

Robust Fault Detection using Set-based Approaches for LPV Systems: Application to Autonomous Vehicles

Shuang Zhang* Vicenç Puig* Sara Ifqir**

* *Institut de Robòtica i informàtica Industrial (CSIC-UPC), Carrer Llorens Artigas, 4-6, 08028 Barcelona, Spain (e-mail: {shuang.zhang, vicenc.puig}@upc.edu).*

** *CRISTAL, UMR CNRS 9189, Centrale Lille Institut, 59651 Villeneuve d'Ascq, France (e-mail: sara.ifqir@centralelille.fr)*

Abstract: This paper addresses the problem of robust fault detection for Linear Parameter Varying (LPV) systems using set-based approaches. Two approaches are proposed, based respectively on set-based state and parameter estimation methods, for implementing direct and inverse test for robust fault detection (FD). The uncertainties are assumed to be unknown but bounded and their effect is propagated using zonotopic sets. These robust FD test methods aim at checking the consistency between the measured and estimated behaviour obtained from estimator in the parameter or output space considering the effect of the uncertainty. When an inconsistency is detected, a fault can be indicated. A case study based on an autonomous vehicle is employed to compare the performance of proposed FD tests.

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1. INTRODUCTION

Autonomous vehicles recently receive much attention and are widely studied, with the advantages of saving energy consumption, improving traffic efficiency and at the same time reducing traffic crashes by taking humans out of driving. The most attractive and essential is avoiding crashes on the roads under all circumstances, which will lead to a huge reduction in deaths. However, as a complex system composed by a series of modules, such as sensing network (GPS, cameras, etc.), it may encounter sensor faults or/and actuator faults which can lead to catastrophic consequences if they are not diagnosed and handled in time.

Fault Detection (FD) plays an important role in improving the safety and reliability of automatic control systems. Model-based fault detection checks the consistency based on generating the residuals between the estimated behaviors and the observations obtained from sensors. Ideally, the residuals should only be affected by faults. However, the built model is always affected by modelling uncertainty, unknown noise and disturbance which lead to mismatch between actual and estimated behaviours. As a consequence, the fault detection must be robust against these undesired uncertainties. And set-based approaches, known as a class of deterministic robust methods, assume an unknown-but-bounded description to model the uncertainties with application to state estimation (Alamo et al., 2005; Combastel, 2015) and parameter estimation (Bravo et al., 2006).

The robust fault detection research focused on two distinct approaches. One family of approaches, called active, aims

to decouple the effect of uncertainty (Chen and Patton, 1999). The other family is called passive, and allows to enhance the robustness of the fault detection system at the residual value evaluation stage (Frank and Ding, 1994). The passive approach leads to two possible implementations known as direct and inverse test, respectively. The direct test is based on verifying if the residual or the measurement output is inside the interval of possible values (Ploix et al., 2000; Puig et al., 2002). The inverse test is based on verifying if there exists a value inside the nominal parameter set that can explain the measured output of the system (Puig et al., 2007). This paper focuses on the use of passive robust fault detection methods to nonlinear systems (e.g. autonomous vehicles) represented in linear parameter varying (LPV) form. The paper also compares the performance of direct and inverse fault detection implementations for different type of faults.

In our previous research, see Zhang and Puig (2021), the comparison between direct and inverse fault detection tests was performed for linear time invariant (LTI) systems. As we know, the best representation of autonomous vehicles systems is driven by a set of nonlinear ordinary differential equations. But, the non-linear systems are complex and difficult to study. An appealing way to deal with non-linear systems is through LPV systems by incorporating non-linearities in the varying parameters. There are a few researches successfully applying LPV framework to solve control and/or fault diagnosis problems (Alcalá et al., 2019; Guzmán-Rabasa et al., 2019; Delgado-Aguíñaga et al., 2021). Furthermore, polytopic LPV system provides a more convenient way to represent and analyze LPV design problems.

The main contribution of this paper is the development of a Set-Membership Approach (SMA) for state/parameter estimation applicable to LPV systems. This is achieved by solving a set of Linear Matrix Inequalities (LMIs) obtained from the minimization of Frobenius norm of the obtained zonotope that approximates the set of states/parameters. Thus, using such estimation approaches, direct and inverse test for fault detection are introduced. The results presented in this paper extend preliminary results presented in Pourasghar et al. (2019) where SMA for state/set estimation via LMI for LTI systems was introduced. This paper is based on extending this research to parameter estimation and LPV systems, which is more challenging due to the varying parameters.

This paper is organized as follows: Section 2 presents the problem formulation. Section 3 describes the two ways to detect faults using the passive approach. Section 4 introduces a SMA for state estimation for LPV systems as a way to implement the direct test. Section 5 proposes a SMA for parameter estimation for LPV systems to implement the inverse test. In Section 6, a case study based on autonomous vehicles for fault detection using direct and inverse tests are presented allowing to assess the proposed approaches. Finally, conclusions are summarised in Section 7.

2. PROBLEM FORMULATION

LPV systems are linear dynamical systems whose mathematical description depends on parameters that change values over time. These parameters are generally considered as bounded and taking values inside a set, often assumed to be a compact and convex polytope (e.g. a box). The non-linear embedding approach is used for obtaining an LPV formulation from a non-linear physical model of the system represented in state-space (LPV-SS):

$$\begin{aligned} x_{k+1} &= A(\theta_k)x_k + B(\theta_k)u_k + w_k \\ y_k &= Cx_k + v_k \end{aligned} \quad (1)$$

where $u_k \in \mathbb{R}^{n_u}$, $y_k \in \mathbb{R}^{n_y}$, $x_k \in \mathbb{R}^{n_x}$, $w_k \in \mathbb{R}^{n_x}$, $v_k \in \mathbb{R}^{n_y}$ are the input, output, state, disturbance and measurement noise vectors, respectively. Moreover, the disturbance and measurement noises are assumed to be unknown but bounded. Besides, $A(\theta_k) \in \mathbb{R}^{n_x \times n_x}$, $B(\theta_k) \in \mathbb{R}^{n_x \times n_u}$, $C \in \mathbb{R}^{n_y \times n_x}$ are the state-space matrices, where $\theta_k \in \mathbb{R}^{n_\theta}$ is the vector of time-varying parameters belonging to a known set Θ . The dependence of state matrices, i.e. $A(\theta_k)$ and $B(\theta_k)$, with respect to θ_k can take many forms. Among the available representation, the polytopic decomposition is of special importance when it comes to analysis and design.

The polytopic framework offers an elegant and convenient way for representing and analyzing LPV and uncertain systems (Briat, 2014). In addition, based on Rotondo et al. (2015), polytopic LPV and Takagi-Sugeno (TS) have strong analogies and close connection. The polytopic LPV system can be represented by matrices $A(\theta_k)$ and $B(\theta_k)$, where the scheduling vector θ_k ranges over a fixed polytope resulting in the following representation

$$x_{k+1} = \sum_{i=1}^N \mu_i(\theta_k) (A_i x_k + B_i u_k) \quad (2)$$

where the system matrices (A_i, B_i) define the so-called vertex systems, $N = 2^{n_\theta}$ leads to a polytope of N vertices, n_θ represents the number of varying parameters and $\mu(\cdot)$ is known as the vertex membership function (or coefficients of the polytopic decomposition), which satisfies

$$\sum_{i=1}^N \mu_i(\theta_k) = 1, \quad \mu_i(\theta_k) \geq 0, \forall \theta_k \in \Theta \quad (3)$$

where $\mu_i(\theta_k) = \prod_{j=1}^{n_\theta} \xi_{ij}(\eta_0^j, \eta_1^j)$, $\forall i = 1, \dots, N$. And each membership function is given by

$$\begin{aligned} \eta_0^j &= \frac{\bar{\theta}^j - \theta_k^j}{\bar{\theta}^j - \underline{\theta}^j} \\ \eta_1^j &= \frac{\theta_k^j - \underline{\theta}^j}{\bar{\theta}^j - \underline{\theta}^j} \end{aligned} \quad (4)$$

where $\theta_k^j \in [\underline{\theta}^j, \bar{\theta}^j]$ is the j^{th} component of θ_k , $\xi_{ij}(\eta_0^j, \eta_1^j)$ is the function that performs N possible combinations.

3. DIRECT VS INVERSE TESTS

The main idea of fault detection methods is to compare the estimated behavior with the behavior measured using sensors of the physical system. The inconsistencies between them are called residuals, usually the residuals are evaluated in the output space or parameter space. When the detection is conducted in output space, it is called direct test. Thereby, inverse test is in parameter space.

In output space, the residuals are calculated as follows

$$\mathbf{r}(k) = \mathbf{y}_k - \hat{\mathbf{y}}_k \quad (5)$$

where $\mathbf{r}(k)$ is the residuals vector, \mathbf{y}_k is the system output measurement, and $\hat{\mathbf{y}}_k$ is the estimated output. In order to consider the uncertainties, the detection test is based on checking if $\mathbf{r}(k) \in [-\sigma, \sigma]$. Then the fault detection test in output space yields to check if

$$\mathbf{y}_k \in [\hat{\mathbf{y}}_k - \sigma, \hat{\mathbf{y}}_k + \sigma] \quad (6)$$

where σ is the noise bound defined such that $|v_k| \leq \sigma$. In output space, the test can be implemented by zonotopic state estimation presented in Section 4. In this case, the estimated output is generated from the estimated state bounded by a zonotope. Then, the direct test involves checking whether the measured output is contained in the estimated output interval.

In the parameter space, the residuals are generated as follows

$$\mathbf{r}(k) = \theta_k - \hat{\theta}_k \quad (7)$$

where θ_k is from the nominal parameters set Θ , and $\hat{\theta}_k$ is the estimated parameter from parameter estimation approach. Thus, the inverse test is alternatively based on checking if there exists a parameter in the set of nominal parameters that enables the model to be consistent with the measurements

$$\exists \hat{\theta}_k \in \Theta \mid \hat{\mathbf{y}}(k, \theta_k) \in [\mathbf{y}_k - \sigma, \mathbf{y}_k + \sigma] \quad (8)$$

where $\hat{\mathbf{y}}(k, \theta_k)$, as e.g., in case of a LTI system, is obtained by using the shift operator q^{-1} and assuming zero initial conditions, as follows:

$$\hat{\mathbf{y}}(k, \theta_k) = M(q^{-1}, \theta)u_k \quad (9)$$

with

$$M(q^{-1}, \theta) = C(\theta)(qI - A(\theta))^{-1}B(\theta).$$

The above formula (8) can be transformed to check if exists an intersection between parameter zonotope and strip (Puig, 2010)

$$\Theta_{k+1} = \Theta_k \cap \mathbb{F}_k \quad (10)$$

where

$$\mathbb{F}_k = \{\theta \in \mathbb{R}^{n_\theta} \mid \mathbf{y}_k - \sigma \leq c_k^T \theta \leq \mathbf{y}_k + \sigma\} \quad (11)$$

is the strip of parameters consistent with the current measurements. c_k^T is the regressor introduced in Section 5. Due to the recursive formulation (10), the inverse test can be implemented by parameter estimation procedure in Section 5. Furthermore, if $\Theta_{k+1} = \emptyset$, a fault is indicated.

4. DIRECT TEST IMPLEMENTATION

As it is mentioned before, state estimation methods is a possible way to implement direct test through obtaining the nominal estimation state plus the uncertainty interval based on measurements data. In this section, we extend a SMA for LTI systems to LPV systems through the polytopic representation and an LMI formulation.

4.1 Set-Membership State Estimation Approach

Based on previous research results (Zhang and Puig, 2021), the SMA is a less conservative approach compared with Interval Observer Approach (IOA) for estimating the state of a LTI system with state-space matrices (A, B, C) . It describes the estimated state by a bounding zonotope \hat{X}_k^{sm} with the center $c_{x,k}^{sm}$ and the shape matrix $R_{x,k}^{sm}$ (Alamo et al., 2005; Pourasghar-Lafmejani, 2019).

$$\hat{X}_k^{sm} = \langle c_{x,k}^{sm}, R_{x,k}^{sm} \rangle \quad (12)$$

Considering that the disturbance and noise are unknown but bounded using zonotopes: $w_k \in Ew_{I_k}, v_k \in Fv_{I_k}$, where $E \in \mathbb{R}^{n_x \times n_w}, F \in \mathbb{R}^{n_y \times n_v}$ are the known constant distribution matrices of the disturbance and noise, $w_{I_k} \in \langle 0, I_{n_w} \rangle, v_{I_k} \in \langle 0, I_{n_v} \rangle, I_{n_w} \in \mathbb{R}^{n_w \times n_w}, I_{n_v} \in \mathbb{R}^{n_v \times n_v}$ denote the identity matrices. The estimated state can be obtained from the intersection between the prediction state set $P_k^{sm} = \langle c_{p,k}^{sm}, R_{p,k}^{sm} \rangle$ and k^{th} instant output strip, where $c_{p,k}^{sm}$ and $R_{p,k}^{sm}$ denote the center and shape of the zonotope P_k^{sm} , respectively. For the sake of simplified notations, the index $k \pm 1$ will be replaced by \pm and k will be omitted, therefore the zonotopic state of LTI systems estimated by SMA can be given as

$$c_x^{sm} = c_p^{sm} + \lambda (y - Cc_p^{sm}) \quad (13a)$$

$$R_x^{sm} = [(I - \lambda C) R_p^{sm} \quad -\lambda F] \quad (13b)$$

with

$$c_p^{sm} = Ac_{x,-}^{sm} + Bu_- \quad (14a)$$

$$R_p^{sm} = [AR_{x,-}^{sm} \quad E] \quad (14b)$$

where the optimal λ can be obtained minimising $\|R_x^{sm}\|_F^2$

$$\lambda^* = \frac{R_p^{sm} R_p^{smT} C}{CR_p^{sm} R_p^{smT} C^T + FF^T} \quad (15)$$

4.2 LMI-based State Estimation Approach

To adapt this approach to the LPV systems, the following improvements will be considered. The estimated state is described by the bounding zonotope $\hat{X}_k^{lmi} = \langle c_x^{lmi}, R_x^{lmi} \rangle$.

$$c_x^{lmi} = c_p^{lmi} + \sum_{i=1}^N \mu_i(\theta_{k-1}) \lambda_i (y - Cc_p^{lmi}) \quad (16a)$$

$$R_x^{lmi} = \sum_{i=1}^N \mu_i(\theta_{k-1}) [(I - \lambda_i C) R_p^{lmi} \quad -\lambda_i F] \quad (16b)$$

with

$$c_p^{lmi} = \sum_{i=1}^N \mu_i(\theta_{k-1}) (A_i c_{x,-}^{lmi} + B_i u_-) \quad (17a)$$

$$R_p^{lmi} = \sum_{i=1}^N \mu_i(\theta_{k-1}) [A_i R_{x,-}^{lmi} \quad E] \quad (17b)$$

It is worth remarking that this method is conducted by "offline+online computation", i.e., solving LMI optimization problem offline to obtain λ_i and conducting online combination $\lambda = \sum_{i=1}^N \mu_i(\theta_{k-1}) \lambda_i$.

In order to obtain the vertex λ_i , we need to construct and solve LMIs with the same aim of minimizing the Frobenius radius of the zonotope shape matrix by following theorem 1.

Theorem 1. Given the polytopic LPV system (2), the polytopic LPV parameter $\lambda_i (i = 1, \dots, N)$ can be designed as

$$\lambda_i = \Upsilon^{-1} W_i \quad (18)$$

if there exist positive scalars $\gamma, \Upsilon = \Upsilon^T$ and W_i by solving the following optimization problem:

$$\min \gamma \quad (19a)$$

subject to the following LMIs:

$$\begin{bmatrix} \gamma I_{n_x} & I_{n_x} \\ I_{n_x} & \Upsilon \end{bmatrix} > 0 \quad (19b)$$

$$\begin{bmatrix} -\Upsilon & \Upsilon A_i - W_i C & \Upsilon E & W_i \\ A_i^T \Upsilon - C^T W_i^T & -\Upsilon & 0 & 0 \\ E^T \Upsilon & 0 & -I & 0 \\ W_i^T & 0 & 0 & -R^{-1} \end{bmatrix} < 0 \quad (19c)$$

where $Q = Q^T \geq 0$ and $R = R^T > 0$ are the covariance matrices of disturbances and noise, respectively.

Proof. To obtain the LMI design conditions (19) for the LPV SMA optimal parameter λ , the duality principle between filtering and LQR optimal control will be used (Ostertag, 2011). Applying duality principle ($A \Rightarrow A^T, B \Rightarrow C^T, K \Rightarrow \lambda^T$) to LQR inequality derived in Ostertag (2011) and using polytopic representation, the optimal λ_i should satisfy the following inequalities:

$$P < \gamma I_{n_x} \quad (20a)$$

$$(A_i^T - C^T \lambda_i^T)^T P (A_i^T - C^T \lambda_i^T) - P + Q + \lambda_i R \lambda_i^T < 0 \quad (20b)$$

where $Q = EE^T = Q^T \geq 0$ and $R = R^T > 0$ are the covariance matrices of disturbances and noise, respectively. Now, applying the Schur complement, (20a) can be reshaped into the LMI form as (19b). And (20a) can be rearranged as follows:

$$-P + (A_i - \lambda_i C) P (A_i - \lambda_i C)^T + Q + \lambda_i R \lambda_i^T < 0 \quad (21)$$

Then, we do left and right multiplications of (21) by $\Upsilon = P^{-1}$ and introduce $W_i = \Upsilon \lambda_i$, the second constraint becomes

$$\begin{aligned} & -\Upsilon + (\Upsilon A_i - W_i C) \Upsilon^{-1} (A_i^T \Upsilon - C^T W_i^T) \\ & + \Upsilon E E^T \Upsilon + W_i R W_i^T < 0 \end{aligned} \quad (22)$$

Now, by reformulating (22) as

$$-\Upsilon - \begin{bmatrix} \Upsilon A_i - W_i C & \Upsilon E & W_i \\ -\Upsilon^{-1} & 0 & 0 \\ 0 & -I & 0 \\ 0 & 0 & -R \end{bmatrix} \begin{bmatrix} A_i^T \Upsilon - C^T W_i^T \\ E^T \Upsilon \\ W_i^T \end{bmatrix} < 0 \quad (23)$$

Finally, by applying the Schur complement, (23) yields the LMI form as (19c). Thus, the i^{th} parameter λ_i is obtained as $\lambda_i = \Upsilon^{-1} W_i$. To obtain the optimal parameter λ , the scalar parameter $\gamma > 0$ should be minimized subject to the LMIs (19b) and (19c) leading to the optimization problem.

5. INVERSE TEST IMPLEMENTATION

5.1 Set-Membership Parameter Estimation Approach

Parameter estimation methods allow implementing the inverse test and are require a set of inputs and measurement outputs to obtain estimated parameters. As it is well-known, state estimation algorithms (as e.g. Kalman filters) can be used for the parameter estimation by means of the following reformulation:

$$A = I, B = 0, C = c_k^T, D = 0, E = 0 \quad (24)$$

Besides, the system (1) can be expressed as another LPV system in the regressor form after using (24), as follows:

$$y_k = c_k^T \theta_k + v_k = \hat{y}_k + v_k \quad (25)$$

where c_k is the regressor vector with dimension of n_θ which contains functions of inputs u_k and outputs y_k ; v_k is measurement noise where $v_k \in Fv_{I_k}$; $\theta_k \in \Theta_k$ is the parameter vector of dimension n_θ and Θ_k is the set that bounds parameter values, which can be described by a zonotope as follows (Bravo et al., 2006):

$$\Theta_k = P_k \oplus H_k B^n = \{P_k + H_k \mathbf{z} : \mathbf{z} \in B^n\} \quad (26)$$

where $P_k \in \mathbb{R}^{n_\theta}$ represents the nominal parameter, $H_k \in \mathbb{R}^{n_\theta \times n}$ is the shape matrix, and $B^n \in \mathbb{R}^{n \times 1}$ is a unitary box.

Thus, the inverse test implementation is as follows:

$$P_+ = P + \lambda(y_+ - c^T P) \quad (27a)$$

$$H_+ = [(I - \lambda c^T) H, -\lambda F] \quad (27b)$$

where the optimal λ can be obtained by minimising $\|H\|_F^2$

$$\lambda^* = \frac{HH^T c}{c^T HH^T c + FF^T} \quad (28)$$

5.2 LMI-based SMA

Since the previous SMA for parameter estimation already works for LPV systems, LMI-based formulation mainly aims to implement the optimal parameter λ^* by constructing and solving LMIs. Based on the Theorem 1, this design consists in considering a varying shape matrix H for parameter estimation to make it less conservative and improve the estimation accuracy. Thus, the process is totally online.

Theorem 2. If there exist positive scalars γ , the optimal λ^* is computed as

$$\lambda^* = \Upsilon^{-1} W \quad (29)$$

where Υ and W are the feasible solutions of the following optimization problem:

$$\min \gamma \quad (30)$$

subject to the following LMIs:

$$\begin{bmatrix} \gamma I_{n_x} & I_{n_x} \\ I_{n_x} & \Upsilon_k \end{bmatrix} > 0 \quad (31)$$

$$\begin{bmatrix} -P^{-1} & \Upsilon I_{n_x} - WC & W \\ I_{n_x} \Upsilon - C^T W^T & -\Upsilon & 0 \\ W^T & 0 & -R^{-1} \end{bmatrix} < 0 \quad (32)$$

Proof. The proof procedure is similar to Theorem 1. Note that in this case (21) becomes

$$P = (I - \lambda C) \bar{P}_+ (I - \lambda C)^T + \lambda F F^T \lambda^T \quad (33)$$

by considering a varying P and the matrices definition (24). Then, the condition (32) could be obtained after replacing $\Upsilon = P_+^{-1}$, $P = H H^T$ from last time instant and applying Schur complement.

6. APPLICATION TO AUTONOMOUS VEHICLE

In this section, we consider the application to an autonomous vehicle to illustrate the effectiveness of the proposed method. The following state-space model from (Ifqir et al., 2020) is considered:

$$\begin{bmatrix} \dot{\beta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} -\frac{c_f + c_r}{mv_x} & \frac{c_r l_r - c_f l_f}{mv_x^2} - 1 \\ \frac{c_r l_r - c_f l_f}{I_z} & -\frac{c_r l_r^2 + c_f l_f^2}{I_z v_x} \end{bmatrix} \begin{bmatrix} \beta \\ \psi \end{bmatrix} + \begin{bmatrix} c_f \\ c_f l_f \\ I_z \end{bmatrix} \delta_f \quad (34)$$

where m, I_z , are the mass and the yaw moment, v_x is the longitudinal velocity, β and ψ are vehicle sideslip angle and yaw rate, l_f, l_r are distances from front and rear axle to the center of gravity, F_{yf} and F_{yr} are lateral tire force of front and rear tires, c_f, c_r are the cornering stiffness of front and rear tires, δ_f is front steering angle. The available measurements are yaw rate $\dot{\psi}$, longitudinal velocity v_x , sideslip angle β and front steering angle δ_f . Then, the model is first discretized using zero-order hold method with a sampling period T . Then, the nonlinear vehicle model can be equivalently transformed into a discrete-time LPV model:

$$x_{k+1} = A x_k + B u_k \quad (35)$$

with

$$x_k = \begin{bmatrix} \beta_k \\ \phi_k \end{bmatrix}, u_k = \delta_{f_k} \quad (36)$$

$$A = \begin{bmatrix} 1 - T \frac{c_f + c_r}{mv_x} & T \frac{c_r l_r - c_f l_f}{mv_x^2} - T \\ T \frac{c_r l_r - c_f l_f}{I_z} & 1 - T \frac{c_r l_r^2 + c_f l_f^2}{I_z v_x} \end{bmatrix}, B = \begin{bmatrix} T \frac{c_f}{I_z} \\ T \frac{c_f l_f}{I_z} \end{bmatrix} \quad (37)$$

where ϕ_k denotes ψ in discrete-time model. Furthermore, disturbances and non-modelled effects are added to the vehicle model through additive state disturbance and measurement noise vectors w_k and v_k , which satisfy $|w_k| \leq [0.002 \ 0.01]^T$ and $|v_k| \leq 0.03$.

6.1 State/Parameter Estimation

Case of state estimation. To get a polytopic LPV system, we select v_x as scheduling variable leading to polytopic model with two vertices considering the following

limits $v_x \in [10.6, 15.3]$. After denoting the state and input variables in (36), it yields

$$x_{k+1} = \sum_{i=1}^2 \mu_i(v_{xk}) (A_i x_k + B_i u_k) \quad (38)$$

The results of the proposed LMI-based and classical state estimation methods are shown below in Fig 1(a). It can be seen the performances of the proposed and classical method for state estimation are almost the same.

Case of parameter estimation. The LPV model for parameter estimation is expressed in regressor formation as follows:

$$\begin{aligned} y_k &= \phi_k, \theta_k = [A_{21}, A_{22}, B_2]^\top, \\ c_k^\top &= [\beta_{k-1}, r_{k-1}, \delta_{f_{k-1}}], \end{aligned} \quad (39)$$

where A_{21}, A_{22}, B_2 are the elements in (37) with the corresponding subscript. It is noted that parameter A_{22} is not constant due to the affect of velocity v_x . Therefore, we design the residual $\mathbf{r}(k) = \theta_k - \hat{\theta}_k$ to avoid the variation of velocity. In this way, only faults can affect the residual. More precisely, considering the velocity bounds, the parameter varies in $A_{22} \in [0.9544, 0.9684]$.

Based on Fig 1(b), the performances for parameter estimation of the proposed method and classical method are quite close. Furthermore, the proposed method could converge faster than the classical method. As a conclusion, the results reveal that the proposed LMI-SMA method is a good alternative to the classical method.

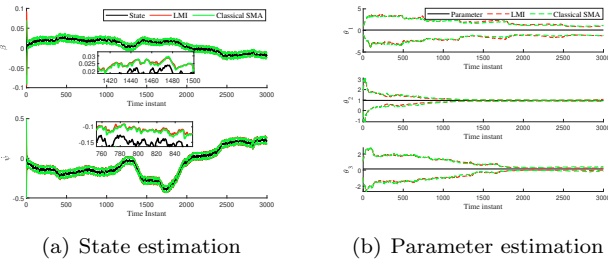


Fig. 1. State/Parameter estimation using classical SMA and LMI-based SMA

6.2 Consistency Test

This part tends to compare the performance of direct test and inverse test for fault detection based on LMI-based estimation method considering the LPV vehicle model. For this purpose, the faults are firstly classified into additive and multiplicative faults depending on whether they are independent of the observable variables, see Patton et al. (2013). Additive faults occur outside of the system, can be divided into actuator fault f_a , input sensor fault f_{in} , and output sensor fault f_{out} . Multiplicative faults affect the parameters of the plant, and we introduce this type of faults on f_{cf} and f_{I_z} to the vehicle system.

For direct test, if the measured output is outside of the estimated output bound, it indicates a fault, see Fig. 2. It is worth remarking that the fault can be detected in inverse test if there is no intersection between the support hyperplane of the zonotope and the strip, see Fig. 5(a).

Moreover, the position relationship between the zonotope and strip in figure is not as the criterion of detection, only for reference, since they are shown in 2D coordinates.

On the one hand, we can see that direct test is effective for all additive faults while partially effective for multiplicative faults, see Fig. 4(a). Even if the direct test could indicate the f_{I_z} , the indicated time instant $k = 1750$ is not consistent with the instant $k = 1601$ when fault occurs. On the other hand, it is can be concluded that inverse test is valid for all multiplicative faults, while partially valid for additive faults, see Fig. 3(b) and Fig. 3(c). It is noted that the inverse test cannot detect this f_{in} due to the existence of intersection, although they are separated in (θ_1, θ_2) space. Overall, direct test is more appropriate for detecting additive faults, while inverse test achieves better performance on the detection of multiplicative faults.

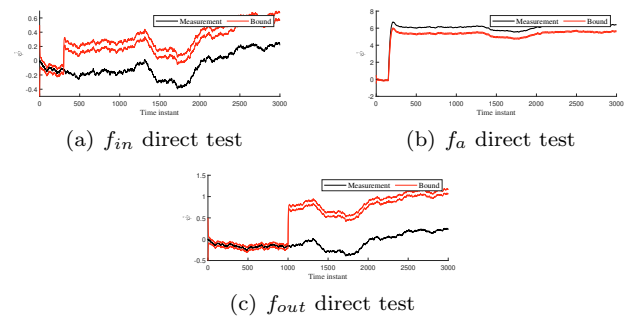


Fig. 2. Additive faults detection by direct test

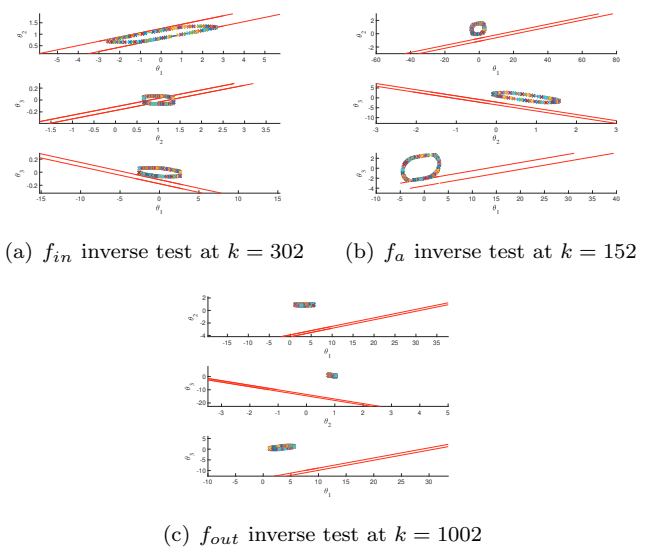


Fig. 3. Additive faults detection by inverse test

7. CONCLUSION

This paper has presented the LMI-based state/parameter estimation methods for LPV systems and applied for fault detection. Firstly, a polytopic LPV system is considered to describe nonlinear properties. Then, regarding fault detection in state space and parameter space, direct and inverse tests are implemented by LMI-based methods, respectively. Moreover, the LMI-based SMA for state estimation is a good alternative to the classical SMA. While the

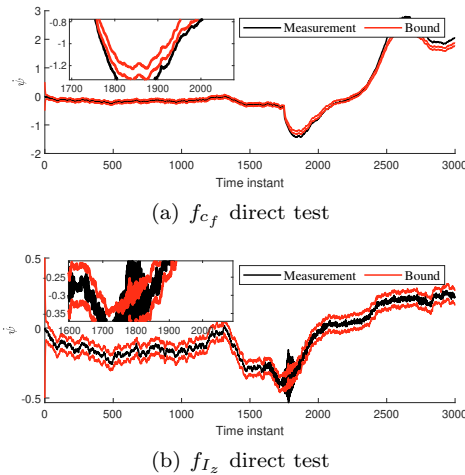
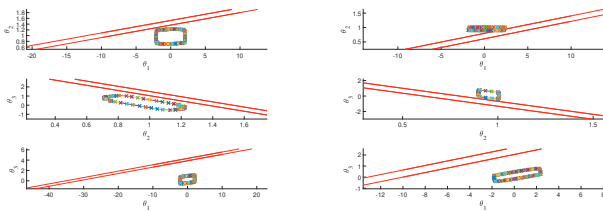


Fig. 4. Multiplicative faults detection by direct test



(a) f_{c_f} inverse test at $k = 1752$ (b) f_{I_z} inverse test at $k = 1602$

Fig. 5. Multiplicative faults detection by inverse test

LMI-based SMA for parameter estimation has kind of low efficiency but high response with quick convergence. Based on the case study of autonomous vehicles, we could arrive the same conclusion as classical state/parameter estimation methods, namely, direct test has better performance on additive faults, inverse test has better performance on multiplicative faults.

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