Institut de Robòtica i Informàtica Industrial

CSIC

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Collocation methods for the synthesis of efficient and graceful robot motions

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Collocation Methods

To discretize the problem, the evolution of its variables are modeled as polynomials. Then, the dynamics equation and other constraints are applied at certain Collocation Points.

- Piecewise Methods: Solution is modeled as a concatenation of low degree polynomials
- Pseudospectral Methods: Solution is modelled as a single domain-spanning high degree polynomial

Space State inconsistencies

- **Collocation Methods** are designed for first order systems. However, in robotics, our systems are usually described by **second order** differential equations.
- The standard procedure is to cast the problem into a first order **State Space** equation
- The combination of the discretization techniques and the first order casting generates unexpected inconsistencies
- We have developed new Collocation schemes that avoid them by modelling the second order system directly

Usual methods

(for 1st order systems)

Trapezoidal: $x_{k+1} = x_k + \frac{h}{2}(f_{k+1} + f_k)$

Hermite Simpson: $x_{k+1} = x_k + \frac{h}{6}(f_k + 4f_c + f_{k+1})$

Meant for 1st order systems

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u}, t),$$

but in robotics we have

$$\ddot{\mathbf{q}} = \mathbf{g}(\mathbf{q}, \dot{\mathbf{q}}, \mathbf{u}, t)$$

Usual workaround:

Define → $\mathbf{x} = (\mathbf{q}, \mathbf{v})$

Add → $\mathbf{v} = \dot{\mathbf{q}}$

to convert to 1st order form

$$\begin{cases} \dot{\mathbf{q}} = \mathbf{v} \\ \dot{\mathbf{v}} = \mathbf{g}(\mathbf{q}, \mathbf{v}, \mathbf{u}, t) \end{cases}$$

But since

$\mathbf{q}(t)$, $\mathbf{v}(t)$ are approximated by polynomials of the same degree,

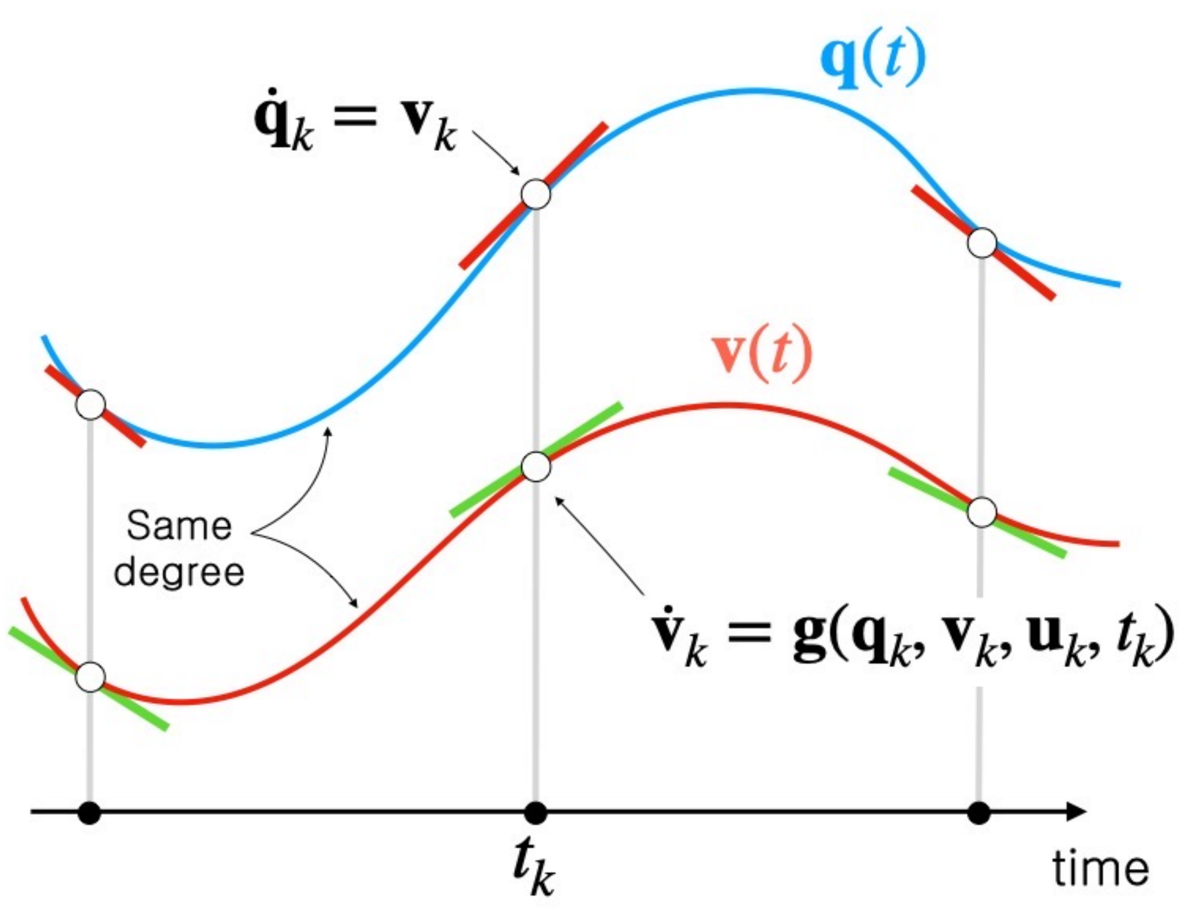
in these polynomials it will be

$\dot{\mathbf{q}}(t) \neq \mathbf{v}(t)$ (except at coll. points)

$\ddot{\mathbf{q}}(t) \neq \dot{\mathbf{v}}(t)$ (even at coll. points)

yields

Inconsistency in usual collocation



Since

$\ddot{\mathbf{q}}_k \neq \dot{\mathbf{v}}_k$

then

$\dot{\mathbf{v}}_k = \mathbf{g}(\mathbf{q}_k, \mathbf{v}_k, \mathbf{u}_k, t_k)$

$\Downarrow$

$\ddot{\mathbf{q}}_k \neq \mathbf{g}(\mathbf{q}_k, \dot{\mathbf{q}}_k, \mathbf{u}_k, t_k)$

so the 2nd order dynamics is not satisfied

Increases dynamic error, so extra control effort is needed to track

$\mathbf{x}(t) = (\mathbf{q}(t), \mathbf{v}(t))$

New methods

(for 2nd order systems)

Trapezoidal:

$$q_{k+1} = q_k + v_k h + \frac{h^2}{6}(g_{k+1} + 2g_k)$$
$$v_{k+1} = v_k + \frac{h}{2}(g_{k+1} + g_k)$$

Hermite Simpson:

$$q_{k+1} = q_k + v_k h + \frac{h^2}{6}(g_k + 2g_c)$$
$$v_{k+1} = v_k + \frac{h}{6}(g_k + 4g_c + g_{k+1})$$

Advantages

- Guarantee $\dot{\mathbf{q}}(t) = \mathbf{v}(t) \quad \forall t$
- Impose actual 2nd order dynamics $\ddot{\mathbf{q}} = \mathbf{g}(\mathbf{q}, \dot{\mathbf{q}}, \mathbf{u}, t)$ at the collocation points
- Reduce dynamic error in more than one order of magnitude
- Do not increase the computation time significantly
- Trajectories will be tracked with less control effort
- Yield twice differentiable trajectories

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Research Plan defense: December 2020

Research collaborations and research stays

- Research stay in 2023 in University of Pisa, Italy

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Publications

[1] S. Moreno, L. Ros and E. Celaya. (2022). ***Collocation methods for second order systems.*** XVIII Robotics: Science and Systems Conference, 2022, New York, pp. 1-11.

[2] S. Moreno, L. Ros and E. Celaya. (2022). ***A Legendre-Gauss pseudospectral collocation method for trajectory optimization in second order systems.*** 2022 IEEE/RSJ International Conference on Intelligent Robots and Systems, 2022, Kyoto, pp. 13335-13340

[2] S. Moreno, L. Ros and E. Celaya. (2024). ***Collocation methods for second and higher order systems.*** Autonomous Robots, 48(2): 1-20, 2024, to appear