Robust Periodic Economic Model Predictive Control using Probabilistic Set Invariance for Descriptor Systems ⋆

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Abstract: This paper proposes a robust periodic economic model predictive control (MPC) using probabilistic set invariance for discrete-time descriptor systems. The considered systems are affected by additive disturbances that are bounded and follow a zero-mean Gaussian distribution. To guarantee robustness of the proposed robust economic MPC, a probabilistic invariant set with a given probability is used to contract the state and input constraints. An auxiliary input variable is defined to satisfy algebraic equations from descriptor systems in presence of disturbances. Finally, an application of the proposed MPC strategy to a smart micro-grid is provided to illustrate the performance.

Keywords: Economic MPC, robustness, periodic operation, probabilistic set invariance, descriptor systems, smart grids.

1. INTRODUCTION

During the past decade, economic model predictive control (MPC) has been widely investigated and attracted much attention (Mayne, 2014). Economic MPC has been also proved to be useful for a large amount of industrial processes, such as drinking/waster-water networks (Puig et al., 2017; Wang et al., 2017; Zeng and Liu, 2015) and electrical grids (Pereira et al., 2015). Considering endogenous and exogenous signals, an optimal periodic operation can be achieved in economic MPC (see, e.g. Huang et al. (2011); Limon et al. (2014); Müller and Grüne (2016)).

From an application point of view, disturbances often affect performance negatively and can even lead to constraint violations and possible loss of feasibility of MPC controllers. In literature, several robust economic MPC controllers have been developed (see e.g. Broomhead et al. (2015); Bayer et al. (2016, 2018)), where the tube-based technique introduced in Mayne et al. (2005) is usually used to refine the state and input constraints.

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Among increasing applications, the mathematical model includes not only differential/difference equations describing system dynamics but also algebraic equations describing static relations among the chosen variables due to mass balance and energy conservation laws. For instance, in water distribution networks, inflows and outflows through an interconnected non-storage node are built in an algebraic equation that must be used to describe the system behavior. This class of systems, modeled by differential/difference and algebraic equations, is called *descriptor*, *singular* or *differential/difference-algebraic* systems (Dai, 1989). The control design and optimization for descriptor systems have been discussed in Biegler et al. (2012) and some applications to water systems and electrical grids can be found in Wang et al. (2017); Pereira et al. (2017).

It is worth mentioning that if disturbances are not taken into account, algebraic equations can be regarded as additional equality constraints in the prediction model. However, for descriptor systems affected by unknown disturbances, algebraic equations cannot be satisfied in a robust manner for all possible perturbations. For linear systems, it is possible to guarantee robust constraint satisfaction of linear algebraic equations by reducing the degrees of freedom of the controller, including in the prediction model an explicit feedback to compensate the effect of the perturbations (Pereira et al., 2017). In this paper, we use the probabilistic set invariance introduced in Kofman et al. (2011) and Kofman et al. (2012), where unknown disturbances follows a zero-mean probability distribution. The probabilistic invariant set will be applied in the robust economic MPC design following a tube-based technique (Mayne et al., 2005).

The main contribution of this paper is to propose a robust economic MPC of descriptor systems for periodic operation following the economic MPC formulation in Wang et al. (2018). The additive disturbances are considered in the system, which is assumed to be bounded and follows a zero-mean Gaussian distribution. We use a tube-based technique with the probabilistic invariant set to refine the constraints of the corresponding MPC problem. Taking into account that linear algebraic equations appear in the descriptor system, a new input variable is defined to satisfy these algebraic equations affected by additive disturbances. Finally, we apply the proposed robust economic MPC strategy to a smart micro-grid including three nanogrids to show its performance.

The remainder of this paper is structured as follows. The problem statement is presented in Section 2. The probabilistic set invariance is introduced in Section 3. The robust periodic economic MPC using probabilistic set invariance is proposed in Section 4. An application of the proposed robust periodic economic MPC to a smart microgrid is provided in Section 5. Finally, conclusion remarks are drawn in Section 6.

Notation. $\mathbb{R}^n$ is a set of real numbers with the order n and $\mathbb{N}$ is a set of natural numbers. For a vector z, we use $\text{diag}(z)$ to denote a diagonal matrix with elements in diagonal defined by z. For two matrices A and B, we use $\text{blkdiag}(A, B)$ to denote a block diagonal matrix with elements in diagonal defined by A and B. For two scalars a and b, $\text{mod}(a, b)$ returns the modulo operation of a and b. For two sets $\mathcal{X}, \mathcal{Y} \subset \mathbb{R}^n$, we use $\oplus$ to denote the Minkowski sum as $\mathcal{X} \oplus \mathcal{Y} := \{x+y \in \mathbb{R}^n : x \in \mathcal{X}, y \in \mathcal{Y}\}$, and use $\ominus$ to denote the Pontryagin difference as $\mathcal{X} \ominus \mathcal{Y} := \{z \in \mathbb{R}^n : z+y \in \mathcal{X}, \forall y \in \mathcal{Y}\}$.

2. PROBLEM STATEMENT

Consider the following class of discrete-time linear descriptor systems subject to additive disturbances

$$x(k+1) = Ax(k) + Bu(k) + B_d d(k) + B_w w(k), \tag{1a}$$
$$0 = E_x x(k) + E_u u(k) + E_d d(k) + E_w w(k), \tag{1b}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, $d \in \mathbb{R}^{m_d}$, $w \in \mathbb{R}^{m_w}$ denote the vectors of system states, control inputs, endogenous demands and unknown disturbances, respectively. Besides, $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $B_d \in \mathbb{R}^{n\times m_d}$, $B_w \in \mathbb{R}^{n\times m_w}$, $E_x \in \mathbb{R}^{n_r\times n}$, $E_u \in \mathbb{R}^{n_r\times m}$, $E_d \in \mathbb{R}^{n_r\times m_d}$ and $E_w \in \mathbb{R}^{n_r\times m_w}$ are system matrices depending on the system topology.

The system states and control inputs of (1) are constrained in the following convex sets:

$$x(k) \in \mathcal{X}, \;\; u(k) \in \mathcal{U}, \; \forall k \in \mathbb{N}. \tag{2}$$

Before the discussion of designing the robust periodic economic MPC, the following assumptions are made:

Assumption 1. The signal $d(k)$ is a known input following a periodic behavior, that is $d(k) = d(k+T)$, $\forall k \in \mathbb{N}$ with a period T.

Assumption 2. The disturbance $w(k)$ is formed by m_w independent noise elements, where each element $w_i(k)$ for $i = 1, \dots, m_w$ is a white noise that follows a zero-mean Gaussian distribution with the variance Σ_{w_i}. The variance matrix of $w(k)$ is a constant diagonal matrix, that is $\Sigma_w = \text{diag}\left(\Sigma_{w_1}, \dots, \Sigma_{w_{m_w}}\right)$. Therefore, with a confidence level $0 < \alpha_w < 1$, the confidence interval of $w(k)$ can be found as $w(k) \in \mathcal{W}$, $\forall k \in \mathbb{N}$ with the probability $\Pr(w(k) \in \mathcal{W}) = \alpha_w$.

Assumption 3. The system state $x(k)$ in (1) is fully measurable at each time step $k \in \mathbb{N}$.

The performance of the system (1) is measured by a time-varying economic cost function:

$$\ell\left(x(k), u(k), c_i\right), \; i = \text{mod}(k, T), \tag{3}$$

where $c = \{c_0, \dots, c_{T-1}\}$ is a periodic sequence with a period T. In practice, the signal c could represent the time-varying economic price pattern, for instance the price of the electricity usages in electrical grids. In general, the control objective is to minimize a closed-loop economic cost function, that can be formulated as

$$J_\infty\left(x_\infty, u_\infty, c\right) = \lim_{M\to\infty} \frac{1}{M} \sum_{k=0}^{M-1} \ell\left(x(k), u(k), c_i\right), \tag{4}$$

for $i = \text{mod}(k, T)$.

Since $w(k)$ is unknown, the descriptor model in (1) cannot be directly used as the prediction model in the MPC design. In this paper, we will propose a robust periodic economic MPC by means of the combination of the tube-based technique and the probabilistic set invariance and a reduction of the degrees of freedom of the controller to guarantee the satisfaction of the equality constraints.

3. PROBABILISTIC SET INVARIANCE

In this section, we introduce the probabilistic set invariance proposed in Kofman et al. (2011) and Kofman et al. (2012). The probabilistic invariant set will be further used in the design of the robust periodic economic MPC.

For a dynamical system described in (1a) with $u \equiv 0$ and $d \equiv 0$, based on Kofman et al. (2012), the probabilistic invariant set is defined in the following.

Definition 1. (Probabilistic invariant sets). Consider a dynamical system described in (1a) with $u \equiv 0$ and $d \equiv 0$, a probability p with $0 < p \leq 1$. $\mathcal{S} \subset \mathbb{R}^n$ is a probabilistic invariant set with a probability p if for any $x(k) \in \mathcal{S}$, the probability $\Pr\left[x(k+M) \in \mathcal{S}\right] \geq p$ for each $M \in \mathbb{N}$ with $M > 0$.

Since the system state vector $x \in \mathbb{R}^n$, given a probability p with $0 < p < 1$, let us define n parameters $\tilde{p}_i$ for $i = 1, \dots, n$ such that

$$0 < \tilde{p}_i < 1, \; \sum_{i=1}^{n} \tilde{p}_i = 1 - p. \tag{5}$$

Therefore, we address the computation result of the probabilistic invariant set in the following lemma.

Lemma 1. Consider a dynamical system described in (1a) with $u \equiv 0$ and $d \equiv 0$, where the matrix A is Schur stable and diagonalizable, and the Jordan decomposition of A can be given by $\Lambda = \text{diag}(\lambda_1, \ldots, \lambda_n) = V^{-1}AV$. Suppose that the disturbance vector $w(k)$, $\forall k \in \mathbb{N}$ follows a zero-mean Gaussian distribution with the variance matrix Σ_w. Given a probability p and the corresponding $\tilde{p}_i$ for $i = 1, \ldots, n$ satisfying (5). Then, $\mathcal{S} := \{x \in \mathbb{R}^n : |V^{-1}x| \leq b\}$ is a probabilistic invariant set for this system with a probability p, where the vector b is composed by $b = [b_1, \ldots, b_n]^T$ and b_i for $i = 1, \ldots, n$ can be computed by

$$b_i := \frac{\sqrt{2\left[\Sigma_v\right]_{i,i}}}{(1 - |\lambda_i|)} \text{erf}^{-1}(1 - \tilde{p}_i), \tag{6}$$

where $[\Sigma_v]_{i,i}$ is the i-th diagonal element of $\Sigma_v = V^{-1}B_w\Sigma_w B_w^T(V^{-1})^T$. erf is the error function: $\text{erf}(z) := \frac{2}{\sqrt{\pi}}\int_0^z e^{-\zeta^2}d\zeta$ and erf^{-1} is the inverse erf function.

Proof. The proof of this lemma can be found in (Kofman et al., 2011, Theorem 2) and is omitted at here. □

From Definition 1, with a given probability p, the set $\mathcal{S}$ obtained from Lemma 1 is also RPI such that

$$A\mathcal{S} \oplus B_w\mathcal{W} \subseteq \mathcal{S}. \tag{7}$$

4. ROBUST PERIODIC ECONOMIC MPC USING PROBABILISTIC SET INVARIANCE

In this section, we propose the robust periodic economic MPC using probabilistic set invariance for the descriptor system (1). The goal is to use a robust tube-based technique based on nominal predictions that drives the closed-loop system trajectory to a neighborhood of an optimal periodic steady trajectory. A local controller is used to reduce the difference between nominal predictions and closed-loop trajectory. This local controller is designed to stabilize the nominal dynamical model of (1a) and simultaneously to satisfy the algebraic equation (1b).

4.1 Refined state and input constraints

Based on Pereira et al. (2017), an auxiliary input signal $v \in \mathbb{R}^{(m-n_r)}$ is used. Therefore, the control input $u(k)$ is structured from the solution of (1) satisfying the algebraic equation (1b) for any $w(k)$ as

$$u(k) = M_x x(k) + M_d d(k) + M_w w(k) + M_v v(k), \tag{8}$$

where the matrices $M_x \in \mathbb{R}^{m\times n}$, $M_d \in \mathbb{R}^{m\times m_d}$, $M_w \in \mathbb{R}^{m\times m_w}$ and $M_v \in \mathbb{R}^{m\times m_v}$ should be designed.

By combining (1b) and (8), we have that the following condition holds:

$$\begin{aligned} E_x x(k) + E_d d(k) + E_w w(k) = &-E_u M_x x(k) - E_u M_d d(k) \\ &- E_u M_w w(k) - E_u M_v v(k). \end{aligned}$$

which gives

$$E_x + E_u M_x = 0, \tag{9a}$$
$$E_d + E_u M_d = 0, \tag{9b}$$
$$E_w + E_u M_w = 0, \tag{9c}$$
$$E_u M_v = 0. \tag{9d}$$

From the condition (9), we can obtain matrices M_x, M_d and M_w. Note that there are infinity solutions of these matrices M_x, M_d and M_w. Specifically, M_v is the null space (kernel) of E_u, and M_x, M_d and M_w can be obtained in a generalized solution with pseudo-inverse matrices.

Besides, by combining (1a) and (8), we have

$$x(k+1) = \tilde{A}x(k) + \tilde{B}_v v(k) + \tilde{B}_d d(k) + \tilde{B}_w w(k), \tag{10}$$

where $\tilde{A} = A + BM_x$, $\tilde{B}_v = BM_v$, $\tilde{B}_d = B_d + BM_d$ and $\tilde{B}_w = B_w + BM_w$.

Let us define the nominal dynamical model of (10) as

$$\bar{x}(k+1) = \tilde{A}\bar{x}(k) + \tilde{B}_v\bar{v}(k) + \tilde{B}_d d(k), \tag{11}$$

where $\bar{x} \in \mathbb{R}^n$ and $\bar{v} \in \mathbb{R}^{(m-n_r)}$, and define the error between closed-loop states and state predictions as $e(k) = x(k) - \bar{x}(k)$. With (10) and (11), the error dynamics is given by

$$e(k+1) = \tilde{A}e(k) + \tilde{B}_v\left(v(k) - \bar{v}(k)\right) + \tilde{B}_w w(k). \tag{12}$$

To attenuate the effect of this error along the prediction horizon, a local control law $K \in \mathbb{R}^{(m-n_r)\times n}$ is proposed for $v(k) = Kx(k)$ and $\bar{v}(k) = K\bar{x}(k)$ such that the matrix $\tilde{A}_K = \tilde{A} + \tilde{B}_v K$ is Schur stable. Therefore, (12) is simplified as

$$e(k+1) = \tilde{A}_K e(k) + \tilde{B}_w w(k). \tag{13}$$

If Assumption 2 holds, $w(k) \in \mathcal{W}$, $\forall k \in \mathbb{N}$ with a confidence level α_w. Moreover, by applying Lemma 1 with a probability p to the closed-loop error system (13), we obtain the probabilistic invariant set $e(k) \in \mathcal{S}$ and the set $\mathcal{S}$ is RPI satisfying $\tilde{A}_K\mathcal{S} \oplus \tilde{B}_w\mathcal{W} \subseteq \mathcal{S}$. Therefore, for any $k \in \mathbb{N}$, $e(k) \in \mathcal{S}$ is equivalent to $x(k) - \bar{x}(k) \in \mathcal{S}$. With $x(k) \in \mathcal{X}$, we have

$$\bar{x}(k) \in \mathcal{X} \ominus \mathcal{S}. \tag{14}$$

Based on (8), let us also denote the nominal input $\bar{u} \in \mathbb{R}^m$ as

$$\bar{u}(k) = M_x\bar{x}(k) + M_d d(k) + M_v\bar{v}(k). \tag{15}$$

By combining (8) and (15), we derive

$$\begin{aligned} u(k) - \bar{u}(k) &= M_x\left(x(k) - \bar{x}(k)\right) + M_w w(k) \\ &\quad + M_v v(k) - M_v\bar{v}(k) \\ &= M_x e(k) + M_w w(k) + M_v K e(k). \end{aligned}$$

Since $u(k) \in \mathcal{U}$, $e(k) \in \mathcal{S}$ and $w(k) \in \mathcal{W}$, we obtain

$$\bar{u}(k) \in \mathcal{U} \ominus M_x\mathcal{S} \ominus M_w\mathcal{W} \ominus M_v K\mathcal{S}. \tag{16}$$

As a result, the constraints on the nominal state and input vectors are refined as in (14) and (16), which will be used in the robust economic MPC design.

4.2 Robust periodic economic MPC planner

An approximation of the optimal periodic trajectory can be obtained solving the following open-loop optimization problem [1]. This optimization problem is set with free initial state and a periodicity constraint for achieving periodic operation.

$$\underset{\substack{\bar{x}(0),\ldots,\bar{x}(T),\\ \bar{u}(0),\ldots,\bar{u}(T-1)}}{\text{minimize}} \; J_T\left(\bar{x}, \bar{u}, c\right) = \sum_{i=0}^{T-1} \ell\left(\bar{x}(i), \bar{u}(i), c_i\right), \tag{17a}$$

[1] It is an approximation because the optimal periodic trajectory would profit from feedback at each sampling time, while in this optimization problem, the predictions are carried out in open-loop for the whole period.

subject to

$$\bar{x}(i+1) = A\bar{x}(i) + B\bar{u}(i) + B_d d(i), \tag{17b}$$
$$0 = E_x\bar{x}(i) + E_u\bar{u}(i) + E_d d(i), \tag{17c}$$
$$\bar{x}(i) \in \mathcal{X} \ominus \mathcal{S}, \tag{17d}$$
$$\bar{u}(i) \in \mathcal{U} \ominus M_x\mathcal{S} \ominus M_w\mathcal{W} \ominus M_v K\mathcal{S}, \tag{17e}$$
$$\bar{x}(0) = \bar{x}(T). \tag{17f}$$

The feasible solutions of the optimization problem (17) define the optimal periodic steady trajectory of system states and control inputs as

$$\bar{x}^p = \{\bar{x}(0), \ldots, \bar{x}(T)\}, \tag{18a}$$
$$\bar{u}^p = \{\bar{u}(0), \ldots, \bar{u}(T)\}. \tag{18b}$$

The closed-loop system with the proposed controller will be driven close to a neighborhood of this trajectory.

4.3 Robust periodic economic MPC formulation

The proposed controller is based on modifying the optimization problem (17) to take into account the current measurement of the state. The current state $x(k)$ at time $k \in \mathbb{N}$ is set into a shift position $j = \mathrm{mod}\,(k, T)$ taken into account the period T. The advantage of this setting is to keep the constraints in (17) that are independent to the current state $x(k)$. In general, the robust periodic economic MPC controller is proposed by implementing the following optimization problem:

$$\underset{\substack{\bar{x}(0),\ldots,\bar{x}(T),\\ \bar{u}(0),\ldots,\bar{u}(T-1)}}{\text{minimize}} \quad J_T(\bar{x}, \bar{u}, c), \tag{19a}$$

subject to

$$\bar{x}(i+1) = A\bar{x}(i) + B\bar{u}(i) + B_d d(i), \tag{19b}$$
$$0 = E_x\bar{x}(i) + E_u\bar{u}(i) + E_d d(i), \tag{19c}$$
$$\bar{x}(i) \in \mathcal{X} \ominus \mathcal{S}, \tag{19d}$$
$$\bar{u}(i) \in \mathcal{U} \ominus M_x\mathcal{S} \ominus M_w\mathcal{W} \ominus M_v K\mathcal{S}, \tag{19e}$$
$$\bar{x}(0) = \bar{x}(T), \tag{19f}$$
$$\Pr\left[x(k) - \bar{x}(j) \in \mathcal{S}\right] \geq p, \quad j = \mathrm{mod}\,(k, T). \tag{19g}$$

From the feasible solutions of the optimization problem (19), the control action at time k is chosen to be

$$u(k) = \bar{u}(j) + M_w w(k) + (M_x + M_v K)(x(k) - \bar{x}(j)), \tag{20}$$

with $j = \mathrm{mod}\,(k, T)$. Note that in the closed-loop simulation, at time $k \in \mathbb{N}$, the current state $x(k)$ and the disturbance $w(k)$ are measurable to implement the control action $u(k)$ chosen in (20).

5. APPLICATION TO A SMART MICRO-GRID

In this section, we apply the proposed robust control strategy into a smart micro-grid chosen from (Pereira et al., 2017). Periodic operation has been proved to be suitable for this system taking into account the potential periodicity of signals in the system.

5.1 Description

The micro-grid system includes three nano-grids placed in parallel. Each nano-grid consists of a cluster of batteries and a fuel cell. These batteries are used to compensate the voltage peaks from the fast system dynamics. Therefore, two system state variables are chosen as the state of charge of batteries and the storage level of hydrogen in the metal hydride tank. Besides, control inputs in each nano-grid are the power of exchange with the electric utility, the power of exchange with the hydrogen and the load power. The control-oriented model of this micro-grid is built by difference-algebraic equations in the form of (1) with the sampling time of 30 minutes, where system matrices are defined by

$$A = \begin{bmatrix} 1&0&0&0&0&0\\ 0&1&0&0&0&0\\ 0&0&1&0&0&0\\ 0&0&0&1&0&0\\ 0&0&0&0&1&0\\ 0&0&0&0&0&1 \end{bmatrix}, \; B = \mathrm{blkdiag}\,(B_n, B_n, B_n),$$

$$B_d = \begin{bmatrix} 5.5847&0&0&0\\ 0&0&0&0\\ 0&5.5847&0&0\\ 0&0&0&0\\ 0&0&5.5847&0\\ 0&0&0&0 \end{bmatrix}, \; B_w = B_d,$$

with $B_n = \begin{bmatrix} 5.5847 & 5.5847 & -5.5847\\ -3.4495 & 0 & 0 \end{bmatrix}$. And $E_x = 0$, $E_u = [0\ 0\ 1\ 0\ 0\ 1\ 0\ 0\ 1]$, $E_d = [0\ 0\ 0\ -1]$, $E_w = E_d$.

The constraint sets on the state vector $x(k) \in \mathcal{X}$ and the input vector $u(k) \in \mathcal{U}$, $\forall k \in \mathbb{N}$ are considered as follows:

$$\mathcal{X} = \left\{ x \in \mathbb{R}^6 : \begin{bmatrix}40\\20\\40\\20\\40\\20\end{bmatrix} \leq x \leq \begin{bmatrix}95\\95\\95\\95\\95\\95\end{bmatrix} \right\},$$

$$\mathcal{U} = \left\{ u \in \mathbb{R}^9 : \begin{bmatrix}-0.9\\-1.5\\0\\-0.9\\-1.5\\0\\-0.9\\-1.5\\0\end{bmatrix} \leq u \leq \begin{bmatrix}0.9\\1\\2\\0.9\\1\\2\\0.9\\1\\2\end{bmatrix} \right\}.$$

The patterns of the periodic signal $d(k)$ with the period $T = 48$ are shown in Fig. 1. This periodic signal is repeated along the simulation time. The variance matrix Σ_w for the Gaussian white disturbance $w(k)$ is given by $\Sigma_w = \mathrm{diag}\,([0.0339, 0.0264, 0.0189, 0.0532])$ and with the 95% confidence level, the set $w(k) \in \mathcal{W}$, $\forall k \in \mathbb{N}$ can be obtained. The initial state $x(0)$ is chosen as $x(0) = [67.2513, 47.4267, 67.0940, 47.4985, 67.3972, 47.0535]^T$.

According to Pereira et al. (2017), the main control objectives for the management of this micro-grid are considered:

- To optimize the economic costs by maximizing the benefit of the energy exchange taken into account a time-varying electricity prices presented in c;
- To minimize the usage damages of equipments.

Based on these two objectives, the cost functions are defined as follows:

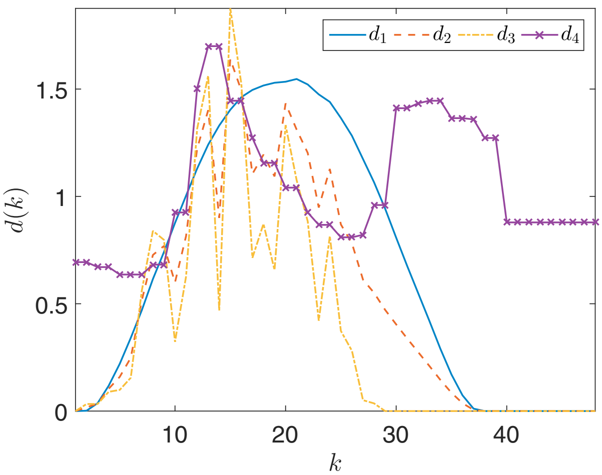


Fig. 1. The patterns of the periodic signal $d(k)$.

$$\begin{aligned} \ell_1(\bar{u}(i), c_i) &:= \lambda_1(c_i - P_1\bar{u}(i))^2, \\ \ell_2(\bar{u}(i)) &:= \lambda_2\bar{u}_1(i)^2 + \lambda_2\bar{u}_2(i)^2 \\ &\quad + \lambda_2\bar{u}_4(i)^2 + \lambda_2\bar{u}_5(i)^2 \\ &\quad + \lambda_2\bar{u}_7(i)^2 + \lambda_2\bar{u}_8(i)^2, \end{aligned}$$

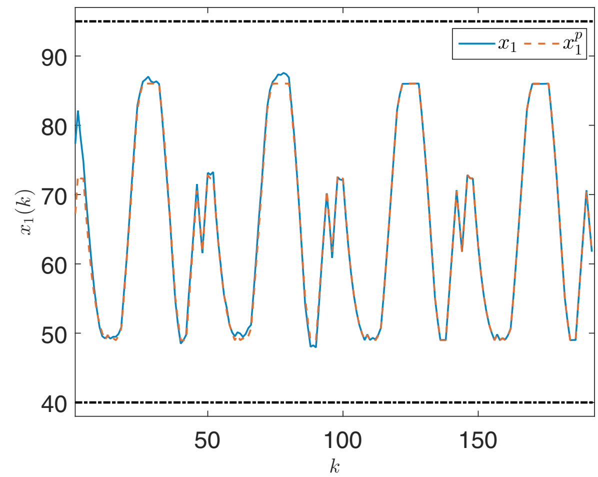
where $P_1 = [0\ 1\ 0\ 0\ 1\ 0\ 0\ 1\ 0]$. Moreover, Λ_1 and Λ_2 are prioritization weights. In this simulation, they are chosen as $\Lambda_1 = 10$ and $\Lambda_2 = 40$.

5.2 Simulation results

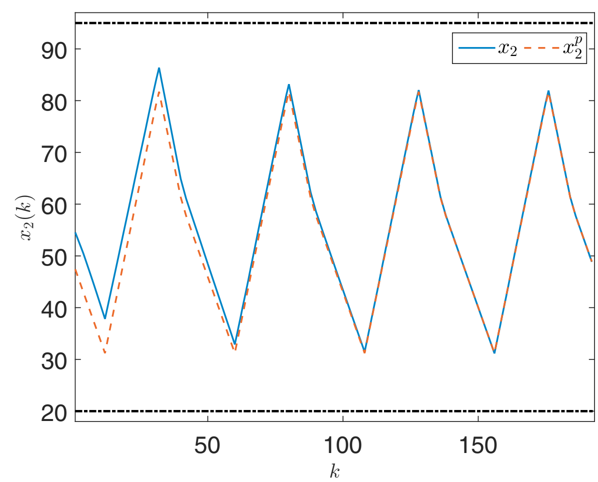
By satisfying the condition in (9), we can obtain M_v as the null space of E_u and M_d is equal to M_w. For the design of the local control law K, the LQR technique is used with the weighing matrices $Q = \text{diag}([0.0182, 0.0133, 0.0182, 0.0133, 0.0182, 0.0133])$, $R = \text{diag}([0.5556, 0.4, 0.5, 0.5556, 0.4, 0.5, 0.5556, 0.4, 0.5])$. With a chosen probability $p = 0.99$, the probabilistic RPI set $\mathcal{S}$ can be obtained using Lemma 1.

The simulation has been carried out for 2 hours (192 sampling time steps). The optimization problems (17) and (19) are solved using the linear programming technique. Note that the planner implemented (17) is only solved once to find the optimal periodic steady trajectory (18). And the closed-loop simulation considers the system (1) with the robust periodic economic MPC controller in (19). The Gaussian white disturbances $w(k) \in \mathcal{W}$ are sampled in a customized way as shown in Fig. 2(c). With these disturbances, the closed-loop system is recursively feasible. Besides, some simulation results of the closed-loop state and input trajectories are shown in Fig. 2 and Fig. 3.

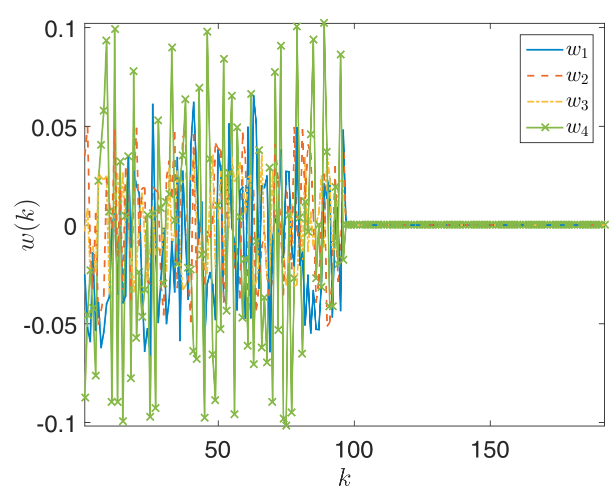
Since this micro-grid consists of three nano-grids, we show the results of the first nano-grid. As shown in Fig. 2, the blue lines represent the closed-loop states $x_1(k)$ and $x_2(k)$ while the red dashed lines represent the optimal periodic steady states x_1^p and x_2^p. When the disturbances are present in Fig. 2(c), both closed-loop states reach a neighborhood of the optimal periodic steady states and when the disturbances vanish, they converge to their optimal periodic steady states. For this nano-grid, as also shown in Fig. 3 control inputs are close to a neighborhood of the optimal periodic steady inputs.



(a)

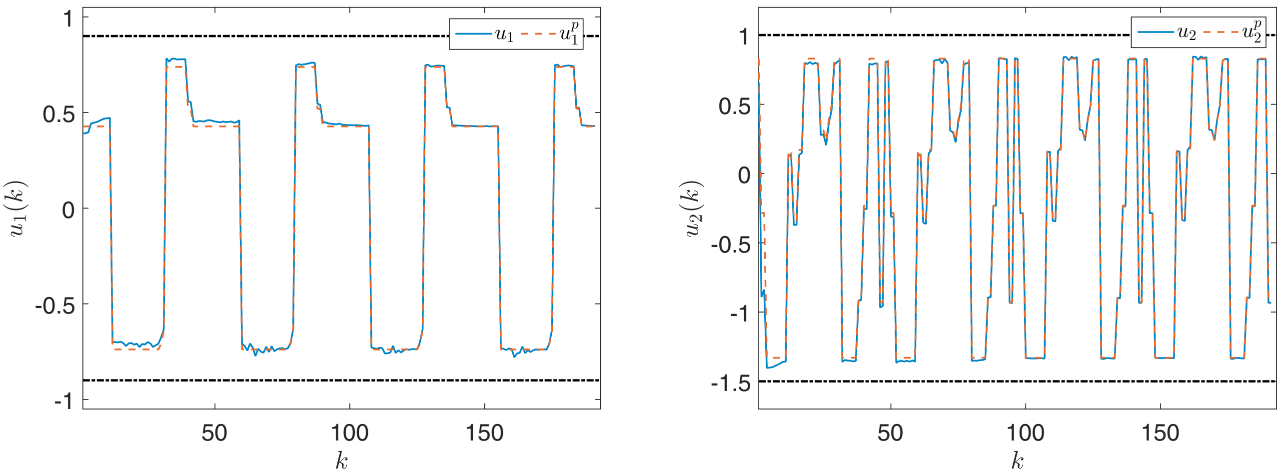


(b)



(c)

Fig. 2. The closed-loop state trajectory and the sampled Gaussian white disturbances of the smart micro-grid.



(a) (b)

Fig. 3. The closed-loop input trajectory of the smart micro-grid.

6. CONCLUSION

In this paper, we have presented a robust economic MPC of discrete-time linear descriptor systems based on the probabilistic set invariance for periodic operation. The considered descriptor system is affected by unknown disturbances following a zero-mean Gaussian distribution. The tube-based technique is used with the probabilistic RPI set, where a local controller is proposed by choosing an auxiliary input signal. Finally, the proposed robust periodic economic MPC controller is applied to a smart micro-grid. The closed-loop simulation results show that the closed-loop trajectory is close to a neighborhood of the optimal periodic steady trajectory obtained by the proposed robust periodic economic MPC planner. In particular, for this application, when the disturbances are zero, the closed-loop trajectory converges to the optimal periodic steady trajectory obtained by the planner. As a future research, we will investigate the closed-loop properties using the Karush-Kuhn-Tucker optimality conditions.

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