Interval observer fault detection ensuring detectability and isolability by using a set-invariance approach ⋆

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Abstract: This paper proposes the design of an interval observer-based Fault Detection and Isolation (FDI) algorithm using the set-invariance approach. Using this approach, both fault detectability and isolability properties of the proposed interval observer based FDI algorithm can be characterized in steady-state operation of the system. The effect of the uncertainty is taken into account using zonotopic-set representations. Finally, a simulation example based on a two-tanks system is employed to both illustrate and discuss the effectiveness of the proposed approach.

Keywords: Fault detection, bounded uncertainties, observers-based approach, set-invariance approach, zonotopes

1. INTRODUCTION

Model-based Fault Detection and Isolation (FDI) is one of the most developed families of approaches that relies on the use of a mathematical model describing the system behavior (Gertler, 1998). Generally speaking, model-based Fault Detection (FD) is based on checking the consistency of the system behavior measured using sensors against a model of the system in non-faulty situation (Gertler, 1998; Puig, 2010). Any inconsistency between the measured outputs using sensors and the estimated behavior that is computed by using the system model, called residual, is considered to be due to a fault occurrence (Gertler, 1998; Chen and Patton, 1999). However, the mismatch between the actual process behaviour and its mathematical model is non-negligible because of the existence of uncertainties (Puig, 2010). Thus, model-based approaches should be robust against uncertainties. One way to include robustness in FD is by evaluating the residual with an adaptive threshold value that takes into account the uncertainties (Puig, 2010).

In recent years, several methods have been developed and introduced to explicitly consider such uncertainties in the models (Pourasghar et al., France, 2017). These methods can be classified into stochastic and deterministic classes of approaches. In the former class, the uncertainties are represented using random variables, and in the latter class, the uncertainties are assumed unknown but bounded (Puig, 2010; Pourasghar et al., 2016b). Moreover, when the uncertainties are taken into account deterministically, following the so-called set-membership approach, the estimation produces a set of possible states that can be bounded using different geometrical structures, e.g., interval boxes, polyhedrons/polytopes, ellipsoids and zonotopes (Puig, 2010; Alamo et al., 2005).

Therefore, detecting the fault depends on how residuals are generated. On top of them, observer-based approach is one of the most widely used approach to generate the residual. The fundamental concept of observer-based approaches is to estimate the states from the measurements using either stochastic (e.g., Kalman filters) or deterministic approaches (e.g., Luenberger observers) for modeling the uncertainties (Puig, 2010).

On the other hand, set-invariance approach is another technique in FDI framework. The general concept of set-invariance approach relies on computing the invariant residual set for the healthy and faulty operation of the system. As long as healthy and faulty sets are separated, the FDI can be performed (Seron et al., 2008; Ocampo-Martinez et al., 2010). The major drawback of set-invariance approach is related to the limitation of computing the finite description of its boundary in some cases. Moreover, this method of analysis has another limitation of only being able to characterise the steady state operation of the system. Thus, set-invariance approach can only ensure the separation of healthy and faulty residual sets in steady state (Seron et al., 2008).

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Recently, there has been an increasing interest in using the set-invariance capability in FDI framework for transient operation of the system. In Xu et al. (2013), the relationship between the classical observer-based approach and set-invariance approach is proposed. However, the analysis of FDI is not considered. Furthermore, in Pourasghar et al. (2016a) and Kodakkadan et al. (2017), the characterization of the minimum magnitude of the fault that can be detected is proposed using both the observer-based and set-invariance-based approaches.

So far, there have been few discussions about the combination of the observer-based and set-invariance-based approaches. Hence, they are still considered two different techniques. In this regard, the main contribution of this paper is to integrate the interval observer-based and set-invariance-based approaches in order to propose an FDI algorithm that can be used in both transient and steady-state operation of a linear system. Finally, a well-known benchmark based on the two-tank system is used as a case study to illustrate the results obtained in the paper and show the effectiveness of the proposed approach.

The structure of the paper is the following. The problem formulation, observer structure and the general framework of set-invariance approach are discussed in Section 2. Online propagation of the residual set and the FDI design integrating the observer-based and set-invariance-based approaches are proposed in Section 3. The application of a two-tank system is used in order to illustrate the effectiveness of the proposed approach in Section 4. Finally, the general conclusion is drawn in Section 5.

Notation

Throughout this paper, $\mathbb{R}^n$ denotes the set of n-dimensional real numbers and $\oplus$ denotes the Minkowski sum. The matrices are written using capital letter, e.g., A, the calligraphic notation is used for denoting sets, e.g., $\mathcal{X}$, $\|.\|_s$ denotes the s-norm, $k \in \mathbb{N}$ indicates the discrete time and the subscript or superscript io and is denote the interval observer and set-invariance approaches, respectively.

2. PROBLEM FORMULATION

2.1 Problem set-up

A discrete linear uncertain system to be monitored in this paper is described in state space as

$$x_+ = Ax + Bu + E_\omega \omega, \tag{1a}$$
$$y = Cx + E_\upsilon \upsilon, \tag{1b}$$

where $u \in \mathbb{R}^{n_u}$, $y \in \mathbb{R}^{n_y}$ and $x \in \mathbb{R}^{n_x}$ are the input, the output and the state vectors, respectively. Furthermore, $A \in \mathbb{R}^{n_x \times n_x}$, $B \in \mathbb{R}^{n_x \times n_u}$ and $C \in \mathbb{R}^{n_y \times n_x}$ are the state-space matrices. State disturbance and process noise vectors are defined by $\omega \in \mathbb{R}^{n_x}$ and $\upsilon \in \mathbb{R}^{n_y}$, respectively. Moreover, E_ω and E_υ are the associated distribution matrices with appropriate dimensions. Notice that the subscript $k+1$ is replaced by $+$ and k is omitted for the sake of simplified notations in dynamical model (1).

Furthermore, the additive uncertainties, i.e., ω and υ, are assumed unknown but bounded, i.e.,

$$\mathcal{W} = \{\omega_k \in \mathbb{R}^{n_x} : |\omega_k - c_\omega| \le \bar{\omega}, c_\omega \in \mathbb{R}^{n_x}, \bar{\omega} \in \mathbb{R}^{n_x}\}, \tag{2a}$$
$$\mathcal{V} = \{\upsilon_k \in \mathbb{R}^{n_y} : |\upsilon_k - c_\upsilon| \le \bar{\upsilon}, c_\upsilon \in \mathbb{R}^{n_y}, \bar{\upsilon} \in \mathbb{R}^{n_y}\}, \tag{2b}$$

where c_ω, $\bar{\omega}$, c_υ and $\bar{\upsilon}$ are constant vectors. Both $\mathcal{W}$ and $\mathcal{V}$ sets can be written as two zonotopes, i.e.,

$$\mathcal{W} = \langle c_\omega,\ R_\omega \rangle, \tag{3a}$$
$$\mathcal{V} = \langle c_\upsilon,\ R_\upsilon \rangle, \tag{3b}$$

where c_ω and c_υ denote the centers of the state disturbance and the process noise zonotopes, respectively, with their generator matrices $R_\omega \in \mathbb{R}^{n_\omega \times n_\omega}$ and $R_\upsilon \in \mathbb{R}^{n_\upsilon \times n_\upsilon}$.

2.2 Observer structure

Monitoring the healthy functioning of a system with the dynamical model (1) can be done by designing a linear Luenberger observer of the form

$$\hat{x}_+ = A\hat{x} + Bu + L(y - \hat{y}), \tag{4a}$$
$$\hat{y} = C\hat{x}, \tag{4b}$$

where $\hat{x}$ and $\hat{y}$ are the state estimation and the output prediction vectors, respectively. Furthermore, L denotes the observer gain that should be chosen such that $(A-LC)$ is a Schur matrix.

Generally speaking, the FD test is done based on testing the consistency of measurements and the behavior of the system obtained by the system model. Any inconsistency will be considered as a fault.

Assumption 2.1. The pair $\{A, C\}$ is detectable. □

Assumption 2.2. Additive uncertainties represented in (3) are assumed to be bounded by a unitary hypercube centered at the origin, i.e., $\forall\ k \ge\ 0,\ \omega = [-1,\ 1] = \langle 0,\ I_{n_\omega} \rangle$ and $\upsilon = [-1,\ 1] = \langle 0,\ I_{n_\upsilon} \rangle$, where $I_{n_\omega} \in \mathbb{R}^{n_\omega \times n_\omega}$ and $I_{n_\upsilon} \in \mathbb{R}^{n_\upsilon \times n_\upsilon}$ denote the identity matrices. □

Assumption 2.3. The initial state x_0 belongs to the zonotopic set $\mathcal{X}_0 = \langle c_0,\ R_0 \rangle$, where $c_0 \in \mathbb{R}^{n_x}$ denotes the center and $R_0 \in \mathbb{R}^{n_x \times n_{R_0}}$ is a non-empty matrix containing the generators matrix of the initial zonotope $\mathcal{X}_0$. □

Then, using the zonotopic-set representation of the uncertainties, the state bounding observer and the output prediction of the dynamical model (1) can be obtained by using Proposition 2.1.

Proposition 2.1. Considering Assumptions 2.2, 2.3 and the Luenberger observer structure in (4), c and R of the state estimation zonotope $\hat{\mathcal{X}}$, and c_y and R_y of the predicted output zonotope $\hat{\mathcal{Y}}$ can be computed as

$$c_+ = (A - LC)c + Bu + Ly, \tag{5a}$$
$$R_+ = \left[(A - LC)\bar{R}\ E\right], \tag{5b}$$
$$c_y = Cc, \tag{5c}$$
$$R_y = \left[C\bar{R}\ E_\upsilon\right], \tag{5d}$$

where $E = [E_\omega\ -LE_\upsilon]$ and the reduction operator that is defined in Property A.2 is introduced as $\bar{R} =\downarrow_q \{R\}$. Moreover, the state inclusion property $x \in \langle c, R \rangle$ in Property A.3 holds for all $k \ge 0$.

Proof. Assume $x \in \langle c,\ R \rangle$, $\omega \in \langle 0,\ I_{n_\omega} \rangle$ and $\upsilon \in \langle 0,\ I_{n_\upsilon} \rangle$ for all $k \ge 0$, where the inclusion property is preserved by using the reduction operator, i.e., $x \in \langle c,\ \bar{R}\rangle$. Thus, (4a) can be written using the set representation as

$$x_+ \in \langle c_+,\ R_+\rangle = \langle (A-LC)c,\ (A-LC)\bar{R}\rangle \oplus \langle B_u u,\ 0\rangle \oplus \langle 0,\ E_\omega\rangle \oplus \langle Ly,\ 0\rangle \oplus \langle 0,\ -LE_v\rangle . \tag{6}$$

Consequently, (4b) can be written using the set representation as

$$y \in \langle c_y,\ R_y\rangle = \langle Cc,\ C\bar{R}\rangle \oplus \langle 0,\ E_v\rangle . \tag{7}$$

Then, based on Definition A.2 and Property A.1, c_+, R_+, c_y and R_y in (6) and (7) can be expressed as in (5). ■

Generally speaking, the fault can be detected by generating the residual $r = y - \hat{y}$. In this case, the residual can be obtained as a zonotope since using the zonotopic definition of a set for propagating the uncertainty. Therefore, the residual zonotope can be generated as

$$c_{r^{io}} = y - Cc, \tag{8a}$$

$$R_{r^{io}} = \left[-C\bar{R}\ -E_v\right]. \tag{8b}$$

Hence, the fault can be detected by checking the satisfaction of $0 \notin \langle c_{r^{io}},\ R_{r^{io}}\rangle$. The computational burden can be reduced by checking whether or not 0 is inside an aligned box enclosing the zonotope $\langle c_{r^{io}},\ R_{r^{io}}\rangle$ as

$$0 \notin \langle c_{r^{io}},\ b(R_{r^{io}})\rangle , \tag{9}$$

where $\langle c_{r^{io}},\ b(R_{r^{io}})\rangle$ is enclosed by an aligned box denoted by $b(R_{r^{io}})$. In the case that (9) is satisfied, the existence of the fault will be detected. Otherwise, the system operation is considered healthy.

2.3 Set-invariance approach

Given a system with the dynamical model (1) and considering the observer (4), there exists an invariant set such that the trajectories of the residual will ultimately converge to this set. In this regard, the residual can be generated as

$$r = y - \hat{y} = C\tilde{x} + E_v v, \tag{10}$$

where $\tilde{x} = x - \hat{x}$ is the state estimation error whose dynamics can be described using (1) and (4) as

$$\tilde{x}_+ = (A - LC)\tilde{x} + Ez, \tag{11}$$

where $z = [\omega\ v]^T$.

According to Blanchini (1999), the set $\Phi^{\tilde{x}}$ is an Robust Positive Invariant (RPI) set for (11) iff for all $\omega \in \mathcal{W}$ and $v \in \mathcal{V}$,

$$(A - LC)\tilde{x} + Ez\ \in \Phi^{\tilde{x}}. \tag{12}$$

There is a large volume of published studies describing the construction of the invariant set $\Phi^{\tilde{x}}$ (see Kofman (2005)). One of the most used methods is known as Ultimate Bound (UB) method that is reported in Kofman et al. (2007) and recalled in Theorem 2.1.

Theorem 2.1. According to Kofman et al. (2007), the Jordan Conical form of matrix $(A - LC)$ can be obtained as $J = V(A - LC)V^{-1}$, where J is a diagonal matrix corresponding to the Jordan-normal form of $(A-LC)$ and V is a non-singular transformation matrix. Thus, the state estimation error $\tilde{x}$ in (11) will ultimately converge to a polyhedral RPI set that is constructed as

$$\Phi^{\tilde{x}} = \left\{\tilde{x} \in \mathbb{R}^{n_x} : |V^{-1}\tilde{x}| \le (I - |J|)^{-1}|V^{-1}|\overline{z} + \varepsilon\right\}, \tag{13}$$

where ε can be any arbitrary small vector with strictly positive components.

Proof. The proof follows from the results presented in (Kofman et al., 2007). ■

Then, considering $\tilde{x} \in \Phi^{\tilde{x}}$ and $v \in \mathcal{V}$, the projection of $\tilde{x}$ into the residual space can be obtained as

$$\Phi^r = C\Phi^{\tilde{x}} \oplus \mathcal{V}. \tag{14}$$

Thus, if $\tilde{x}$ is inside $\Phi^{\tilde{x}}$, then, r is inside Φ^r. Therefore, the existence of the fault will be detected based on the set-invariance approach whenever

$$r \notin \Phi^r. \tag{15}$$

3. FDI DESIGN

Given the observer (4) and considering unknown but bounded uncertainties, two main approaches are presented in this paper to detect the fault: i) zonotopic interval observer and ii) set-invariance approach. Each approach has its own advantages and drawbacks. In the former class, the FD principle leads to detect the fault in both transient state and steady state since its residual generation is performed on-line. The latter only works during the steady-state operation of the system since the off-line analysis is used to generate the invariant set characterizing the residual, i.e.,

$$r = \underbrace{y - \hat{y}}_{\text{on-line}} = \underbrace{C(x - \hat{x}) + E_v v}_{\text{off-line}}. \tag{16}$$

Therefore, the most serious weakness of the set-invariance approach in comparison with the zonotopic interval observer is related to its FD limitation in transient state. On the other hand, one important characteristic of the set-invariance approach is the ability of providing both detectability and isolability by means of the off-line computation of an invariant residual set that characterizes the healthy functioning of the system. In the set-invariance approach, the fault isolability can be obtained by guaranteeing the separability of faulty residual sets, not being possible using the zonotopic interval observer alone. Therefore, the purpose of this paper is to propose an FDI observer-based approach that integrates interval observer and set-invariance approaches with the following features:

- to be used during whole time range (transient and steady states),
- to ensure both detectability and isolability properties.

Coming back to the main issue discussed at the beginning of this section, since the set in (13) is symmetric around the origin and considering the zonotopic representation of sets, Proposition 3.1 implies that $\tilde{\mathcal{X}}$ can also be denoted as a zonotope.

Proposition 3.1. Considering the steady-state operation, the interval hull (see Definition A.3) of the RPI set of the state estimation error in (11) can be computed using

$$c_{\tilde{x}_\infty} = 0, \tag{17a}$$

$$\left\| R_{\tilde{x}_{\infty_i}} \right\|_1 = \left\| R_{\infty_i} \right\|_1 , \tag{17b}$$

where $c_{\tilde{x}_\infty}$ and $R_{\tilde{x}_{\infty_i}}$ are the center and the generator matrix of the zonotopic state estimation error set $\tilde{\mathcal{X}}$, respectively. Furthermore, i denotes the i-row of matrices $R_{\tilde{x}}$ and R when k tends to infinity.

Proof. Considering the dynamical model of the state estimation error in (11) and assuming the initial state estimation error $\tilde{x}_0$ belongs to the zonotopic set $\tilde{\mathcal{X}}_0 = \langle c_{\tilde{x}_0},\ R_{\tilde{x}_0}\rangle$ that is defined as an RPI set, it can be written for all $k \geq 0$ if $\tilde{x} \in \langle c_{\tilde{x}},\ R_{\tilde{x}}\rangle$, $\omega \in \langle 0,\ I_{n_\omega}\rangle$ and $\upsilon \in \langle 0,\ I_{n_\upsilon}\rangle$ that

$$\tilde{x}_{j+1} \in \left\langle c_{\tilde{x}_{j+1}},\ R_{\tilde{x}_{j+1}}\right\rangle = \left\langle (A-KC)c_{\tilde{x}_j},\ (A-KC)R_{\tilde{x}_j}\right\rangle \oplus \langle 0,\ E_\omega\rangle \oplus \langle 0,\ -KE_\upsilon\rangle \tag{18}$$

is another RPI set with arbitrarily expected precision enclosing the minimal RPI (mRPI) set of the state estimation error in (11), where j denotes the j-th element of the set sequence (18). Thus, the center and the generator matrix of the set in (18) can be calculated as

$$c_{\tilde{x}_{j+1}} = (A-LC)\, c_{\tilde{x}_j}, \tag{19a}$$
$$R_{\tilde{x}_{j+1}} = \left[(A-LC)R_{\tilde{x}_j}\ \ E\right]. \tag{19b}$$

Furthermore, the state estimation error will converge towards the RPI set in steady state. Thus, the RPI set of $\tilde{x}$ can be computed by recursive propagation of the zonotopic set (19) starting from the initial set $\tilde{\mathcal{X}}_0 = \langle c_{\tilde{x}_0},\ R_{\tilde{x}_0}\rangle$. $\tilde{\mathcal{X}}_0$ belongs to the RPI set that can be computed using the any available method, e.g., UB method in Section 2.3. Furthermore, it can be written that if j tends to infinity (i.e., steady state), the following conditions will be satisfied:

$$c_{\tilde{x}_{j+1}} = c_{\tilde{x}_j}, \tag{20a}$$
$$\left\|R_{\tilde{x}_{j+1}}\right\|_1 = \left\|R_{\tilde{x}_j}\right\|_1. \tag{20b}$$

Therefore, the same formulations as in (17) for the center and the shape matrix of $\tilde{\mathcal{X}}$ can be obtained by the substitution of conditions (20) in (19). ■

Therefore, considering Proposition 3.1, the residual set in steady state is invariant and can be considered as an invariant set that combines the polytopic UB method with the zonotopic iterative approximation. Then, when the system is working in either healthy or faulty operations, the residual set characterizing healthy or faulty system operations can be computed. The benefit of generating the residual in this way is to track the residual trajectories not only in steady state but also in transient state. Furthermore, in the case of having several types of faults, as long as the faulty and the healthy sets are separated, the proposed FDI approach will be able to work correctly.

In this paper, sensor and actuator faults will be considered. Including their effect, the dynamical model (1) can be rewritten as

$$x_+ = Ax + Bu + E_\omega\omega + F_a f_a, \tag{21a}$$
$$y = Cx + E_\upsilon\upsilon + F_y f_y, \tag{21b}$$

where the vectors $f_a \in \mathbb{R}^{n_u}$ and $f_y \in \mathbb{R}^{n_y}$ denote the actuator and output sensor faults with their associated matrices $F_a \in \mathbb{R}^{n_x \times n_u}$ and $F_y \in \mathbb{R}^{n_y \times n_y}$, respectively. Furthermore, the other type of fault that is considered in this paper is known as input sensor fault. The effect of the input sensor fault on the observer (4) is considered as

$$\hat{x}_+ = A\hat{x} + B\left(u + F_u f_u\right) + L(y - \hat{y}), \tag{22}$$

where $f_u \in \mathbb{R}^{n_u}$ represents the input sensor fault with its associated matrix $F_u \in \mathbb{R}^{n_u \times n_u}$. Figure 1 shows the schematic graphical interpretation of the different actuator and sensors faults.

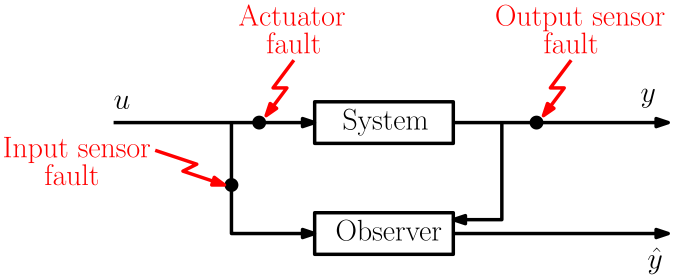


Fig. 1. Graphical interpretation of the different actuator and sensors faults.

Assumption 3.1. The additive fault represented in (21) and (22) is assumed to be bounded by a unit hypercube expressed as centered zonotopes, i.e., for all $k \geq 0$, $f_\bullet = [-1,\ 1] = \left\langle 0,\ I_{n_{f_\bullet}}\right\rangle$, where the subscript $\bullet$ can be respectively assigned to y, u or a associated with the considered output sensor, input sensor and actuator faults, respectively. □

Furthermore, the dynamics of state estimation error in (11) can be rewritten in faulty operation of the system as

$$\tilde{x}_+ = (A-LC)\tilde{x} + Ez + Ff, \tag{23}$$

where

$$F = \left[F_a\ -LF_y\ -BF_u\right], \tag{24a}$$
$$f = \left[f_a\ f_y\ f_u\right]^T. \tag{24b}$$

Furthermore, the effect of uncertainties and faults can be decomposed as in Proposition 3.2.

Proposition 3.2. Considering the dynamical model (21) and the observer (22), the effect of uncertainties and fault can be decomposed considering the state estimation error dynamics in (23) as

$$\tilde{x}_+ \in \left\langle c_{\tilde{x}_{d_+}},\ R_{\tilde{x}_{d_+}}\right\rangle \oplus \left\langle c_{\tilde{x}_{f_+}},\ R_{\tilde{x}_{f_+}}\right\rangle, \tag{25}$$

with

$$c_{\tilde{x}_{d_+}} = (A-LC)\, c_{\tilde{x}_d}, \tag{26a}$$
$$R_{\tilde{x}_{d_+}} = \left[(A-LC)\bar{R}_{\tilde{x}_d}\ \ E\right], \tag{26b}$$
$$c_{\tilde{x}_{f_+}} = (A-LC)\, c_{\tilde{x}_f}, \tag{26c}$$
$$R_{\tilde{x}_{f_+}} = \left[(A-LC)\bar{R}_{\tilde{x}_f}\ \ F\right], \tag{26d}$$

where the subscripts d and f denote the effect of uncertainties (i.e., state disturbance and measurement noise) and the fault, respectively.

Proof. Assuming $\tilde{x} \in \langle c_{\tilde{x}_d},\ R_{\tilde{x}_d}\rangle \oplus \left\langle c_{\tilde{x}_f},\ R_{\tilde{x}_f}\right\rangle$ and considering Assumptions 2.2 and 3.1, the zonotopic form of the state estimation error in (19) can be expressed as

$$x_+ \in \langle (A-LC)c_{\tilde{x}_d},\ (A-LC)R_{\tilde{x}_d}\rangle \oplus \left\langle (A-LC)c_{\tilde{x}_f},\ (A-LC)R_{\tilde{x}_f}\right\rangle \oplus \langle 0,\ E\rangle \oplus \langle 0,\ F\rangle. \tag{27}$$

Furthermore, consider that the superposition principle can be explicitly invoked in the considered linear setting. Therefore, using Definition A.2, the center and the generator matrices in (27) can be reorganized as in (19) and (26). Thus, $\tilde{x}_+ \in \left\langle c_{\tilde{x}_{d_+}},\ R_{\tilde{x}_{d_+}}\right\rangle \oplus \left\langle c_{\tilde{x}_{f_+}},\ R_{\tilde{x}_{f_+}}\right\rangle$. ■

Consequently, the state estimation error can be projected into the residual space using (16) to obtain the effect of the fault in the residual set. Thus, Proposition 3.2 allows to derive the residual set as

$$c_{r_{d_+}} = Cc_{\tilde{x}_{d_+}}, \tag{28a}$$
$$R_{r_{d_+}} = \left[CR_{\tilde{x}_{d_+}} \;\; E_v\right], \tag{28b}$$
$$c_{r_{f_+}} = Cc_{\tilde{x}_{f_+}}, \tag{28c}$$
$$R_{r_{f_+}} = \left[CR_{\tilde{x}_{f_+}}\right]. \tag{28d}$$

Therefore, generating the on-line residual set following (28), it can be used in both transient and steady states to ensure the detectability and isolability in the case of satisfaction of the conditions in Theorems 3.1 and 3.2.

Theorem 3.1. (Detectability condition) Considering Definition A.3 and the decomposed form of the residual in (28), the fault will be detected if

$$c_{r_{f_{\bullet_i}}} \pm \left\|R_{r_{f_{\bullet_i}}}\right\|_1 \notin 0 \pm 2\left\|R_{r_{d_i}}\right\|_1, \tag{29}$$

where i corresponds to the i-th component of the vector c and matrix R. Furthermore, the subscript $\bullet$ can be assigned by y, u and a for the considered output sensor, input sensor and actuator faults, respectively. Moreover, the factor 2 appears because the worst-case scenario is considered, where the uncertainties have a maximum influence in the opposite direction compared to that of the fault occurrence.

Proof. Consider (28) in faultless scenario (i.e., $f = 0$), $r \in \langle c_{r_d},\, R_{r_d}\rangle$. But, in the faulty operation of the system $r \notin \langle c_{r_d},\, R_{r_d}\rangle$. Therefore, it can be written that

$$\langle c_{r_d},\, R_{r_d}\rangle \oplus \langle c_{r_f},\, R_{r_f}\rangle \notin \langle c_{r_d},\, R_{r_d}\rangle. \tag{30}$$

Then, (30) can be rewritten using Definition A.3 as

$$\left(c_{r_{d_i}} \pm \left\|R_{r_{d_i}}\right\|_1\right) + \left(c_{r_{f_i}} \pm \left\|R_{r_{f_i}}\right\|_1\right) \notin \left(c_{r_{d_i}} \pm \left\|R_{r_{d_i}}\right\|_1\right). \tag{31}$$

Finally, the condition (29) is obtained by manipulating (31). This gives the proof of Theorem 3.1. ■

The condition in Theorem 3.1 is sufficient only for detecting the fault. Moreover, the fault can be isolated if the intersection between the residual sets obtained based on different type of faults is empty. Therefore, the conditions in Theorem 3.2 should also be satisfied together with condition (29) in order to ensure both detection and isolation of the fault.

Theorem 3.2. (Isolability condition) Considering the decomposed form of the residual set in (28), a necessary condition for isolating a fault $f_{\bullet_p}$ from a fault $f_{\bullet_k}$ is

$$c_{r_{f_{\bullet_p}}} \pm \left\|R_{r_{f_{\bullet_p}}}\right\|_1 \notin c_{r_{f_{\bullet_q}}} \pm \left\|R_{r_{f_{\bullet_q}}}\right\|_1, \tag{32}$$

where the subscripts p and q are used to distinguish the different actuator and different sensor faults.

Proof. The proof is similar to the one in Theorem 3.1. Considering (28) and (29), it can be written that a fault $f_{\bullet_p}$ is isolable from a fault $f_{\bullet_q}$ if

$$\left\langle c_{r_{f_{\bullet_p}}},\, R_{r_{f_{\bullet_p}}}\right\rangle \notin \left\langle c_{r_{f_{\bullet_q}}},\, R_{r_{f_{\bullet_q}}}\right\rangle. \tag{33}$$

Then, (33) can be rewritten using Definition A.3 as in (32). This gives the proof of Theorem 3.2. ■

4. CASE STUDY

4.1 Plant Description

The proposed interval observer-based FDI scheme will be tested using a two-tank system based on the well-known benchmark proposed in Johansson (2000). A schematic of the system can be seen in Figure 2.

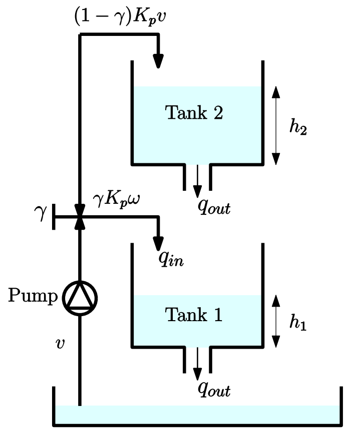


Fig. 2. Schematic diagram of the two-tanks system.

The input of the two-tanks system is the pump flow rate that is determined when applying voltage v to the pump. Therefore, the action of the pumps is to pour the tanks by extracting the water from the basin. Moreover, Tank 1 is placed below Tank 2. Furthermore, the outputs of the process are the water levels in both upper and lower tanks that are obtained as voltages from the measurement devices.

Additionally, Tank 1 is being affected by an additional disturbance ω that is generated by the uncertain position of the valve γ that can vary between 0 and 1 based on experimental apparatus, i.e., the position of the valve is the ratio modeling how the output of the pump is divided between upper and lower tanks. Thus, the water flow to each tank is controlled by the position of the valve considered as $\gamma \in (0, 1)$. Furthermore, both upper and lower tanks are made from plexiglas tubes with the height of 20 cm that are connected by the pipe with a diameter of 6 cm.

Given all the physical features, the mathematical model of the process can be determined based on the mass balance relations and Bernoullis law as

$$\frac{dh_1(t)}{dt} = -\frac{a_1}{A_1}\sqrt{2gh_1(t)} + \frac{a_2}{A_1}\sqrt{2gh_2(t)} + \frac{\gamma K_p}{A_1}\omega(t), \tag{34a}$$
$$\frac{dh_2(t)}{dt} = -\frac{a_2}{A_2}\sqrt{2gh_2(t)} + \frac{(1-\gamma)K_p}{A_2}v(t), \tag{34b}$$

where

- K_p is the pump constant,
- v is the velocity of the water flow through the pump,
- A_i is the cross section of Tank i, with $i = 1, 2$,
- a_i is the cross sectional area of the outlet pipes,
- g is acceleration due to gravity,
- h_i is the level of the water in Tank i, with $i = 1, 2$,
- $K_p v$ is the flow through the pump,
- $(1-\gamma)K_p v(t)$ is the flow towards the Tank 1 according to the valve position,

- $\gamma K_p \omega(t)$ is the flow towards the Tank 2 according to the valve position.

The non-linear model (34) is linearized around the following working point:

- $h_1^* = 12.4$ cm,
- $h_2^* = 1.8$ cm,
- $v = 3.00$ V,

with the following parameter values:

- $K_p = 3.35$ cm^3/Vs,
- $\gamma = 0.60$.

Hence, introducing the variables $\tilde{h}_i = h_i - h_i^*$ and $\tilde{v}_i = v_i - v_i^*$, the linearized model of (34) can be written as follows:

$$\dot{\tilde{h}}(t) = \begin{bmatrix} -\frac{1}{T_1} & \frac{A_2}{A_1 T_2} \\ 0 & -\frac{1}{T_2} \end{bmatrix} \tilde{h}(t) + \begin{bmatrix} \frac{\gamma K_p}{A_1} & 0 \\ 0 & \frac{(1-\gamma)K_p}{A_2} \end{bmatrix} \tilde{v}(t), \quad (35a)$$

$$y(t) = \begin{bmatrix} K_c & 0 \\ 0 & K_c \end{bmatrix} \tilde{h}(t), \quad (35b)$$

where K_c is measured laboratory parameter. Moreover, $T_i = \frac{A_i}{a_i}\sqrt{\frac{2h_i^*}{g}}$ with $i = 1, 2$.

According to Johansson (2000), the parameters of the model in (35) are $A_1 = A_2 = 28$ cm^2, $a_1 = a_2 = 0.071$ cm^2, $K_c = 0.50$ V/cm and $g = 981$ cm/s^2. Therefore, $T_1 = 62.7034$ s and $T_2 = 23.8900$ s.

Using the Euler discretization with a sampling time of 1s, the linearized model of this system can be rewritten in the state-space form as

$$\tilde{h}_+ = A\tilde{h} + B\tilde{v} + E_\omega \omega, \quad (36a)$$

$$y = C\tilde{h} + E_v v, \quad (36b)$$

where $A = \begin{bmatrix} 0.9842 & 0.0407 \\ 0 & 0.9590 \end{bmatrix}$, $B = \begin{bmatrix} 0.0831 & 0.0007 \\ 0 & 0.0352 \end{bmatrix}$ and $C = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}$. Furthermore, taking into account the state disturbance and measurement noise, E_ω and E_v are simulated in (36) with

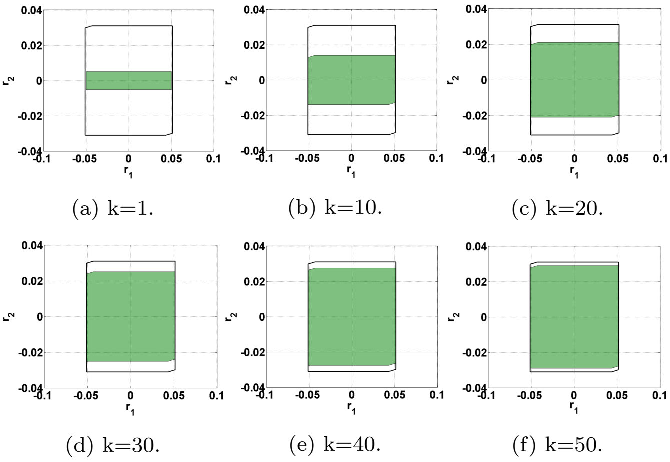
$$E_\omega = \begin{bmatrix} 0.05 & 0 \\ 0 & 0.05 \end{bmatrix}, \qquad E_v = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.05 \end{bmatrix}. \quad (37)$$

As it can seen in (37), E_ω is used to define a disturbance influencing all the states and the measurement noise is modeled through E_v.

Based on (36), it can be noticed that the range of the measured output is 10 V since the height of the each tank is 20 cm and $K_c = 0.5$ V/cm. Moreover, the incremental value of the measured output around the working point of the lower tank is 4 V to 8 V (or 8 cm to 16 cm).

4.2 Performing FDI

As discussed in Section 2, there are two different approaches for bounding the effect of uncertainty in the residual. On the one hand, the on-line interval observer that is able to generate the residual set in both transient-state and steady-state. On the other hand, the set-invariance approach that is an off-line procedure to compute the residual set in steady-state (see (16)). Figure 3 presents the residual set based on on-line interval observer that is obtained from the transient operation of the healthy system. The obtained residual zonotopes at time instants $k = 1$, $k = 10$, $k = 20$, $k = 30$, $k = 40$ and $k = 50$ are shown in Figure 3 for the healthy functioning of the system in (36). From the results in Figure 3, it can be seen that the residual generated using the proposed on-line zonotopic observer (the green zonotopes) ultimately converges to the one that is shown by the black solid line.



(a) k=1. (b) k=10. (c) k=20.

(d) k=30. (e) k=40. (f) k=50.

Fig. 3. On-line propagation of residual set using zonotopic interval observer during transient state and considering the healthy functioning of the system.

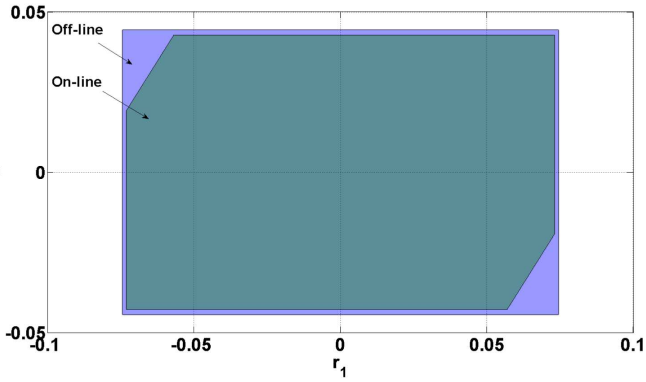


Fig. 4. Comparing the residual set using on-line and off-line approaches in steady state.

Figure 4 shows the comparison of the obtained residual set from the on-line zonotopic observer and the off-line set-invariance approach during steady state. A comparison of the two results reveals that no significant differences were found between the size of the residual zonotopes in steady state. Therefore, the obtained zonotopic set for the residual based the proposed on-line zonotopic interval observer is confirmed by the well-known set-invariance approach.

As it is explained in the description of the case study, the electrical actuator is used for controlling the position of the valve. The position of the valve during the experiment is related to the parameter γ with the range between 0 and 1. In this regard, the flow to the lower and upper tanks is influenced by the valve position through $\gamma K_p \omega$ and $(1-\gamma)K_p v$, respectively. The outputs of the system are the water levels in Tanks 1 and 2 that can be measured using the measurement devices as voltages.

Furthermore, considering (21) and (22), the system is influenced by the fault through matrices F_a, F_y and F_u that are chosen as

$$F_a = 5B, \qquad F_y = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}, \qquad F_u = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}, \quad (38)$$

where matrix F_a is selected to simulate the actuator fault, matrix F_y is defined with the whole range of the measurement and matrix F_u is defined with the whole range of the input. Moreover, the direction of the faults are simulated through the vectors f_a, f_y and f_u with suitable dimensions.

As mentioned before, the most interesting advantage of the proposed zonotopic interval observer is related to its capability of being used during the transient state. In this regard, Figure 5 presents the FDI using zonotopic interval observer the faults are simulated using the matrices in (38) at $k = 15$ to show the benefit of using this approach in transient.

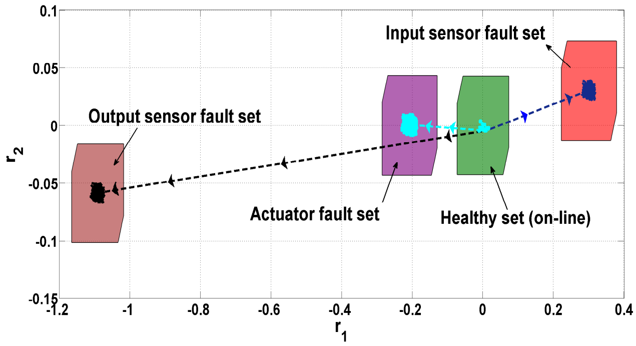


Fig. 5. FDI using proposed on-line approach using interval observer when fault occurs at $k = 15$ to simulate the transient operation of the system.

Figure 5 shows four different residual sets. The residual set that is indicated by the green color is computed based on the healthy operation of the system. Furthermore, the red, brown and purple residual sets are computed when considering the actuator, output and input sensor faults, respectively. As can be seen from Figure 5, the separation of the healthy and faulty sets is obtained after occurrence the fault at $k = 15$. This separation can be understood as the fault occurrence. Moreover, if the sets for the given fault will not overlap, it is possible to distinguish the different faults. Therefore, as can be seen in Figure 5, FDI is provided by obtaining the separated sets for the healthy and faulty functioning of the case study. It means, in the case of entering the residual trajectories into particular set that is introduced in Figure 5, the type of the fault can be identified.

Further analysis is done for the case study when the fault occurs at $k = 500$ to simulate the steady-state operation of the system as can be seen in Figure 6. Furthermore, considering the system in steady state, it allows to use the set-invariance approach for computing the residual set (invariant set) during healthy functioning of the system. The separated healthy and faulty sets are obtained for the case study as can be seen in Figure 6. Therefore, the fault detectability and isolability are provided by both the interval observer and set-invariance approaches. Finally,

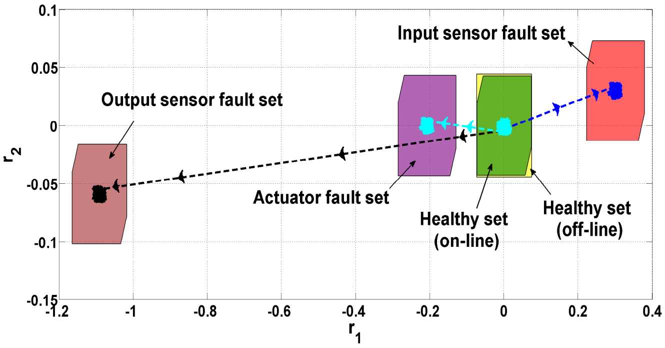


Fig. 6. FDI using proposed on-line approach using interval observer when fault occurs at $k = 500$ to simulate the steady-state operation of the system.

Figure 6 confirms through simulations the obtained results previously presented in Figure 5 for the transient state. This illustrates that the proposed interval observer scheme is well able to ensure the fault detectability and isolability properties.

5. CONCLUSION

This paper has proposed an interval observer-based Fault Detection and Isolation (FDI) algorithm integrated with the set-invariance approach. As a novelty, in the proposed FDI design, fault detectability and fault isolability can be ensured in both transient and steady states. In the proposed algorithm, the influences of all possible state disturbance and measurement noise are addressed using the zonotopic-set representation of a set. The obtained results show the capability of the proposed algorithm not only in steady state but also in transient state. Finally, a case study based on two-tank system is used to illustrate the obtained results. As a future research, the characterization of the minimum magnitude of the fault that can be detected and also isolated will be further analyzed in order to improve the performance the proposed approach in FDI framework.

Appendix A. BACKGROUND ON ZONOTOPES

Definition A.1. (Zonotope) A zonotope $\langle c, R\rangle \subset \mathbb{R}^n$ with the center $c \in \mathbb{R}^n$ and the generator matrix $R \in \mathbb{R}^{n\times p}$ is a polytopic set defined as a linear image of the unit hypercube $[-1, 1]^n$, i.e.,

$$\langle c, R\rangle = \{c + Rs, \|s\|_\infty \le 1\}. \qquad (A.1)$$

Moreover, a centered zonotope is denoted by $\langle R\rangle = \langle 0, R\rangle$. Any permutation of the columns of R leaves it invariant. □

Definition A.2. (Minkowski sum) Considering two sets $\mathcal{A}$ and $\mathcal{B}$, their Minkowski sum is a set defined as $\mathcal{A} \oplus \mathcal{B} = \{a + b | \, a \in \mathcal{A}, \, b \in \mathcal{B}\}$. Furthermore, the Minkowski sum of the zonotopes $\mathcal{Z}_1 = \langle c_1, R_1\rangle$ and $\mathcal{Z}_2 = \langle c_2, R_2\rangle$ is $\mathcal{Z}_1 \oplus \mathcal{Z}_2 = \langle c_1 + c_2, [R_1 \; R_2]\rangle$. □

Definition A.3. (Interval hull) The interval hull of a given zonotope $\mathcal{Z} = \langle c, R\rangle$, denoted by $\Box\mathcal{Z}$, is the smallest interval box that contains $\mathcal{Z}$ and can be evaluated for all $i = 1, 2, \ldots, n$ as

$$\Box\mathcal{Z} = \{z : |z_i - c_i| \le \|R_i\|_1\}, \qquad (A.2)$$

where R_i indicates the i-th line of matrix R, and z_i and c_i are the i-th components of z and c, respectively. □

Definition A.4. (Invariant set) The invariant set $\Omega \subseteq \mathcal{Z}$ is the set which its existence allowed the evolution of a constrained system, where $z_0 \in \Omega \subseteq \mathcal{Z}$ and then, $z_K \in \Omega \subseteq \mathcal{Z}$ for all time steps k. □

Property A.1. (Linear image) The linear image of a zonotope $\mathcal{Z} = \langle c, R\rangle$ by a compatible matrix L is $L \odot \langle c, R\rangle = \langle Lc, LR\rangle$. □

Property A.2. (Reduction operator) A reduction operator, denoted $\downarrow_q$, permits to reduce the number of generators of a zonotope $\langle c,\ R\rangle$ to a fixed number q while preserving the inclusion property $\langle c,\ R\rangle \subset \langle c,\ \downarrow_q \{R\}\rangle$. A simple yet efficient solution to compute $\downarrow_q\ \{R\}$ is given in Combastel (2003). It consists in sorting the columns of R on decreasing Euclidean norm and enclosing the influence of the smaller columns only into an easily computable interval hull, so that the resulting matrix $\downarrow_q\ \{R\}$ has no more than q columns. □

Property A.3. (Zonotope inclusion) Given a zonotope $\mathcal{Z} = \langle c,\ R\rangle \subset \mathbb{R}^n$, a zonotope inclusion, indicated by $\diamond(\mathcal{Z})$, is defined as $\diamond(\mathcal{Z}) = \langle c, [mid(R)\ S]\rangle$, where S is a diagonal matrix that satisfies $S_{ii} = \sum_{j=1}^{m} \dfrac{diam(R_{ij})}{2}$, $i = 1, 2, \cdots, n$, with $mid(.)$ and $diam(.)$ being the center and diameter of the interval matrix, respectively. □

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