Robust optimization based energy dispatch in smart grids considering demand uncertainty

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Abstract: In this study we discuss the application of robust optimization to the problem of economic energy dispatch in smart grids. Robust optimization based MPC strategies for tackling uncertain load demands are developed. Unexpected additive disturbances are modelled by defining an affine dependence between the control inputs and the uncertain load demands. The developed strategies were applied to a hybrid power system connected to an electrical power grid. Furthermore, to demonstrate the superiority of the standard Economic MPC over the MPC tracking, a comparison (e.g average daily cost) between the standard MPC tracking, the standard Economic MPC, and the integration of both in one-layer and two-layer approaches was carried out. The goal of this research is to design a controller based on Economic MPC strategies, that tackles uncertainties, in order to minimise economic costs and guarantee service reliability of the system.

*Keywords:*Smart Grid, Model Predictive Control, Demand Uncertainty, Robust Optimization

1. Introduction

Due to model uncertainty and noise, the development of robust MPC strategies is not a trivial task [2,10]. The problem of model uncertainty and noise in linear time-invariant (LTI) systems can be solved through enforcement of computational constraints as reported in [2,11,12]. However, this approach is usually raising the issue of tractability.
There are other techniques based on optimization techniques such as Minimax MPC [12,16], that can be used to tackle model uncertainties.
In [13,14,16] Adjustable Robust Solutions have been proposed, which assume that adjustable control inputs can be made to depend affinely on the uncertain parameters of the problem. This approach is more flexible, and is most of the time expected to result in a computationally tractable problem [15].
In this paper, we follow the technique of affine dependence to solve the problem of demand uncertainty in a smart micro-grid.
In [3] uncertainties were ignored, we showed that the standard Economic MPC (EMPC) was not only superior to the standard MPC tracking, but also to the integration of both in a two-layer approach.
In this paper, we repeat the study in [3] by considering unexpected additive load demands in the development of control strategies. Additionally we have also analysed the integration of EMPC and MPC tracking into a single layer.
Several studies [1,3,4] have dealt with the issue of tackling uncertainties using stochastic approaches.
In this work, a deterministic approach namely robust optimization is used. Each uncertain variable in the system is splitted into a non-adjustable one and an adjustable one. After that, using affine dependence method, some relationship between the control inputs (desired input) and the unexpected inputs (additive disturbances) are established.
By setting some bounds on the uncertain load demand, it is easier, realistic and robust to develop a

controller capable of guaranteeing satisfaction of load demands. We have explicitly included the uncertainty information into the optimization problem, by replacing uncertain constraints of states and control inputs with robust counterpart ones, and by substituting the uncertain variables with their robust counterparts in the objective function.
We consider a smart grid consisting of ***G*** fossil fuel generators, ***S*** clean energy generators, and ***B*** storage devices integrated through a DC Bus into a power grid.
The DC Bus collects the energy generated by the subsystems and delivers it to the load, and if necessary to the storage devices.
We have developed some MPC control strategies by using both single-layer and two-layer approaches.
In the one-layer approach, we started by applying MPC tracking and EMPC strategies separately to the hybrid system. After that, we integrated both the economic optimisation and the tracking formulation in a single layer.
In the two-layer approach, the upper layer consists of an EMPC controller viewed as the supervisory unit, which is in charge of scheduling the operation of the subsystems, and computing their power references. At the lower layer, standard MPC tracking controllers responsible for implementing the computed reference values for each subsystem were used.
The main goal of this work is the development of a robust optimization based Economic MPC strategy capable of satisfying uncertain load demands in smart grids.

2. Control-oriented modelling

A typical smart grid architecture can be represented with a directed graph consisting of n_s energy sources, n_x storage elements, n_q intersection Buses, and n_d loads. Smart grids could be viewed as instances of generalized flow-based networks.

2.1. Component modeling

Smart grids could be considered as examples of generalized flow-based networks. Basically, every flow-based network is made up of some components e.g: flow sources, links, nodes, storage, flow handling, and sink elements. Considering [1, 5, 7, 26], we can model the components as follows:

2.1.1. Flow source elements:

Flow sources consist of the generators and the storage elements, which supply the required energy to the load. Since everything has got its limits, flow sources should also be constrained. Some lower and upper limits can be defined.
For any given generator we can write: $P^{min} \leq P(k) \leq P^{max}$; where $P(k)$ the supplied energy at time k, and P^{min} and P^{max} are the lower and upper limits respectively.

2.1.2. Flow handling elements:

Inverters are used to convert direct current (DC) power into alternating current (AC) power. They usually connect the batteries to an AC Bus. Inverters being a type of rectifiers have the characteristic of modifying the nature of an input signal. Hence, they can be considered as flow handling elements. They can amplify or decrease an input power if necessary. Therefore, we can define the maximum and minimum power that they can handle. $P^{min} \leq P(k) \leq P^{max}$
Moreover, inverters could even be considered as "*Active Nodes*". To some extend flow handling elements can basically be considered as links that can modify the nature of the flowing substance i.e current.

2.1.3. Link elements:

Links are basically made up of converters and inverters that connect the generators to the AC/DC Bus. Contrary to inverters, converters do not change the nature of an input power i.e (AC to AC, or DC to DC). They could be termed as "*Passive nodes*". They can also be constrained by setting the maximum and minimum energy allowed to flow through those links. $P^{min} \leq P(k) \leq P^{max}$

2.1.4. Node elements:

Node elements are generally the buses or other elements (e.g transformers), which interconnects the subsystems and the load. They are also known as junctions, where energy flows are either merged or propagated [26] to other elements of a system. For any given node, we can write: $\sum_i P_{\text{in},i}(k) = \sum_i P_{\text{out},i}(k)$

2.1.5. Storage elements:

Storage elements are the batteries that store energy. Their storage capacity should also be constrained as follows:

$SOC^{min} \leq SOC(k) \leq SOC^{max}$

where SOC^{min} and SOC^{max} are the maximum and minimum values of the stored energy, and SOC(k) the stored energy at time k.

Moreover we can describe the dynamic process of storing the energy with the following equation:

$SOC(k+1) = SOC(k) + \Delta t(P_{in}(k) - P_{out}(k))$

2.1.6. Sink elements:

Loads are the sink elements, because they consume the flowing energy. Loads should not consume more than what is produced: $P_L(k) \leq Tp^{max}(k)$, where $P_L(k)$ is the load demand, and $Tp^{max}(k)$ is the total energy flow at time k.

2.2. Control-oriented model

We consider a smart grid consisting of n_x storage elements, n_u energy flow handling and sources elements, n_d sinks and n_q intersection nodes. The n_s source elemets are considered as inflows.

2.2.1. State space model

The hybrid power system is an example of a MIMO (multiple-input multiple-output) system, whose linear state space modeling is given by the following equations:

$$\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{B}\mathbf{u}(k) + \mathbf{B}_p\mathbf{d}(k) \qquad \text{with } k = 0,1,2,\ldots,N\text{-}1 \tag{1a}$$

$$\mathbf{E}_u\mathbf{u}(k) + \mathbf{E}_d\mathbf{d}(k) = 0 \qquad \text{with } k = 0,1,2,\ldots,N \tag{1b}$$

where:

$\mathbf{x} \in \mathbb{R}^{nx}$ is the state vector, $\mathbf{u} \in \mathbb{R}^{nu}$ stands for the vector of control inputs, $\mathbf{d} \in \mathbb{R}^{nd}$ denotes the disturbances vector. $\mathbf{A} \in \mathbb{R}^{nx \times nx}$, $\mathbf{B} \in \mathbb{R}^{nx \times nu}$, $\mathbf{B}_p \in \mathbb{R}^{nx \times nd}$ are system matrices.

$\mathbf{E}_u \in \mathbb{R}^{nq \times nu}$ and $\mathbf{E}_d \in \mathbb{R}^{nq \times nd}$ are matrices of suitable dimensions relating the supply and the load demand on the DC Bus(ses).

2.2.2. Constraints

Control inputs are subject to some bounds (upper and lower limits):

$$\mathbf{u}_i^{min}(k) \leq \mathbf{u}_n(k) \leq \mathbf{u}_i^{max}(k), \quad \forall k \tag{2}$$

where i denotes a subsystem, and ($\mathbf{u}_i^{min}(k)$ is in this case always zero, because energy flow from the generators is positive).

The state of charge (SOC) of each storage element is subject to the following constraint:

$$\mathbf{x}^{min} \leq \mathbf{x}(k) \leq \mathbf{x}^{max}, \qquad \forall k \tag{3}$$

where $\mathbf{x}^{min}$ and $\mathbf{x}^{max}$ are the lower and upper limit values of the state of charge respectively.

To guarantee availability of energy in the batteries we set: $\mathbf{x}^{min} \geq \boldsymbol{\delta}$ (4)

where $\boldsymbol{\delta}$ is the minimum quantity of energy that should always be available in the batteries.

3. Robustness in MPC strategy

Instead of using probabilistic methods, we opted for a deterministic approach namely robust optimization, which offers some possibilities of bounding uncertain load demands. The adjustable

robust optimization (ARO) method [13,14] is used.
Let's assume that, load demands can be subject to the following constraint:

$$\mathbf{d}_i^{min}(k) \leq \mathbf{d}(k) \leq \mathbf{d}_i^{max}(k) \qquad \text{where } i=1,...,n_d \tag{5}$$

where $\mathbf{d}_i^{min}$ *and* $\mathbf{d}_i^{max}$ are the lower and upper bounds of load demands respectively.

3.1. Affine dependence

The main objective of MPC is to generate some optimal control inputs, which should be fed to the plant in order to produce desired outputs.
If the control inputs could be adjusted according to the uncertain variables, the MPC strategy would be robust. To achieve this aim, an affine dependence between the control input variables and the load demand is developed. The dependence is able to adjust the control inputs according to past measured disturbances.
In fact, the adjustable robust optimization method [13,14] relies on affine dependence technique, because the decision variables are made to be "wait and see" variables, which depend on the past measured disturbances.
Mathematically, the dependence can be derived as follows:

$$\mathbf{u}(k) = \mathbf{u}^k(k)$$
$$\mathbf{u}(k+1) = \mathbf{u}^{k+1}(k+1) + \mathbf{L}_{10}\mathbf{d}(k)$$
$$\mathbf{u}(k+2) = \mathbf{u}^{k+2}(k+2) + \mathbf{L}_{21}\mathbf{d}(k+1) + \mathbf{L}_{20}\mathbf{d}(k)$$
$$\mathbf{u}(k+3) = \mathbf{u}^{k+3}(k+3) + \mathbf{L}_{32}\mathbf{d}(k+2) + \mathbf{L}_{31}\mathbf{d}(k+1) + \mathbf{L}_{30}\mathbf{d}(k)$$
....
$$\mathbf{u}(k+N) = \mathbf{u}^{k+N}(k+N) + \mathbf{L}_{N(N-1)}\mathbf{d}(k+N-1) + ... + \mathbf{L}_{N0}\mathbf{d}(k)$$

where
$\mathbf{u}^k(k)$ is the computed control input at time k and $\mathbf{L}_{ij} \in \mathbb{R}^{nu \times nd}$.

In compact matricial form:

$$\underbrace{\begin{Bmatrix} \mathbf{u}(k) \\ \mathbf{u}(k+1) \\ \mathbf{u}(k+2) \\ \mathbf{u}(k+3) \\ . \\ . \\ \mathbf{u}(k+N) \end{Bmatrix}}_{\underline{\mathbf{u}}} = \underbrace{\begin{Bmatrix} \mathbf{u}^k(k) \\ \mathbf{u}^{k+1}(k+1) \\ \mathbf{u}^{k+2}(k+2) \\ \mathbf{u}^{k+3}(k+3) \\ . \\ . \\ \mathbf{u}^{k+N}(k+N) \end{Bmatrix}}_{\underline{\mathbf{v}}} + \underbrace{\begin{Bmatrix} 0 & 0 & 0 \;\ldots\ldots\ldots & 0 \\ \mathbf{L}_{10} & 0 & 0 \;\ldots\ldots\ldots & 0 \\ \mathbf{L}_{20} & \mathbf{L}_{21} & 0 \;\ldots\ldots\ldots & 0 \\ \mathbf{L}_{30} & \mathbf{L}_{31} & \mathbf{L}_{32} \ldots\ldots\ldots & 0 \\ . & . & . \;\ldots\ldots & 0 \\ . & . & & \\ \mathbf{L}_{N0} & \ldots\ldots & \ldots\ldots & \mathbf{L}_{N(N-1)} \end{Bmatrix}}_{\mathbf{W}} \underbrace{\begin{Bmatrix} \mathbf{d}(k) \\ \mathbf{d}(k+1) \\ \mathbf{d}(k+2) \\ . \\ . \\ \mathbf{d}(k+N-1) \end{Bmatrix}}_{\underline{\mathbf{d}}}$$

Then, equations can be compacted as follows:

$$\underline{\mathbf{u}} = \underline{\mathbf{v}} + \mathbf{W}\underline{\mathbf{d}} \tag{6}$$

The new optimization variables are contained in the vector $\underline{\mathbf{v}}$ and in the matrix $\mathbf{W}$.
The adjusted input vector $\underline{\mathbf{u}}$ can now be used in the formulation of the MPC problem. It can also be plugged into (1), as well as into the objective function.

It should be noticed that, the vectors of control input $\mathbf{u}$ and disturbances $\mathbf{d}$ might not have the same size. Hence, elements of vector $\mathbf{d}$ must be combined.

3.2. Problem formulation

All the problems are formulated using the worst-case robust optimization approach, namely the minimax format of the Wald's maximin model. The goal is to minimize the costs of energy production in the presence of uncertain load demand.

The optimization problem is formulated as follows:

$$\min_{\mathbf{x},\mathbf{u}} \max_{\mathbf{d}} \mathrm{f}(\mathbf{x},\mathbf{u},\mathbf{d})$$
$$\text{s.t } \mathbf{g}(\mathbf{x},\mathbf{u},\mathbf{d}) \preceq 0 \quad \forall\, \mathbf{d} \in \mathrm{D} = \{\mathbf{d}:\ \mathbf{d}^{\mathbf{min}} \leq \mathbf{d} \leq \mathbf{d}^{\mathbf{max}}\}, \tag{7}$$
where:
$\mathrm{f}(\mathbf{x},\mathbf{u},\mathbf{d})$ is the cost function, D is the uncertainty set defined as a box−bounded set,
and $\mathbf{g}(\mathbf{x},\mathbf{u},\mathbf{d})$ represents the constraints

3.2.1. MPC tracking

MPC tracking operates by following some reference trajectories from time k over a span of N steps. The objective function of MPC tracking is usually given in Quadratic Program (QP) form.
In this study, we use a QP of the following form:

$$\boldsymbol{J_{MPC}} = (\mathbf{x}-\mathbf{x}^{\mathrm{r}})^{\mathrm{T}}\mathbf{Q}(\mathbf{x}-\mathbf{x}^{\mathrm{r}}) + (\underline{\mathbf{u}}-\mathbf{u}^{\mathrm{r}})^{\mathrm{T}}\mathbf{R}(\underline{\mathbf{u}}-\mathbf{u}^{\mathrm{r}}) + (\mathbf{x}_{\mathbf{Np}}-\mathbf{x}_{\mathbf{Np}}{}^{\mathrm{r}})^{\mathrm{T}}\mathbf{S}_{\mathrm{x}}(\mathbf{x}_{\mathbf{Np}}-\mathbf{x}_{\mathbf{Np}}{}^{\mathrm{r}}) \tag{8}$$

where:
$\mathbf{x}=[\boldsymbol{x}(k+1), \boldsymbol{x}(k+2), ..., \boldsymbol{x}(k+N)]^T$ is the state vector,
$\mathbf{x}^{\mathrm{r}}=[\boldsymbol{x}^r(k+1), \boldsymbol{x}^r(k+2), ..., \boldsymbol{x}^r(k+N)]^T$ is the reference trajectory vector of the states,
$\underline{\mathbf{u}}$ is the adjusted control input vector,
$\mathbf{u}^{\mathrm{r}}=[\boldsymbol{u}^r(k+1), \boldsymbol{u}^r(k+2), ..., \boldsymbol{u}^r(k+N)]^T$ is the reference trajectory vector of the control inputs,
$\mathbf{x}_{\mathbf{Np}}$ is the vector of terminal state and $\boldsymbol{x}_{Np}{}^r$ its reference trajectory,
Q, R and $\mathbf{S}_{\mathrm{x}}$ are weights on the states, control inputs, and terminal state respectively.

$$\mathbf{Q} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
$$\mathbf{R} = \mathrm{diag}(\mathrm{c_b}, \mathrm{c_b}, \mathrm{c_h}, \mathrm{c_h}, \mathrm{c_g}, \mathrm{c_g}, \mathrm{c_d}, \mathrm{c_{hy}}, \mathrm{c_w}, \mathrm{c_{pv}})$$
$$\mathbf{Sx} = \mathrm{W_c}.\mathbf{Q};\quad \text{where } \mathrm{W_c} \text{ is a positive scalar}$$

c_b, c_h, c_g, c_d c_d, c_{pv}, c_w, c_{hy} , are positive weight coefficient (≤ 1) for the lead-acid battery, hydrogen battery, grid connection, diesel, solar, wind, and hydroelectric generators respectively.

The reference trajectory for the input controls were computed using the following formula:

$$\mathbf{u}^{\mathrm{r}}(k) \;= \rho \cdot \sigma(k) \tag{9}$$

where:
ρ is the nominal maximum power of each generator.
$\sigma(k)$ is the effective generated power at time k. It is measured directly from the generators.
This formula is a result of some trial and error simulations based on the idea that, the reference trajectory is expected to be proportional to the control inputs.

For instance, the updating of the states can be done by substituting the adjusted control input in (1a), and in the objective function as follows:

$$\mathbf{x}(k+1)=\mathbf{A}\mathbf{x}(k) + \mathbf{B}\underline{\mathbf{u}}(k) + \mathbf{B_p}\mathbf{d}(k) = \mathbf{A}\mathbf{x}(k) + \mathbf{B}(\underline{\mathbf{v}} + \mathbf{W}\underline{\mathbf{d}})(k) + \mathbf{B_p}\mathbf{d}(k)$$
$$\boldsymbol{J_{MPC}} = (\mathbf{x}-\mathbf{x}^{\mathrm{r}})^{\mathrm{T}}\mathbf{Q}(\mathbf{x}-\mathbf{x}^{\mathrm{r}}) + ((\underline{\mathbf{v}} + \mathbf{W}\underline{\mathbf{d}})-\mathbf{u}^{\mathrm{r}})^{\mathrm{T}}\mathbf{R}((\underline{\mathbf{v}} + \mathbf{W}\underline{\mathbf{d}})-\mathbf{u}^{\mathrm{r}}) + (\mathbf{x}_{\mathbf{Np}}-\mathbf{x}^{\mathbf{r}}_{\mathbf{Np}})^{\mathrm{T}}\mathbf{Sx}(\mathbf{x}_{\mathbf{Np}}-\mathbf{x}^{\mathbf{r}}_{\mathbf{Np}})$$

The MPC tracking problem can be formulated as follows:

$$\min_{\mathbf{u}(k)\mathbf{x}(k)} \max_{\mathbf{d}(k)} [\sum_{k=0}^{N-1} J_{MPC}(k)]$$
$$\text{s.t } (1),(2),(3),(4),\ \mathbf{d}(k) \in \mathrm{D} = \{\mathbf{d}:\ \mathbf{d}^{\mathbf{min}} \leq \mathbf{d} \leq \mathbf{d}^{\mathbf{max}}\}\ \forall k \tag{10}$$

3.2.2. Economic MPC

Economic MPC does not require a reference trajectory. Three operational goals have been considered: economic, safety, and smoothness as described in [3,5].

Economic cost:

The total cost is given by: $f_E(k) = (\boldsymbol{\alpha}_1 + \boldsymbol{\alpha}_2(k))\mathbf{u}(k)\Delta t$

where $\mathbf{u}(k)$ is a vector of control actions at time k;

$\boldsymbol{\alpha}_1$ is a known vector related to economic costs of maintenance of generators and its accessories; $\boldsymbol{\alpha}_2(k)$ is a known time-varying vector associated to the economic cost of power flows related to transmission and distribution. The time dependence of $\boldsymbol{\alpha}_2$ is given by the power distribution, which varies along the time. Δt is the sampling time.

Safety Storage Measures:

The safety measures are defined as: $f_S(k) = \varepsilon(k)^T\boldsymbol{\varepsilon}(k)$

where $\varepsilon(k)$ is the amount of soft constraint violation. $\boldsymbol{\varepsilon} = 0$ means there is no violation.

Smoothness/Stability of the control action:

The rate of change of the control action must be made smooth, in order to ensure that, consecutive control inputs are either continuously increasing or decreasing.

$f_{\Delta u}(k) = \Delta\mathbf{u}(k)^T\Delta\mathbf{u}(k)$

where $\Delta\mathbf{u}(k)$ is the rate of change of control signal, defined as $\Delta\mathbf{u}(k)=\mathbf{u}(k) - \mathbf{u}(k-1)$.

Let $F_1 = f_E(k)$, $F_2 = f_S(k)$, $F_3 = f_{\Delta u}(k)$; substituting the control input $\mathbf{u}$ with (6), we get:

$\Delta\mathbf{u}(k)=\mathbf{u}(k) - \mathbf{u}(k-1)= \underline{\mathbf{u}}(k) - \underline{\mathbf{u}}(k-1)$

The objective function is given by:

$$J_{EMPC}= \lambda_1 F_1 + \lambda_2 F_2 + \lambda_3 F_3 = \lambda_1 (\boldsymbol{\alpha}_1 + \boldsymbol{\alpha}_2(k))\underline{\mathbf{u}}(k)\Delta t + \lambda_2 \varepsilon(k)^T\boldsymbol{\varepsilon}(k) + \lambda_3 [\underline{\mathbf{u}}(k) - \underline{\mathbf{u}}(k-1)]^T[\underline{\mathbf{u}}(k) - \underline{\mathbf{u}}(k-1)] \quad (11)$$

The Economic MPC optimization problem is formulated as follows:

$$\min_{\mathbf{u}(k)\,\mathbf{x}(k)} \max_{\mathbf{d}(k)} \left[\sum_{k=0}^{N-1} J_{EMPC}(k)\right] \qquad (12)$$

$$\text{s.t } (1),(2),(3),(4),\ \mathbf{d}(k)\in \mathrm{D}=\{\mathbf{d}:\ \mathbf{d}^{\min}\le\mathbf{d}\le\mathbf{d}^{\max}\}\ \forall k$$

3.2.3. Two-layer approach: EMPC and MPC tracking

The main idea behind the use of a two-layer approach is to overcome the problem of non-reachable reference trajectories. We integrate the EMPC and the MPC tracking in a cascaded fashion.

a) Upper layer: Economic MPC

This layer comprises the EMPC described in (*3.2.2*). The problem to be solved is expressed in equation (12).

In this layer the prediction horizon is set to *2N*-1, in order to be able to track *N* time-varying set points with the receding horizon strategy of the lower layer.

b) Lower layer: MPC tracking

The lower layer consists of the MPC tracking described in (*3.2.1*). Instead of using manually selected reference trajectories, the computed states and control inputs by the upper layer are used.

The problem to be solved is expressed in equation (11).

3.2.4. Single-layer approach: EMPC and MPC tracking

The main idea behind the use of a two-layer approach is to overcome the problem of non-reachable reference trajectories (feasibility).

Contrary to the two-layer approach as defined above, the economic optimisation (EMPC) and the tracking formulation (MPC tracking) are integrated in a single layer.

The problem to be solved is given as follows:

$$\min_{\mathbf{u}(k)\,\mathbf{x}(k)} \max_{\mathbf{d}(k)} \left[\sum_{k=0}^{N-1} J_{EMPC}(k)+J_{MPC}(k)\right] \qquad (13)$$

$$\text{s.t } (1),(2),(3),(4),\ \mathbf{d}(k)\in \mathrm{D}=\{\mathbf{d}:\ \mathbf{d}^{\min}\le\mathbf{d}\le\mathbf{d}^{\max}\}\ \forall k$$

4. Stability analysis

Weights on the terminal state (where MPC tracking is involved) are used to improve (guarantee) stability of the system. However, in the works ahead we will use asymptotic stability method to analyse stability conditions of the controllers. As for the EMPC strategy, the stability of the system around a feasible region can be guaranteed following the results in [18] for periodic systems, whereby a Lyapunov function was developed.

The infeasibilty issue was overcome using the following two strategies:

a) reusing previously predicted solutions,

b) online adjustment of the upper and lower limits of the control inputs and states by considering the highest computed control input and state respectively.

5. Case study

5.1. Description

This subsection presents a smart micro-grid that consists of: two storage elements (batteries), three sinks (loads) and one virtual sink (external grid connection), one node (DC Bus), four sources (PV, Wind, Hydroelectric, and Diesel generators), and one virtual source (external grid connection). A grid connection is a sink when buying energy, and a source when selling energy. Since all the components (excluding sinks) are connected to a single node (DC Bus) through flow handling elements, they are all considered as manipulated inputs. The state of charge of the storage elements are the states of the micro-grid. A block diagram of the micro-grid is shown in Figure 1.

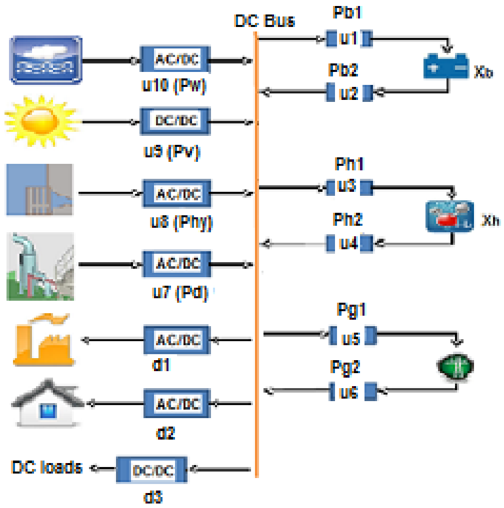

Figure 1. block diagram of the micro-grid

5.2. Control-oriented model

State variables:

x_b and x_h are the state of charge of the batteries (lead-acid and hydrogen respectively).

$\mathbf{x}(k) \triangleq [x_b(k), x_h(k)]^T$

Control input variables:

P_{b1} and P_{b2} are charged power and discharged power of the lead-acid battery;

P_{h1} and P_{h2} are the charged and discharged power of the hydrogen battery;

P_{g1} and P_{g2} are the exported and imported power into/from the external grid;

P_d, P_{hy}, P_{pv}, and P_w stand for the power supplied to the DC Bus by the diesel, hydroelectric, wind, and photovoltaic generators respectively;

$\mathbf{u}(k) \triangleq [P_{b1}(k), P_{b2}(k), P_{h1}(k), P_{h2}(k), P_{g1}(k), P_{g2}(k), P_d(k), P_{hy}(k), P_w(k), P_{pv}(k)]^T$

Disturbance variables:

d_1, d_2, d_3 are the industrial load, the residential load, and the DC-load respectively. The disturbance vector **d** comprises the three loads. $\mathbf{d}(k) \triangleq [d_1(k), d_2(k), d_3(k)]^T$

The matrices and vectors that define the system and theconstraints are given as follows:

$$A=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} \quad B=\begin{bmatrix}\eta_{bc} & -\eta_{bd} & 0 & 0 & 0\;0\;0\;0\;0\;0\\ 0 & 0 & \eta_{hc} & -\eta_{hd} & 0\;0\;0\;0\;0\;0\end{bmatrix} \quad B_p=\begin{bmatrix}-1 & -1 & -1\\ -1 & -1 & -1\end{bmatrix}$$

$\mathbf{x}^{\min} = [0\ 0]^T$, $\mathbf{x}^{\max} = [100\ 100]^T$

$\mathbf{u}^{\min} = [0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0]^T$, $\mathbf{u}^{\max} = [35\ 10\ 35\ 18\ 18\ 18\ 18\ 18\ 18\ 18]^T$

$\mathbf{E_u} = [-1\ 1\ -1\ 1\ -1\ 1\ 1\ 1\ 1\ 1]$, $\mathbf{E_d} = [-1\ -1\ -1]$

We have considered a scenario (days with bad weather conditions) where all the generators are required to be in operation, in order to satisfy load demands. If they cannot, then the batteries, the external grid, and the diesel generator will compensate the shortage one after the other. Initial values of the subsystems as well as the state of charge of the batteries are set to zero. The simulations were made for 96 hours (4 days).

The diesel generator (1-1.3 kWh) was operated in summer during the first six hours of the day, and in winter six hours in the afternoon. It supplied 1.3 kWh in the first hour, and 1 kWh in the remaining hours. The batteries were used during the first two hours of the day. They delivered 2 kWh in the first hour and 1 kWh in the second hour.

1 kWh was bought from the external grid during the second hour of the day.

Each additive uncertain load demand (residential, industrial and DC-load) is bounded between -1 and +1 kWh. Figure 2. shows the profiles of the load demands. The additive uncertain demand is represented with the shadowed area.

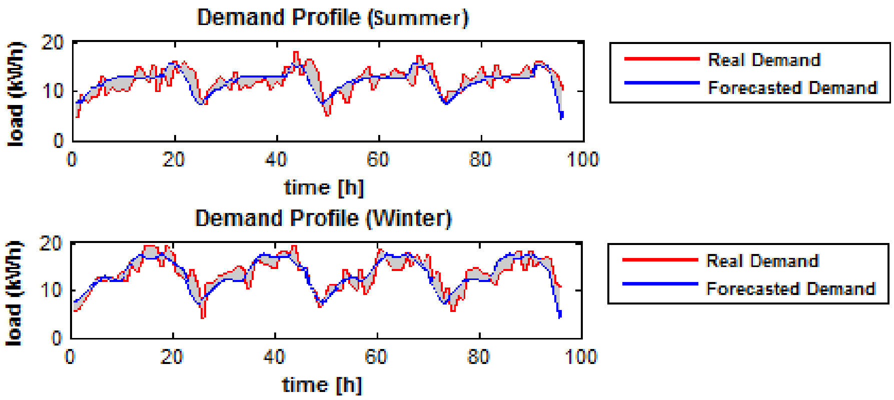

Figure 2. Profile of the total load demand

Generators' profiles are displayed in Figure 3.

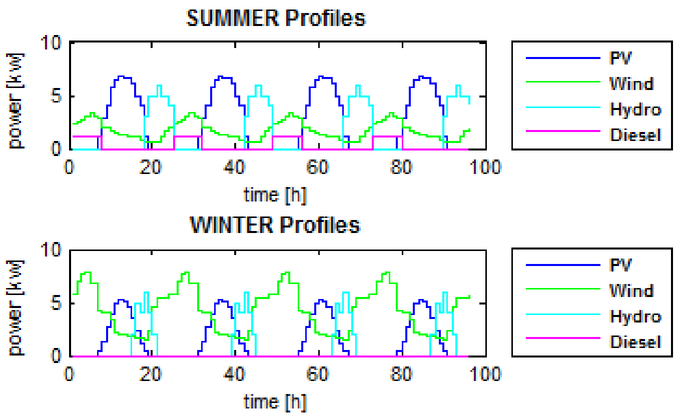

Figure 3. Generators' profiles

System parameters		Control parameters		Energy prices	
Parameters	Values in kW	Parameters	Values	Lead-acid battery charging	2.2
P^{max}_{pv}	15	N_p	24	Lead-acid battery discharging	2.2
P^{max}_{w}	15	N_c	24	Hydrogen battery charging	2.2
P^{max}_{hy}	15	c_{pv}	0.2	Hydrogen battery discharging	2.2
P^{max}_{d}	15	c_w	0.3	External grid selling	3
P^{max}_{b1}	35	c_{hy}	0.4	External grid buying	3
P^{max}_{b2}	15	c_b	0.75	Diesel	3.3
P^{max}_{h1}	35	c_h	0.75	Hydroelectric	2.1
P^{max}_{h2}	15	c_d	1	Wind	2.1
P^{max}_{g1}	15	c_g	0.75	Solar	2
P^{max}_{g2}	15	$\mathbf{Q}$	as defined previously		
η_{bc}	0.95	$\mathbf{R}$	as defined previously		
η_{bd}	1				
η_{hc}	0.85				
η_{hd}	1.0				
$\boldsymbol{\delta}$	$[35\ 35]^T$				

Table 2. System and control parameters, and energy prices

5.3. Simulation results

All the plots have been made using YALMIP (CPLEX and QuadProg solvers) [16] within the Matlab environment.

The purpose of including all the simulations figures is to enable a visual comparison of the four MPC strategies.

5.3.1. One-layer: MPC tracking

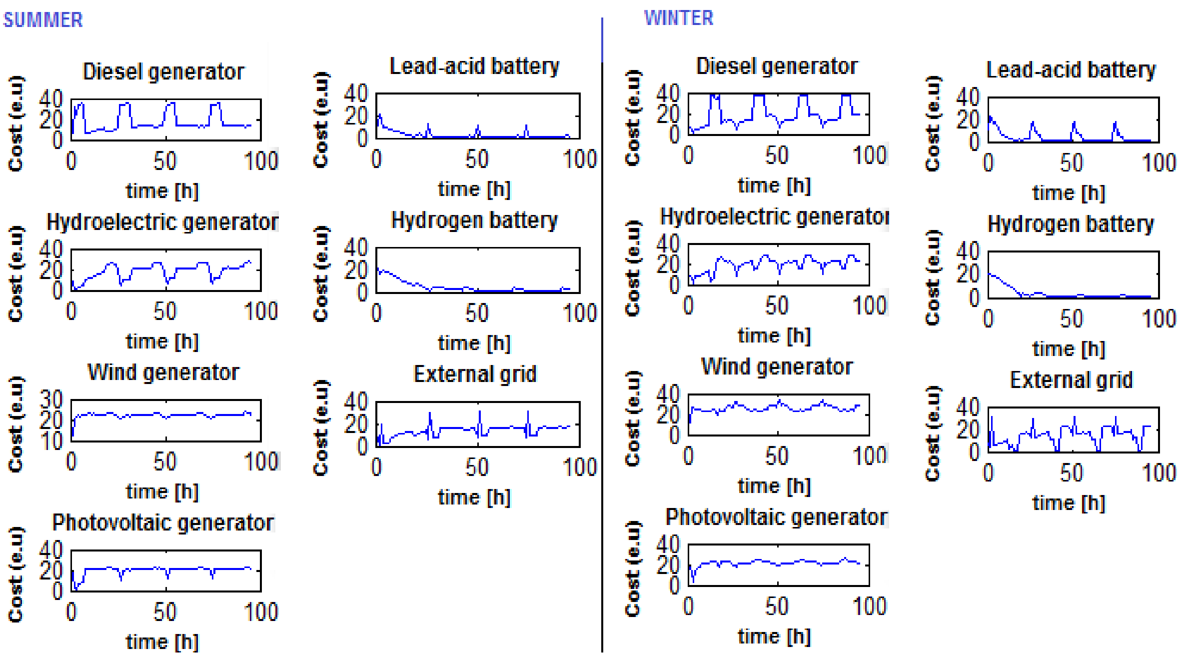

Figure 4. Plots of the economic costs

5.3.2. One-layer: EMPC

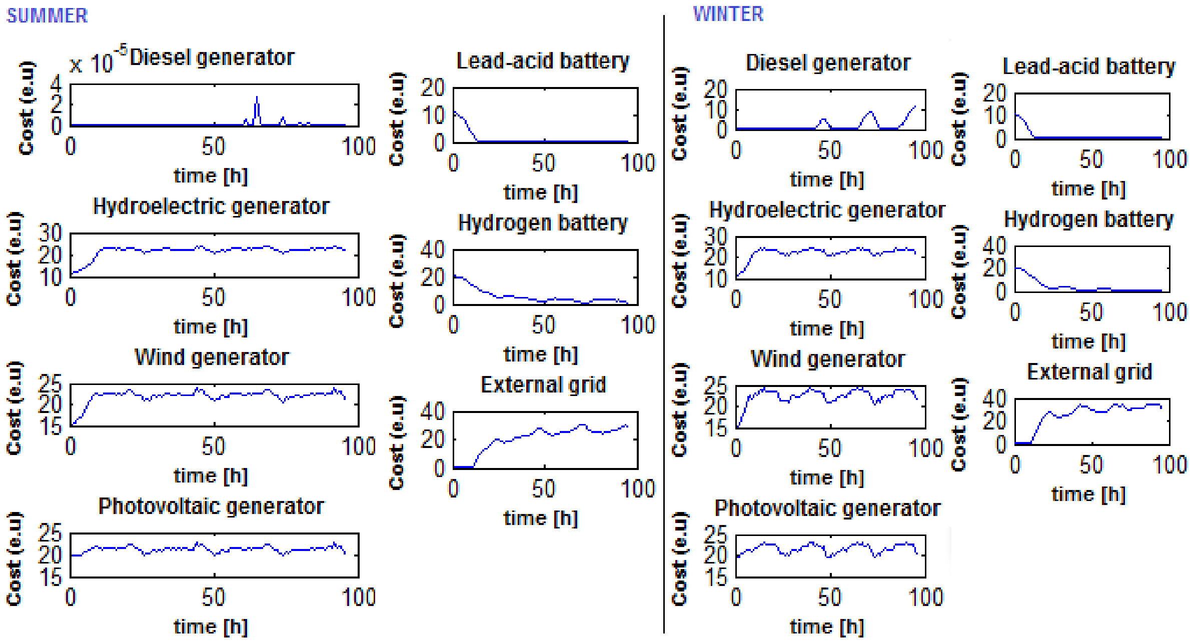

Figure 5. Plots of the economic costs

5.3.3. Two-layer approach: EMPC and MPC tracking

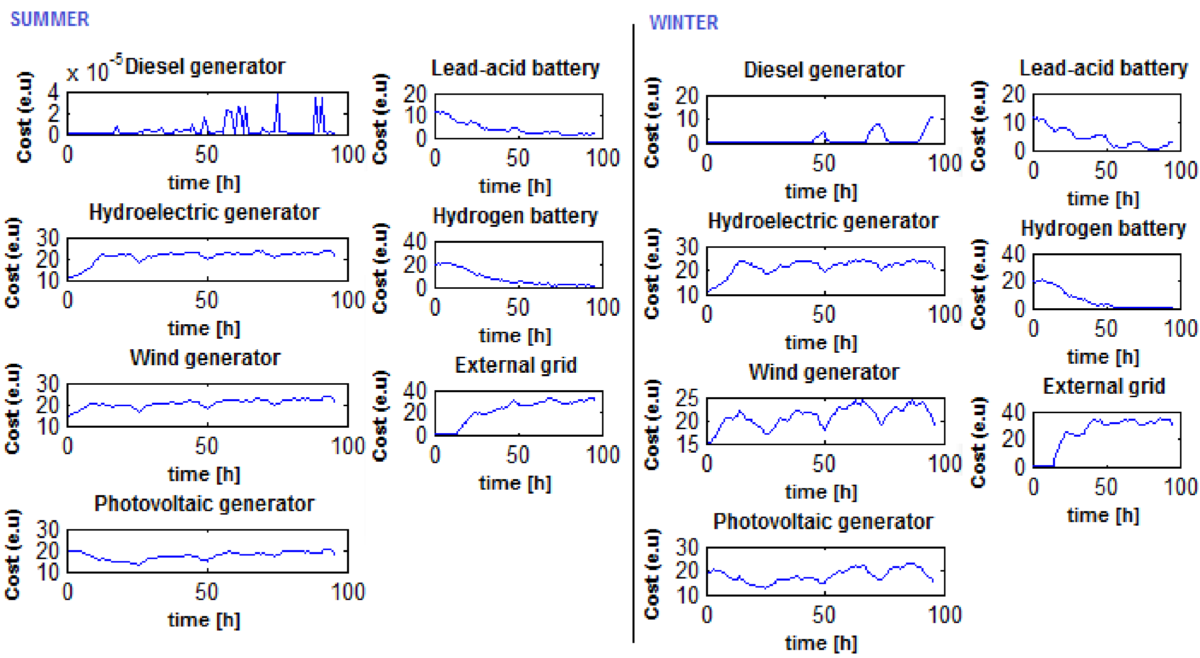

Figure 6. Plots of the economic costs

5.3.4. One-layer: EMPC and MPC tracking

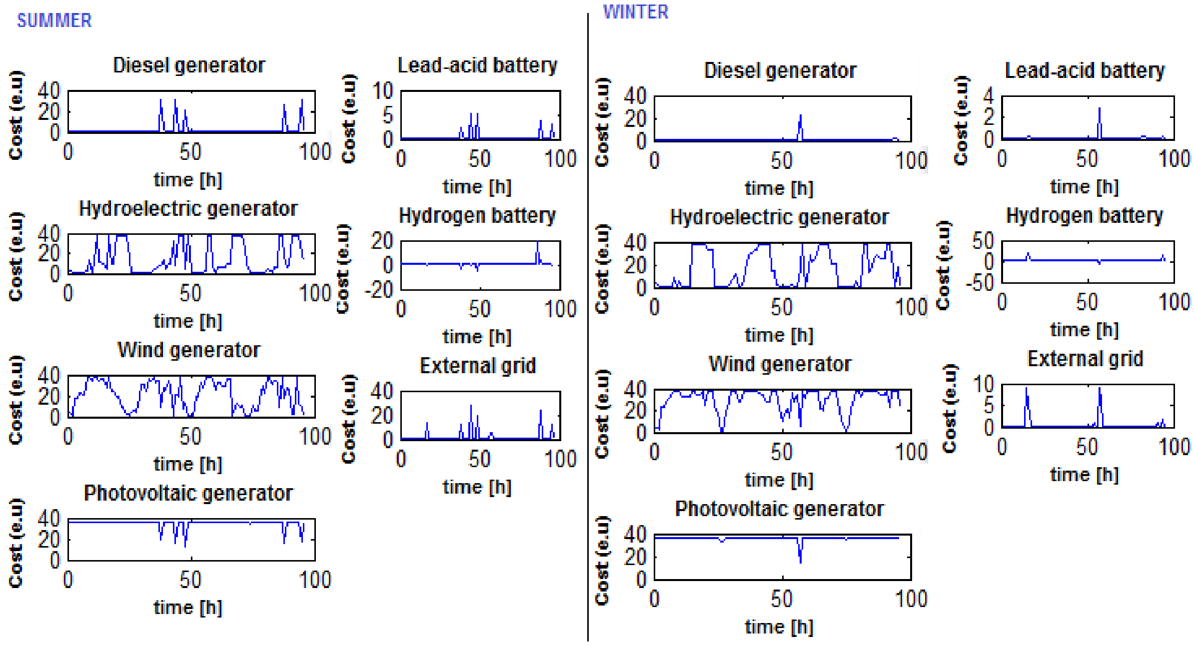

Figure 7. Plots of the economic costs

Remark:

It has been found that, one-layer approach is economically superior to the two-layer hierarchical scheme. Similar result was obtained in [1,4]. Moreover, it can also be seen (table 2) that, the EMPC produces the lowest economic costs, thereby proving its superiority to the MPC tracking.

Table 2. presents a comparison of the four MPC approaches' economic costs in economic unit (e.u).

	EMPC	EMPC + MPC tracking (single-layer)	EMPC + MPC tracking (two-layer)	MPC tracking
Summer economic cost	2934.4	2963.2	3761.5	4175.1
Winter economic cost	3369.0	3378.7	3802.0	4451.5

Table 2. Quantitative comparison of the economic costs

6. Conclusion and future work

In this study, we have presented the application of four variations of robust optimization based MPC strategies for controlling energy dispatch in a grid connected smart micro-grid. An affine dependence between the control inputs and the load demand has been derived. The standard EMPC, the MPC tracking, and their combination in a single and two-layer approaches have been discussed and compared. It has been found that, the standard Economic MPC yields a better economic result, because the energy consumption is the lowest. The next steps for completing this research will be devoted to tackling uncertainties of energy prices and renewable energy sources.

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References

[1] Grosso J M, Ocampo-Martinez C, and Puig V 2012 Chance-constrained model predictive control for large-scale Networks, *Transactions on Control Systems Technology,* UPC (Barcelona, Spain)

[2] Abate A, Laurent El G 2004 Robust model predictive control through adjustable variables: an application to path planning *43rd IEEE Conference on Decision and Control*, (Atlantis, Paradise Island, Bahamas)

[3] Nassourou M 2016 Model Predictive Control Strategies for Energy Dispatch of a Smart Micro-grid System UPC (Barcelona, Spain)

[4] Ocampo-Martinez C, Puig V, Cembrano G, Creus R, and Minoves M 2009 Improving water management efficiency by using optimization-based control strategies: The Barcelona case study *Water Science & Technology: Water supply*, vol. **9**, no. 5, pp. 565–575

[5] Grosso J M 2014 On model predictive control for economic and robust operation of generalised flow-based networks, *PhD thesis UPC* (Barcelona, Spain)

[6] Rosero J L 2012 Estrategia de control de modelo predictivo para el despacho y almacenamiento de energia renovable en sistemas hibridos *Master thesis*, *Instituto Tecnologico y de Estudios Superiores de Monterrey*

[7] Grosso G, Ocampo-Martinez C, and Puig V 2012 A service reliability model predictive control with dynamic safety stocks and actuators health monitoring for drinking water networks *in Proceedings of the IEEE Conference on Decision and Control*

[8] Pereira M, Limon D, Muñoz de la Peña D, Valverde L, Alamo T 2015 Periodic economic control of a non-isolated micro-grid, *Transactions on Industrial Electronics 62*, Institute of Electrical and Electronics Engineers, 5247-5255

[9] Limon D, Pereira M, Muñoz de la Peña D, Alamo T, Grosso J.2014 Single-layer economic model predictive control for periodic operation *Journal of Process Control* **24**, 1207–1224

[10] Pereira M, Limon D, Alamo T, Valverde L 2015 Application of periodic economic MPC to a Grid-connected micro-grid, *conference poster, 5th IFAC Nonlinear Model Predictive Control Conference International Federation of Automatic control (*Sevilla, España)

[11] Bemporad A and Morari M 1999 Robust model predictive control: A survey hybrid systems *Computation and Control, F.W. Vaandrager and J.H. van Schuppen,* vol. **1569**, pp. 31–45,1999, lecture Notes in Computer Science.

[12] Loefberg J 2003 Minimax approaches to robust model predictive control *Ph.D. dissertation, Univ. of Linkoping*

[13] Ben-Tal A, Goryashko A, Guslitzer E, Nemirovski A 2004 Adjustable solutions of uncertain linear programs *Mathematical Programming*, **Vol. 99**, 2004, pp. 351–376

[14] Ben-Tal A and Nemirovski 1998 A Robust convex optimization *Mathematics of Operations Research*, Vol. **23**, No. 4, Nov. 1998, pp. 769–805

[15] Vandenberghe L and Boyd S 1996 Semidefinite programming *SIAM Rev.*, Vol. **38(1)**, 49–95

[16] Löfberg J 2012 Automatic robust convex programming *Optimization methods and software*, **27(1)**:115-129, 2012

[17] Prodan I, Enrico Z 2013 Predictive control for reliable microgrid energy management under uncertainties *22nd European Safety and Reliability (ESREL 2013) (*Amsterdam, Netherlands) <hal-00912003>

[18] Zanon M, Gros S, and Diehl M 2013 A Lyapunov function for periodic economic optimizing model predictive control *In Proc. 52nd IEEE Annual Conference on Decision and Control (CDC),* pages 5107–5112*,* (Firenze, Italy)