Modeling and Control of Interacting Irrigation Channels

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Abstract—One of the most important activities in agriculture is irrigation, which has a high ecological impact and expends around 85% of the freshwater that humanity consumes. In the irrigation process, most of the water is transported by networks of open-channel irrigation systems (OCIS) to each farmer. Despite the importance of the process, in Colombia these systems are not automated in order to reduce the waste of water. Additionally, several studies address the control problem in non-interactive channels, however in OCIS the water is usually transported by interactive channels. In this paper, a modeling and control strategy for interactive channels are proposed. The model is based on mass balance and considers interactions, delays, and potential energy difference along the channels. The control strategy used is backstepping. The model is compared with EPA-SWMM, and the control strategy is tested over the proposed model. The results show that the model is suitable for control systems design and the control strategy is able to overcome parameter variation, channel interaction, nonlinearities, and internal delays.

Index Terms—Control systems, feedback control, backstepping, open channel, irrigation, water control, hydraulic, Model.

I. INTRODUCTION

The world population has been growing continuously. As a matter of fact, according to the world population prospects prepared by the Population Division of the Department of Economic and Social Affairs of the United Nations [1], in 2050 the number of people could increase by 37%. Therefore, it is necessary to think on new ideas to increase food production. However, these ideas must be sustainable, which means that the natural resources used to cover the future demand must be less or almost equal to the resources that we are currently spending. Undoubtedly, the most important natural resource used in food production is water. This activity spends nearly 85% of freshwater consumed in the world [2] and most of this resource is transported by OCIS. OCIS are the fundamental part of irrigation districts, which are communities of farmers that dispose of the water from natural sources. Usually, the fresh water is taken from a river and it is transported by intricate networks of OCIS to each farm. The water flow and level into the OCIS is regulated by gates, and each gate aperture is calculated by an administrator of the district, who is the person that assigns the appropriate amount of water resource to each farmer. Finally, each gate is manually adjusted by operators, who perform these tasks throughout kilometers of channels and hundreds of gates. Nevertheless, in the normal operation of the system, disturbances are common (e.g., flow variation at the source, channel obstructions, leaks, overflows, and even water robbery), causing low efficiency with a high environmental impact.

Despite the problems associated with OCIS, and the environmental impact of these problems, in the world, few OCIS have the necessary technology to ensure the water resource of the farmers and to increase the efficiency of the system, which is the relationship between the taken and the used water. One of the causes of the lack of technology could be associated to the complexity of the modeling and control strategies for this kind of system. Therefore, the research around modeling and control of OCIS is required to contribute to the system's technification.

In the field of modeling, the fundamental laws that describe the dynamics of the flow are the Saint-Venant Equations (SVE), which are differential equations with partial derivatives that, for one direction, describe the water movement along a channel. These equations are derived from conservation of mass and energy analysis [3]. However, due to the impracticality of the SVE for control systems design [4], in the literature there are different kinds of models that approximate the dynamics of irrigation channels to linear systems with delays such as [5]–[14], and non-linear systems with delays such as [4], [15]–[17], where the nonlinearities are associated with the nonlinear relationship between the water level and the flow, and the delay is associated with the time that the water spends traveling along the channel. It is important to mention that most of the research has dealt with the problem of modeling and control of OCIS without channel interaction. For instance, Cantoni et al. [4] propose a model-
based control, a decentralized control strategy, and an optimal centralized and distributed control for irrigation networks without channel interaction. Herrera et al. [18] obtain by identification a dynamical model and propose a control strategy with Smith Predictor, where the identification and control strategy is tested over an OCIS without channel interaction. Overloop et al. [19] develop a model predictive control by linearization of a non-interactive irrigation network. In [20], a control technique based on the Hayami model equations is proposed. In this research a parametrization of trajectories of the input for a desired output is developed, which is called a Flat-Based Control and the design is made with noninteractive network assumption. In [21], based on a linear discrete and non-interactive model, a predictive control with hierarchical controller is designed. Finally, research reported in [9], [15], [16] has shown the development of conventional control techniques for OCIS without interaction. On the other hand, OCIS with undershoot gates have interactions between channels, and the control design for interactive systems has always been a challenging problem. Additionally, according to the United States Department of Agriculture [22] undershoot gates are the most common discharge structures in open channels. This can be seen in the irrigation districts in Tolima-Colombia, where only undershoot gates are used to regulate flows and levels along the channels. Recently, the research around modeling and control of interactive OCIS has earned significant relevance and some research such as [23]–[26] has been developed in the field. However, these works avoid the nonlinear interaction by linearization such as [23], where the desired behavior is restricted around the equilibrium point, or by the strong assumption that there is a perfect flow control for each discharge structure [24]–[26]. On the other hand, there are nonlinear control techniques that have shown suitable behavior controlling interactive nonlinear systems with delays but had not been used in OCIS. In a specific way, the nonlinear control technique known as backstepping has shown a successful performance over nonlinear systems with delays such as [27] where the control technique is applied to a two-stage chemical reactor with delayed recycle streams. The work in [28] shows a control technique where backstepping is used to stabilize nonlinear systems with large delays. Finally, [29] shows the implementation of backstepping for nonlinear large-scale systems with delayed interactions.

As a contribution, this paper proposes: i) the development of a nonlinear model that describes in a broad operation range the dynamics of OCIS with nonlinear channel interaction; and ii) the development of a nonlinear control strategy that is useful to overcome nonlinearities, delays, and interactions. The nonlinear model is formulated with a mass balance per channel with the assumptions that there are time delays for water transport, potential energy difference along the channels, and undershoot gates. The nonlinear control strategy is a nonlinear model-based control that ensures the system stability despite nonlinearities, internal delays, and channel interaction. The model is compared with the specialized software (Storm Water Management Model of the Environmental Protection Agency of the United States (EPA-SWMM) showing accurate behavior. The control strategy is tested over the proposed model showing proper performance in the presence of parameter variation.

The remainder of the paper is organized as follows. In Section II, the proposed model is explained. In Section III, an introduction to the backstepping control technique and its implementation in the proposed model is performed. In Section IV, an OCIS with two channels is modeled and simulated. The modeling results are compared with the results of the two channels system simulated in the specialized software EPA-SWMM. Furthermore, in this section, the control strategy results are presented and discussed. Finally, in Section V, the conclusions are presented.

II. Model

OCIS are frequently modeled with the SVE, which are derived from the fundamental laws of mass and energy conservation. The work in [3] presents a methodology for their derivation obtaining the equations given by

\[
\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = q_i
\]

\[
\frac{\partial V}{\partial t} + g \frac{\partial}{\partial x} \left( \frac{V^2}{2g} + y \right) = g(S_0 - S_f),
\]

where \( A \) and \( Q \) are the water flow area and the flow along the area, respectively. The evaporation, rain or filtration flow are represented by \( q_i \). \( g \) is the constant of gravity, \( S_0 \) is the slope, and \( S_f \) is related to friction and slope. SVE are partial differential equations, which implies that their use for control is impractical [4]. On the other hand, the OCIS could be described with a mass balance per channel by

\[
\dot{V}_i = q_i - q_{i+1},
\]

where, for a channel \( i \), \( V_i \) is the liquid volume and, \( q_i \) and \( q_{i+1} \) are inflows and outflows, respectively. According to [30], the flow \( q_i \) depends on the kind of gate that regulates it, leading to three cases:

- Overshoot gate in free flow
  \[
  q_i = \gamma_i y_{dn,i-1}^{3/2}.
  \]

- Undershoot gate
  \[
  q_i = \gamma_i \sqrt{y_{dn,i-1} - y_{up,i}}.
  \]

- Overshoot gate in submerged flow
  \[
  q_i = \gamma_i y_{dn,i-1}^{3/2} \left( 1 - \frac{y_{up,i}}{y_{dn,i-1}} \right)^{0.385}.
  \]

Where \( i \) is used to indicate the channel for which the equation is valid, then \( i - 1 \) and \( i + 1 \) are used to indicate the upstream and downstream channels, respectively, the \( \gamma_i \) constant is associated with characteristics such as gate opening and the discharge coefficient. \( y_{dn,i} \) is the depth at downstream point, and \( y_{up,i} \) is the depth at upstream point.
A. Proposed Model

We propose to model OCIS with a mass balance per channel. In our model, we assume that the discharge structures are submerged undershoot gates and that there is a time delay for water transport; additionally, we assume a potential energy difference along the channels. Figure 1 shows a representation of the model.

![Graphical description for the proposed modeling approach](image)

In the model, it is assumed that the upper part of the channel \(i\) has a depth \(y_{up,i}\) and the lower part of the channel has a depth \(l_i y_{up,i}(t - \tau_i)\). That means that, after a time delay, the level at the lower part of the channel is similar to the level at the upper part but attenuated by a constant \(l_i\), which is associated with a difference of potential along the channel. As a result, for a channel \(i\), the mass balance is described by

\[
A_i \dot{y}_{up,i} = w u_{i-1} \sqrt{2 g c \sqrt{l_i y_{up,i-1}(t - \tau_{i-1}) - y_{up,i}}} - w u_i \sqrt{2 g c \sqrt{l_i y_{up,i}(t - \tau_i) - y_{up,i+1}} + Q_i},
\]

where the parameter \(A_i\) is the channel area, and the mass balance is given by a volume change \(A_i \dot{y}_{up,i}\) that is equal to the inflow from the channel \(i - 1\) (a) minus the outflow to the channel \(i + 1\) (b), plus a flow \(Q_i\) associated to intakes, outtakes, rain, evaporation, and leaks. In our model, (a) and (b) obey the Torricelli’s principle [22], where \(c\) is the discharge coefficient, \(w\) is the discharge structure width, \(g\) is the gravity constant, and \(\tau_i\) is the discharge structure height. Besides, \(u_i\) is the control input to the system, which means that the flows and heights along the channels are regulated with the variation of \(u_i\).

As a manner of conclusion, if the proposed model is used for \(n\) channels, a highly interactive model of \(n\) differential equations with \(n\) inputs and \(n\) outputs will be obtained and could be described by

\[
\dot{y}_{up} = f(y_{up}, u_{up}(t - \tau), u),
\]

with \(y_{up} \in \mathbb{R}^n\), and \(u \in \mathbb{R}^n\).

It is important to mention that parameters such as area, discharge structure width, and the discharge coefficient could be obtained by structural channel information. The other parameters such as the time delay, and the potential energy difference could be obtained with methodologies from experimental data, e.g. using step response.

III. Control Strategy

The purpose of an OCIS is to deliver the appropriate amount of water to each farmer. In a ideal system, the intake water must be equal to the used water, or in other words, the waste of water should be reduced to a minimum [15]. This should be an easy task in an ideal and non-dynamic system. However, as it was shown before, OCIS are complex systems with nonlinearities, delays, and high channel interactions. Additionally, in these systems disturbances such as intermittent demands, level source variation, obstructions, and leaks are quite common. Usually, in OCIS, a constant upstream or downstream depth is set at each channel, and with the variation of the outtake structure, the flow is regulated to each farmer [4]. In the system operation, the use of the terms upstream control and downstream control is common, where with the variation of the discharge structures height \((\eta_i)\), a fixed level upstream or downstream of the channel is maintained. Therefore, our control objective is to maintain the depth upstream at a constant value overcoming the challenges given by the nonlinearities, delays, interactions, and disturbances. In order to accomplish this objective, the use of the backstepping control technique is proposed, and the control design is outlined below.

A. Backstepping

With the aim to ensure zero stationary error for the controlled system, the open loop system is augmented with an integral action. Therefore, for the model that describes a channel in (6), with regularization error \(e_i = y_{ref,i} - y_{up,i}\), where \(y_{ref,i}\) is the desired depth for a channel \(i\), the system is augmented with \(\dot{\eta}_i = e_i\). Hence, the resultant system is given by

\[
\dot{\eta}_i = y_{ref,i} - y_{up,i},
\]

\[
\dot{y}_{up,i} = f_\alpha(y_{up,i}, \beta_i) + f_\beta(y_{up,i}(t - \tau_i), \beta_i)u_i,
\]

where \([\eta_i, y_{up,i}]^T \in \mathbb{R}^2\), \(\eta_i \in \mathbb{R}\), and \(y_{up,i} \in \mathbb{R}\) are the state variables, \(u_i \in \mathbb{R}\) is the control signal, \(f_\alpha : D \rightarrow \mathbb{R}\), \(f_\beta : D \rightarrow \mathbb{R}\), for that the operator \(\sqrt{\cdot}\) is changed by \(sgn(\cdot)\sqrt{\cdot}\), and \(\beta_i\) is a vector with external variables that can be measured and used for control purpose, \(\beta_i = (y_{up,i-1}, y_{up,i-1}(t - \tau_{i-1}), y_{up,i+1}, u_{i-1})\).

As shown in Fig. 2, the augmented system could be seen as a cascade system, where the objective is to find a control strategy \(u_i\) that ensures the stability of the origin \((\eta_i = 0, y_{up,i} = 0)\), for all \(y_{ref,i} = 0\).
The control technique could be seen as a step-by-step method, where:

1) A change of variable \( y_{up,i} = \phi_i(\eta_i) \), \( \phi(0) = 0 \) is proposed. Then, in (8), \( \phi_i(\eta_i) \) is assumed as a control variable with the objective that \( V_{i,1} < -W_{i,1}(\eta) \), where \( W_{i,1}(\eta) \) is positive definite and \( V_{i,1}(\eta) \) is a known Lyapunov function. The control variable is,

\[
\phi_i(\eta_i) = y_{ref,i} - \lambda_{i,a}\eta_i, (10)
\]

where \( \lambda_{i,a} \in \mathbb{R} : \lambda_{i,a} < 0 \) is used as a tuning constant. Then, according to the control law, now (8) is given by

\[
\dot{\eta}_i = \lambda_{i,a}\eta_i, \quad \text{and with } V_{i,1}(\eta_i) = \frac{1}{2}\eta_i^2, \quad V_{i,1} = \lambda_{i,a}\eta_i^2 \].

Hence, the origin of (8) is globally exponentially stable.

2) According to the last assumption, a new variable emerges in the system. This variable is the error between the control signal \( \phi_i(\eta_i) \) and the variable \( y_{up,i} \); this error is denoted by \( z_i = y_{up,i} - \phi_i(\eta_i) \), then the dynamical behavior of the error is given by

\[
\dot{z}_i = f_b(y_{up,i}, \beta_i)u_i - \frac{d}{dt}[y_{ref,i} - y_{up,i}], \quad i = 1, 2 \].

Then, \( \dot{V}_i(\eta_i, y_{up,i}) = \lambda_{i,a}\eta_i^2 + z_i^2 + z_i^2 \). Hence, the origin \( (\eta = 0, z = 0) \) is asymptotically stable and due to the fact that \( \phi(0) = 0 \), the origin \( (\eta = 0, y = 0) \) is also asymptotically stable.

3) The dynamical behavior of the controlled system is given by

\[
\dot{\eta}_i = y_{ref,i} - y_{up,i} \]

\[
y_{up,i} = \lambda_{i,a}\lambda_{i,b}\eta_i + (\lambda_{i,a} + \lambda_{i,b})y_{up,i} - \lambda_{i,a}\lambda_{i,b}y_{ref,i}, (13)
\]

which is an uncoupled second order system with characteristic polynomial given by \( P_i(s) = s^2 + s(-\lambda_{i,a} - \lambda_{i,b}) + \lambda_{i,a}\lambda_{i,b} \).

IV. SIMULATION, COMPARISON AND DISCUSSION

In order to validate the modeling strategy and assess the control technique performance, an OCIS with two channels is proposed. The model is simulated in the SWMM software, which is a free software of the Environmental Protection Agency (EPA) of the United States, specializing in hydraulic simulation which numerically solves the SVE [31]. In this software, the model is codified by a graphic description of the system as presented in Fig. 3, and the most relevant parameters of the simulation are listed in Table I.

![Diagram of an OCIS with two channels in the SWMM software](image)

Fig. 3. Simulation diagram of an OCIS with two channels in the SWMM software.

### Table I

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Junction 1</td>
<td>Inflow</td>
</tr>
<tr>
<td>Channel 1.2</td>
<td>Shape</td>
</tr>
<tr>
<td>Length</td>
<td>1m</td>
</tr>
<tr>
<td>Roughness</td>
<td>0.005</td>
</tr>
<tr>
<td>Category</td>
<td>Orifice</td>
</tr>
<tr>
<td>Type</td>
<td>Side</td>
</tr>
<tr>
<td>Gate 1.2</td>
<td>Shape</td>
</tr>
<tr>
<td>Width</td>
<td>1m</td>
</tr>
<tr>
<td>Discharge Coeff.</td>
<td>0.9</td>
</tr>
<tr>
<td>Height</td>
<td>[0.35m]</td>
</tr>
</tbody>
</table>

A. Adjustment and Model Validation

Fig. 4 shows the dynamic behavior of the depth at the upstream and downstream of the channels simulated in SWMM. This test is developed with null initial conditions and with an intake of 0.6m³/s. The parameters for the proposed model are obtained from the experimental results shown in Fig. 4, where \( l_1 \) and \( l_2 \) are obtained from the stationary system behavior, with equilibrium points in \( y_{up,1} = 2.58m, \dot{y}_{up,1} = 2.56m, \dot{y}_{up,2} = 3.25m, \) and \( l_2 \dot{y}_{up,2} = 2.31m. \) To obtain \( \tau_1, \tau_2, \) the dynamical behavior is analyzed as shown in Fig. 4, where the
first intersection corresponds to the value of $\tau_1 \approx 320$ s, and the second is used to obtain the value of $\tau_2 \approx 1205$ s $- \tau_1$.

![Junction Depth](image1)

**Fig. 4.** Dynamic depth behavior in SWMM.

Then, the values assigned to the proposed model are collected in Table II.

**TABLE II**

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$, $A_2$</td>
<td>[1000 m²], [1000 m²]</td>
</tr>
<tr>
<td>$t_1$, $t_2$</td>
<td>[0.99], [0.99]</td>
</tr>
<tr>
<td>$Q_1$, $Q_2$</td>
<td>[0.6 m²/s], [0.8 m³/s]</td>
</tr>
<tr>
<td>$u_1$, $u_2$</td>
<td>[0.35 m], [0.1 m]</td>
</tr>
</tbody>
</table>

![Junction Depth](image2)

**Fig. 5.** SWMM vs proposed model for a step in the intake flow.

![Junction Depth](image3)

**Fig. 6.** Model simulated in SWMM vs proposed model for changes in gates 1 and 2.

Figure 5 shows the comparison between the model simulated in SWMM and the proposed model for a step of 0.6 m³/s at the intake flow. In this figure, it is observed that the proposed model presents similar behavior to the model simulated in the specialized software SWMM. On the other hand, Figure 6 shows a comparison between both systems. In this case, a step variation is made in the height of the first gate ($u_1$) and the height of the second gate ($u_2$). According to the results of the comparison (Figures 5, 6), the proposed model have similar dynamical and statical behavior than the model simulated in the specialized software. Additionally, the procedure presented to obtain the parameters of the proposed model is suitable to obtain the parameters of any real system.

**B. Performance of the closed-loop scheme**

The backstepping control strategy is implemented in the proposed system. As a result, a linear system with eigenvalues in $\lambda_{1,a}$, $\lambda_{1,b}$, $\lambda_{2,a}$, and $\lambda_{2,b}$ is obtained. The control system is tuned with $\lambda = -0.0002$ with the purpose of ensuring a constant time close to the open-loop system in Fig. 4. In the test, some changes in the references are performed; Figure 7 shows the tracking performance for each channel depth, and each control signal. This figure reveals that the controlled system presents the expected behavior with zero-stationary error, time response equal to 1/0.0002, and smooth control signal. Moreover, the presented overshoot corresponds to the time solution of the closed-loop system, for $\lambda_{1,a}$, $\lambda_{1,b}$, $\lambda_{1,c}$, $\lambda_{1,e}$ = $\lambda$, which is described by a diagonal matrix of identical transfer functions given by $u_{G_{c1}}(s) = \frac{1}{(s + 1)^2}$.

![Channels Depth](image4)

**Fig. 7.** Tracking test.

Finally, the robustness of the controlled system is tested with an increment of 10% in the parameters (area, gate width, outtake flow, and delay). Figure 8 shows the closed-loop system behavior in the presence of these disturbances, where the controlled system maintains stability and desired dynamical behavior. This shows the robustness of the control technique, which implies that changes in areas and delays due to diverse events like sedimentation, cause small disturbances in the performance of the controlled system. On the other hand, changes in the flow due to gate width changes, and intake or outtake flow, cause a significant effort in the control signal. However, the controlled system overcomes the problem and return to an equilibrium state.
V. CONCLUSIONS

A modeling strategy and a nonlinear control technique have been presented for open channel irrigation systems. The proposed model can be adjusted to several irrigation systems. According to the comparisons with the specialized software EPA-SWMM, it is shown that the proposed model can be adjusted with structural data and simple tests, with high accuracy. However, it is necessary to increase the number of tests and evaluate a possible application in a real system. The backstepping control strategy is suitable for the proposed model. With this strategy it is possible to guarantee null steady state error, to adjust the speed response, and to overcome the problems of non-linearities, interaction between channels, and internal delays.

REFERENCES