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Reconfiguration of flow-based networks with back-up components using robust economic MPC[☆]

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ABSTRACT

This paper addresses the post-fault selection of an actuators configuration for flow-based networks with back-up components. The proposed reconfiguration methodology consists of an offline and an online phase. On the one hand, an offline analysis looks for the minimal configurations for which the economic cost of the (best) steady-state trajectory that can be achieved using a robust model predictive control (MPC) policy is admissible. On the other hand, at fault detection time, an online search for the best actuators configuration to cope with the transient induced by the fault is conducted in the superset of each minimal configuration calculated offline. With this strategy, the final new configuration is computed by sequentially solving a set of mixed-integer programs whose constraints are derived from single-layer robust MPC schemes coupled with local controllers designed for the *a priori* minimal configurations identified offline. A portion of a water transport network is used to show the performance the proposed solution.

1. Introduction

Generalized flow-based networks (FNs) model many safety–critical infrastructures such as water distribution networks, power distribution networks, etc [1]. Accordingly, it is of paramount importance the implementation of secure control techniques for this type of systems from both: the design phase, with the installation of redundant physical components; and the operational phase, devising fault-tolerant control (FTC) algorithms that take full advantage of system redundancy to maintain an admissible performance after a fault. Notably, the detection of a component fault in FNs is often followed by the isolation of the affected area to prevent the spread of potentially damaging effects, as well as to repair the faulty elements. In this scenario, response procedures typically consist of a set of heuristic rules where different interventions comprising the activation of back-up elements are carried out, e.g. opening normally-closed valves to distribute the flow through secondary pipes or activating auxiliary pumps to feed certain tanks/storage elements. Consequently, the main aim of this paper is to propose an automated procedure for selecting the appropriate back-up actuators after a component fault (thus selecting a new system configuration) for constrained FNs subject to uncertain periodic disturbances such as exogenous flow demands and periodically varying power prices.

In nominal operation, a large number of solutions use model predictive control (MPC) schemes to control FNs due to their ability to efficiently control complex processes [2,3]. In particular, economic MPC strategies [4] have been presented as a convenient approach to regulate the high economic costs associated with the operation of large-scale FNs [5,6]. Two main economic MPC architectures have been proposed: (I) double-layer schemes composed of an upper layer dynamic real-time optimizer that plans the optimal system steady-state trajectory and a low-level predictive controller that tracks the previous Ref. [7]; (II) single-layer schemes where the economic cost function is included in the computation of the control law, thus allowing to assess the economic cost during the transients [8]. In addition, the periodic nature of the disturbances that normally affected FNs causes that, in some cases, the best way to operate the network is the imposition of a cyclic steady-state operation [9,10].

Concerning the analysis of the post-fault *configuration selection* problem within the automatic control field, the study of the FTC capabilities granted by different configurations of actuators/sensors was mainly developed by Staroswiecki in the context of unconstrained systems [11–13]. Nevertheless, the consideration in this case of constrained systems precludes a direct application of the above techniques, since structural and performance methods must be extended by considering feasibility issues. On the other hand, the system reconfiguration with back-up components has been investigated through the three-tank

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Algorithm 1 Search of minimal configurations over a BUM property.

Input: $\mathcal{I}, \mathcal{P}$
Output: $\mathcal{M}$

$\mathcal{M} \leftarrow \emptyset$ ▷ Initialize the empty set
if $\neg\mathcal{P}(\mathcal{I}(1))$ **then**
  Stop ▷ No solution
end if
$C_x \leftarrow \mathcal{I}(1)$
while $\mathcal{I} \neq \emptyset$ **do**
  $C_y \leftarrow$ `next_strict_successor`$(\mathcal{I}, C_x)$
  if $C_y = \emptyset$ **then**
    $\mathcal{M} \leftarrow \{\mathcal{M}, C_x\}$ ▷ C_x is minimal
    Remove $\mathbb{P}(C_x)$ from $\mathcal{I}$
    $C_x \leftarrow \mathcal{I}(1)$
  else
    if $\mathcal{P}(C_y)$ **then**
      $C_x \leftarrow C_y$
      Remove $\mathbb{P}(C_y)$ from $\mathcal{I}$
    else
      Remove $\mathbb{S}(C_y)$ from $\mathcal{I}$
    end if
  end if
end while

function `next_strict_successor`$(\mathcal{I}, C_x)$
  $i \leftarrow 1$
  while all(is_member($\mathcal{I}(i)$, C_x)) = False **do**
    $i \leftarrow i + 1$
    if $i > |\mathcal{I}|$ **then**
      error('No C_x in $\mathcal{I}$')
    end if
  end while
  if $i = |\mathcal{I}|$ **then**
    $C_y \leftarrow \emptyset$ ▷ C_x is the last conf.
  else
    $j \leftarrow i + 1$
    while all(is_member($\mathcal{I}(j)$, $\mathcal{I}(i)$)) = False **do**
      $j \leftarrow j + 1$
      if $j > |\mathcal{I}|$ **then**
        $C_y \leftarrow \emptyset$
        Break all ▷ Breaks all the loops
      end if
    end while
    $C_y \leftarrow \mathcal{I}(j)$
  end if
  return C_y
end function

- *Offline*: the set of minimal configurations that yield an admissible steady-state cost (i.e. such that $\mathcal{A}^{pf}(C_l^m)$), are identified offline by means of Algorithm 1. A local controller $K^{[l]}$ is computed for $C_l^m \in \mathcal{M}^{[f]}$.
- *Online*: given a fault in C_{out}, the search for the optimal admissible configuration (6) that allows to handle the transient induced by the fault is conducted online in the remaining predecessors of each minimal configuration identified offline, that is, in $\mathbb{P}(C_l^m|C_{out})$.

6.1. Online MIP optimization

Given a fault in C_{out}, the online configuration search in the remaining predecessors of a minimal configuration can be posed as an MIP that selects the optimal configuration that satisfies the constraints of a single-layer robust MPC (14). To that end, let $\mathcal{X}_m^{[l]}(j), \mathcal{U}_m^{[l]}(j)$ represent the tightened set of state and input constraints for the minimal configuration C_l^m computed according to (15). Moreover, let $\delta_m^{[l]}$ denote a binary vector with: the elements in C_l^m activated (and thus not considered as variables in the optimization problem), and the elements in $(C_0 \setminus C_{out}) \setminus C_l^m$ as binary optimization variables. Consistently, the selection of a new configuration $C_{new}^{(l)}$ in the set the set $\mathbb{P}(C_l^m|C_{out})$ can be posed as the following MIP

$$C_{new}^{(l)} = \underset{\delta_m^{[i]}, \boldsymbol{u}_N, x_0^v, \boldsymbol{u}_N^v}{\arg\min} \; J(C_i),$$

$$\begin{aligned}
&\text{s.t.}\quad x(0) = x, && (18a)\\
&x(j+1) = Ax(j) + Bz(j) + B_d d(k+j), && (18b)\\
&0 = Ez(j) + E_d d(k+j), \qquad j \in \mathbb{I}_{N-1}, && (18c)\\
&x(N) = x^v(N), && (18d)\\
&x^v(j+1) = Ax^v(j) + Bz^v(j) + B_d d(k+j), && (18e)\\
&0 = Ez^v(j) + E_d d(k+j), \qquad j \in \mathbb{I}_{N-1}, && (18f)\\
&x^v(T) = x^v(0) = x_0^v, && (18g)\\
&x(j) \in \mathcal{X}_m^{[l]}(j), \qquad x^v(j) \in \mathcal{X}_m^{[l]}(N), \qquad j \in \mathbb{I}_N, && (18h)\\
&u(j) \in \mathcal{U}_m^{[l]}(j), \qquad u^v(i) \in \mathcal{U}_m^{[l]}(N), \qquad j \in \mathbb{I}_{N-1}, && (18i)\\
&z(j) = diag(\delta_m^{[l]})u(j), && (18j)\\
&z^v(j) = diag(\delta_m^{[l]})u^v(j), \qquad j \in \mathbb{I}_{N-1}, && (18k)
\end{aligned}$$

where the product of continuous and logic variables in (18j)–(18k) can be transformed into equivalent linear integer inequalities [38].

Note that, by means of the single-layer MPC scheme, the integer program (18) updates coherently the model used for control and the model used for the virtual planner. This ensures the generation of a reachable trajectory for the new configuration (retrieved from the binary vector $\delta_m^{[l]}$), since the terminal ingredient used for stability (18d) is also modified coherently with the virtual planner. Consequently, the system in configuration $C_{new}^{(l)}$ will satisfy the stability admissibility $\mathcal{A}^{st}(C_{new}^{(l)}|k_d)$, whereas, on the other hand, since $C_{new}^{(l)} \in \mathbb{P}(C_l^m|C_{out})$, from Lemma 1 it follows that $\mathcal{A}^{pf}(C_{new}^{(l)})$, and thus $C_{new}^{(l)}$ is admissible.

Remark 5. The satisfaction of $\mathcal{A}^{pf}(C_i)$ for a given C_i, requires the tightened sets of constraints $\mathcal{X}^{[i]}(N)$ and $\mathcal{U}^{[i]}(N)$ to be non empty. In the case that this is not satisfied, then the offline search for minimal configurations that satisfy $\mathcal{A}^{pf}(\cdot)$ will filter out C_i as non admissible. Besides, from (15), it follows that $\mathcal{X}^{[i]}(N) \subseteq \mathcal{X}^{[i]}(j)$, $\forall j \in \mathbb{I}_{N-1}$ (similarly for $\mathcal{U}^{[i]}(j)$), and thus, if $\mathcal{A}^{pf(\cdot)}$ is satisfied, then the tightened set of constraints are guaranteed to be non empty for all $j \in \mathbb{I}_N$.

Remark 6. The proposed methodology implicitly assumes that the whole system reconfiguration takes less than one sampling time, and thus delays are negligible. If that is not the case, the online optimization (18) should be adapted to deal with the (previously modeled) input-delays caused by the reconfiguration as proposed in [39–41].

6.2. Decision between sets

Section 6.1 formulates an MIP for obtaining (if it exists) a new configuration $C_{new}^{(l)}$ for each one of the sets $C_l^m \in \mathcal{M}^{[f]}$. Below, a

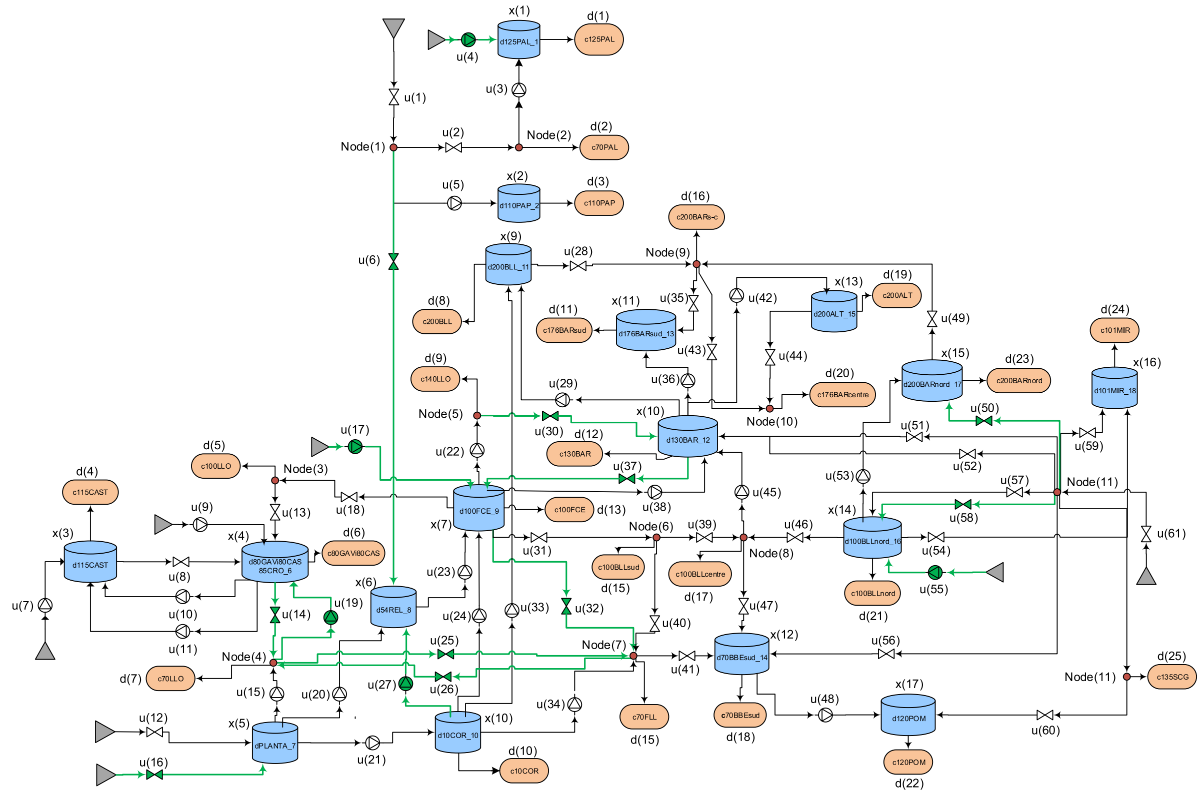


Fig. 2. DWTN description. Nominal actuators (black); back-up actuators (green).

Algorithm 2 Sequential configuration search

Input: $P_l, \;\; l \in \{1, \ldots, n_l\}$
Output: C_{new}
$J^* \leftarrow 10^{10}$ ▷ Initialize large value
$C_{new} \leftarrow \emptyset$ ▷ Initialize empty configuration
for $l = 1$ to n_l **do**
  $\tilde{\mathrm{P}}_l \leftarrow \min\{J(\psi_l) \;:\; \psi_l \in \Psi_l \wedge J(\psi_l) \leq J^*\}$
  if is_feasible($\tilde{\mathrm{P}}_l$) **then**
    $J^* \leftarrow$ solution($\tilde{\mathrm{P}}_l$)
    $C_{new} \leftarrow$ argmin($\tilde{\mathrm{P}}_l$)
  end if
end for
if is_empty(C_{new}) **then**
  No solution
end if

sequential method is followed in order to decide how to conduct the search among the different sets and select a final C_{new}. In this sequential approach, the information retrieved from solving the configuration selection problem in one set is used for limiting the search space in the remaining sets.

For simplicity, let us characterize each of the n_l optimizations (18) as the optimization problem

$$(\mathrm{P}_l) \quad \min\{J(\psi_l) \;:\; \psi_l \in \Psi_l\}, \tag{19}$$

where the vector ψ_l encompasses the different decision variables and Ψ_l is the feasibility set obtained for C_l^m. Hence, using the notation in (19), Algorithm 2 reflects the steps followed to compute the final configuration C_{new}. Note that, any intermediate solution C_{new} found by Algorithm 2 is an admissible solution to Eq. (6). This allows, if necessary, to interrupt the search if some time restrictions must be met.

7. Case study

The proposed case study is based on the aggregated version of the drinking water transport network (DWTN) of the city of Barcelona. This DWTN consists of: 9 water sources, 17 water tanks, 61 actuators (37 valves and 24 pumps), 12 nodes and 25 demands (cf. Fig. 2 for a schematic representation of the network). The details of the system can be found in the technical report [42], including network equations as well as tanks and actuators limits. Besides, the predicted water demand used in the simulations has been obtained from historical water consumption data, and it can be found in the supplementary material of [43]. The robustness of the network model has been enhanced by considering uncertainties in the water demand predictions. In this regard, similarly to [43], the prediction error is bounded in the set

$$\mathcal{W} = \{w(k) \in \mathbb{R}^{25} \;:\; |w(k)| \leq \bar{w}\},$$

where the maximum prediction error $\bar{w} \in \mathbb{R}^{25}$ is set as the 5% of the maximum expected demand during the tests, i.e. $\bar{w}_i = 0.05 \max_k d_i(k)$. In the simulations presented below, the values of $w(k)$ have been randomly generated following a uniform distribution bounded within $\mathcal{W}$.

Following the problem statement presented in Section 2, the network actuators have been randomly partitioned into nominal C_n and back-up components C_a, with $|C_n| = 46$ and $|C_a| = 15$ (see Table 1). Note that this artificial division has been carried out with illustrative purposes, however, in real-world applications this partition is specific of each system design. The considered division is illustrated in Fig. 2, where the back-up elements appear highlighted in green.

7.1. Management criteria

The stage cost used to asses the FN performance (Section 2.3) operating in configuration C_i takes into account the following criteria [44,45]:

Table 1
Elements partition.

	Pumps (a_i)	Valves (a_i)
Nominal ($i =$)	3, 5, 9, 10, 11, 15, 20, 21, 22, 23, 24, 29, 33, 34, 36, 38, 42, 48, 53	1, 2, 7, 8, 12, 13, 18, 28, 31, 32, 35, 39, 40, 41, 43, 44, 45, 46, 47, 49, 51, 52, 54, 56, 57, 59, 60, 61
Back-up ($i =$)	4, 17, 19, 27, 55	6, 14, 16, 25, 26, 30, 32, 37, 50, 58

1. **Minimizing water production and transport costs:** This term accounts for the economic costs associated with the drinking water production (water treatment) and transporting (pumping). The performance index to be minimized is described by

$$f_1(C_i, k) = (\alpha_1^T + \alpha_2^T(k))\Sigma^{[i]}u(k),$$

where α_1 accounts for the fixed economic cost of the water according to its source (treatment plant, dwell, etc.) and $\alpha_2(k)$ is associated with the economic cost of pumping the water. In the simulations, vector $\alpha_2(k)$ presents a $T = 24$ h cyclic pattern that relates with the daily variations in the electricity rate.

2. **Safety storage term:** The satisfaction of water demands has been imposed as a hard constraint in the network model that should be fulfilled at every time instant. As a consequence, the stored water volume is preferably maintained around a given safety value as a risk prevention mechanism. This concept is formulated as

$$f_2(k) = (x(k) - x_{sf})^T W_x(x(k) - x_{sf}),$$

where $x(k) \in \mathbb{R}^{17}$ denotes the water volume in the tanks and $x_{sf} \in \mathbb{R}^{17}$ denotes the safety storage volume. Particularly, the safety volume has been designed as $x_{sf} = 0.75(\bar{x} - \underline{x})$, where $\bar{x}$ and $\underline{x}$ represent the maximum and minimum accepted tank volumes, respectively. Moreover, the weighting matrix is set to $W_x = diag(1/(\bar{x} - \underline{x}))$, in order to penalize the deviation from the safety volume proportionally to the size of each one of the tanks.

3. **Smoothness of the control actions:** The variations of the control signal between consecutive sampling intervals is also penalized. By denoting $\Delta u(k) = u(k) - u(k-1)$, this objective is formulated as

$$f_3(C_i, k) = \Delta u(k)^T \Sigma^{[i]} W_u \Sigma^{[i]} \Delta u(k),$$

where the weighting matrix is designed as $W_u = I_{61}$.

Accordingly, the stage cost is made up of a weighted sum of the previous terms

$$l(C_i, k, x(k), u(k)) = \lambda_1 f_1(C_i, k) + \lambda_2 f_2(k) + \lambda_3 f_3(C_i, k),$$

with $\lambda_1 = 1$, $\lambda_2 = 0.05$ and $\lambda_3 = 0.01$.

7.2. Configuration selection criteria

In the sequel, the selection of a new configuration C_{new} after a fault in the components in C_{out} (see Section 3) is performed by minimizing the following multi-objective criteria $J = [J_1,\ J_2]$, where

1. *Objective 1 (J_1):* minimize number of back-up actuators activated after the fault.
2. *Objective 2 (J_2):* minimize the expected performance-loss during the transient induced by the fault.

The multi-objective optimization is addressed considering a lexicographic ordering among the previous objectives, i.e. the optimization of J_1 is infinitely more important than the optimization of J_2.

7.3. Robust MPC tuning parameters

Here, the computation of the tuning parameters required for computing the single-layer MPC control law in Section 4.3 for an arbitrary configuration C_i, as well as for the pair $(M_w^{[i]}, \mathcal{L}_T^{[i]})$ retrieved from solving the robust planner in Section 4.4, are detailed.

The MPC time horizon set is to $N = T = 24$h. For an arbitrary configuration C_i, matrix $M_v^{[i]}$ is designed as an orthonormal basis to the null space $E\Sigma^{[i]}$. In addition, the controller gain $K^{[i]}$ is computed using an LQR design for matrices $(A, \hat{B}^{[i]}, Q, R^{[i]})$, with

$$\hat{B}^{[i]} = B\Sigma^{[i]}M_v^{[i]}, \qquad Q = I_{17},$$
$$R^{[i]} = M_v^{[i]T} diag(1/u_{max})M_v^{[i]},$$

where u_{max} denotes the maximum flow handling capacity of the actuators. Matrix $R^{[i]}$ has been selected to handle the big differences in the network actuator limits: a_{50} has a maximum value of $15\text{m}^3/\text{s}$, whereas the maximum value of a_7 is of $10^{-5}\text{m}^3/\text{s}$.

7.4. Admissibility criterion

The solution of the robust planner problem in Section 4.4 for the nominal configuration C_n yields the average stage cost $\mathcal{L}_T^{[n]} = 2.2518 \cdot 10^3$. Below, the parameter β that rules the satisfaction of the performance admissibility condition (cf. Section 2.4) is set to $\beta = 1.25$.

7.5. Search for minimal configurations

An offline analysis of the minimal configurations C_l^m that satisfy the performance property after a fault in any of the nominal actuators has been performed. In particular, Algorithm 1 presented in Section 5.1 is used in order to look for the minimal configurations that satisfy $\mathcal{A}^{pf}(C_l^m)$.

The offline tests yielded the following results[1]:

- A fault in $C_{out} = \{a_i\}$ for $i \in \{1, 2, 5, 12, 13, 15, 18, 21, 22, 23, 28, 29, 31, 35, 36, 40, 41, 42, 44, 49, 54, 56, 59, 61\}$ have been identified as critical faults, that is, either there are no combinations of back-up elements for which (17) generates a feasible trajectory, or the attained trajectory has associated cost lower than the threshold. The total time for the offline identification of the above set of critical faults is $t_{crt} = 578.64$ s.
- For a fault in $C_{out} = \{a_i\}$ with $i \in \{7, 8, 9, 10, 11, 33, 38, 39, 43, 45, 46, 48, 52, 60\}$, the solution of (17) is able to generate an admissible trajectory without the need for back-up actuators, i.e. $C_l^m = C_f$. The total time required for the offline identification of the above set of admissible faults is $t_{adm} = 184.94$ s.
- For a fault in $C_{out} = \{a_i\}$ with $i \in \{3, 20, 24, 34, 47, 51, 53, 57\}$, Algorithm 1 must perform a non trivial search for minimal configurations. In this regard, Table 2 shows the number of minimal configurations found, as well as the number (and percentage) of configurations explored by Algorithm 1 out of the possible $2^{15} - 1 = 32.767$ candidate configurations. The total time for the offline search of minimal configurations for the above set of faults is $t_{rec} = 1.332 \cdot 10^4$ s.

The total time for the offline analysis of all possible faults C_{out} in the nominal components is $t_{off} = 1.4086 \cdot 10^4$ s ≈ 3 h 55 min.

[1] Laptop (Intel i7 1.8 GHz, 16 GB RAM) running Windows 10; optimizations using Yalmip parser and Cplex solver.

Table 2
Minimal configurations.

C_{out}	$\|\mathcal{M}^{[f]}\|$	Explored confs.	Percentage
a_3	1	16	0.049
a_{20}	17	200	0.610
a_{24}	11	144	0.439
a_{34}	1	17	0.052
a_{47}	2	34	0.104
a_{51}	1	17	0.052
a_{53}	1	17	0.052
a_{57}	1	16	0.049

Table 3
Set of minimal configurations — $C_{out} = \{a_{24}\}$; $\beta = 1.25$.

l	C_l^m
1	$C_f \cup \{a_6, a_{27}\}$
2	$C_f \cup \{a_{17}, a_{27}\}$
3	$C_f \cup \{a_{27}, a_{37}\}$
4	$C_f \cup \{a_{27}, a_{50}\}$
5	$C_f \cup \{a_{27}, a_{55}\}$
6	$C_f \cup \{a_{27}, a_{58}\}$
7	$C_f \cup \{a_6, a_{17}, a_{50}\}$
8	$C_f \cup \{a_{14}, a_{25}, a_{27}\}$
9	$C_f \cup \{a_{25}, a_{26}, a_{27}\}$
10	$C_f \cup \{a_6, a_{14}, a_{17}, a_{25}\}$
11	$C_f \cup \{a_6, a_{17}, a_{25}, a_{26}\}$

7.6. Fault scenario — fault in actuator 24

Here, a fault in actuator $C_{out} = \{a_{24}\}$ is simulated. For this scenario, the set of minimal configurations $\mathcal{M}^{[f]}$ is shown in Table 3. Notice that in Table 3 the configurations have been sorted by taking into account its cardinality. The fault scenario simulated is the following: a fault in actuator 24 appears at $k_f = 39$ h. This fault causes a performance loss of 25% of the actuator capabilities. Moreover, it is assumed that an FDI block detects the fault at $k_d = 43$ h and that the actuator 24 is turned-off.

For the above scenario, Algorithm 2 in Section 6.2 is used for the online computation of the new configuration C_{new}. In this case, the sequential addition of constraints in Algorithm 2 is only imposed for the first optimization objective J_1. The algorithm behaves as follows:

- **Iteration $i = 1$:** the first optimization is launched for the set $\mathbb{P}(C_1^m|a_{24})$, yielding the solution:

  $$\tilde{\mathrm{P}}_1: \ C_{new} = C_1^m, \ (J_1^* = 2 + |C_f|, \ J_2^* = 2.2641 \cdot 10^3).$$

  Hence, further optimizations are subject to the constraint $J_1 \le 2 + |C_f|$. Notice that, because $|C_l^m| > 2 + |C_f|$ for $l > 6$ (cf. Table 3), these optimizations are guaranteed to be infeasible, and thus the search must only continue in the sets $\mathbb{P}(C_l^m|a_{24})$, $l \in \{2, \dots, 6\}$.
- **Iterations $i \in \{2, \dots, 6\}$:** The results obtained in the successive optimizations are

  $$\tilde{\mathrm{P}}_2: \ C_{new} = C_2^m \quad (J_1^* = 2 + |C_f|, \ J_2^* = 2.2528 \cdot 10^3),$$
  $$\tilde{\mathrm{P}}_3: \ C_{new} = C_2^m \quad (J_1^* = 2 + |C_f|, \ J_2^* = 2.2953 \cdot 10^3),$$
  $$\tilde{\mathrm{P}}_4: \ C_{new} = C_4^m \quad (J_1^* = 2 + |C_f|, \ J_2^* = 2.2419 \cdot 10^3),$$
  $$\tilde{\mathrm{P}}_5: \ C_{new} = C_4^m \quad (J_1^* = 2 + |C_f|, \ J_2^* = 2.2658 \cdot 10^3),$$
  $$\tilde{\mathrm{P}}_6: \ C_{new} = C_4^m \quad (J_1^* = 2 + |C_f|, \ J_2^* = 2.2589 \cdot 10^3).$$

Accordingly, $C_{new} = C_4^m$, since, for the same number of back-up actuators ($J_1^* = 2$), generates the best expected average cost during the transient. Previous optimizations are run online in a total time $t_{on} = t_{J_1} + t_{J_2} = 4.812$ s (well below the 1 h sampling time), where the total times for the computation of J_1 and J_2 are $t_{J_1} = 1.912$ s; $t_{J_2} = 2.90$ s.

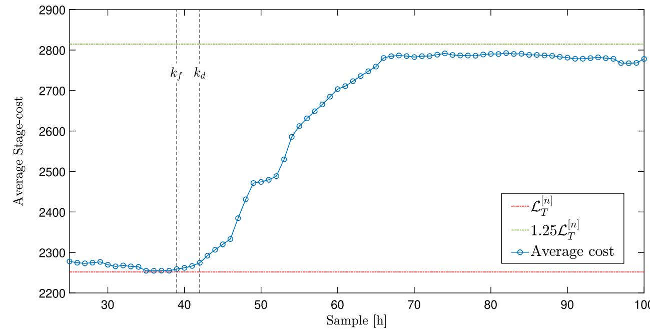


Fig. 3. Average stage cost — Fault in actuator 24.

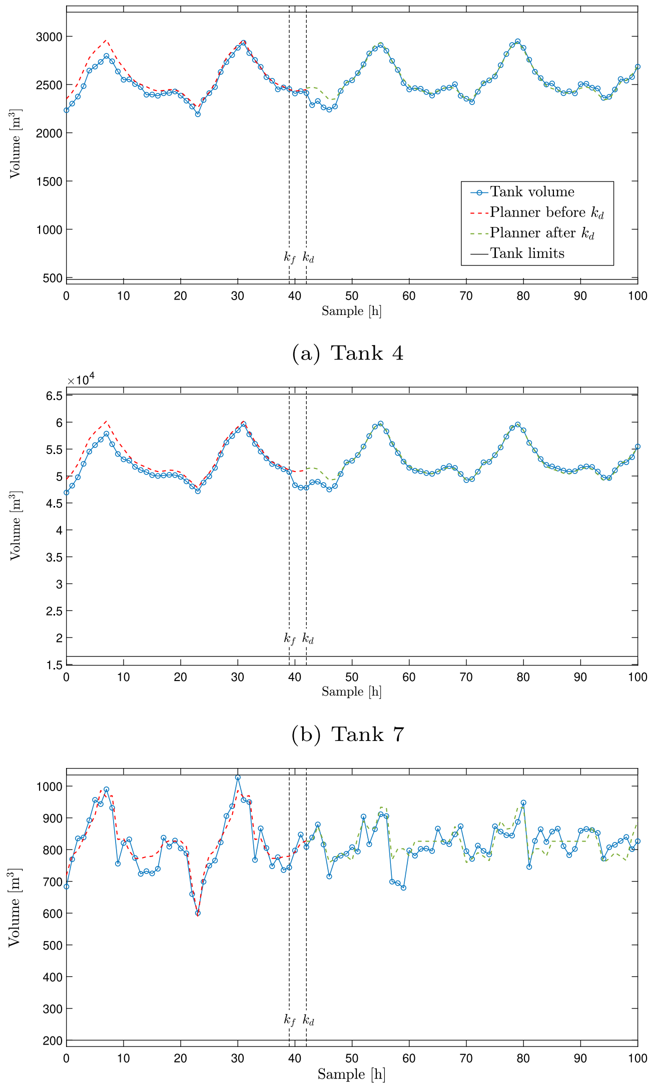
Fig. 3 depicts the evolution of the average stage cost during the fault scenario. This cost has been computed at time k by averaging the cost obtained for the time interval $[k-T+1, k]$. In this figure, it can be seen how, firstly, the cost stabilizes at $\mathcal{L}_T^{[n]}$ and how, after the transient induced by the fault plus reconfiguration, the stage cost stabilizes below the admissibility threshold $\beta\mathcal{L}_T^{[n]}$ ($\beta = 1.25$). It must be remarked that there may be some discrepancies between the cost values obtained by the real system trajectory and the values generated by the robust planner, since the planner is computed for the non-disturbed system.

Besides, Figs. 4(a) to 4(c) show the time evolution of the volume of tanks 4, 7 and 11 during the previous fault scenario, respectively. In these figures, it can be appreciated how: before the fault, the system stabilizes over the nominal configuration reference (red dashed line); and after the fault detection, over the new planner trajectory corresponding with the new configuration C_{new} (green dashed line). Additionally, Fig. 5 displays the evolution of several actuators of the network. In particular, Fig. 5(a) shows how pump 23 increases its power with the new configuration, yielding a worse economic performance. Besides, in Fig. 5(b) it can be seen how the back-up actuator 27 is turned-on after the detection time, whereas Fig. 5(c) shows the modification in the periodic operation of valve 51 before and after the configuration change.

8. Conclusions

This paper presents a methodology for the robust system reconfiguration with back-up components problem in FNs, which combines an offline analysis on the configurations that yield and admissible steady-state operation with an online search for the optimal configuration required to cope with the transient induced by the fault. One of the main difficulties relates with the tuning of the parameters that constitute the control scheme, since small modifications can significantly affect the existence of an admissible control law for a given system configuration. In this regard, the proposed solution seeks for a good trade-off between the optimality in the new configuration selection and the computational complexity of the approach.

Finally, the next natural step in the development of these reconfiguration techniques would be to consider possible non-linearities in the system model. Additionally, partitioning approaches can be used to split the network into physically redundant areas, so the search for hardware-redundant components can be conducted in each area independently.

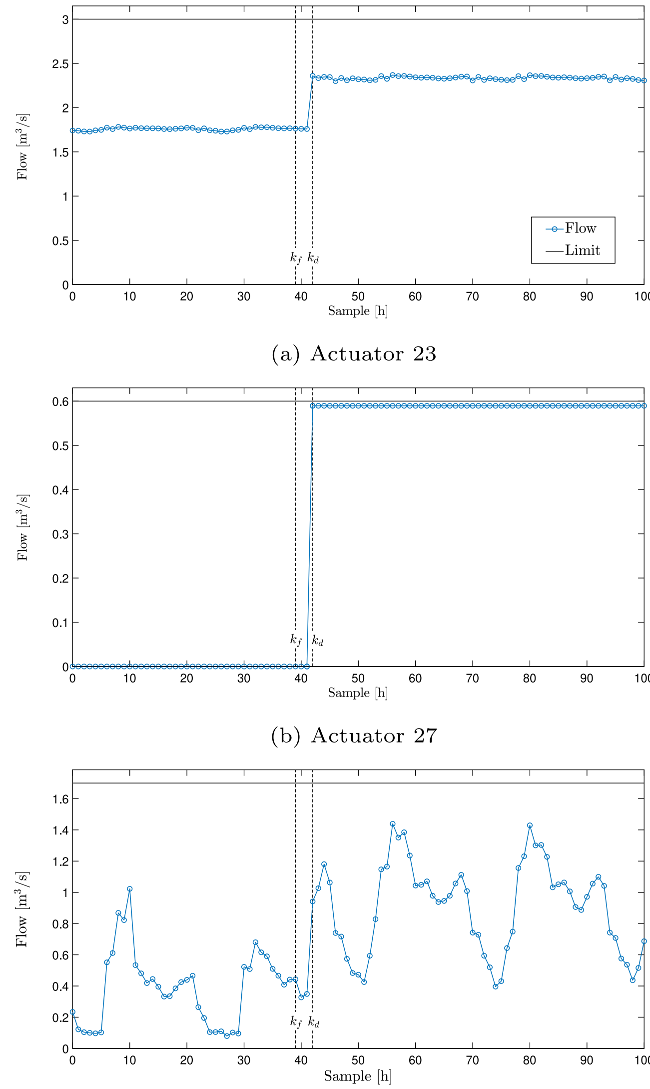


(a) Tank 4

(b) Tank 7

(c) Tank 11

Fig. 4. Tank volumes evolution — Fault in actuator 24.



(a) Actuator 23

(b) Actuator 27

(c) Actuator 51

Fig. 5. Actuators evolution — Fault in actuator 24.

CRediT authorship contribution statement

Carlos Trapiello: Conceptualization, Methodology, Software, Validation, Formal analysis, Writing – original draft, Writing – review & editing. **Vicenç Puig:** Conceptualization, Methodology, Software, Validation, Formal analysis, Writing – original draft, Writing – review & editing. **Gabriela Cembrano:** Conceptualization, Methodology, Software, Validation, Formal analysis, Writing – original draft, Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

The data that has been used is confidential.

Appendix. Proof of Corollary 1

Lemma 2 (*P-difference [46]*). *Given the zonotope $\mathcal{Z} = \langle c, H\rangle \subset \mathbb{R}^n$, with $c \in \mathbb{R}^n$ and $H \in \mathbb{R}^{n\times z}$, and the polyhedron $\mathcal{S} = \{x \in \mathbb{R}^n : Lx \leq l\} \subseteq \mathbb{R}^n$, with $l \in \mathbb{R}^m$ and $L \in \mathbb{R}^{m\times n}$, then $\mathcal{S} \ominus \mathcal{Z} = \{x \in \mathbb{R}^n : Lx \leq l - Lc - |LH|\mathbf{1}_z\}$.*

Proof of Corollary 1. Starting from the robust planner (7) for the configuration C_i. Matrix $M_w^{[i]}$ affects the sets

$$\mathcal{X}^{[i]}(N) = \mathcal{X} \ominus \mathcal{R}^{[i]}(N), \qquad \mathcal{U}^{[i]}(N) = \mathcal{U} \ominus \mathcal{R}_u^{[i]}(N), \tag{A.1}$$

with $\mathcal{R}^{[i]}(N)$ and $\mathcal{R}_u^{[i]}(N)$ the Nth iteration of

$$\mathcal{R}^{[i]}(j) = \bigoplus_{0}^{j-1} \mathcal{Q}^{[i]}(l), \quad \mathcal{Q}^{[i]}(l) = (A + \hat{B}^{[i]}K^{[i]})^l \hat{B}_w^{[i]}\mathcal{W},$$
$$\mathcal{R}_u^{[i]}(j) = M_w^{[i]}\mathcal{W} \oplus M_v^{[i]}K^{[i]}\mathcal{R}^{[i]}(j).$$

By recalling that $\mathcal{W}$ is a unitary zonotope and that $\hat{B}_w^{[i]} = B_w + B\Sigma^{[i]}M_w^{[i]}$, it follows that $\mathcal{R}^{[i]}(j)$ and $\mathcal{R}_u^{[i]}(j)$ are also zonotopic sets which can be rewritten as

$$\begin{aligned} \mathcal{R}^{[i]}(j) &= \langle 0, G_a(j) + G_b(j)(I_j \otimes M_w^{[i]})\rangle, \\ \mathcal{R}_u^{[i]}(j) &= \langle 0, [M_w^{[i]},\ \ M_v^{[i]}K^{[i]}\big(G_a(j) + G_b(j)(I_j \otimes M_w^{[i]})\big)]\rangle, \end{aligned} \tag{A.2}$$

with $G_a(j)$ and $G_b(j)$ the jth elements of the recursion

$$G_a(j+1) = [(A + \hat{B}^{[i]}K^{[i]})G_a(j),\ B_w], \qquad G_a(0) = 0,$$

$$G_b(j+1) = [(A + \hat{B}^{[i]}K^{[i]})G_b(j),\ B], \qquad G_b(0) = 0.$$

Therefore, from $\mathcal{X}$ and $\mathcal{U}$ in (2) and Lemma 2, the sets in (A.1) are rewritten as

$$\begin{aligned} \mathcal{X}^{[i]}(N) &= \{x(k) : H_x x(k) \le h_x^{[i]}\}, \\ \mathcal{U}^{[i]}(N) &= \{u(k) : H_u u(k) \le h_u^{[i]}\}, \end{aligned} \tag{A.3}$$

where

$$\begin{aligned} h_x^{[i]} &= h_x - |H_x G_a(N)|\mathbf{1}_{Nn_w} \\ &\quad - |H_x G_b(N)(I_N \otimes M_w^{[i]})|\mathbf{1}_{Nn_w}, \\ h_u^{[i]} &= h_u - |H_u M_v^{[i]}K^{[i]}G_a(N)|\mathbf{1}_{Nn_w} \\ &\quad - |H_u(\tilde{I} + M_v^{[i]}K^{[i]}G_b(N))(I_N \otimes M_w^{[i]})|\mathbf{1}_{Nn_w}, \end{aligned} \tag{A.4}$$

and $\tilde{I} = [I_{n_u}\ 0_{n_u \times n_u(N-1)}]$.

In addition, (A.4) can be reformulated as linear constraints in $M_w^{[i]}$ by bounding the absolute value of the matrices from above through the introduction of the variable matrices Ω_x and Ω_u as

$$\begin{aligned} h_x^{[i]} &= h_x - |H_x G_a(N)|\mathbf{1}_{Nn_w} - \Omega_x \mathbf{1}_{Nn_w}, \\ h_u^{[i]} &= h_u - |H_u M_v^{[i]}K^{[i]}G_a(N)|\mathbf{1}_{Nn_w} - \Omega_u \mathbf{1}_{Nn_w}, \end{aligned} \tag{A.5}$$

altogether with the set of constraints

$$\begin{aligned} H_x G_b(N)(I_N \otimes M_w^{[i]}) &\le \Omega_x, \\ -H_x G_b(N)(I_N \otimes M_w^{[i]}) &\le \Omega_x, \\ H_u(\tilde{I} + M_v^{[i]}K^{[i]}G_b(N))(I_N \otimes M_w^{[i]}) &\le \Omega_u, \\ -H_u(\tilde{I} + M_v^{[i]}K^{[i]}G_b(N))(I_N \otimes M_w^{[i]}) &\le \Omega_u. \end{aligned} \tag{A.6}$$

Hence, by means of (A.5)–(A.6) and imposing the satisfaction of (10b), then $M_w^{[i]}$ can be set as an optimization variable in the robust-planner while preserving the convexity of the optimization problem. □

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