(s) + k_{u_2} u_2(s)) \tag{14}$$

that after some algebraic manipulations can be written as follows

$$y_d^{(1)}(s) = \frac{P_{21}^{(1)}(s) k_{u_1}}{1 - P_{22}^{(1)}(s) k_d^{(1)}} u_1(s) + \frac{P_{22}^{(1)}(s) k_{u_2}}{1 - P_{22}^{(1)}(s) k_d^{(1)}} u_2(s) \tag{15}$$

Taking into account: $P_{21}^{(1)}(s) = \frac{e^{-\tau_d^{(1)}}}{A_d^{(1)} s}$ and $P_{22}^{(1)}(s) = \frac{-1}{A_d^{(1)} s}$ where A_d is the downstream backwater area and $\tau_d{}^{(1)}$ is the downstream propagation delay, the following model is obtained (with the desired structure)

$$y_d{}^{(1)}(s) = \frac{k_{u_1} e^{-\tau_{11} s}}{A_d{}^{(1)} s + k_d^{(1)}} u_1(s) - \frac{k_{u_2}}{A_d{}^{(1)} s + k_d^{(1)}} u_2(s) = \frac{K_{11} e^{-\tau_{11} s}}{T_{11} s + 1} u_1(s) + \frac{K_{12} e^{-\tau_{12} s}}{T_{12} s + 1} u_2(s) \tag{16}$$

where

$$T_{11} = \frac{A_d^{(1)}}{k_d^{(1)}},\ T_{12} = \frac{A_d^{(1)}}{k_d^{(1)}},\ \tau_{11} = \tau_d^{(1)},\ \tau_{12} = 0,\ K_{11} = \frac{k_{u_1}}{k_d^{(1)}},\ K_{12} = -\frac{k_{u_2}}{k_d^{(1)}}$$

Remark: Equation (16) is composed by two transfer functions that relate the two inputs (opening gates) with the output (downstream level in pool 1). The new notation T_{ij}, τ_{ij} and K_{ij} has been introduced in order to emphasize the output (subscript *i*) and input (subscript *j*).

On the other hand, applying the same modeling methodology to the second pool and its hydraulic structure leads to:

- Pool 2:

$$y_d^{(2)}(s) = P_{21}^{(2)}(s) q_u^{(2)}(s) + P_{22}^{(2)} q_d^{(2)}(s) \tag{17}$$

where $y_d^{(2)}$ is the downstream level of the pool 2, $q_u^{(2)}$ is the downstream flow of the gate 2 and $q_d^{(2)}$ is the downstream flow of the pool 2.

- Spillway of the pool 2:

$$q_d^{(2)}(s) \approx k_d^{(2)} y_d^{(2)}(s) \tag{18}$$

where $k_d^{(2)} = \frac{\partial f_{k_d^{(2)}}}{\partial y_d^{(2)}}$ is a parameter that varies with the operating point too where $q_d^{(2)} = f_{k_d^{(2)}}(y_d^{(2)})$ is the non-linear relation between the downstream flow and level.

Replacing (13) and (18) in equation of (17) produces:

$$y_d^{(2)}(s) = P_{21}^{(2)}(s)(k_d^{(1)} y_d^{(1)}(s) + k_{u_2} u_2(s)) + P_{22}^{(2)}(s)(k_d^{(2)} y_d^{(2)}(s))) \tag{19}$$

that leads to

$$y_d^{(2)}(s) = \frac{P_{21}^{(2)}(s)}{1 - P_{22}^{(2)}(s) k_d^{(2)}} (k_d^{(1)} y_d^{(1)}(s) + k_{u_2} u_2(s)) \qquad (20)$$

Replacing (15) in (20) leads to

$$y_d^{(2)}(s) = \frac{p_{21}^{(2)}(s) k_{u_1}}{1 - p_{22}^{(2)}(s) k_d^{(2)}} \left(k_d^{(1)} \frac{P_{21}^{(1)}(s) k_{u_1}}{1 - P_{22}^{(1)}(s) k_d^{(1)}} u_1(s) + k_d^{(1)} \frac{P_{22}^{(1)}(s) k_{u_2}}{1 - P_{22}^{(1)}(s) k_d^{(1)}} u_2(s) + k_{u_2} u_2(s) \right) \qquad (21)$$

Taking into account that: $P_{21}^{(1)}(s) = \frac{e^{-\tau_d^{(1)}}}{A_d^{(1)} s}$, $P_{22}^{(1)}(s) = \frac{-1}{A_d^{(1)} s}$, $P_{21}^{(2)}(s) = \frac{e^{-\tau_d^{(2)}}}{A_d^{(2)} s}$ and $P_{22}^{(2)}(s) = \frac{-1}{A_d^{(2)} s}$, Eq. (21) leads to:

$$y_d^{(2)}(s) = \frac{e^{-\tau_d^{(2)}}}{A_d^{(2)} s + k_d^{(2)}} \left(k_d^{(1)} \frac{e^{-\tau_d^{(1)}} k_{u_1}}{A_d^{(1)} s + k_d^{(1)}} u_1(s) - k_d^{(1)} \frac{k_{u_2}}{A_d^{(1)} s + k_d^{(1)}} u_2(s) + k_{u_2} u_2(s) \right) \qquad (22)$$

that can be rearranged as follows

$$y_d^{(2)}(s) = \frac{e^{-(\tau_d^{(1)} + \tau_d^{(2)})} k_d^{(1)} k_{u_1}}{(A_d^{(2)} s + k_d^{(2)})(A_d^{(1)} s + k_d^{(1)})} u_1(s) + \frac{e^{-\tau_d^{(2)}} A_d^{(1)} k_{u_2} s}{(A_d^{(2)} s + k_d^{(2)})(A_d^{(1)} s + k_d^{(1)})} u_2(s) \qquad (23)$$

The transfer functions in (23) can be approximated as follows according to (Skogestad, 2004)

- the first transfer function behaves approximately as a first order model with a time constant that is the sum of the time constants of the second order model,
- while the second one leads to a first order model under the assumption $k_d^{(1)} << A_d^{(1)}$ that induces a zero-pole cancellation.

Then, following these approximations Eq. (23) leads to

$$y_d^{(2)}(s) \approx \frac{K_{21} e^{-\tau_{21} s}}{T_{21} s + 1} u_1(s) + \frac{K_{22} e^{-\tau_{22} s}}{T_{22} s + 1} u_2(s) \qquad (24)$$

where

$$T_{21} = \frac{A_d^{(1)}}{k_d^{(1)}} + \frac{A_d^{(2)}}{k_d^{(2)}}, T_{22} = \frac{\frac{A_d^{(2)}}{k_d^{(2)}}\frac{A_d^{(1)}}{k_d^{(1)}}}{\left(\frac{A_d^{(2)}}{k_d^{(2)}} + \frac{A_d^{(1)}}{k_d^{(1)}}\right)}, \tau_{21} = \tau_d^{(1)} + \tau_d^{(2)},\ \tau_{22} = \tau_d^{(2)},\ K_{21} = \frac{k_{u_1}}{k_d^{(2)}},\ K_{22} = \frac{A_d^{(1)}\frac{k_{u_2}}{k_d^{(2)}k_d^{(1)}}}{\left(\frac{A_d^{(2)}}{k_d^{(2)}} + \frac{A_d^{(1)}}{k_d^{(1)}}\right)}$$

Thus, the MIMO control model for the two pool canal system is:

$$\begin{bmatrix} y_d^{(1)} \\ y_d^{(2)} \end{bmatrix} = \begin{bmatrix} \frac{K_{11}e^{-\tau_{11}s}}{T_{11}s+1} & \frac{K_{12}e^{-\tau_{12}s}}{T_{12}s+1} \\ \frac{K_{21}e^{-\tau_{21}s}}{T_{21}s+1} & \frac{K_{22}e^{-\tau_{22}s}}{T_{22}s+1} \end{bmatrix} \begin{bmatrix} u_1(s) \\ u_2(s) \end{bmatrix} \tag{25}$$

As it can be seen from Eq. (25), the relationship among the downstream levels and their gate openings are represented by using first order transfer functions. This can be verified by simulating the two-pool canal system using the Saint-Venant equations for a given operating point is shown in Figure 6.

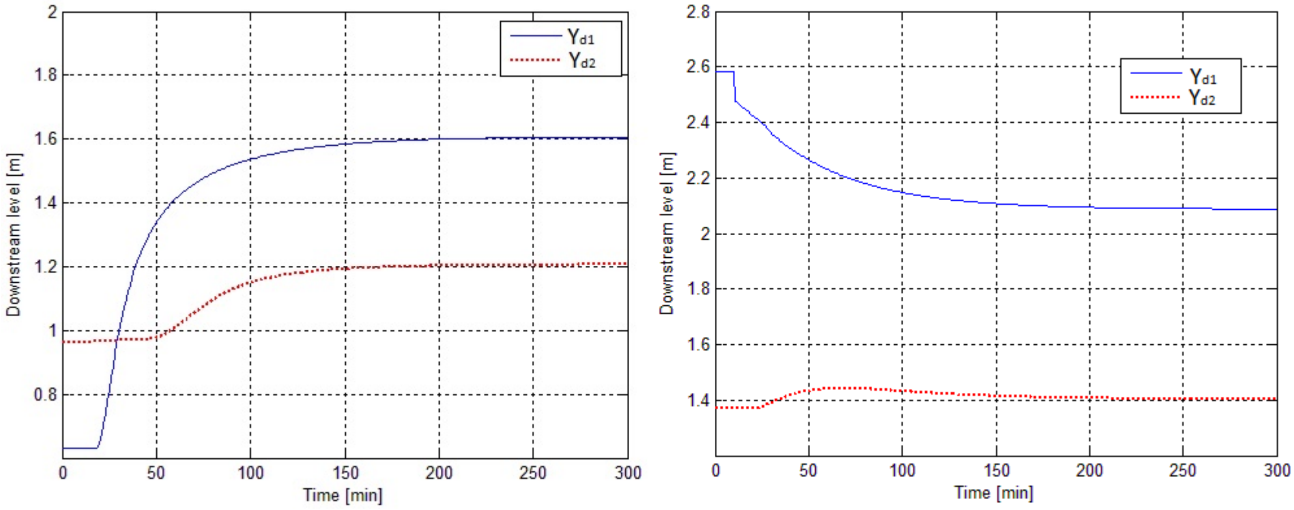


Figure 6. Step time responses for different operating points in two-pool canal. (Left) (U_1 from 0.1 to 0.3m and U_2 remains at 0.5m), (Right) (U_1 remains at 0.5m and U_2 from 0.5 to 0.7m),

IV. LPV Control Model Identification

A. LPV model identification review

In the literature, there exist two main approaches for the identification of LPV (or SDP) models (Norton, 1975; Young, 2002 and 2011; Norton 2005): a global (see e.g. (Lee, 1999; Bamieh, 2002; Verdult, 2005; Felici, 2007) and a local (see e.g. (Steinbuch, 2003; Wassink, 2005; Lovera, 2007; Paijmans, 2008; De Caigny, 2009) one.

The global approach is based on the assumption that it is possible to perform a global identification experiment by exciting the system while the scheduling parameters are persistently changing the system dynamics. This assumption, however, may be difficult to satisfy in many cases. In the case that it is impossible to perform a global experiment, it is appropriate to use the local LPV identification approach, based on the interpolation of a set of local LTI models that are estimated using a set of local measurements, obtained by exciting the system at different fixed operating conditions, that is, for constant values of the scheduling parameters. The local approach has the important practical advantage that many engineers are well-experienced in LTI identification experiments and that the local LTI models can be estimated using a wide variety of well established and widely spread LTI identification algorithms. To properly interpolate these local models, all local LPV identification techniques require that an appropriate methodology is applied to construct an LPV model that interpolates these consistent local models. As the local models can be either continuous- or discrete-time, both continuous- and discrete-time LPV models can be obtained, which is another advantage of the local approach.

B. LPV model identification approach

In this paper, a methodology inspired in the one proposed (De Caigny, 2009) is used. Given the range on input values, a set of *M* operating points is used. For each operating point *i*, the parameters of the model β^i are estimated by solving a least-square parameter estimation problem using a set of *N* input/output data

$$\begin{aligned}
&\min_{\beta^i} \sum_{k=1}^{N} (y(t_k) - \hat{y}(t_k, \beta^i))^2 \\
&s.t. \\
&\hat{y}(t_k, \beta^i) = \Psi^T(t_k)\beta^i \qquad t_k \in \{t_1, \cdots, t_N\}
\end{aligned} \tag{26}$$

where Ψ^T is the regressor vector that contains inputs *u* and outputs *y* as well as their derivatives. The parameter estimation problem formulated in (26) is solved using the MATLAB Identification Toolbox using the '*idproc*' command that allows the system identification in continuous time. Other software especially addressed to system identification in continuous time as the CAPTAIN toolbox could be used (Taylor et al. 2007).

Once the parameters β^i have been obtained, the LPV parameters are obtained by interpolating the parameters of each operating point using a polynomial by solving the following least-square parameter estimation problem

$$\begin{aligned}&\min_{(\alpha_o^i,\alpha_1^i,\cdots,\alpha_{n_\alpha}^i)} \sum_{i=1}^{M}(\beta^i(\theta_i)-\hat{\beta}^i(\theta_i))^2\\&s.t.\\&\hat{\beta}^i(\theta_k)=\alpha_o^i+\alpha_1^i\theta_k+\cdots+\alpha_{n_\alpha}^i\theta_k^{n_\alpha}\qquad \theta_k\in\{\theta_1,\cdots,\theta_M\}\end{aligned} \tag{27}$$

where θ_k is the scheduling variable used to change parameters according to the operating point. The parameter estimation problem formulated in (27) can be solved using the MATLAB Optimization Toolbox.

V. RESULTS

Once the LPV control model for single and multiple reach open-flow canals have been presented they will be applied to the benchmark case studies presented in Section II. The LPV SISO model of the single pool canal represented by (6) and the LPV MIMO model of the multiple pool canal represented by (25). Then, the parameters of both LPV models are estimated by the method described in Section IV. In the single reach case, each linear varying parameter uses as scheduling variable the gate opening (U) while in the MIMO case the LPV model parameters depends on the two gate openings (U_1,U_2).

A. Application to single reach case

In the single reach case, the model structure is given by the LPV FOPDT presented in Eq. (6). Table 1 presents the values of the parameters of the LPV FOPDT models (K,T,τ) for different operating points that corresponds to different gate openings obtained by solving the least squares parameter identification problem (26).

U	**PARAMETERS**		
	K	*T*[sec]	*τ*[sec]
0.1-0.3	1.440	525	564
0.3-0.5	0.966	420	448.40
0.5-0.7	0.728	366	377.73
0.7-0.9	0.576	312	351.97

Table 1. LPV FOPDT in each operating point.

Parameter variation with the operating point (U) can be described using the following polynomials

$$K(U) = a_2\, U^2 + a_1\, U + a_0$$

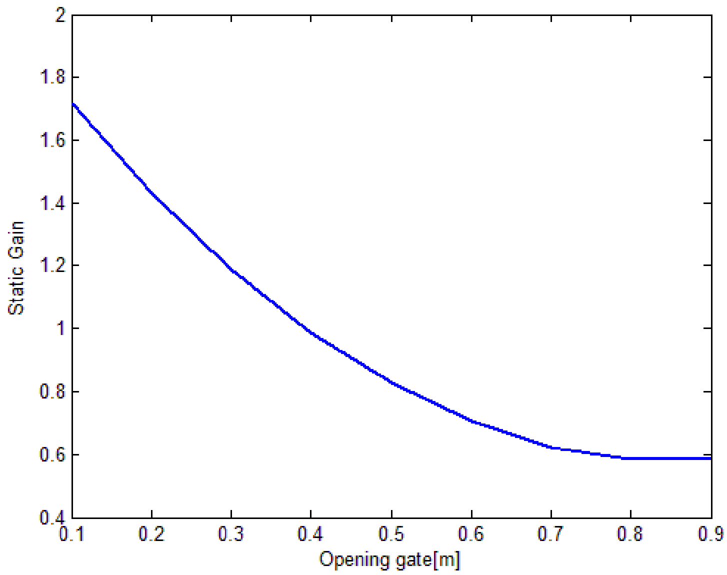
$$T(U) = b_2\, U^2 + b_1 U + b_0 \tag{28}$$

$$\tau(U) = c_2\, U^2 + c_1\, U + c_0$$

whose parameters are determined by interpolating the operating points presented in Table 1 by solving the least square parameter estimation problem (27).

	j=2	j=1	j = 0
a_j	2.01	-3.42	2.04
b_j	9.35	-15.24	12.07
c_j	5.30	-11.08	10.71

Table 2. Coefficients of the polynomials in SISO case.



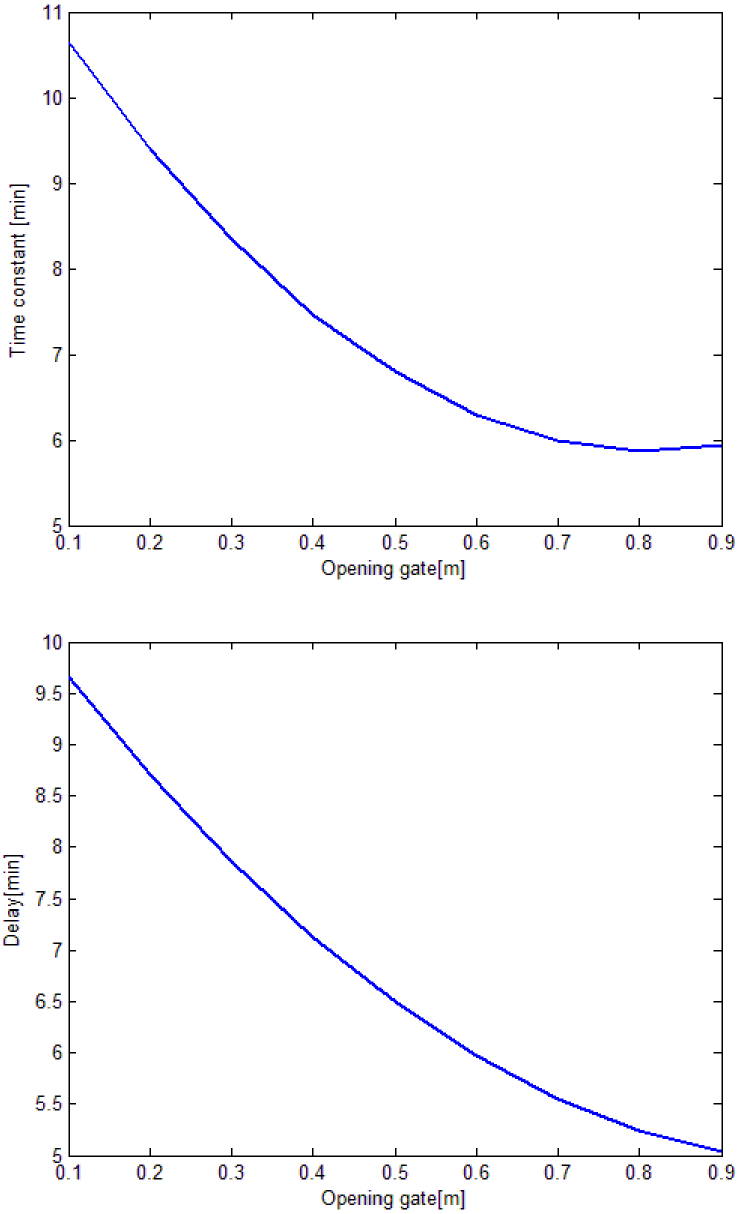


Figure 7. Evolution of the variable parameters of the SISO model. (Up) static gain, (middle), time constant and (down) delay coefficients of the model.

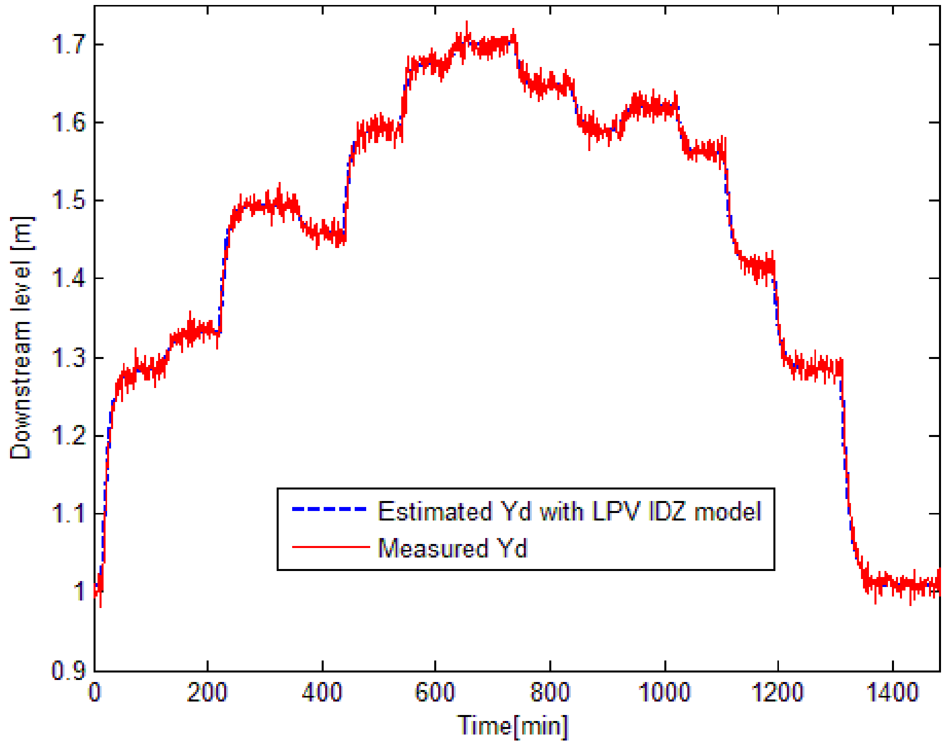


Figure 8. Comparison between the measured an estimated rich temporal responses in SISO case in a scenario model in a scenario that sweeps all the operation range.

The parameters of the SISO model are represented in Figure 7 up, middle and down. The dependency and the evolution of these parameters (static gain, constant time and delay) with the operating point (gate opening) are shown in a clear way in these figures.

Figure 8 shows the comparison between the real (simulated by means of the Saint-Venant equations) and estimated temporal output (downstream) level canal response by the proposed LPV model inside of all the operation range of the single canal. The most prevalent methods for model evaluation are residual methods, which calculate the difference between observed (measured, 'real') data and modeled data. Of the many possible numerical calculations on model residuals, by far the most common are bias and mean square error (MSE). Bias is simply the mean of the residuals, indicating whether the model tends to under-or-over-estimate the measured data, with an ideal value zero. However, positive and negative errors tend to cancel each other out. To prevent such cancelation, the mean square error criterion squares the residuals before calculating the mean, making all contributions positive and penalizing greater errors more heavily, perhaps reflecting the concerns of the user (Bennet *et al*., 2013). For this reason MSE is selected to validate the proposed model in this work, but using the root of the MSE (RMSE) to express the error metric in the same units as the original data. The RMSE of the proposed SISO LPV model is of the order of 10^{-6} m^2, a very suitable result.

B. Application to multiple reach case

In the multiple reach case, the model structure is given by the LPV FOPDT presented in Eq. (25). Varying the operating point by means of the gate openings (U_1,U_2) in the range $U_j \in [0,0.9]$ j=1,2, the parameters of the system vary in these intervals:

$$K_{11}=[5\ ,1.65],\ K_{21}=[1.5,0.15],\ K_{12}=[-4.35,-1.1],\ K_{22}=[1.7,\ 0.12]$$

$$T_{11}=[1900,1550],\ \ T_{21}=[1900,1800],\ T_{12}=[2100,1400], T_{22}=[1500,1430]$$

$$\tau_{11}=[580,300], \tau_{21}=[1650,1200],\ \tau_{12}=0,\ \tau_{22}=[1010,840]$$

with constant time and delay expressed in seconds. Parameters of the LPV FOPDT models (K,T,τ) can be identified in different operating points corresponding to different gate openings by solving the least squares parameter identification problem (27). Then, parameter variation with the operating point (U_1, U_2) can be described using the following polynomials

$$K_{ij}\ (U_j) = a_{ij\ 2}\ U_j^2 + a_{ij\ 1}\ U_j + a_{ij\ 0}$$

$$T_{ij}\ (Uu_j) = b_{ij\ 2}\ U_j^2 + b_{ij\ 1}\ U_j + b_{ij\ 0} \qquad (30)$$

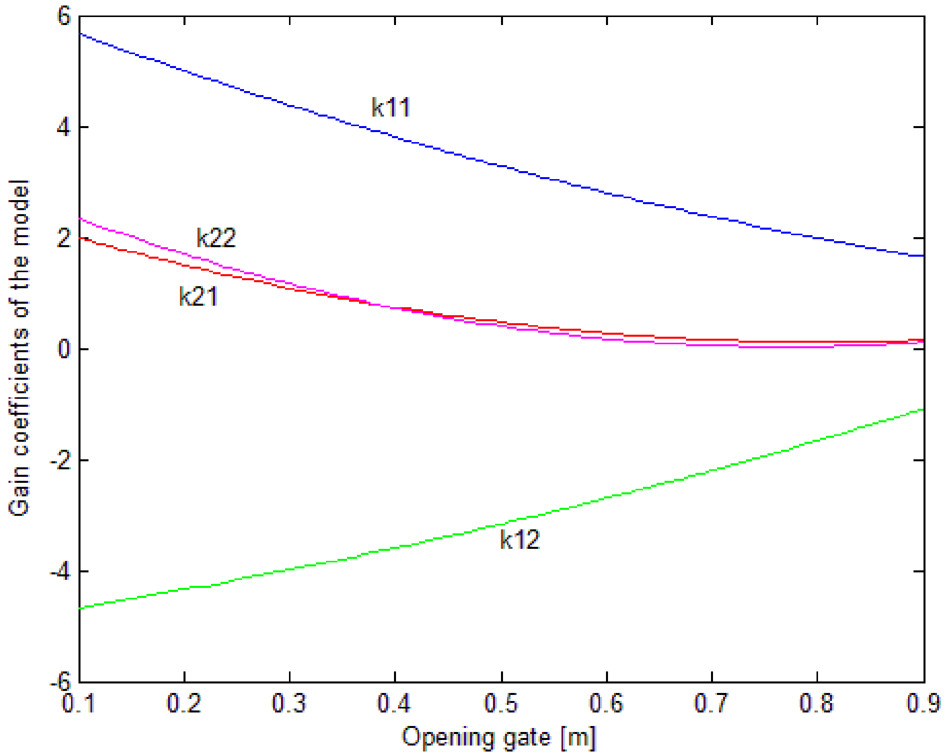
$$\tau_{ij}\ (U_j) = c_{ij\ 2}\ U_j^2 + c_{ij\ 1}\ U_j + c_{ij\ 0}$$

whose parameters are determined by interpolating the operating points presented in Table 3 by solving the least square parameter estimation problem (27) and that are represented graphically in function of (U_1, U_2) in Figure 9. We can see the strong dependence of the values of these parameters with the operating points (opening gates).

	$a_{ij\ 2}$	$a_{ij\ 1}$	$a_{ij\ 0}$	$b_{ij\ 2}$	$b_{ij\ 1}$	$b_{ij\ 0}$	$c_{ij\ 2}$	$c_{ij\ 1}$	$c_{ij\ 0}$

$U_1 \in [0,0.9], j=1$	$i=1$	2.38	-7.40	6.38	1308.45	-1925.01	2222.66	881.42	-1398.14	844.37
	$i=2$	3.77	-6.08	2.56	-296.43	204.64	1855.92	580.47	-1281.38	1883.06
$U_2 \in [0,0.9], j=2$	$i=1$	1.77	2.70	-4.96	989.05	-1502.23	2360.87	0	0	0
	$i=2$	5.20	-7.98	3.10	-5.95	-107.73	1521.78	-550.77	362.99	959.47

Table 3. Coefficients of the polynomials presented in Eq. (30)



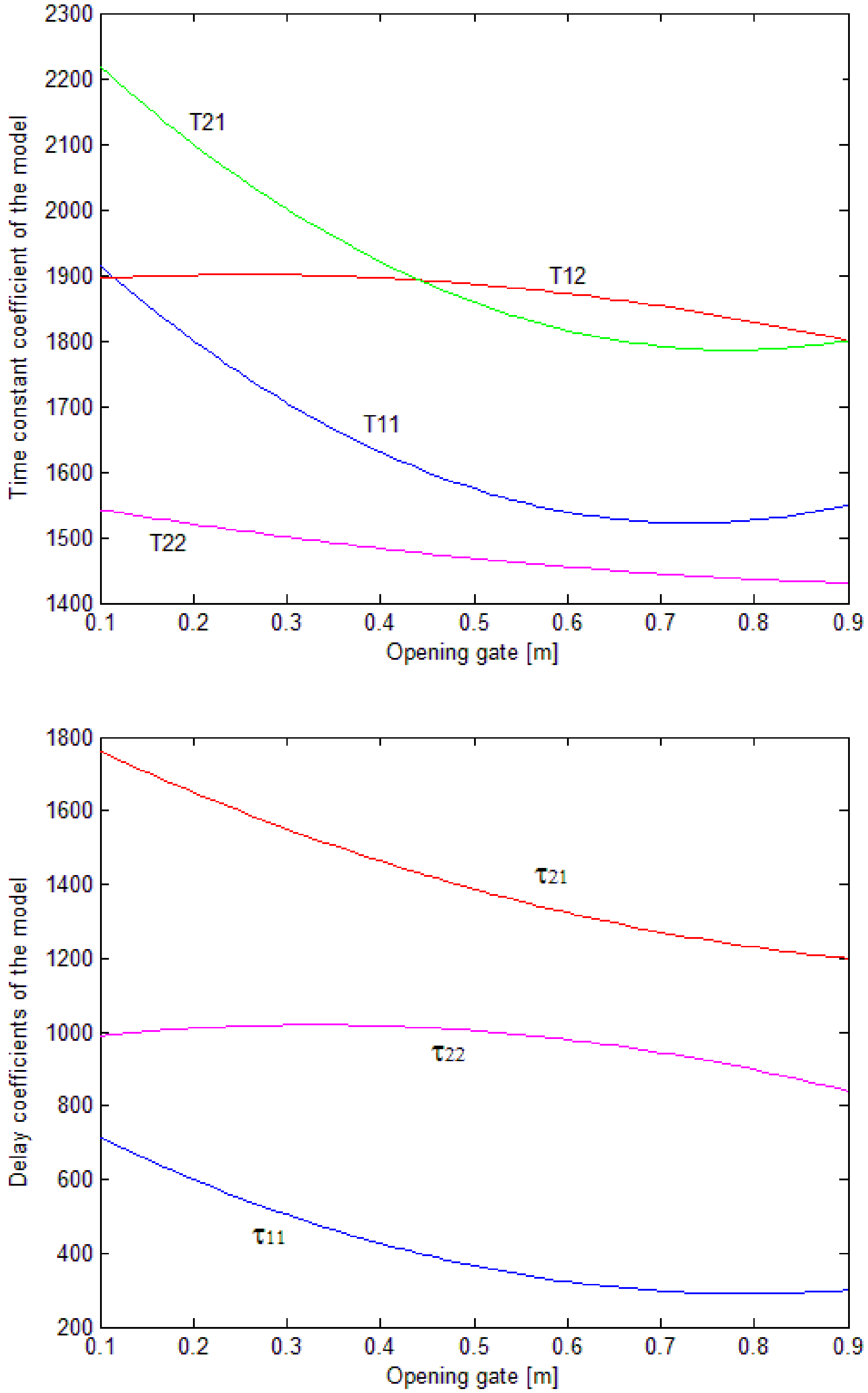
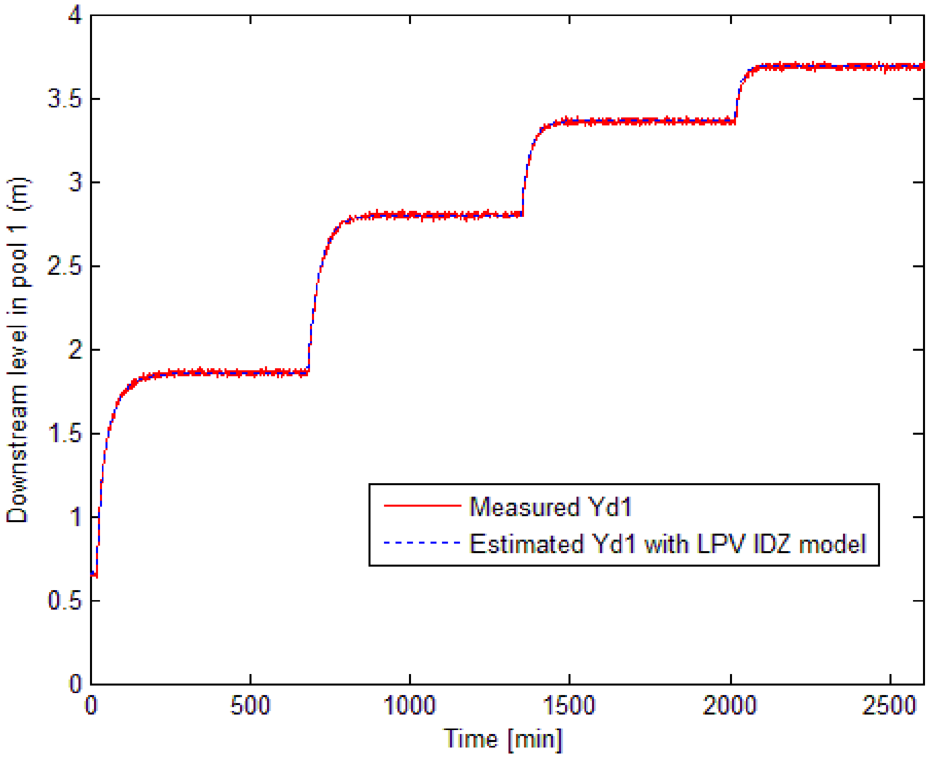
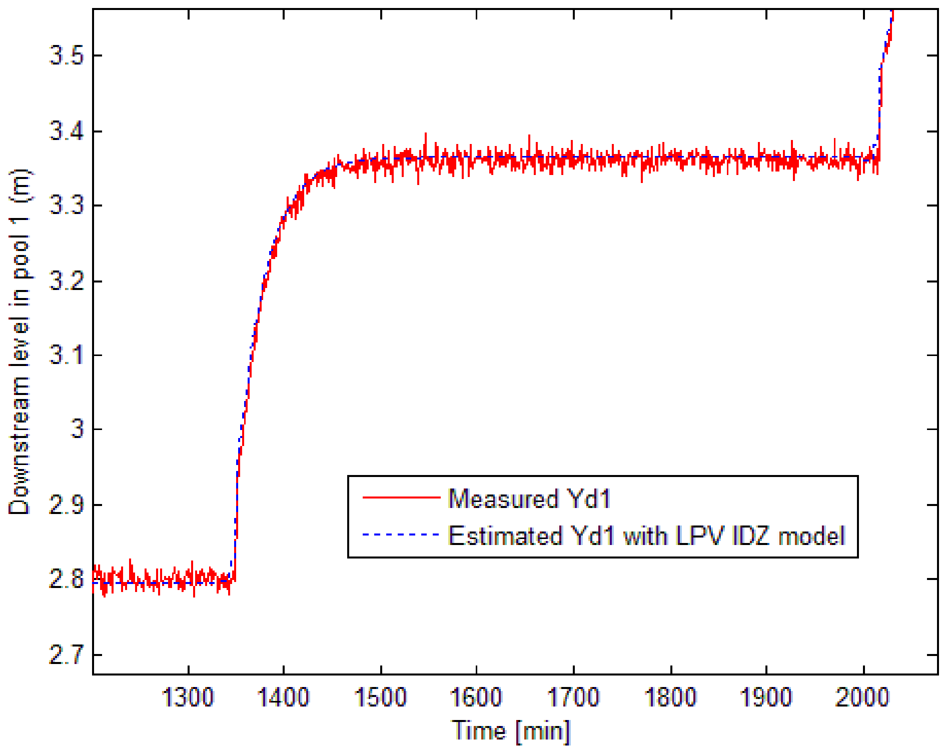


Figure 9. Evolution of the variable parameters of the MIMO model. (Up) gain, (middle), time constant and (down) delay coefficients of the model.

Figure 10 shows the comparison between the real (simulated by means of the Saint-Venant equations) and estimated temporal multiple reach canal response by the proposed LPV model in a scenario that sweeps all the operation range. As in the case of the single reach canal, the root mean square error of the LPV model is of the order of $10^{-6}m^2$.




Downstream level in pool 1 (m)
3.5
3.4
3.3
3.2
3.1
3
2.9
2.8
2.7
1300
1400
1500
1600
1700
1800
1900
2000
Time [min]
Measured Yd1
Estimated Yd1 with LPV IDZ model

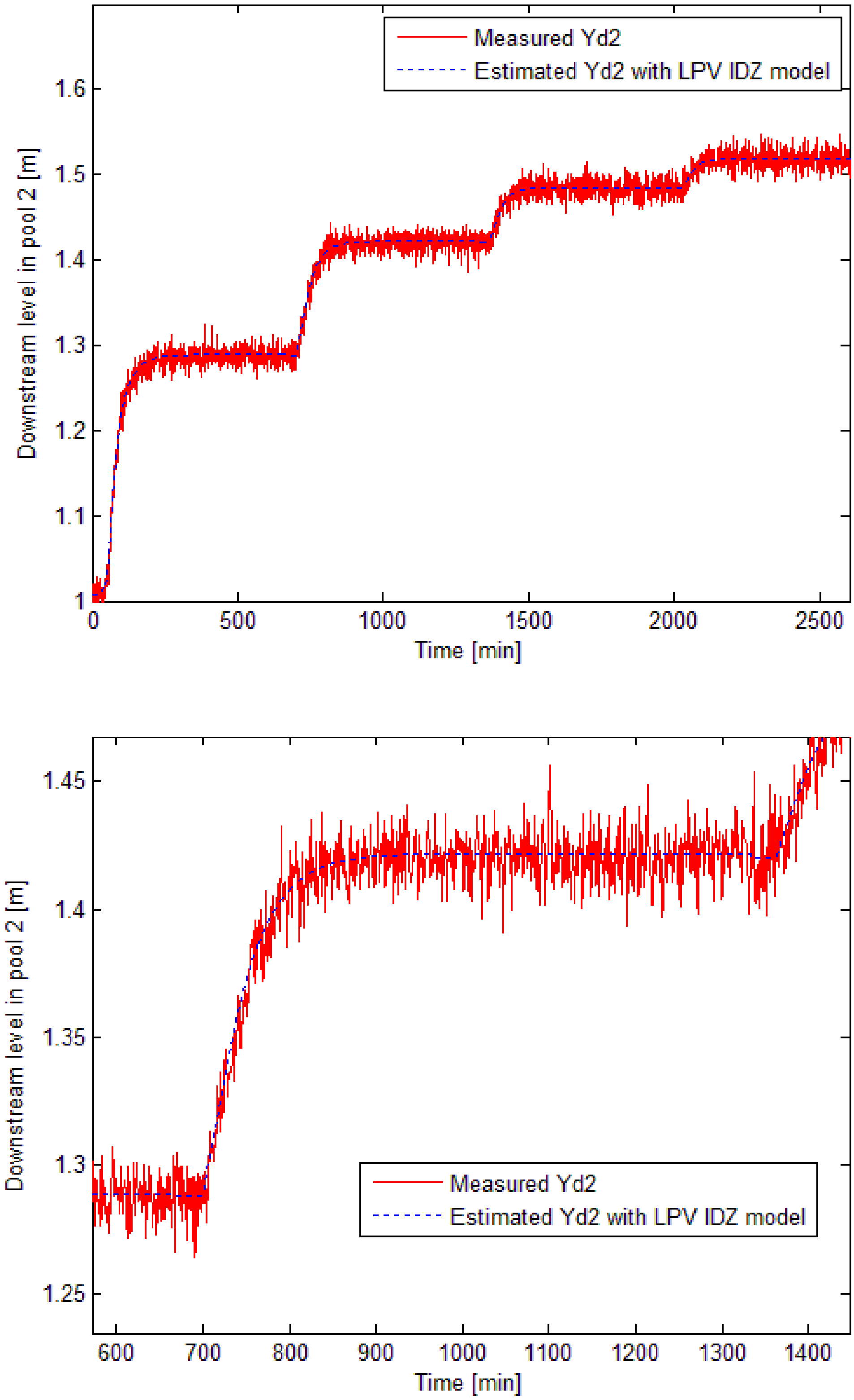


Figure 10. Comparison between the measured an estimated temporal responses in two-pool canal in a scenario that sweeps all the operation range, in right details of the graphics of the left.

VI. Conclusions

Because of open flow canals are systems whose parameters depend on the operating point, it is appropriate to catch such dependence using a LPV model. In this paper, LPV models for open flow canals are obtained by means of a gray-box approach: the structure of an IDZ model is selected to obtain a LPV controlmodel by physical modeling and the estimation of the LPV parameters is carried out by least square optimization methods. Firstly, the LPV model is developed for single pool canal systems, and subsequently an extension for MIMO case is presented. In both cases, hydraulic laws allow a first order plus delay time (FOPDT) structure selection of the model and deriving the parameters dependence with the operating point. The variation of these parameters is approximated in a polynomial way and their parameters are estimated by optimization techniques without using the hydraulic knowledge but only selecting the structure and the parameters dependence with the operating point obtained from the LPV physical model. This aspect is especially important using LPV identification because the functions that describe the parameter dependence with the operating point should be provided at the beginning of the identification process. In particular, in this paper these functions are assumed to be second order polynomials in SISO and MIMO case as well. The obtained LPV model has been tested in a single pool test bench canal and also in a two-pool test bench canal proposed as case studies, obtaining in all the cases satisfactory results.

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