

A comparison of modelling approaches for closed-loop decision making over discrete domains in manufacturing systems

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Abstract—The manufacturing industry is transforming towards smart, flexible, and energy-efficient systems. In this regard, control strategies based on optimisation have been proposed to improve the energy efficiency of manufacturing systems. Usually, the optimisation problem behind the controller design involves decision variables constrained to binary and discrete domains resulting in Mixed-Integer Linear Programming (MILP) problems and require a high computational burden to find their optimal solution. In this paper, three different approaches are proposed for the closed-loop decision making over discrete domains in manufacturing systems, including the constraints and limitations to their implementation. Thus, the number and nature of required variables as well as the additional constraints needed to represent the discrete domains of the decision variables are established. These approaches are tested and compared to solve the optimisation problem behind a predictive-like controller designed to minimise the energy consumption of a manufacturing process line.

I. INTRODUCTION

In many industrial processes, both the optimal design and the implementation of optimisation-based control techniques often require variables constrained to a discrete set of admissible values. Some relevant applications of the use of discrete input signals can be found in chemical processes [1], process planning and scheduling [2], smart grids [3], and on-off systems [4]. For the particular case of manufacturing systems, system actuators can be of on-off nature of having different activation levels or status [5]. Thus, both the problem formulation and discrete optimisation method are crucial to solve these optimisation problems in a reliable and fast way. This last fact is quite important in control applications subject to real-time implementation constraints [6].

The use of discrete decision variables would imply a high computational burden for the optimisation routines. In most of the control applications, decisions involving discrete actuators are removed from the control layer and dealt with logic rules, precomputed control laws, or heuristic approximations [7], [8], [9]. However, most of these approaches are not able to respond effectively to disturbances or changes in the system since they are predefined considering specific operational conditions. For the case of flexible manufacturing systems, in which their operational conditions can change, these

approaches could be not suitable. Thus, with advances in computer performance and optimisation software, the common way to solve such problems is by transforming them into Mixed-Integer Programming (MIP) problems and solving them by using branch-and-bound-based MILP solvers such as those available in packages like `cplex` and `gurobi`. Based on this approach, to choose among n discrete values in an MIP problem, n binary variables that sum up to one are required [10]. Nonetheless, if the optimisation problem is not properly formulated and the decision variables are not restricted effectively to the feasible alternative, this approach could lead to unbalanced branch-and-bound trees and may result in long solution times [11].

Different approaches have been proposed in the literature to reduce the number of decision variables and the computational time to solve the related problem [12]. However, in more complex cases, these approaches imply the product of some polynomial terms, thus losing the linearity. In [13], a set of equations for linearising discrete cross-product terms that can be incorporated into conventional MILP reformulations is proposed. Similarly, a framework for reformulating Mixed-Integer Non-Linear Programming (MINLP) problems to a convex relaxed form is discussed in [14]. As common ideas, trying to avoid both bi-linear and non-linear terms into the problem reformulation is linked to the reduction of the number of decision variables and simplifying the nature of the proposed constraints from the definition stage of the optimisation problem.

Thus, the main contribution of this work is to provide three different ways of representing discrete feasible sets for control applications based on optimisation as well as to present a comparison of such approaches concerning the required computational burden for their implementation. In this regard, mathematical tools and representations such as rounding error and polyhedrons [15], [16] have been employed to constrain the discrete sets and to reduce the number of binary variables required when the optimisation problem is reformulated as a compact 0-1 MILP problem. The main idea is to reformulate the optimisation problem effectively and efficiently to simplify the input to optimisation routines such that the controllers can be suitable for their implementation in real time without the use of precomputed control laws. Other strategies have been excluded from this proposed comparative assessment due to their computational burden, a fact that does not allow their use in real time according to the field of application considered in this paper.

Then, to test the performance of the proposed approaches, an optimisation-based controller proposed in [17] to min-

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imise the energy consumption of a manufacturing process line will be considered. In this case, the optimal activation instants of binary and discrete actuators should be determined considering the operating constraints on both the process line and the system actuators. Thus, the optimisation problem is reformulated as an MILP problem using the proposed approaches, and the closed-loop performance for the controller is compared in terms of the control objective and the computational burden required to find an optimal solution.

The rest of the paper is organised as follows. The energy efficiency control strategy for manufacturing systems and the problem statement are introduced in Section II. Then, in Section III, the proposed approaches to model the discrete sets are presented. Next, the case study and the related decision models are introduced in Section IV. Later, the simulations results and the comparative assessment for the proposed approaches are presented and discussed in Section V. Finally, conclusions from the obtained results are presented in Section VI.

II. ENERGY EFFICIENCY OF MANUFACTURING SYSTEMS

A. Energy Efficiency Control Strategy

A process line is a complex system including several machine tools and peripheral devices that work sequentially and logically, as shown in Figure 1. Machines refer to a set of devices that are directly related to the machining processes (e.g., milling, cutting, turning), while the peripheral devices are those elements that provide the resources required to machines for their proper operation. Thus, there exist several functional relationships between machines and peripheral devices that determine the productivity of the process line.

Machine tools are characterised by periodic behaviour according to the time required to process a piece, which is denoted by T_{M_i} . Besides, due to the different operations performed by machine devices, there are high and low energy consumption stages along T_{M_i} . Then, since peripheral devices supply resources to machines in the process line, such devices might or might not show a periodic behaviour, which may match with T_{M_i} . Thus, to improve the energy efficiency of manufacturing system, a predictive-like controller to manage the activation instants of the peripheral devices in a process line without affecting its productivity has been proposed in [17]. In this case, the activation/deactivation of the machine devices along T_{M_i} , i.e., the machining sequence of the i -th machine (Λ_{M_i}), is considered as fixed and periodic over time. Thus, T_{M_i} are kept the same and machines can handle the same number of pieces as when the control strategy is not implemented. The energy consumption for Λ_{M_i} , i.e., $\beta_{M_i} = \sum_{k=1}^{T_{M_i}} S_{M_i}(k)$, will also be constant over any interval T_{M_i} and periodic over time, being S_{M_i} the instantaneous power consumption of the machine.

The control strategy proposed in [17] focuses on the management of peripheral devices to minimise the total energy consumption and guarantee the supply of the resources required by the machines. Then, considering a fixed number of both machines and peripheral devices in a process line,

their activation sequences can be defined as

$$\Lambda_{M_i}(k) = \{u_{M_{i,1}}(k), u_{M_{i,2}}(k), \dots, u_{M_{i,v}}(k)\}, \quad (1a)$$

$$\Lambda_P(k) = \{u_1(k), u_2(k), \dots, u_m(k)\}, \quad (1b)$$

being $k \in \mathbb{Z}_{\geq 0}$ the discrete-time index, and $u_{M_{i,l}}(k)$ and $u_j(k)$ the activation signals of machining devices of the i -th machine and the peripheral devices in the process line. Then, considering a prediction horizon H_p , the sequence¹ for Λ_P along H_p is defined as

$$\Gamma(k) \triangleq \{\Lambda_P(k|k), \dots, \Lambda_P(k+H_p-1|k)\}, \quad (2)$$

and an MPC controller is designed based on the following open-loop optimisation problem:

$$\min_{\Gamma(k)} \sum_{k=1}^{H_p} S(k) \Delta k \quad (3a)$$

subject to

$$\xi(k+r+1|k) = f(\xi(k+r|k), \Lambda_{M_i}(k+r|k), \dots, u_j(k+r|k)), \quad (3b)$$

$$S(k+r|k) = g(\xi(k+r|k)), \quad (3c)$$

$$Q_j(k+r+1|k) = q_j(Q_j(k+r|k), u_j(k+r|k), \dots, \Lambda_{M_i}(k+r|k)), \quad (3d)$$

$$u_j(k+r|k) \in \Omega_j = \{s_{j_1}, s_{j_2}, \dots, s_{j_n}\}, \quad (3e)$$

$$Q_h(k+r+1|k) \in [\bar{Q}_h \underline{Q}_h], \quad (3f)$$

for $r \in \{0, 1, 2, \dots, H_p - 1\}$, being $S \in \mathbb{R}$ the instantaneous power consumption of the whole process line given by

$$S(k) = \left(\sum_{i=1}^b S_{M_i}(k) \right) + \left(\sum_{j=1}^m S_j(k) \right), \quad (4)$$

with $S_{M_i} \in \mathbb{R}$ and $S_j \in \mathbb{R}$ the power consumption of the machines and peripheral devices, respectively. Besides, in (3), $\xi \in \mathbb{R}^{n_1}$ is the state vector of the energy consumption model of the process line, while $f : \mathbb{R}^{n_1} \times \Omega_j \mapsto \mathbb{R}^{n_1}$ and $h : \mathbb{R}^{n_1} \mapsto \mathbb{R}_{\geq 0}$ are the maps for such a model. Moreover, Q_j corresponds to states of the process dynamics related to the operation of peripheral devices with $q_j : \Omega_j \times \mathbb{R} \mapsto \mathbb{R}_{\geq 0}$ the maps that consider the operational relationships between machines and peripheral devices. According to Figure 1, the q -relations in (3) correspond to the process dynamics for the supply systems of compressed air and coolant.

B. Problem Statement

It is worth noting that (3e) and (3f) refer to the constraints for the domain of decision variables and process variables Q_j , respectively. In this case, $\Omega_j \subset \mathbb{R}$ is the discrete and countable finite set of cardinality n_j describing the feasible domain of u_j , with s_{j_n} the values (or symbols) that form such discrete (or alphabet) set. In the most simple case, u_j will be constrained to a binary set, which represents the on/off status of a device with nominal power consumption.

¹Here, $z(k+r|k)$ denotes the prediction of the variable z at time instant $k+r$ performed at k . The index r will cover the finite prediction window of length H_p .

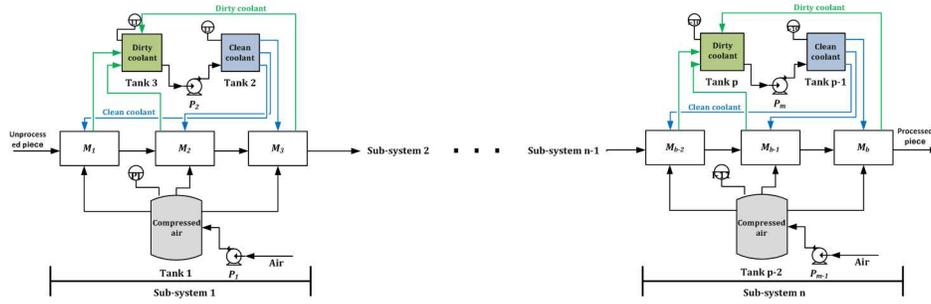


Fig. 1: A b-stage serial process line with its corresponding peripheral devices.

However, in a real plant, peripheral devices are of different nature and usually constrained to different discrete sets based on their design and the configuration of the process line.

Taking into account the admissible domain for the decision variables in (3), the problem addressed in this paper is to determine suitable ways of modelling these domains, fact that contributes to reducing the computational burden required to solve (3). This last fact is important for the real-time implementation of closed-loop control strategies based on optimisation, in which the optimisation problem should be solved fast enough at every instant k . Therefore, to get suitable representations of discrete sets such as Ω_j , the nature of the decision variables and corresponding mathematical expressions to represent the admissible values should be determined and added to the optimisation problem.

III. MODELLING OF DISCRETE SETS

Due to the nature of the decision variables in (3), three different ways to represent the discrete sets are presented below to formulate the optimisation problem as a compact 0-1 MILP problem. Although there are some optimisation routines that model directly such sets, the idea behind the proposed approaches is to further simplify the problem before to use the optimisation solver.

Approach 1: Rounding-error based strategy: In this case, to model Ω_j the following statement is assumed:

Assumption 1: A discrete set $\Omega_j = \{s_{j_1}, s_{j_2}, \dots, s_{j_n}\}$ with $s_{j_n} \in \mathbb{R}$ is an ordered set in which all its elements have a regular spacing ρ . \square

Under this assumption, to model the discrete feasible set Ω_j , one decision variable, i.e., $\tilde{u}_j \in [\underline{\varepsilon}, \bar{\varepsilon}]$ with $\bar{\varepsilon} = \max(\Omega_j)$ and $\underline{\varepsilon} = \min(\Omega_j)$ is required. Besides, to ensure that only the admissible values of Ω_j in $[\underline{\varepsilon}, \bar{\varepsilon}]$ could be selected, the rounding error should be added as a constraint into the optimisation problem in (3) as follows:

$$\left| \rho \left[\left(\frac{\tilde{u}_j}{\rho} \right) \right] - \tilde{u}_j \right| \leq 0, \quad (5)$$

with $[\cdot]$ the round operator and ρ the regular spacing among the elements of Ω . Based on the previous inequality, only the values in $[\underline{\varepsilon}, \bar{\varepsilon}]$ that satisfy $(x \bmod \rho) \equiv 0$ can be chosen.

Remark 1: If zero belongs to Ω_j and its distance to the next element in Ω_j does not satisfy the spacing regularity of the rest of the elements, a new variable and a constraint should be introduced, e.g., when zero represents the off status

and the rest of the elements refer to different activation levels. Thus, zero is considered separately by its nilpotent properties within Ω_j , and an extra binary variable ϕ_j and a new constraint are added as follows:

$$\underline{\varepsilon} \phi_j \leq \tilde{u}_j \leq \bar{\varepsilon} \phi_j, \quad (6)$$

with $\phi_j \in \{0, 1\}$ and $\tilde{u}_j \in [\underline{\varepsilon}, \bar{\varepsilon}]$. Besides, this approach could be extended to the case in which ρ for the elements of Ω_j is not regular. The original set is split into subsets with regular spacing, which are associated with one binary variable, rounding error equation, and extra constraints on binary variables to ensure the selection of only one set. \square

Approach 2: Direct binary approach encoding: Given a set $\Omega_j = \{s_{j_1}, s_{j_2}, \dots, s_{j_n}\}$ with $s_{j_n} \in \mathbb{R}$, one can associate n decision variables, i.e., $\gamma_1, \gamma_2, \dots, \gamma_n \in \{0, 1\}$, and reinforce the alternative of decision with the constraint

$$\gamma_1 + \gamma_2 + \dots + \gamma_p = 1. \quad (7)$$

Indeed, to guarantee the selection of only one of the values in Ω_j , the decision variable u_j is computed according to

$$u_j = s_{j_1} \gamma_1 + s_{j_2} \gamma_2 + \dots + s_{j_n} \gamma_n. \quad (8)$$

Remark 2: It should be noted that if zero belongs to Ω_j , $n - 1$ binary variables are needed, and the constraint in (7) should be relaxed as $\gamma_1 + \gamma_2 + \dots + \gamma_{n-1} \leq 1$. \square

Approach 3: Geometrical representation of the feasible domain: In this case, the discrete set will be modelled by means of a polyhedron given as the intersection of inequalities and equalities (referred to as H-representation). To get a polyhedral representation of a discrete set, auxiliary variables are defined to fit the vertices of the polyhedron. For instance, considering a discrete set $\Omega_j = \{s_{j_1}, s_{j_2}, s_{j_3}, s_{j_4}\}$, two binary variables and one real variable, i.e., $\delta_1, \delta_2 \in \{0, 1\}$, and $u_j \in \mathbb{R}$, are required. Next, the discrete values of Ω_j will be associated to the combinations of variables δ_1 and δ_2 , which define the vertices set V of the polyhedron as

$$V = \{[0, 0, s_{j_1}]; [0, 1, s_{j_2}]; [1, 0, s_{j_3}]; [1, 1, s_{j_4}]\}.$$

Then, using suitable tools, such as the MPT Toolbox [18], the H-representation of the polyhedron $P_v = \{v | A v \leq b, A_e v = b_e\}$, which corresponds to the convex hull of V , can be determined. These expressions should be added as constraints in the optimisation problem such that $[\delta_1, \delta_2, u_j]^T \in P_v$. Based on this formulation, for any combination of the binary variables only the values in

Ω_j satisfy the inequality and equality matrices added as constraints.

IV. BENCHMARK SYSTEM

According to Figure 1, the process line is formed by n sub-systems, which consists of three machines and two shared peripheral devices. Thus, the process description presented below for the one sub-system is straightforwardly extended to the other sub-systems without loss of generality. It is assumed that all machines in each sub-system have the same period, i.e., $T_{M_i} = 28s \forall i = 1, 2, 3$, and that the peripheral devices P_1 and P_2 are shared among these machines. In this case, P_1 is associated to a supply system of compressed air with nominal energy consumption, i.e., $u_1 \in \Omega_1 \triangleq \{0, 1\}$. Thus, the q -relations in (3d) related to P_{G_1} refer to the dynamics for the total change of mass M_{T_1} and pressure P_{T_1} inside a storage tank T_1 , which are expressed as follows:

$$M_{T_1}(k+r+1|k) = M_{T_1}(k+r|k) + \tau_s \sigma(k+r|k), \quad (9a)$$

$$\sigma(k+r|k) = m_{in} u_{G_1}(k+r|k) - \sum_{i=1}^b m_{out_{M_i}}(k+r|k), \quad (9b)$$

$$P_{T_1}(k+r|k) = \frac{M_{T_1}(k+r|k) R T}{V_{T_1} W_{air}}, \quad (9c)$$

with τ_s the sampling time, $m_{out_{M_i}}$ the air consumption from M_i , m_{in} the air flow pumped by P_{G_1} towards the tank T_1 , and, R, T, V_{T_1} , and W_{air} the gas constant, air temperature, volume of T_1 , and the molecular weight of the air, respectively. Besides, P_{T_1} must belong to $[\underline{P}_{T_1}, \bar{P}_{T_1}]$.

Besides, P_2 is related to a coolant supply system for the cutting operations at each machine, and its activation can be modulated at different levels, i.e., $u_2 \in \Omega_2 \triangleq \{0, 100, 120, 140\}$. According to Figure 1, a flow of coolant m_c is pumped by P_2 from T_3 towards T_2 , passing through a filter in which the gross particles are separated. The coolant flows required by the machines are pumped from T_2 . The level dynamics in both tanks are given by

$$L_2(k+r+1|k) = L_2(k+r|k) + \tau_s \gamma(k+r|k) \left(\frac{1}{\rho_c A_{T_2}} \right), \quad (10a)$$

$$\gamma(k+r|k) = m_c(k+r|k) - \sum_{i=1}^b m_{out,c_{M_i}}(k+r|k), \quad (10b)$$

$$L_3(k+r+1|k) = L_3(k+r|k) + \tau_s \theta(k+r|k) \left(\frac{1}{\rho_c A_{T_3}} \right), \quad (10c)$$

$$\theta(k+r|k) = \sum_{i=1}^b m_{in,c_{M_i}}(k+r|k) - m_c(k+r|k), \quad (10d)$$

$$P_{out}(k+r|k) = P_{in}(k+r|k) + \rho_c h_{f_{1 \rightarrow 2}}(k+r) - \eta \left(\frac{W(k+r|k)}{m_c(k+r|k)} \right), \quad (10e)$$

with $m_{out,c_{M_i}}$ the coolant flow required by the i -th machine, and $m_{in,c_{M_i}}$ the flow of the dirty coolant recovered. Besides, P_{in} and P_{out} correspond to the input and output pressure in the pipe system from T_3 towards T_2 , while, ρ_c, η, W and $h_{f_{1 \rightarrow 2}}$ are the coolant density, the pump efficiency, the work supply to P_{G_2} , and the energy losses by friction, respectively.

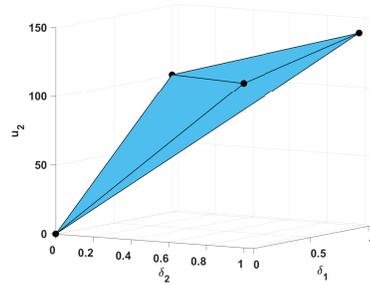


Fig. 2: Polyhedron for the discrete set Ω_2 .

A. Discrete domain for the decision variables

Given the nature of the discrete sets Ω_1 and Ω_2 , the decision variables and constraints required by each one of the approaches proposed in Section III are presented below.

Approach 1: If zero is removed from the set Ω_2 and an extra variable is added for the switching off of P_2 , i.e. $u_2 = 0$, it is possible to fix $\rho = 20$ for the elements of Ω_2 . Thus, three decision variables are required to model Ω_1 and Ω_2 , i.e., $u_1(k), \phi_2(k) \in \{0, 1\}$, and $\tilde{u}_2(k) \in [100, 140]$, with u_1 related to P_1 and, ϕ_2 and \tilde{u}_2 associated to Ω_2 . Then, considering $\bar{\varepsilon} = 100$ and $\underline{\varepsilon} = 140$, constraints in (5) and (6) should be adapted and added to (3).

Approach 2: In this “full” binary encoding, four binary variables are required to represent the domain of decision variables, i.e., $u_1(k), \gamma_1(k), \gamma_2(k), \gamma_3(k) \in \{0, 1\}$, with u_1 related to the device P_1 , and according to Remark 2, three binary variables γ_1, γ_2 and γ_3 are required to model the discrete set Ω_2 together with the following constraint:

$$\gamma_1(k) + \gamma_2(k) + \gamma_3(k) \leq 1. \quad (11)$$

Finally, u_2 is given by

$$u_2(k) = 100\gamma_1(k) + 120\gamma_2(k) + 140\gamma_3(k). \quad (12)$$

Approach 3: In this case, one binary variable for the activation/deactivation of P_1 is required, i.e., $u_1(k) \in \{0, 1\}$, while two binary variables, i.e., $\delta_1(k), \delta_2(k) \in \{0, 1\}$, and one real variable, i.e., $\tilde{u}_2(k) \in \mathbb{R}$, are needed to model Ω_2 . Then, each combination of δ_1 and δ_2 will be related to one element in the set Ω_2 , and a polyhedron with vertices

$$V_{P_{G_2}} = \{[0, 0, 0] [0, 1, 100] [1, 0, 120] [1, 1, 140]\}$$

is defined and shown in Figure 2. Next, using the MPT toolbox, the H-representation of the polyhedron was obtained and, the following inequality matrix is added to (3):

$$\begin{bmatrix} -0.768 & -0.640 & 0.006 & 0 \\ 0.986 & 0.164 & -0.008 & 0 \\ 0.371 & 0.928 & -0.009 & 0 \\ -0.436 & -0.218 & 0.011 & 0.873 \end{bmatrix} \begin{bmatrix} \delta_1 \\ \delta_2 \\ \tilde{u}_2 \\ -1 \end{bmatrix} \leq \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

B. Energy consumption models

Data-driven models were used to identify the power consumption models of machines and peripheral devices. In this case, Subspace Identification (SI) methods were used

since they allow identifying the matrices of a state-space realisation of linear time-invariant systems based on input-output data [19]. According to [17], different sequences of Λ_{M_i} along T_{M_i} were designed and tested in a test bench that emulates the energy consumption of machine tools to collect the data of S_{M_i} . Besides, different sequences of Λ_P were tested to obtain a rich range of the outputs S_{P_j} . From the obtained input-output data sets, energy consumption models were identified by using the `n4sid` routine of the System Identification Toolbox™ provided by Matlab®. Thus, different values of N were tested to identify the matrices that allow the highest fitting degree between the real and modelled outputs. The energy consumption models are detailed in [17].

V. SIMULATION RESULTS

The control strategy was tested using each one of the proposed approaches to compare their performance and computational burden. All simulations were performed using an Intel Core i7-5500U 2.4 GHz processor with 8G RAM and considering a sampling time equal $\tau_s = 0.1$ s. The simulation results were obtained in Matlab by using the software IBM ILOG CPLEX Optimisation Studio integrated to YALMIP toolbox [20]. Since the controller was designed to make decisions every second along $H_p = 28$ s, a faster internal loop (blocking) for both the process and energy consumption models are implemented [21].

The resulting energy consumption profile and the optimal activation/deactivation sequence of peripheral devices are presented in Figures 3a and 3b for the case in which $n = 1$. Besides, in Figure 3c the process dynamics related to both P_{G_1} and P_{G_2} are shown. From these results, some differences can be observed for the peripheral device with a discrete domain, i.e., P_{G_2} , for which even when the device was turned on at the same time instant, the activation value was different according to the approach tested. Thus, although the stopping criteria of optimisation routine were the same for all approaches, these differences could be related to the solver ability to test all possible combinations for each approach before to reach the stopping criteria. This is due to the fact that, for each modelling approach, the nature and the total number of decision variables are different and, therefore, the number of the possible combinations could differ although the feasible domain is similar.

Next, to compare the computational burden when more decision variables are considered, a process line adding sub-systems as shown in Figure 1 is studied. It should be noted that for each one of the sub-systems added, the operating conditions, machine cycles, energy consumption models, and dynamics of peripheral devices are assumed to be the same that for the first subsystem with three machines and two peripheral devices. In Table I, a comparison of the proposed approaches concerning both the computational burden and the control objective is presented when several sub-systems are considered as the process line. In this regard, the decision variables (DV) along H_p , their classification, the number of equality and inequality constraints, the CPU time and the value of the cost function are presented in Table I.

In Table I, the auxiliary variables (AV) refers to the variables added to model the non-linear operators involved in the rounding error expression. Thus, the total number of decision variables considered by each approach corresponds to the set of both the decision and auxiliary variables. Besides, for Approach 1 all the auxiliary variables added to the decision model were defined as integer variables and by the optimisation routine internally. Finally, the total number of continuous variables (CV) for Approaches 1 and 3 are summarised in the seventh column. From these results, it should be noted that the equality constraints are the same for all approaches since they correspond to the initial conditions for both the process model and the machining sequences.

Based on the results in Table I, it is possible to observe that even when the optimal activation sequences of peripheral devices obtained from each approach are different, the total energy consumption is similar for each one of the approaches tested. However, higher differences can be observed in the CPU time spent to solve the optimisation problem at each iteration. Thus, although the Approach 1 has a higher number of total decision variables, the solution is obtained faster than for the rest of cases. This behavior is related to the fact that Approach 1 has a lower number of binary variables, and therefore, fewer combinations should be tested and the completion conditions are achieved faster. Therefore, when the branch-and-bound algorithms are employed, the computational burden to evaluate all (or most of) the combinations is increased as the binary variables increase. In this regard, by implementing the proposed Approach 1 with fewer number of binary variables, the computational burden can be reduced even when auxiliary variables are required. However, although both Approaches 2 and 3 require more binary variables to model Ω_2 than Approach 1, the latter is more restrictive than the other approaches. Thus, when more complex discrete sets with no regular spacing are analysed, Approach 1 could require more binary variables than Approaches 2 and 3, in addition to the auxiliary variables due to both the rounding operator and absolute value.

VI. CONCLUSIONS

In this paper, three different approaches to reformulate optimisation problems involving discrete and continuous variables into a mixed-integer MILP problem have been presented. Based on the proposed formulations, ordered and non-ordered discrete sets could be modelled by using rounding error, binary variables or polyhedral approximations. Besides, the necessary constraints to guarantee the selection of the discrete admissible values according to each approach as well as the conditions for their implementation were presented and discussed. Based on the obtained results, it is possible to conclude that the total amount of binary variables is a crucial factor regarding the computational burden even when the total number of decision variables is lower. However, despite the computational cost, the binary variables lead to optimisation problems that involve non-ordered and non-regularly distributed symbol sets can be solved in an efficient manner and with a manageable computational burden.

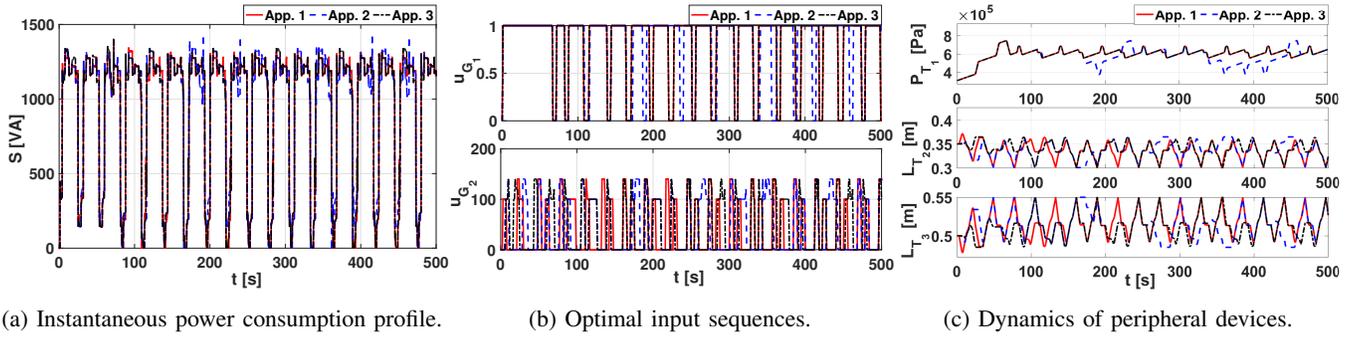


Fig. 3: Simulation results of the control strategy using the proposed approaches.

TABLE I: Comparison of the proposed approaches regarding their performance and computational burden.

App.	Subsystem n	No. Devices	No. DV along H_P	No. AV along H_P	No. BV along H_P	No. CV along H_P	No. inequality constraints	No. equality constraints	max CPU time (s)	CPU time (s)	S [VA]
1	1	2	87	58	58	29	1033	1977	0.157	43.278	1057389.179
2	1	2	116	0	116	0	975	1977	0.563	119.064	1057631.542
3	1	2	116	0	87	29	946	1977	0.390	84.978	1057389.179
1	2	4	174	116	116	58	2066	3954	0.875	241.582	2114885.914
2	2	4	232	0	232	0	1950	3954	1.172	424.487	2114843.857
3	2	4	232	0	174	58	1892	3954	1.609	661.855	2114506.408
1	3	6	261	174	174	87	3099	5931	2.375	581.211	3172410.754
2	3	6	348	0	348	0	2925	5931	3.078	1004.515	3171807.409
3	3	6	348	0	261	87	2838	5931	4.781	1019.072	3171530.959
1	4	8	348	232	232	116	4132	7908	2.953	605.873	4229758.495
2	4	8	464	0	464	0	3900	7908	2.891	1103.728	4229160.288
3	4	8	464	0	348	116	3784	7908	1.983	1105.196	4229503.581
1	5	10	453	290	290	145	5165	9885	2.844	603.630	5286723.512
2	5	10	580	0	580	0	4875	9885	2.562	1274.541	5286230.569
3	5	10	580	0	435	145	4730	9885	2.218	978.293	5286553.431

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