Actuator multiplicative fault estimation in discrete-time LPV systems using switched observers

Damiano Rotondo[a,b], Francisco-Ronay López-Estrada[c,*], Fatiha Nejjari[a], Jean-Christophe Ponsart[d,e], Didier Theilliol[d,e], Vicenç Puig[a,f]

[a]*Department of Automatic Control (ESAII), Technical University of Catalonia (UPC), Rambla de Sant Nebridi 10, 08222 - Terrassa (Spain). Tel: +34 93 739 89 73*
[b]*Center for Autonomous Marine Operations and Systems (NTNU-AMOS), Department of Engineering Cybernetics, Norwegian University of Science and Technology, Trondheim, Norway.*
[c]*Tecnológico Nacional de México. Instituto Tecnológico de Tuxtla Gutierrez (ITTG), Electronic Department, Carretera Panamericana km 1080, Col. Terán, CP 29050, Tuxtla Gutiérrez, Chiapas, Mexico*
[d]*Université de Lorraine, CRAN, UMR 7039, Campus Sciences, B.P. 70239, Vandoeuvre-les-Nancy Cedex 54506*
[e]*CNRS, CRAN, UMR 7039.*
[f]*Institut de Robòtica i Informàtica Industrial (IRI), UPC-CSIC Carrer de Llorens i Artigas, 4-6, 08028 Barcelona, Spain*

Abstract

This paper proposes an observer for the joint state and fault estimation devoted to discrete-time linear parameter varying (LPV) systems subject to actuator faults. The major contribution of this work is that the observer is able to estimate multiplicative faults, contrarily to the existing approaches, that consider additive faults. The main characteristic of this observer is that it is scheduled not only by means of the endogenous varying parameters of the faulty model, but also by the input vector. Another contribution of this paper consists in adding a switching component in order to guarantee the feasibility of the conditions for designing the observer gains. It is proved that, as long as the input sequence satisfies some characteristics, the convergence of the observer error dynamics to zero is assured. A numerical

*Corresponding author: Francisco-Ronay López-Estrada

Email addresses: `damiano.rotondo@yahoo.it` (Damiano Rotondo), `frlopez@ittg.edu.mx` (Francisco-Ronay López-Estrada), `fatiha.nejjari@upc.edu` (Fatiha Nejjari), `Jean-Christophe.Ponsart@univ-lorraine.fr` (Jean-Christophe Ponsart), `didier.theilliol@univ-lorraine.fr` (Didier Theilliol), `vicenc.puig@upc.edu` (Vicenç Puig)

example is used to demonstrate the effectiveness of the proposed strategy.

Keywords: Linear parametrically varying (LPV) methodologies; Multiplicative actuator faults; Fault estimation; Switched observers.

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1. Introduction

In order to increase the reliability and performance of control systems, fault detection and isolation (FDI) has been widely investigated in the last decades (see [1, 2, 3] for recent surveys of the most relevant results). Model-based methods, which use mathematical models to perform FDI in real-time, have proven to be powerful tools for detecting and isolating faults in dynamic systems. Among these methods, observer-based ones attempt to provide an estimation of the fault's magnitude, which is important in many applications, especially when an active fault tolerant control (FTC) strategy is implemented. For example, [4, 5, 6] have proposed to use sliding mode observers to decouple the effects of the faults from the system's estimated outputs. [7] has provided a robust observer for Lipschitz nonlinear descriptor systems with bounded input disturbances. In [8], a bank of unknown input observers (UIOs) has been used to perform robust fault diagnosis in Takagi-Sugeno (TS) descriptor systems. In [9], UIOs have been used for FDI in overactuated systems.

The provided list of references is not exhaustive, and many other observer-based FDI methods have been developed for actuator faults, e.g. [10, 11, 12]. It is worth highlighting that most of the proposed approaches consider the case of additive faults, and there is a lack of results concerning observer-based estimators for actuator multiplicative faults. However, much of the recent research in FTC has considered systems affected by multiplicative faults, see e.g. the sliding mode control-based solutions developed in [13] and [14]. The main difference between an additive and a multiplicative fault is that, as a result of the additive faults, the mean value of the output changes, while if the fault is multiplicative, it generates changes on the system parameters [15]. The design of observers for multiplicative fault estimation is not as straightforward as the case of additive fault estimation, because the effect of the input and the fault are mixed. To the best of the authors' knowledge, [16] and [17] are the only observer-based solutions proposed for the estimation of multiplicative faults. In [16], the multiplicative faults have been reshaped into additive faults, such that a sliding mode observer can be used, while in [17], the control input has been considered as a scheduling parameter, such that the faulty system can be rewritten as a switched linear parameter varying (LPV) system, for which an LPV switched observer is designed. Hence, developing solutions for multiplicative fault estimation remains an open and interesting research issue.

The LPV paradigm is appealing because it can be used efficiently to represent some nonlinear systems [18]. This fact has motivated many researchers from the FDI community to develop model-based methods for LPV systems [19, 20, 21, 22, 23]. In some cases, due to the loss of feasibility of the LMIs, or the inherent switching modes of the system, it may be needed to split the parameter region into subregions, and switch among them during the LPV system operation. Thus, the LPV system is transformed into a new class of system, referred to as *switched LPV system* [24]. Both the study of the observability properties of switched systems [25, 26] and the design of observers for switched systems with unknown inputs [27, 28, 29, 30] have been investigated with interest in recent years. However, only a few works have dealt with the problem of observer design for switched LPV system, e.g. [31], where a Luenberger-like hybrid LPV observer was used for the continuous state estimation, and [32], where an adaptive switched LPV observer was proposed for the joint state and parameter estimation.

The main contribution of this paper is to propose an observer for the joint estimation of the state and multiplicative faults in discrete-time LPV systems. Similarly to [17], the control input is considered as an additional scheduling parameter, such that the proposed observer is scheduled not only by the endogenous varying parameters of the faulty model, but also by the input vector. The design is performed solving matrix inequalities, and it is shown that if any of the system inputs can take a value equal to zero, a problem of feasibility of the matrix inequalities would appear if a non-switching structure is used for the LPV observer. However, the addition of a switching component allows to overcome this issue by considering different feasibility regions generated by the scheduling parameters. It is worth highlighting that, although interesting and innovative, the approach proposed in [17] can be used only with single-input systems, which limits its applicability. On the other hand, the approach presented in this paper can be applied to the more general class of MIMO systems. Moreover, another difference with respect to [17] is that the conditions that the input should satisfy in order to assure convergence to zero of the estimation error dynamics are determined through an average dwell time reasoning.

The paper is structured as follows. Section 2 presents the problem of joint state and actuator multiplicative fault estimation. Section 3 provides the solution proposed for this problem. Section 4 illustrates the proposed approach using a numerical example. Finally, Section 5 outlines the conclusions.

2. Problem Statement

Let us consider a discrete-time LPV system subject to actuator faults

$$\bar{x}(k+1) = \bar{A}\left(\bar{\theta}(k)\right)\bar{x}(k) + \bar{B}\left(\bar{\theta}(k)\right)\mathit{\Gamma}(k)u(k) \tag{1}$$

$$y(k) = \bar{C}\bar{x}(k) \tag{2}$$

where $\bar{x}(k) \in \mathbb{R}^{n_x}$ and $y(k) \in \mathbb{R}^{n_y}$ are the state and output vector, respectively. The input vector, denoted by $u(k)$, takes values in a subset $\Upsilon \subset \mathbb{R}^{n_u}$, defined as follows

$$\Upsilon = \left[u_1^{min}, u_1^{max}\right] \times \ldots \times \left[u_{n_u}^{min}, u_{n_u}^{max}\right] \tag{3}$$

where $u_j^{min} < 0$ and $u_j^{max} > 0$ for all $j = 1, \ldots, n_u$.

The matrices $\bar{A}\left(\bar{\theta}(k)\right) \in \mathbb{R}^{n_x \times n_x}$, $\bar{B}\left(\bar{\theta}(k)\right) \in \mathbb{R}^{n_x \times n_u}$ are scheduled by the vector of varying parameters $\bar{\theta}(k) \in \Theta \subset \mathbb{R}^{n_\theta}$, which is assumed to be known. The matrix $\bar{C} \in \mathbb{R}^{n_y \times n_x}$ is a known constant matrix. On the other hand, the matrix $\mathit{\Gamma}(k)$ is unknown and describes the multiplicative faults, as follows

$$\mathit{\Gamma}(k) = \begin{bmatrix} \gamma_1(k) & 0 & \cdots & 0 \\ 0 & \gamma_2(k) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \gamma_{n_u}(k) \end{bmatrix} \tag{4}$$

where each $\gamma_j(k)$, $j = 1, \ldots, n_u$, represents the loss of effectiveness of the j-th actuator, i.e. its degree of degradation. For example, $\gamma_j(k) = 0.7$ would denote that the j-th actuator is degraded by 30% (the delivered action is 70% of the nominal one). On the other hand, $\gamma_j(k) = 1$ denotes the nominal operation (no fault), while $\gamma_j(k) = 0$ would denote a 100% degradation, which corresponds to a total loss. This type of representation for describing faults is quite common in the literature, see e.g. [17, 33].

Remark 1. *The assumption of constant output matrix is quite common in the literature, and could be relaxed either by increasing the mathematical complexity of the solution proposed in the following, or by post-filtering the output vector $y(k)$, as proposed by [34].*

The problem addressed in this paper is the joint estimation of the system states and the multiplicative faults using the model (1) and the available measurements

(2). In order to achieve this objective, let us notice that, thanks to the diagonal structure of $\varGamma(k)$, which implies

$$\varGamma(k)u(k) = U\left(u(k)\right)\gamma(k) \tag{5}$$

$$U\left(u(k)\right) = \mathsf{diag}\left(u_1(k), \ldots, u_{n_u}(k)\right) \tag{6}$$

$$\gamma(k) = \begin{bmatrix} \gamma_1(k) & \gamma_2(k) & \ldots & \gamma_{n_u}(k) \end{bmatrix}^T \tag{7}$$

it is possible to rewrite the system state equation (1) as

$$\bar{x}(k+1) = \bar{A}\left(\bar{\theta}(k)\right)\bar{x}(k) + \bar{B}\left(\bar{\theta}(k)\right)U\left(u(k)\right)\gamma(k) \tag{8}$$

Under the assumption of slow-varying faults, i.e. $\gamma(k+1) \approx \gamma(k)$, and by considering the augmented state vector $x(k) \triangleq [\bar{x}(k)^T\, \gamma(k)^T]^T$ and the scheduling vector $\theta(k) \triangleq [\bar{\theta}(k)^T\, u(k)^T]^T$, the following augmented system is obtained

$$x(k+1) = A(\theta(k))x(k) \tag{9}$$

$$y(k) = Cx(k) \tag{10}$$

with

$$A(\theta(k)) = \begin{bmatrix} \bar{A}(\bar{\theta}(k)) & \bar{B}(\bar{\theta}(k))U\left(u(k)\right) \\ 0 & I \end{bmatrix}, C = \begin{bmatrix} \bar{C} & 0 \end{bmatrix}$$

Remark 2. *The assumption of slow variation of the faults could appear very restrictive. Nevertheless, this assumption could be relaxed from a practical point of view, as stated by [35] and [36].*

Under the assumption that the augmented system (9)-(10) is observable, the following observer for the joint state and fault estimation could be proposed

$$\hat{x}(k+1) = A(\theta(k))\hat{x}(k) + L(\theta(k))\left(\hat{y}(k) - y(k)\right) \tag{11}$$

$$\hat{y}(k) = C\hat{x}(k) \tag{12}$$

In this case, the problem would reduce to find the observer gain $L(\theta(k))$ such that $\lim_{k\to\infty} e(k) = \lim_{k\to\infty}\left(\hat{x}(k) - x(k)\right) = 0$.

Taking into account the augmented system (9)-(10) and the state/fault observer (11)-(12), the dynamics of the estimation error $e(k)$ is given as follows

$$e(k+1) = \left(A\left(\theta(k)\right) + L\left(\theta(k)\right)C\right)e(k) \tag{13}$$

Let us recall the following lemma.

Lemma 1. *[37]* (Lyapunov condition for the stability of discrete-time LPV systems) *Consider an autonomous discrete-time LPV system*

$$x(k+1) = A\left(\theta(k)\right)x(k), \quad \theta \in \Theta \subset \mathbb{R}^{n_\theta} \tag{14}$$

If there exists a matrix $P = P^T > 0$ *such that* $\forall\theta \in \Theta$ *the following holds*

$$\begin{bmatrix} P & PA\left(\theta\right) \\ A\left(\theta\right)^T P & P \end{bmatrix} > 0 \tag{15}$$

then the system (14) *is stable in the sense of Lyapunov.*

Proof: The proof is straightforward by considering the Lyapunov function $V\left(k\right) = x(k)^T Px(k)$ and imposing that the difference $V(k+1) - V(k)$ is negative. $\square$

Then, sufficient conditions for guaranteeing the stability of (13) are obtained by applying Lemma 1, and considering the change of variables $\Xi(\theta) = PL(\theta)$, which leads to the following matrix inequalities

$$\begin{bmatrix} P & PA\left(\theta\right) + \Xi\left(\theta\right)C \\ \star & P \end{bmatrix} > 0 \quad \forall\theta \in \Theta \times \Upsilon \tag{16}$$

However, due to the structure of the matrices $A\left(\theta\right)$ and C, if any of the inputs can take a value equal to zero, then a problem of feasibility of the matrix inequalities appears due to the loss of observability of the pairs $\left(A\left(\theta\right), C\right)$. For example, if $u(k) = 0$ were an admissible input, the observability matrix for this value, defined as

$$\mathcal{O} = \begin{bmatrix} C \\ CA(\theta) \\ \vdots \\ CA\left(\theta\right)^{n_x+n_u-1} \end{bmatrix} = \begin{bmatrix} \bar{C} & 0 \\ \bar{C}\bar{A}\left(\bar{\theta}\right) & 0 \\ \vdots & \vdots \\ \bar{C}\bar{A}\left(\bar{\theta}\right)^{n_x+n_u-1} & 0 \end{bmatrix} \tag{17}$$

is such that $\mathrm{rank}(\mathcal{O}) < n_x + n_u$.

In order to guarantee the feasibility of the conditions for designing the observer gains, a switched LPV state/fault observer, which is the main contribution of this work, is proposed in the next section.

3. Main result

Before introducing the switched LPV state/fault observer, let us define the switching signal that will be used by the observer for achieving a correct estimation. In order to do so, let us define the following subsets of the input space

$$\mathcal{R}_{s_1 s_2 \ldots s_{n_u}} = \left\{ u : u_1 \overset{s_1}{\gtreqless} (-1) s_1 \varepsilon_1, \ldots, u_{n_u} \overset{s_{n_u}}{\gtreqless} (-1) s_{n_u} \varepsilon_{n_u} \right\} \tag{18}$$

$$\mathcal{Q}_{s_1 s_2 \ldots s_{n_u}} = \left\{ u : u_1 \overset{s_1}{\gtreqless} s_1 \varepsilon_1, \ldots, u_{n_u} \overset{s_{n_u}}{\gtreqless} s_{n_u} \varepsilon_{n_u} \right\} \tag{19}$$

$$\mathcal{R} \backslash \mathcal{Q}_{s_1 s_2 \ldots s_{n_u}} = \left\{ u : u \in \mathcal{R}_{s_1 s_2 \ldots s_{n_u}}, u \notin \mathcal{Q}_{s_1 s_2 \ldots s_{n_u}} \right\} \tag{20}$$

where the operators $\overset{s_j}{\gtreqless}$, $j = 1, \ldots, n_u$, are a shorthand notation for

$$\overset{s_j}{\gtreqless} = \begin{cases} \geq & if \ s_j = + \\ \leq & if \ s_j = - \end{cases} \tag{21}$$

and ε_j, $j = 1, \ldots, n_u$, are given small scalars. The piecewise constant switching signal

$$\sigma(k) = s_1(k) \ldots s_{n_u}(k) \tag{22}$$

defines at each time sample whether the index s_j of the active subsets $\mathcal{R}_{s_1 s_2 \ldots s_{n_u}}$ and $\mathcal{Q}_{s_1 s_2 \ldots s_{n_u}}$ is + or −, as given in (21). In particular, the switching rule that provides the switching signal is chosen to be dependent on the values of the inputs, as follows

$$s_j(k) = \begin{cases} s_j(k-1) & if \ u_j(k) \overset{s_j(k-1)}{\gtreqless} (-1) s_j(k-1) \varepsilon_j \\ -s_j(k-1) & if \ u_j(k) \overset{s_j(k-1)}{\lessgtr} (-1) s_j(k-1) \varepsilon_j \end{cases} \tag{23}$$

where the operators $\overset{s_j}{\lessgtr}$ are a shorthand notation for

$$\overset{s_j}{\lessgtr} = \begin{cases} < & if \ s_j = + \\ > & if \ s_j = - \end{cases} \tag{24}$$

In order to clarify the notation, let us consider a system with two available inputs $u_1(k)$ and $u_2(k)$, i.e. $n_u = 2$. Eq. (18)-(20) define the following subsets of the input space

$$\begin{array}{ll} \mathcal{R}_{++} = \{u : u_1 \geq -\varepsilon_1, u_2 \geq -\varepsilon_2\} & \mathcal{R}_{+-} = \{u : u_1 \geq -\varepsilon_1, u_2 \leq \varepsilon_2\} \\ \mathcal{R}_{-+} = \{u : u_1 \leq \varepsilon_1, u_2 \geq -\varepsilon_2\} & \mathcal{R}_{--} = \{u : u_1 \leq \varepsilon_1, u_2 \leq \varepsilon_2\} \end{array} \tag{25}$$

$$\begin{array}{ll} \mathcal{Q}_{++} = \{u : u_1 \geq \varepsilon_1, u_2 \geq \varepsilon_2\} & \mathcal{Q}_{+-} = \{u : u_1 \geq \varepsilon_1, u_2 \leq -\varepsilon_2\} \\ \mathcal{Q}_{-+} = \{u : u_1 \leq -\varepsilon_1, u_2 \geq \varepsilon_2\} & \mathcal{Q}_{--} = \{u : u_1 \leq -\varepsilon_1, u_2 \leq -\varepsilon_2\} \end{array} \tag{26}$$

$$\begin{array}{ll} \mathcal{R}\backslash\mathcal{Q}_{++} = \{u : u \in \mathcal{R}_{++}, u \notin \mathcal{Q}_{++}\} & \mathcal{R}\backslash\mathcal{Q}_{+-} = \{u : u \in \mathcal{R}_{+-}, u \notin \mathcal{Q}_{+-}\} \\ \mathcal{R}\backslash\mathcal{Q}_{-+} = \{u : u \in \mathcal{R}_{-+}, u \notin \mathcal{Q}_{-+}\} & \mathcal{R}\backslash\mathcal{Q}_{--} = \{u : u \in \mathcal{R}_{--}, u \notin \mathcal{Q}_{--}\} \end{array} \tag{27}$$

Then, $\sigma(k-1) = ++$ would indicate that the active subset at time sample $k-1$ is $\mathcal{R}_{++}$, while $\sigma(k-1) = +-$, $\sigma(k-1) = -+$ and $\sigma(k-1) = --$ would indicate $\mathcal{R}_{+-}$, $\mathcal{R}_{-+}$ and $\mathcal{R}_{--}$, respectively. Then, if the active subset at time sample $k-1$ were $\mathcal{R}_{++}$, the switching rule (23) would be as follows

$$\sigma(k) = \begin{cases} ++ & if\ u_1(k) \geq -\varepsilon_1, u_2(k) \geq -\varepsilon_2 \\ +- & if\ u_1(k) \geq -\varepsilon_1, u_2(k) < -\varepsilon_2 \\ -+ & if\ u_1(k) < -\varepsilon_1, u_2(k) \geq -\varepsilon_2 \\ -- & if\ u_1(k) < -\varepsilon_1, u_2(k) < -\varepsilon_2 \end{cases}$$

On the other hand, if the active subset at time sample $k-1$ were $\mathcal{R}_{+-}$, the switching rule would be

$$\sigma(k) = \begin{cases} ++ & if\ u_1(k) \geq -\varepsilon_1, u_2(k) > \varepsilon_2 \\ +- & if\ u_1(k) \geq -\varepsilon_1, u_2(k) \leq \varepsilon_2 \\ -+ & if\ u_1(k) < -\varepsilon_1, u_2(k) > \varepsilon_2 \\ -- & if\ u_1(k) < -\varepsilon_1, u_2(k) \leq \varepsilon_2 \end{cases}$$

Similar switching rules are obtained for the remaining active subsets $\mathcal{R}_{-+}$ or $\mathcal{R}_{--}$.

Taking into account the definition of the switching signal provided in (22)-(23), the following switched LPV state/fault observer is proposed in order to guarantee that in each subset of the input space $\mathcal{R}_{s_1 s_2 \ldots s_{n_u}}$, the design conditions for the gains are feasible

$$\hat{x}(k+1) = A(\theta(k))\hat{x}(k) + L_{\sigma(k)}(\theta(k))\left(\hat{y}(k) - y(k)\right) \tag{28}$$

$$\hat{y}(k) = C\hat{x}(k) \tag{29}$$

where $L_{\sigma(k)}(\theta(k))$ corresponds to the active LPV observer gain, that is defined by the value of the switching signal $\sigma(k)$. For comparison with LPV observers and switched observers, see [38] and [39], respectively. The design problem becomes to find the possible LPV observer gains $L_{s_1 \ldots s_{n_u}}(\theta(k))$ for all the subsets defined in (18), such that the dynamics of the estimation error

$$e(k+1) = \left(A\left(\theta(k)\right) + L_{\sigma(k)}\left(\theta(k)\right)C\right)e(k) \tag{30}$$

satisfies some stability condition.

The following theorem provides a sufficient condition for the stability of the estimation error dynamics (30). This proof resembles the reasoning used to prove the stability of switched LPV systems with average dwell time [24].

Theorem 1. *If there exist* 2^{n_u} *positive definite matrices* $P_l = P_l^T \in \mathbb{R}^{(n_x+n_u)\times(n_x+n_u)}$, 2^{n_u} *matrices* $\Xi_l(\theta) \in \mathbb{R}^{(n_x+n_u)\times n_y}$*, scalars* $0 \leq a \leq 1$*,* $b \geq 0$ *and* $\mu > 1$ *such that the following conditions hold*

$$\begin{bmatrix} P_l & P_l A\left(\theta\right) + \Xi_l\left(\theta\right) C \\ \star & aP_l \end{bmatrix} > 0, \quad \forall \theta \in \Theta \times \left(\Upsilon \cap \mathcal{Q}_l\right) \tag{31}$$

$$\begin{bmatrix} P_l & P_l A\left(\theta\right) + \Xi_l\left(\theta\right) C \\ \star & bP_l \end{bmatrix} > 0, \quad \forall \theta \in \Theta \times \left(\Upsilon \cap \mathcal{R}_l\right) \tag{32}$$

$$\frac{1}{\mu} P_m \leq P_l \leq \mu P_m \tag{33}$$

with l and m equal to all the possible combinations of index $s_1 \ldots s_{n_u}$ *as defined in* (18)-(19)*, then the error dynamics* (30) *converges asymptotically to zero as long as the LPV observer gains are calculated as*

$$L_l\left(\theta(k)\right) = P_l^{-1}\Xi_l\left(\theta(k)\right) \tag{34}$$

and the input sequence $u(k)$ *is such that for any* $k_0 \geq 0$*, it is possible to find a* $k_f > k_0$ *such that*

$$\left(\mu^N\right)\left(b^{N\bar{k}_{\mathcal{R}\setminus\mathcal{Q}}}\right)\left(a^{N\bar{k}_{\mathcal{Q}}}\right) < 1 \tag{35}$$

where N is the number of switches in $[k_0, k_f]$ *given by* (23)*,* $\bar{k}_{\mathcal{Q}}$ *is the average number of samples per switch during which* $u(k)$ *belongs to one of the subsets* $\mathcal{Q}_{s_1 \ldots s_{n_u}}$*, and* $\bar{k}_{\mathcal{R}\setminus\mathcal{Q}}$ *is the average number of samples per switch during which* $u(k)$ *belongs to the regions that belong to a subset* $\mathcal{R}_{s_1 \ldots s_{n_u}}$ *but does not belong to any subset* $\mathcal{Q}_{s_1 \ldots s_{n_u}}$.

Proof: First of all, for each possible combination of indices $s_1 \ldots s_{n_u}$ in (18)-(19), let us define the corresponding Lyapunov function

$$V_l\left(e(k)\right) = e(k)^T P_l e(k) \tag{36}$$

It is also assumed that over an interval $[k_0, k_f]$, the input $u(k)$ changes the active subset as $\mathcal{Q}_0 \to \mathcal{R}_0\setminus\mathcal{Q}_0, \ldots, \mathcal{Q}_l \to \mathcal{R}_l\setminus\mathcal{Q}_l, \ldots, \mathcal{Q}_{N-1} \to \mathcal{R}_{N-1}\setminus\mathcal{Q}_{N-1}$ at time samples

$k_{\mathcal{Q}}^0, \ldots, k_{\mathcal{Q}}^l, \ldots, k_{\mathcal{Q}}^{N-1}$, and as $\mathcal{R}_0 \backslash \mathcal{Q}_0 \to \mathcal{Q}_1, \ldots, \mathcal{R}_l \backslash \mathcal{Q}_l \to \mathcal{Q}_{l+1}, \ldots, \mathcal{R}_{N-1} \backslash \mathcal{Q}_{N-1} \to \mathcal{Q}_N$ at time samples $k_{\mathcal{R}\backslash\mathcal{Q}}^0, \ldots, k_{\mathcal{R}\backslash\mathcal{Q}}^l, \ldots, k_{\mathcal{R}\backslash\mathcal{Q}}^{N-1}$.

Then, considering conditions (31), at $k = k_{\mathcal{Q}}^0, \ldots, k_{\mathcal{Q}}^l, \ldots, k_{\mathcal{Q}}^{N-1}$, the Lyapunov functions $V_0, \ldots, V_l, \ldots, V_N$ satisfy

$$V_0\left(e(k_{\mathcal{Q}}^0)\right) < \left(a^{k_{\mathcal{Q}}^0 - k_0}\right) V_0\left(e(k_0)\right) \tag{37}$$

$$V_1\left(e(k_{\mathcal{Q}}^1)\right) < \left(a^{k_{\mathcal{Q}}^1 - k_{\mathcal{R}\backslash\mathcal{Q}}^0}\right) V_1\left(e(k_{\mathcal{R}\backslash\mathcal{Q}}^0)\right) \tag{38}$$

$$\vdots$$

$$V_l\left(e(k_{\mathcal{Q}}^l)\right) < \left(a^{k_{\mathcal{Q}}^l - k_{\mathcal{R}\backslash\mathcal{Q}}^{l-1}}\right) V_l\left(e(k_{\mathcal{R}\backslash\mathcal{Q}}^{l-1})\right) \tag{39}$$

$$\vdots$$

$$V_N\left(e(k_f)\right) < \left(a^{k_f - k_{\mathcal{R}\backslash\mathcal{Q}}^{N-1}}\right) V_N\left(e(k_{\mathcal{R}\backslash\mathcal{Q}}^{N-1})\right) \tag{40}$$

On the other hand, the conditions (32) guarantee that at $k = k_{\mathcal{R}\backslash\mathcal{Q}}^0, \ldots, k_{\mathcal{R}\backslash\mathcal{Q}}^l, \ldots, k_{\mathcal{R}\backslash\mathcal{Q}}^{N-1}$, the Lyapunov functions $V_0, \ldots, V_l, \ldots, V_{N-1}$ satisfy

$$V_0\left(e(k_{\mathcal{R}\backslash\mathcal{Q}}^0)\right) < \left(b^{k_{\mathcal{R}\backslash\mathcal{Q}}^0 - k_{\mathcal{Q}}^0}\right) V_0\left(e(k_{\mathcal{Q}}^0)\right) \tag{41}$$

$$V_1\left(e(k_{\mathcal{R}\backslash\mathcal{Q}}^1)\right) < \left(b^{k_{\mathcal{R}\backslash\mathcal{Q}}^1 - k_{\mathcal{Q}}^1}\right) V_1\left(e(k_{\mathcal{Q}}^1)\right) \tag{42}$$

$$\vdots$$

$$V_l\left(e(k_{\mathcal{R}\backslash\mathcal{Q}}^l)\right) < \left(b^{k_{\mathcal{R}\backslash\mathcal{Q}}^l - k_{\mathcal{Q}}^l}\right) V_l\left(e(k_{\mathcal{Q}}^l)\right) \tag{43}$$

$$\vdots$$

$$V_{N-1}\left(e(k_{\mathcal{R}\backslash\mathcal{Q}}^{N-1})\right) < \left(b^{k_{\mathcal{R}\backslash\mathcal{Q}}^{N-1} - k_{\mathcal{Q}}^{N-1}}\right) V_{N-1}\left(e(k_{\mathcal{Q}}^{N-1})\right) \tag{44}$$

In addition, the conditions (33) guarantee that at $k = k_{\mathcal{R}\backslash\mathcal{Q}}^0, \ldots, k_{\mathcal{R}\backslash\mathcal{Q}}^l, \ldots, k_{\mathcal{R}\backslash\mathcal{Q}}^{N-1}$

$$V_1\left(e(k_{\mathcal{R}\backslash\mathcal{Q}}^0)\right) < \mu V_0\left(e(k_{\mathcal{R}\backslash\mathcal{Q}}^0)\right) \tag{45}$$

$$\vdots$$

$$V_{l+1}\left(e(k_{\mathcal{R}\backslash\mathcal{Q}}^l)\right) < \mu V_l\left(e(k_{\mathcal{R}\backslash\mathcal{Q}}^l)\right) \tag{46}$$

$$\vdots$$

$$V_N\left(e(k_{\mathcal{R}\backslash\mathcal{Q}}^{N-1})\right) < \mu V_{N-1}\left(e(k_{\mathcal{R}\backslash\mathcal{Q}}^{N-1})\right) \tag{47}$$

Therefore, linking all the inequalities (37)-(47), the following is true

$$V_N\left(e(k_f)\right) < \left(\mu^N\right)\left(b^{\sum_{l=0}^{N-1}\left(k_{R\backslash Q}^l - k_Q^l\right)}\right)\left(a^{k_f - k_{R\backslash Q}^{N-1} + \sum_{l=1}^{N-1}\left(k_Q^l - k_{R/Q}^{l-1}\right) + k_Q^0 - k_0}\right) V_0\left(e(k_0)\right) \tag{48}$$

By considering

$$\bar{k}_Q = \frac{k_f - k_{R\backslash Q}^{N-1} + \sum_{l=1}^{N-1}\left(k_Q^l - k_{R/Q}^{l-1}\right) + k_Q^0 - k_0}{N} \tag{49}$$

$$\bar{k}_{R\backslash Q} = \frac{\sum_{l=0}^{N-1}\left(k_{R\backslash Q}^l - k_Q^l\right)}{N} \tag{50}$$

the inequality (48) can be rewritten as

$$V_N\left(e(k_f)\right) < \left(\mu^N\right)\left(b^{N\bar{k}_{R\backslash Q}}\right)\left(a^{N\bar{k}_Q}V_0\left(e(k_0)\right)\right) \tag{51}$$

Then, if the input $u(k)$ is such that for each $k_0 \geq 0$ it is possible to find $k_f > k_0$ that satisfies (35), the Lyapunov functions will be decreasing to zero, and therefore the estimation error $e(k)$ will converge asymptotically to zero, completing the proof. □

From a practical point of view, Theorem 1 cannot be used because it relies on the satisfaction of infinite constraints. However, the number of constraints can be reduced to a finite number by choosing the observer gain L to depend only on $\bar{\theta}(k)$, and by considering a polytopic representation of $A\left(\theta(k)\right)$ and $L\left(\bar{\theta}(k)\right)$, as follows

$$A\left(\theta(k)\right) = \sum_{i=1}^{N_{\bar{\theta}}}\alpha_i\left(\bar{\theta}(k)\right)\sum_{j=1}^{N_u}\beta_j^l\left(u(k)\right)A_{ij}^{\mathcal{Q}_l} \quad \forall\theta \in \Theta \times \left(\Upsilon \cap \mathcal{Q}_l\right) \tag{52}$$

$$A\left(\theta(k)\right) = \sum_{i=1}^{N_{\bar{\theta}}}\alpha_i\left(\bar{\theta}(k)\right)\sum_{j=1}^{N_u}\chi_j^l\left(u(k)\right)A_{ij}^{\mathcal{R}_l} \quad \forall\theta \in \Theta \times \left(\Upsilon \cap \mathcal{R}_l\right) \tag{53}$$

$$L_l\left(\bar{\theta}(k)\right) = \sum_{i=1}^{N_{\bar{\theta}}}\alpha_i\left(\bar{\theta}(k)\right)L_i^l \quad \forall\theta \in \Theta \times \left(\Upsilon \cap \mathcal{R}_l\right) \tag{54}$$

with

$$\sum_{i=1}^{N_{\bar{\theta}}}\alpha_i\left(\bar{\theta}(k)\right) = \sum_{j=1}^{N_u}\beta_j^l\left(u(k)\right) = \sum_{j=1}^{N_u}\chi_j^l\left(u(k)\right) = 1 \tag{55}$$

and $\alpha_i \geq 0\ \forall i = 1,\ldots,N_{\bar{\theta}}, \beta_j \geq 0\ \forall j = 1,\ldots,N_u, \chi_j \geq 0\ \forall j = 1,\ldots,N_u$.

Then, the following corollary can be obtained easily from Theorem 1.

Corollary 1. *Choose scalars $0 \leq a \leq 1$, $b \geq 0$, and find 2^{n_u} positive definite matrices $P_l = P_l^T \in \mathbb{R}^{(n_x+n_u)\times(n_x+n_u)}$ and $2^{n_u} N_{\bar{\theta}}$ matrices $\Xi_i^l \in \mathbb{R}^{(n_x+n_u)\times n_y}$ such that*

$$\begin{bmatrix} P_l & P_l A_{ij}^{\mathcal{Q}_l} + \Xi_i^l C \\ \star & a P_l \end{bmatrix} > 0 \tag{56}$$

$$\begin{bmatrix} P_l & P_l A_{ij}^{\mathcal{R}_l} + \Xi_i^l C \\ \star & b P_l \end{bmatrix} > 0 \tag{57}$$

and such that there exists $\mu > 1$ for which

$$\frac{1}{\mu} P_m \leq P_l \leq \mu P_m \tag{58}$$

with l and m equal to all the possible combinations $s_1, \ldots, s_{n_u}$ as defined in (18)-(19)*, $i = 1, \ldots, N_{\bar{\theta}}$ and $j = 1, \ldots, N_u$.*

Then, the error dynamics (30) *converges asymptotically to zero as long as the LPV observer gain is given by* (54) *with*

$$L_i^l = P_l^{-1} \Xi_i^l \tag{59}$$

and the input sequence $u(k)$ is such that for any $k_0 \geq 0$, it is possible to find a $k_f > k_0$ such that (35) *holds.*

Proof: The proof is based on a basic property of matrices [40], which establishes that any linear combinations of (56) and (57) with non-negative coefficients (of which at least one different from zero) is positive definite. Using the coefficients $\alpha_i\left(\bar{\theta}(k)\right)$ and $\beta_j^l\left(u(k)\right)$, (56) becomes

$$\sum_{i=1}^{N_{\bar{\theta}}} \alpha_i\left(\bar{\theta}(k)\right) \sum_{j=1}^{N_u} \beta_j^l\left(u(k)\right) \begin{bmatrix} P_l & P_l A_{ij}^{\mathcal{Q}_l} + \Xi_i^l C \\ \star & a P_l \end{bmatrix} > 0 \tag{60}$$

that, taking into account $\Xi_i^l = P_l L_i^l$, and (52), (54) and (55), becomes (31).

If the same process is applied to (57) using the coefficients $\alpha_i\left(\bar{\theta}(k)\right)$ and $\chi_j^l\left(u(k)\right)$ and taking into account (55)-(57), (32) would be obtained.

Since (58) corresponds to (33), the conditions provided by Theorem 1 are recovered, completing the proof. □

4. Illustrative Example

Consider the discrete-time LPV system subject to multiplicative actuator faults (1)-(2) with state and input matrices described by

$$\bar{A}(\bar{\theta}(k)) = \begin{bmatrix} 0.3 & 0.2 & \bar{\theta}_2(k) \\ 0.6 & \bar{\theta}_1(k) & 0.1 \\ 2\bar{\theta}_2(k) & 0.3 & 0.5 \end{bmatrix}, \bar{B} = \begin{bmatrix} 0.8 + \bar{\theta}_1(k) & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}, \bar{C} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

with the varying parameter $\bar{\theta}_1(k), \bar{\theta}_2(k) \in [0.1, 0.3]$ for all k, and with the inputs $u_1(k)$ and $u_2(k)$ taking values in $[-10, 10]$. By considering (5)-(8), under the assumption of slow-varying faults, the augmented system (9)-(10) is obtained as follows

$$A(\theta) = \left[\begin{array}{ccc|cc} 0.3 & 0.2 & \bar{\theta}_2(k) & (0.8 + \bar{\theta}_1(k))u_1(k) & 0 \\ 0.6 & \bar{\theta}_1(k) & 0.1 & 0 & u_2(k) \\ 2\bar{\theta}_2(k) & 0.3 & 0.5 & 0 & 0 \\ \hline 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{array}\right], C = \left[\begin{array}{ccc|cc} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{array}\right]$$

The subsets $\mathcal{R}_{\dots}$, $\mathcal{Q}_{\dots}$ and $\mathcal{R}\backslash\mathcal{Q}_{\dots}$ are defined as in the example provided in the previous section, with a choice of $\varepsilon_1 = \varepsilon_2 = 1$. By taking into account the limits of $\bar{\theta}_1(k)$, $\bar{\theta}_2(k)$ and $u(k)$, 16 vertex matrices are obtained for each subset.

By choosing $a = 0.9$ and $b = 2$, the LMIs (56)-(57) have been solved using the YALMIP toolbox [41] with SeDuMi solver [42], verifying that, for the obtained Lyapunov matrices, the condition (58) holds with $\mu = 5$. According to Corollary 1, if the LPV observer vertex gains are calculated as in (59), then the estimation error would converge to zero as long as the input sequence satisfies condition (35). It is worth remarking that, since the Lyapunov-based conditions are always sufficient for convergence, and not necessary, it is possible that the estimation error would still converge to zero even though the input sequence does not satisfy (35).

The results shown in the following refer to multiplicative faults assumed to occur in both inputs. The initial conditions and the scheduling parameter trajectory are as follows:

$$x(0) = \begin{bmatrix} 0.2 & 0.5 & 1 \end{bmatrix}^T \tag{61}$$

$$\hat{x}(0) = \left[\begin{array}{ccc|cc} -0.2 & 0 & -0.5 & 1 & 1 \end{array}\right]^T \tag{62}$$

$$\bar{\theta}_1(k) = 0.2 + 0.1\sin(0.005k) \tag{63}$$

$$\bar{\theta}_2(k) = 0.2 + 0.1\cos(0.01k) \tag{64}$$

The input sequences are chosen as follows (see Fig. 1)

$$u_1(k) = \begin{cases} 0 & \text{if } k \in [500, 1500] \\ 5\sin(0.001k) & \text{else} \end{cases} \tag{65}$$

$$u_2(k) = -5\cos(0.0046k) \tag{66}$$

Fig. 2 shows the states and their estimation, while Fig. 3 shows the state estimation errors obtained from the simulation. On the other hand, the faults and their estimation are depicted in Fig. 4. At the time instant $k = 500$, $u_1(k)$ becomes zero, such that the condition (35) is not satisfied anymore. However, the fault/state estimation errors have already converged to zero, so no problems arise. On the other hand, when the fault appears at sample $k = 1000$, the lack of excitation of the input causes the multiplicative fault in the first actuator γ_1 not to be correctly estimated. By recalling that the Lyapunov functions are quadratic functions of the state/fault estimation errors, it is reasonable that they start diverging as well, as shown in Fig. 5. However, when satisfactory input sequences enter into the system, i.e. starting from sample $k = 1500$, the switched LPV observer behaves as expected, such that both the faults and the states are correctly and rapidly estimated, and the Lyapunov functions converge again to zero.

Notice that, although in the proposed scenario $\gamma_i(k+1) = \gamma_i(k)$, $i = 1, 2$, does not hold at sample $k = 1000$ and in the interval $k \in [2500, 4000]$, the proposed approach is able to achieve correctly the goal of estimating jointly the states and faults in the system. It is worth stating that in the case of abrupt faults, the resulting stepwise change in the fault profile would cause a sudden change in the value of the Lyapunov function (e.g. at $k = 2500$ in Fig. 5). However, due to the actuator effectiveness being constant in the subsequent samples, the convergence of the Lyapunov function to zero would be guaranteed by the theoretical results provided by Theorem 1 (in other words, in the case of abrupt faults, the assumption of slow-varying faults would not affect the estimation performance). On the other hand, in the case of incipient faults (e.g. linear changes), undesired and not expected effects could appear, e.g. the bumps in the state estimation errors and fault estimates that are visible in the interval $k \in [2500, 4000]$ in Figs. 3-4. Although the results presented in this Section have shown that the proposed approach is able to work correctly for a wider range of multiplicative fault profiles than the ones satisfying the slow-varying assumption, it is clear that this point should be further investigated by future research.

5. Conclusions

This paper has proposed a method for estimating simultaneously the states and the actuator faults in discrete-time LPV systems, using a switched LPV observer. Differently from the existing approaches that consider additive faults, the proposed observer is able to estimate multiplicative faults. The proposed approach considers switching rules between different regions and design LMIs that take into account the properties of observability and non-observability of different regions. Sufficient conditions to design the observer gains were provided in the form of a set of LMIs. Moreover, it has been shown that if the input sequence has some characteristics in terms of numbers of switching in an interval, and average number of samples per switch in every switching region, the convergence of the error dynamics to zero is assured. Simulation results have shown and validated the relevant characteristics of the proposed method.

Although the simulation results have shown that the proposed approach can be successfully applied to faults with different profiles (e.g. stepwise or linear), it should be underlined that the theoretical guarantees provided by the design would hold strictly only for the case of slow-varying faults, i.e. the case when $\gamma(k+1) \approx \gamma(k)$. Future research will aim at overcoming this limitation, so as to obtain a design approach which provides theoretical guarantees about the convergence to zero of the estimation error for a wider range of possible fault change rates. Also, the design of input sequences which satisfy the required characteristics will be considered, with the aim of developing an *active* fault estimation technique.

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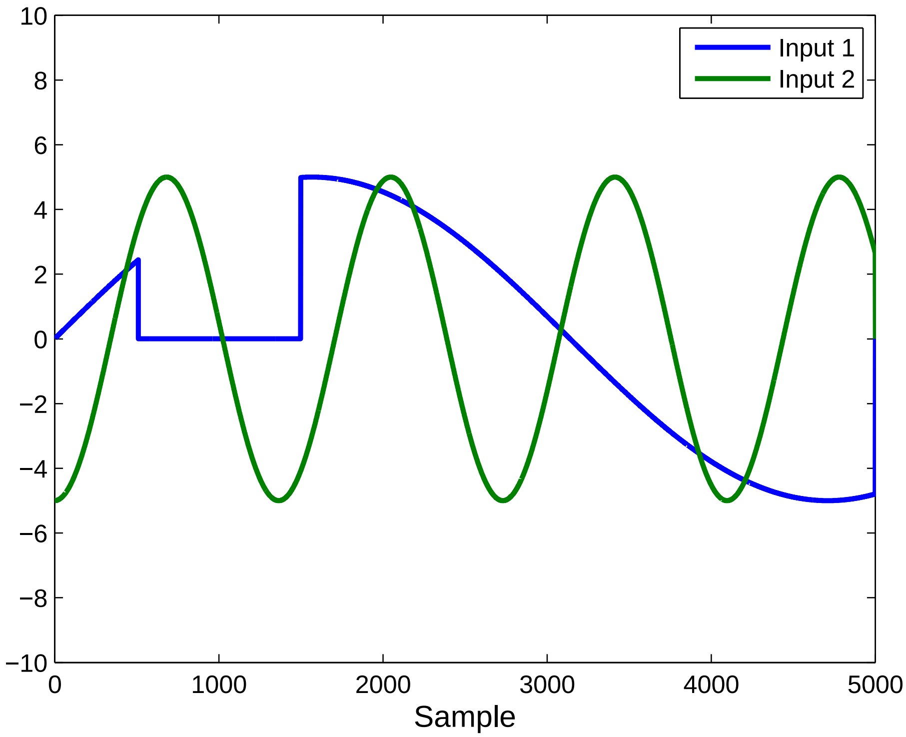

Figure 1: Applied inputs $u_1(k)$ and $u_2(k)$.

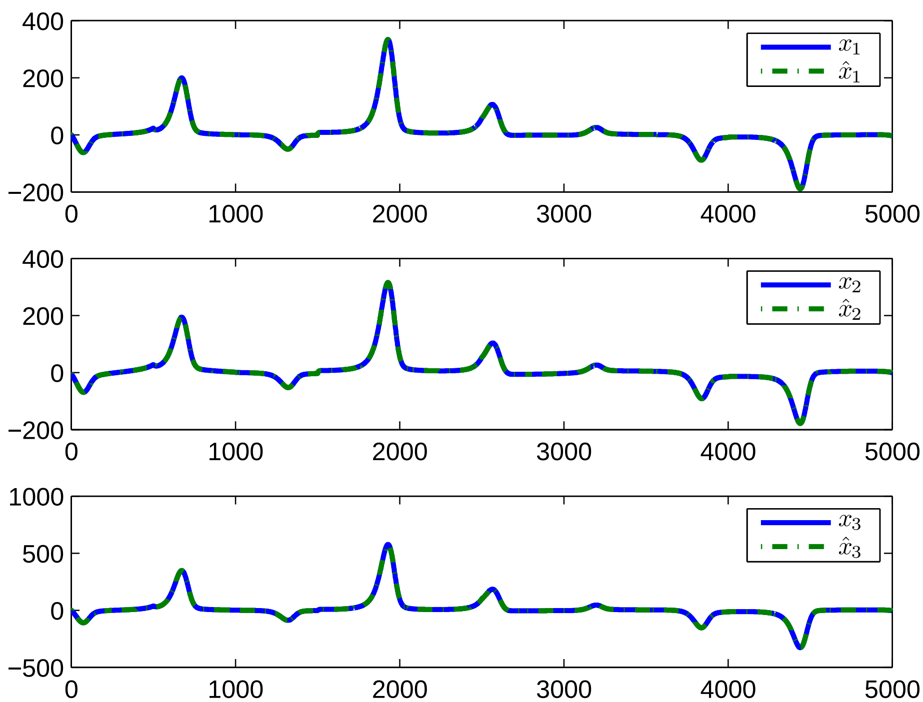

Figure 2: States and their estimation.

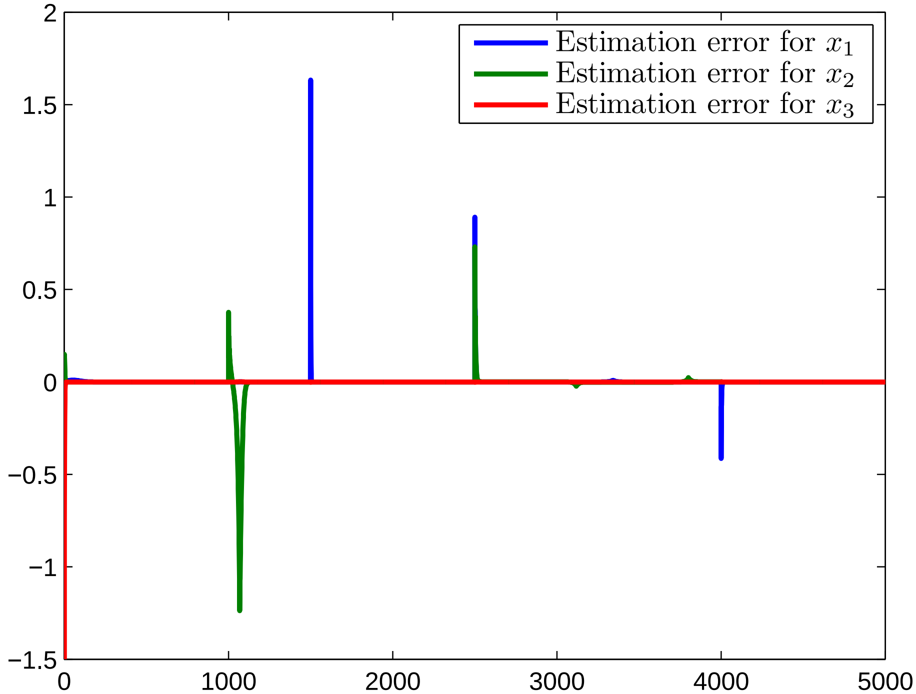

Figure 3: State estimation errors.

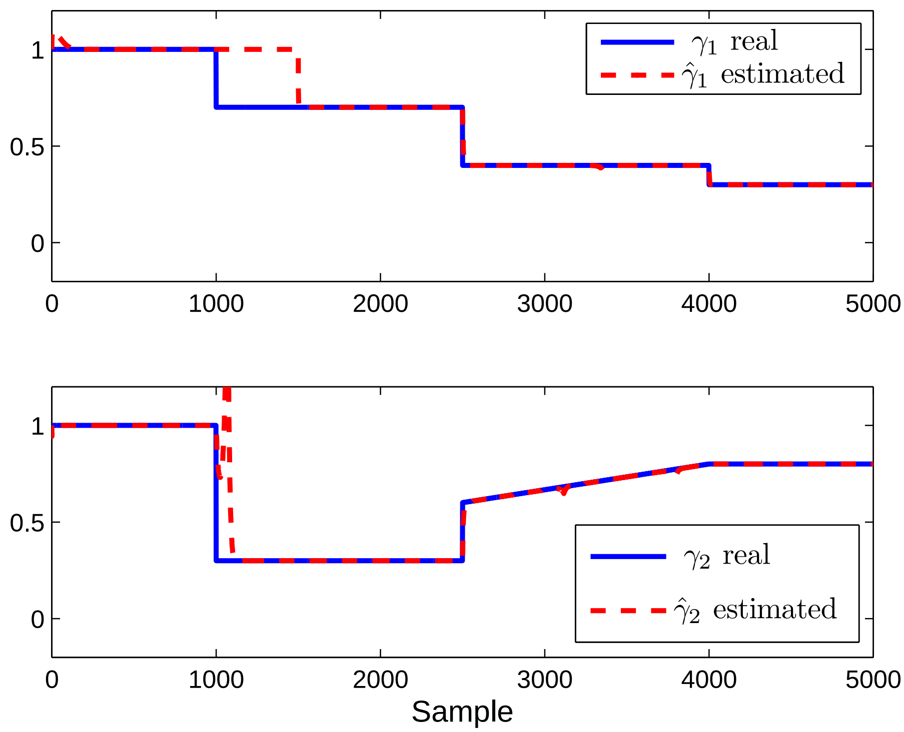

Figure 4: Faults and their estimation.

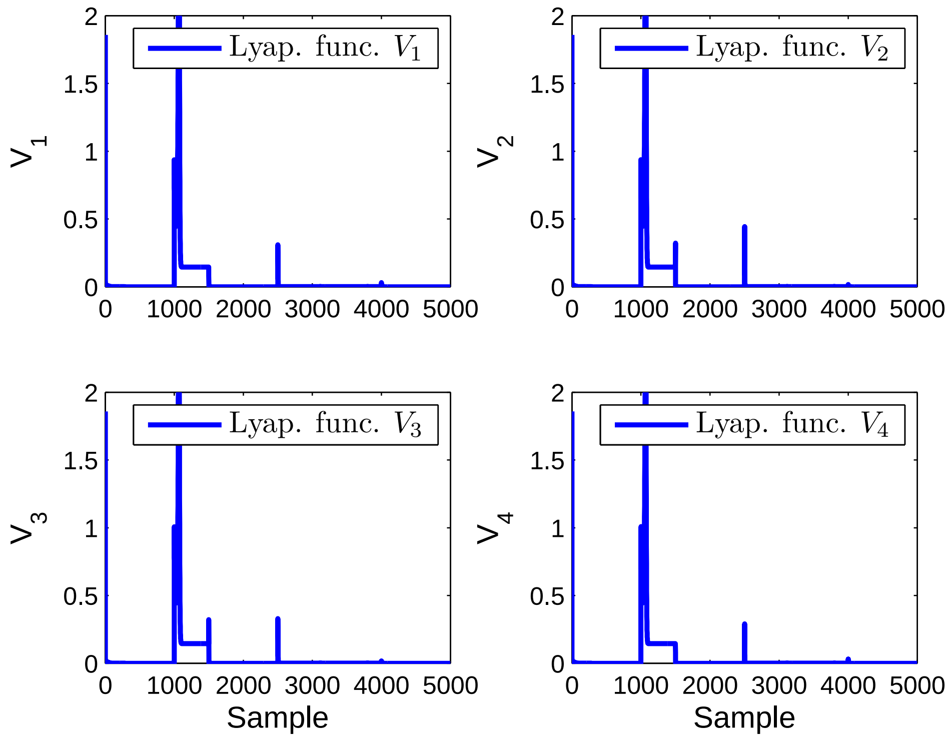

Figure 5: Lyapunov functions.

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