Interval Luenberger-like Observer Design for Fault Detection and State Estimation in One-Sided Lipschitz Quadratically Inner-Bounded Systems

Juan Pablo Arango[1,2], Lucien Etienne[1], Eric Duviella [1], K. Langueh[1], Pablo Segovia [2,3] and Vicenç Puig[2,3]

***Abstract*— This paper presents the design of an interval observer (IO) for systems characterized by nonlinearities that are assumed to fulfill one-sided Lipschitz quadratically inner-bounded (OSL-QIB) conditions. This novel approach extends conventional design of IO for Lipschitz systems to OSL-QIB systems with the aim of improving observer design and convergence. Additionally, this new IO design is applied to fault detection, and state estimation which is crucial to guarantee normal process operation. A bioreactor for biomass production and a FHN system are considered as case studies to test the performance of the approach for fault detection and state estimation purposes, respectively.**

I. INTRODUCTION

Modern control systems must be robust against possible failures that may occur during process operation. These algorithms, which are known as fault-tolerant control strategies, mainly consist of three types of layers: fault detection, fault diagnosis, and fault-tolerant control [1]. Fault detection, which is the first step, is tasked with determining the existence of a fault, without indicating its nature and magnitude. Then, fault isolation and diagnosis contribute the remaining information. The focus of this paper is on fault detection, and more specifically fault detection using interval observers (IO) [2]. This is a robust strategy, as parameter uncertainty is assumed to be unknown but bounded, which allows to determine the uncertainty interval using the mathematical model of the process and the available measurements [3]. Then, whatever lies outside the uncertainty interval is classified as a fault.

The literature on the design of interval observers for fault detection is vast, and multiple approaches exist for linear systems [4]–[6], linear parameter-varying systems (LPV) [7]–[9], and nonlinear systems [10]–[12]. Zooming in on the nonlinear systems case, most papers consider the particular class of Lipschitz systems. However, these systems are very restrictive when designing observers, as they can only be bounded by a single positive constant [13], and it is not always possible to find convergence conditions [14]. In order to overcome their limitations, One-Sided Lipschitz Quadratically Inner-Bounded (OSL-QIB) systems have been proposed, as these offer more flexibility than Lipschitz systems: both positive and negative constants can be used, and convergence conditions can be found for observer synthesis [13], [14].

According to the previous exposition, the goal of this paper is to design an interval observer for fault detection in OSL-QIB systems. To the best knowledge of the authors, this constitutes a novel result. Since with this approach, systems to which interval estimation and simultaneous fault detection (as in the Lipschitz case, e.g. FHN) could not be applied will now be possible (Since OSL-QIB conditions make possible observer design when in Lipschitz case it is not). This estimation algorithm is novel in that it provides a threshold for detection in a wide variety of nonlinear systems.

The structure of the paper is the following: Section II presents the problem of designing IO for OSL-QIB nonlinear systems, and the synthesis design procedure is detailed in Section III. Section IV tests the proposed approach using two illustrative case studies: fault detection applied to a bioreactor, and state estimation applied to the FitzHugh-Nagumo (FHN) system. The main conclusions and future research avenues are discussed in Section V.

Notations and definitions

- v^T denote the transpose of v for either a matrix or a vector.
- $\mathcal{H}(A) = A + A'$
- For a symmetric matrix the symbol $*$ denotes the elements induced by symmetry: $\begin{bmatrix} A & B \\ B^T & C \end{bmatrix}$ will be denoted $\begin{bmatrix} A & B \\ * & C \end{bmatrix}$
- I_n is the identity of size n
- For a square matrix $P \succ 0$ (resp. $P \prec 0$) means that P is positive definite (resp. negative definite).
- For a square matrix $P \succeq 0$ (resp. $P \preceq 0$) means that P is positive semi-definite (resp. negative semi-definite).

*The work of this paper is partially supported by the Spanish project SEAMLESS: Sustainable learning-based Management of Multi-resource Large-scale Systems (ref. PID2023-148840OB-I00), funded by MCIN/AEI/10.13039/501100011033/FEDER, UE.

[1]J.P. Arango, L. Etienne, E. Duviella and K. Langueh are with IMT Nord Europe, CERI Digital Systems, Lille, France (e-mail: {juan-pablo.arango, lucien.etienne, eric.duviella, kokou.langueh}@imt-nord-europe.fr)

[2] J.P. Arango, P. Segovia and V. Puig are with the Department of Automatic Control, Universitat Politècnica de Catalunya - BarcelonaTECH, Barcelona, Spain (e-mail: {pablo.segovia, vicenc.puig}@upc.edu)

[3] P. Segovia and V. Puig are with the Institut de Robòtica i Informàtica Industrial, CSIC-UPC, Barcelona, Spain

II. Problem statement

Consider the following nonlinear dynamical system:

$$\begin{aligned} \dot{x} &= Ax(t) + Bu(t) + D_f f(Hx) + Dv(t), \\ y &= Cx(t) + Rh(t) + Ef_a(t), \end{aligned} \tag{1}$$

where $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}^m$ is the input, $y \in \mathbb{R}^q$ is the output, $v \in \mathbb{R}^m$ is the actuator fault, $f_a \in \mathbb{R}^j$ is is the actuator fault, $h \in \mathbb{R}^j$ is the measurement perturbation, and $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{q\times n}$, $E \in \mathbb{R}^{q\times j}$, $D_f \in \mathbb{R}^{n\times n}$, $H \in \mathbb{R}^{n\times n}$, $D \in \mathbb{R}^{n\times q}$ and $R \in \mathbb{R}^{q\times j}$ are the system matrices. The nonlinearity $f(Hx)$ is assumed to fulfill the following conditions [15].

Condition 1: The nonlinearity $f(Hx)$ satisfies the OSL condition if the following inequality is valid:

$$\langle f(H\hat{x}) - f(Hx), H(\hat{x} - x)\rangle \leq \rho \|H(\hat{x} - x)\|^2. \tag{2}$$

Defining $\Delta f = f(H\hat{x}) - f(Hx)$ and $e = \hat{x} - x$, Equation (2) can be rewritten as

$$\Delta f^T He \leq \rho e^T H^T He, \tag{3}$$

where $\rho \in \mathbb{R}$ is the OSL constant.

Condition 2: The nonlinearity $f(Hx)$ satisfies the QIB condition if the following inequality is valid:

$$\begin{aligned} \|f(H\hat{x}) - f(Hx)\|^2 \leq \beta \|H(\hat{x} - x)\|^2 \\ + \gamma \langle f(H\hat{x}) - f(Hx), H(\hat{x} - x)\rangle. \end{aligned} \tag{4}$$

Defining $\Delta f = f(H\hat{x}) - f(Hx)$ and $e = \hat{x} - x$, Equation (4) can be rewritten as

$$\Delta f^T \Delta f \leq \beta e^T H^T He + \gamma \Delta f^T He, \tag{5}$$

and the scalars γ, β in (5) are the QIB constants.

In order to design an interval observer, the following assumptions must be met:

Assumption 1: The nonlinear system (1) is cooperative [16], that is:

$$A \overset{M}{\succeq} 0, \quad B \geq 0, \quad C \geq 0, \tag{6}$$

where the operator $\overset{M}{\succeq} 0$ indicates that the matrix is Metzler [11], i.e., all elements outside its main diagonal are non-negative, and the operator ≥ 0 for the previous matrices means that all its elements are nonnegative.

Assumption 2: [10] There exists a partial ordering between the initial conditions $\underline{x}_0 \leq x_0 \leq \overline{x}_0$ such that $\underline{x} \leq x \leq \overline{x}$ for some $\underline{x}, \overline{x} \in \mathbb{R}^n$, then $\underline{f}(t, \underline{x}, \overline{x}, u) \leq f(t, \underline{x}, \overline{x}, u) \leq \overline{f}(t, \underline{x}, \overline{x}, u)$ for all $t \geq 0$, $\|u\| \leq U$,$U \in \mathbb{R}$.

The necessary ingredients to design the IO are already stated. However, the following definition is required to include noise in the formulation of the observer.

Definition 1: (Continuous time robust observer) [17]: An observer $\hat{x}$ of x with $\mathcal{L}_2$ gain μ is said to be robust with respect to perturbation v, if :

$$\int_0^\infty (e(s)^T e(s) - \mu^2 v(s)^T v(s))ds < 0, \tag{7}$$

where $e(s)$ corresponds to the estimation error $(\hat{x}(s) - x(s))$.

III. IO design for OSL-QIB systems

This section covers the design of the interval observer for OSL-QIB systems considering a noise-free and a noise-contaminated measurement. This part of the paper will provide LMI-type conditions for IO synthesis in OSL-QIB systems for fault detection purposes, thus extending the limitation of the Lipschitz systems in this area.

A. Interval Luenberger Observer free of noise ($R = 0$)

Consider the following Luenberger-like IO:

$$\dot{\overline{x}} = A\overline{x} + Bu + \overline{f}(\overline{x}, \underline{x}, t, u) + KC(x - \overline{x}), \tag{8}$$

$$\dot{\underline{x}} = A\underline{x} + Bu + \underline{f}(\overline{x}, \underline{x}, t, u) + KC(x - \underline{x}). \tag{9}$$

Knowing the structure of the observer, the dynamics of the upper $\overline{e}$ and lower error $\underline{e}$ are given by

$$\underline{e} = x - \underline{x}, \tag{10}$$

$$\overline{e} = \overline{x} - x. \tag{11}$$

Taking the time derivative of the previous expressions, and considering the nominal system, i.e., fault-free, or $v = 0$, the error dynamics for the lower and upper parts are:

$$\dot{\overline{e}} = A\overline{e} + \Delta\overline{f}(\overline{x}, \underline{x}, t, u) - KC\overline{e}, \tag{12}$$

$$\dot{\underline{e}} = A\underline{e} + \Delta\underline{f}(\overline{x}, \underline{x}, t, u) - KC\underline{e}, \tag{13}$$

with

$$\Delta\underline{f} = f(\overline{x}, \underline{x}, t, u) - \underline{f}(\overline{x}, \underline{x}, t, u)$$

and

$$\Delta\overline{f} = \overline{f}(\overline{x}, \underline{x}, t, u) - f(\overline{x}, \underline{x}, t, u).$$

Theorem 1: If there exist matrices $P \succ 0$, K, and positive scalars τ_1, τ_2, λ such that the following LMIs are feasible:

$$\begin{bmatrix} \mathcal{H}(A^T P - C^T M + (\tau_1\rho + \tau_2\beta)H^T H) & * \\ -\tau_1 H + \tau_2\gamma H + D_f^T P & -2\tau_2 I_n \end{bmatrix} \prec 0$$

$$\begin{aligned} A^T P - C^T M + PA - M^T C &\prec 0 \\ PA - M^T C + \lambda P &\geq 0 \\ \lambda &\geq 0 \\ M &= K^T P \end{aligned}$$

Then the observer given by Eqs. (8) and (9) is an IO for system (1), and its gain is given by K.

Proof: Using the quadratic Lyapunov Function (LF) $V = V_1 + V_2$, with $V_1 = \underline{e}^T P \underline{e}$ and $V_2 = \overline{e}^T P \overline{e}$, it is possible to obtain the LF dynamics

$$\dot{V}_1 = \dot{\underline{e}}^T P\underline{e} + \underline{e}^T P\dot{\underline{e}}, \tag{14}$$

$$\dot{V}_2 = \dot{\overline{e}}^T P\overline{e} + \overline{e}^T P\dot{\overline{e}}. \tag{15}$$

Expanding the previous expressions leads to

$$\dot{V}_1 = \underline{e}^T \mathcal{H}(A^T P - C^T K^T P)\underline{e} + \mathcal{H}(\Delta\underline{f}^T D_f^T P\underline{e}), \tag{16}$$

$$\dot{V}_2 = \overline{e}^T \mathcal{H}(A^T P - C^T K^T P)\overline{e} + \mathcal{H}(\Delta\overline{f}^T D_f^T P\overline{e}). \tag{17}$$

Pre-multiplying conditions (3) and (5) with the constants $2\tau_1 \in \mathbb{R}$ and $2\tau_2 \in \mathbb{R}$, respectively, it is possible to find an upper bound for the LF derivative:

$$\Psi = \rho e^T H^T H e - \Delta f^T H e > 0, \quad (18)$$

$$\Gamma = -\Delta f^T \Delta f + \gamma e^T H^T \Delta f + \beta e^T H^T H e > 0, \quad (19)$$

$$\dot{V}_1 \leq \dot{V}_1 + 2\tau_1 \mathcal{H}(\Psi) + 2\tau_2 \mathcal{H}(\Gamma). \quad (20)$$

$$\dot{V}_2 \leq \dot{V}_2 + 2\tau_1 \mathcal{H}(\Psi) + 2\tau_2 \mathcal{H}(\Gamma). \quad (21)$$

$$\dot{V} = \dot{V}_1 + \dot{V}_2 \quad (22)$$

By introducing the variables $\underline{\zeta} = [\underline{e} \quad \Delta\underline{f}]$, $\overline{\zeta} = [\overline{e} \quad \Delta\overline{f}]$, Equation (39) can be rewritten as

$$\dot{V} \leq \overline{\zeta}^T \Lambda \overline{\zeta} + \underline{\zeta}^T \Lambda \underline{\zeta} \quad (23)$$

$$\Lambda = \begin{bmatrix} \mathcal{H}(A^T P - C^T M + (\tau_1\rho + \tau_2\beta) H^T H) & * \\ -\tau_1 H + \tau_2 \gamma H + D_f^T P & -2\tau_2 I_n \end{bmatrix} \prec 0$$

Assumption 1 is fulfilled as long as the observer's closed-loop matrix for the linear part, i.e., $A - KC$, is Metzler, and also Hurwitz at the same time to guarantee convergence [3], [16]. Then, the following conditions must hold, along with the property that the matrix Λ in Eq. (23) is negative definite:

$$A^T P - C^T M + PA - M^T C \prec 0 \quad (24)$$

$$PA - M^T C + \lambda P \geq 0 \quad (25)$$

$$\lambda \geq 0 \quad (26)$$

$$M = K^T P \quad (27)$$

■

Remark 1: The Metzler condition embedded in Eq. (25) turns out to be infeasible when solved together with the Hurwitz condition and the OSL-QIB stability condition in some cases. Transforming the observer's closed-loop matrix $(A - KC)$ into Metzler requires to perform a coordinate change $x = Sz$, where the S matrix converts the $(A - KC)$ matrix into a Metzler matrix on the new coordinate axis. This can be achieved as shown in the literature [1], [10], [18].

B. Interval Luenberger Observer considering noise ($R \neq 0$)

To incorporate the effect that sensor noise has on the system, it is possible to start from the error dynamics previously obtained, and include the fact that $y = Cx + Rh$, leading to

$$\dot{\overline{e}} = A\overline{e} + \Delta\overline{f}(\overline{x}, \underline{x}, t, u) - KC\overline{e} + KRh, \quad (28)$$

$$\dot{\underline{e}} = A\underline{e} + \Delta\underline{f}(\overline{x}, \underline{x}, t, u) - KC\underline{e} + KRh. \quad (29)$$

Theorem 2 states the conditions of existence of an IO that robust to sensor noise.

Theorem 2: If there exist matrices $P \succ 0$, K, and positive scalars $\tau_1, \tau_2, \tau_3, \lambda$ such that the following convex optimization problem is feasible, and $A - KC$ is Metzler:

$$\begin{aligned} \min_{P,M} \quad & -\tau_3 \\ \text{s.t.} \quad & \Lambda_2 \prec 0, \quad P \succ 0, \end{aligned} \quad (30)$$

$$\beta_1 = \mathcal{H}(A^T P - C^T M + (\tau_1\rho + \tau_2\beta) H^T H) + \tau_3 I$$

$$\Lambda_2 = \begin{bmatrix} \beta_1 & * & * \\ -\tau_1 H + \tau_2 \gamma H + D_f^T P & -2\tau_2 I_n & * \\ MR & 0 & -\epsilon I \end{bmatrix}$$

$$\epsilon = \tau_3 \mu^2$$

$$A^T P - C^T M + PA - M^T C \prec 0$$

$$PA - M^T C + \lambda P \geq 0$$

$$\lambda \geq 0$$

$$M = K^T P$$

Then the observer given by Eqs. (8) and (9) is a robust IO for system (1), and its gain is given by K.

Proof: Using the quadratic Lyapunov Function (LF) $V = V_1 + V_2$, with $V_1 = \underline{e}^T P \underline{e}$ and $V_2 = \overline{e}^T P \overline{e}$, it is possible to obtain the LF dynamics

$$\dot{V}_1 = \dot{\underline{e}}^T P \underline{e} + \underline{e}^T P \dot{\underline{e}}, \quad (31)$$

$$\dot{V}_2 = \dot{\overline{e}}^T P \overline{e} + \overline{e}^T P \dot{\overline{e}}. \quad (32)$$

Expanding the previous expressions leads to

$$\dot{V}_1 = \underline{e}^T \mathcal{H}(A^T P - C^T K^T P)\underline{e} + \mathcal{H}(\Delta\underline{f}^T D_f^T P \underline{e}) + e^T P K R h, \quad (33)$$

$$\dot{V}_2 = \overline{e}^T \mathcal{H}(A^T P - C^T K^T P)\overline{e} + \mathcal{H}(\Delta\overline{f}^T D_f^T P \overline{e}) + e^T P K R h. \quad (34)$$

Pre-multiplying conditions (3) and (5) with the constants $2\tau_1$ and $2\tau_2 \in \mathbb{R}$, respectively, it is possible to find an upper bound for the LF derivative:

$$\Psi = \rho e^T H^T H e - \Delta f^T H e > 0, \quad (35)$$

$$\Gamma = -\Delta f^T \Delta f + \gamma e^T H^T \Delta f + \beta e^T H^T H e > 0, \quad (36)$$

$$\dot{V}_1 \leq \dot{V}_1 + 2\tau_1 \mathcal{H}(\Psi) + 2\tau_2 \mathcal{H}(\Gamma) + \mathcal{H}(e^T P K R h). \quad (37)$$

$$\dot{V}_2 \leq \dot{V}_2 + 2\tau_1 \mathcal{H}(\Psi) + 2\tau_2 \mathcal{H}(\Gamma) + \mathcal{H}(e^T P K R h). \quad (38)$$

$$\dot{V} = \dot{V}_1 + \dot{V}_2 \quad (39)$$

■

Considering h as a perturbation, it is possible to determine the $\mathcal{L}_2$ gains for both the lower and upper error as

$$\int_0^\infty (\underline{e}(t)^T \underline{e}(t) - \mu^2 h(t)^T h(t)) dt < 0, \quad (40)$$

$$\int_0^\infty (\overline{e}(t)^T \overline{e}(t) - \mu^2 h(t)^T h(t)) dt < 0. \quad (41)$$

Pre-multiplying Eqs. (40) and (41) with τ_3, and adding these to Eq. (39), together with the definition of variables $\underline{\omega} = \begin{bmatrix} \underline{e} & \Delta\underline{f} & h \end{bmatrix}$, $\overline{\omega} = \begin{bmatrix} \overline{e} & \Delta\overline{f} & h \end{bmatrix}$, it is possible to write the following optimization problem with LMI constraints:

$$\begin{aligned} \min_{P,M} \quad & -\tau_3 \\ \text{s.t.} \quad & \Lambda_2 \prec 0, \quad P \succ 0 \end{aligned} \quad (42)$$

$$\beta_1 = \mathcal{H}(A^T P - C^T M + (\tau_1\rho + \tau_2\beta)H^T H) + \tau_3 I$$

$$\Lambda_2 = \begin{bmatrix} \beta_1 & * & * \\ -\tau_1 H + \tau_2\gamma H + D_f^T P & -2\tau_2 I_n & * \\ MR & 0 & -\epsilon I \end{bmatrix}$$

$$\epsilon = \tau_3\mu^2$$

$$\begin{aligned} A^T P - C^T M + PA - M^T C &\prec 0 \\ PA - M^T C + \lambda P &\geq 0 \\ \lambda &\geq 0 \\ M = K^T P. \end{aligned}$$

C. Interval Luenberger Observer for fault detection

After building the interval observer for OSL-QIB systems, it is desired to incorporate the ability to detect faults, and do interval state estimation. To this end, the upper and lower error residuals are constructed as [19]

$$\overline{r}(t) = C\overline{x}(t) - Cx(t), \tag{43}$$

$$\underline{r}(t) = Cx(t) - C\underline{x}(t). \tag{44}$$

Once the values of the residuals are known, the following detection criterion is established:

$$\begin{cases} \underline{r}(t) \leq 0 \;\&\; \overline{r}(t) \leq 0 \;\&\; |\overline{r}(t)| < \eta \;\&\; |\underline{r}(t)| < \eta, \text{ no fault} \\ \text{otherwise the system is faulty.} \end{cases} \tag{45}$$

IV. Examples of Application

A. IO for Fault Detection

1) Actuator fault scenario: Bioreactors are essential devices in biotechnology and chemical engineering because they allow the controlled cultivation of microorganisms, animal or plant cells for the production of various biotechnological products. Within its components, the valves play a crucial role because they are responsible for supplying the necessary amount of nutrients, gases or reagents. Despite their usefulness, they can fail due to biomass accumulation, clogging, ingress of solid particles, etc. Therefore, it is of high interest to implement a failure detection system for the operation of a bioreactor. Due to the above reasons, it is decided to implement the interval observer previously designed for fault detection in a bioreactor for the production of biomass from substrate [20]. The bioreactor's mathematical model is given by the following equations:

$$\dot{X} = -D_r X + a_1\mu_r, \tag{46}$$

$$\dot{S}_u = D_r S_M - D_r S_u - a_3 a_1 \mu_r, \tag{47}$$

$$\mu_r = \frac{S_u X}{a_2 + S_u} \quad y = S_u \quad u = D_r, \tag{48}$$

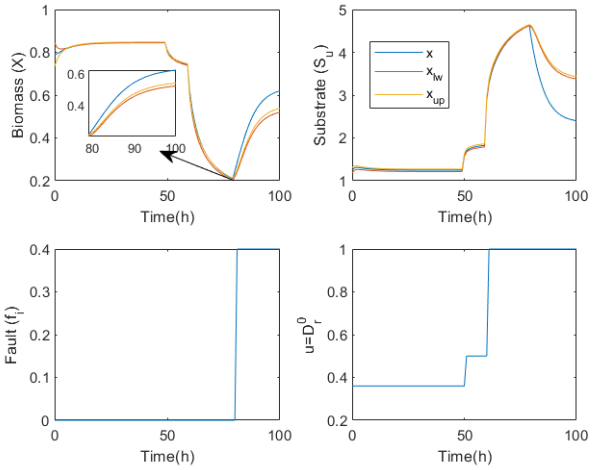

Fig. 1. First scenario of fault detection: valve obstruction $x_{lw} = \underline{x}$, $x_{up} = \overline{x}$

$$A = \begin{bmatrix} 0.0057 & 0.1 \\ -1.95 & -1.41 \end{bmatrix}, \; C = \begin{bmatrix} 1 & 0 \end{bmatrix}, \tag{49}$$

$$H = D_f = I_2, \tag{50}$$

with $D_r = 0.36$, $a_1 = \begin{bmatrix} 1.15 & 1.25 \end{bmatrix}$, $a_2 = \begin{bmatrix} 6.9 & 7.15 \end{bmatrix}$, $a_3 = 10.53$.

In order to calculate the optimal observer's gain (Theorem 2 with Metzler LMI condition), the OSL-QIB constants $\rho = 100, \gamma = 0, \beta = -1$ were taken into account, together with the matrices reported in Equations (49)-(50). To simulate the first fault scenario, the bioreactor dilution rate is modified as follows:

$$D_r = D_r^0 - f_i D_r^0, \tag{51}$$

where D_r^0 is the nominal value of the control action, and $f_i \in (0, 1)$ is the fault, where $f_i = 1$ indicates that the valve is completely obstructed, and $f_i = 0$ means that no obstruction is present.

The scenario includes different movements in the control action in order to determine whether the detection algorithm is able to distinguish between fault and control efforts. Moreover, the constant η for the fault detection criterion in Eq. (45) is equal to 0.147 and was determined from simulations with $f_i = 0$.

Figure 1 shows the result obtained considering a 40% of obstruction in the valve. It can be inferred that the observer distinguishes control efforts from faults, as modifying the valve position causes the state trajectories to remain within the detection threshold. On the other hand, activating the fault (at time 80h) causes the biomass and the substrate to exit the lower and upper bounds of the observer, making it possible to detect the fault.

2) Sensor fault scenario: Various types of sensors are often used to monitor the operation of a bioreactor, including concentration sensors. These are useful for observing biomass production in real time, and thus being able to find

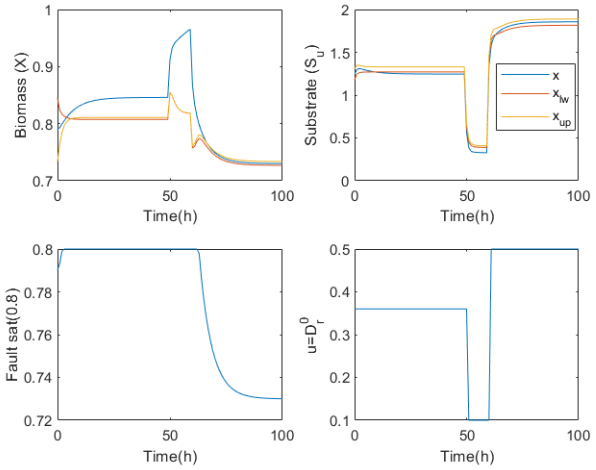

Fig. 2. First scenario of fault detection: sensor saturation $x_{lw} = \underline{x}$, $x_{up} = \overline{x}$

an estimate for the current state of the substrate consumption inside the reactor. However, these can fail, so it is interesting to test the OSL-QIB interval observer when these types of situations arise.

In this second simulation case, it will be assumed that the sensor is not capable of reading concentrations higher than 0.8, and the control efforts will be modified in order to maintain a high biomass concentration and to be able to evaluate observer performance in this situation.

As shown in Figure 2, the sensor begins to read the correct concentration at t= 62h. During this interval, the trajectories remain outside the threshold established by the observer, concluding that the observer can detect the occurrence of the sensor fault. The gains were obtained with the help of LMIlab from MATLAB.

B. IO for state estimation

FitzHugh-Nagumo (FHN) systems are widely used to model neural behavior, chemical reaction kinetics, artificial neural networks and electronic oscillators, and also to study external stimulation (such as deep brain stimulation) therapies for effective treatment of brain disorders [21].

The following state-space model represents two coupled synchronous FHN neurons [21]:

$$A = \begin{bmatrix} -g_1 - 1 & -v_1 & g_1 & 0 & 0 \\ b_1 & 0 & 0 & 0 & 0 \\ g_2 & 0 & -g_2 - 1 & -v_2 & 0 \\ 0 & 0 & b_2 & 0 & 0 \\ 1 & 0 & -1 & 0 & -1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \end{bmatrix}^T, \tag{52}$$

$$R = C, \quad D = C^T, \tag{53}$$

$$H = D_f = diag(\begin{bmatrix} 1 & 0 & 1 & 0 & 0 \end{bmatrix}), \tag{54}$$

$$f = \begin{bmatrix} -rx_1^3 + (1+r)x_1^2 + \frac{a}{\omega} cos\omega t \\ 0 \\ -rx_3^3 + (1+r)x_1^3 + \frac{a}{\omega} cos\omega t \\ 0 \\ 0 \end{bmatrix}, \tag{55}$$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}, \quad y = x_1. \tag{56}$$

The parameters are chosen as reported in [21]: $r = 10, a = 1, \omega = 0.26\pi, g_1 = 0.7, g_2 = 1, b_1 = 0.8, b_2 = 1.2, v_1 = b_1^{-1}$ and $v_2 = b_2^{-1}$. Moreover, the OSL-QIB constants take the following values: $\rho = \gamma = 0.1, \beta = 0.2$.

Knowing the dynamics of the system to be studied, the optimization problem (42) is solved, leading to

$$P = \begin{bmatrix} 10.14 & 3.69 & 3.35 & 0.54 & 2.02 \\ 3.69 & 19.90 & -0.04 & -0.08 & -0.54 \\ 3.35 & -0.04 & 10.60 & 5.24 & -2.67 \\ 0.54 & -0.08 & 5.24 & 16.07 & 0.35 \\ 2.02 & -0.54 & -2.67 & 0.35 & 10.28 \end{bmatrix},$$

$$K = \begin{bmatrix} 0.2054 & -0.0401 & -0.0949 & 0.0253 & -0.0828 \end{bmatrix}^T.$$

Analysis of the results reveals that the observer's closed-loop matrix is Hurwitz and the LMI for attenuating the nonlinearity is satisfied. However, this matrix is not Metzler, so the change of coordinates $x = Sz$ proposed in Algorithm 1 of [18] is applied. The resulting matrix S is

$$S = \begin{bmatrix} -1.67 & -1.04 & 2.33 & 0.97 & 0.00 \\ 0.36 & 0.10 & -0.09 & -0.02 & 0.00 \\ -0.07 & -0.34 & 0.22 & -0.01 & 0.00 \\ -0.07 & -0.34 & 0.22 & -0.01 & 0.00 \\ -0.02 & -0.03 & 1.01 & 0.95 & 0.001 \end{bmatrix}$$

After obtaining each of the matrices required for the design of the observer, the following simulation scenario is designed: the uncertain parameter will be $r = 10$, with lower and upper values equal to $r_{min} = 5$,$r_{max} = 15$. Having identified the parametric uncertainty, the case study is complemented by contaminating the sensor with noise, obtaining the results shown in Figure 3. It can be seen that the observer manages to wrap most of the states effectively. Regarding state 4, there is a small startle due to noise above the upper limit imposed by the observer. However, it is not significant in terms of magnitude for the application under study.

From the results obtained, it can be inferred that the Luenberger type observer for OSL-QIB systems is a valid alternative to perform interval estimation. The FHN system is used to test estimation algorithms within the OSL-QIB family, since the determination of the Lipschitz constant and the observer synthesis becomes very complex in the Lipschitz case [14], [21].

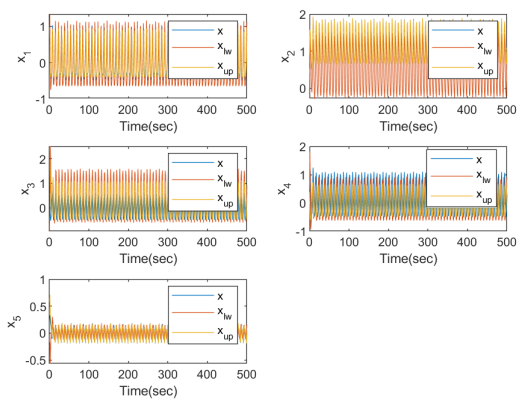

Fig. 3. Interval Observer applied to FHN system, $x_{lw} = \underline{x}$,$x_{up} = \overline{x}$

V. Conclusions

This paper deals with the design of an interval observer for fault detection and state estimation in OSL-QIB systems. From the results obtained, it can be concluded that the observer has the ability to detect faults in both sensors and actuators (bioreactor case), and to perform state estimation in high complexity systems (FHN system). The LMI conditions proposed in the present paper turn out to be novel in the field of application, since most of the synthesis of nonlinear interval observers is devoted to Lipschitz systems.

As future work, high uncertainty scenarios can be considered, where additional constraints are required to guarantee robustness, in different conditions that may occur in practice (parametric uncertainty, unknown inputs, etc.), or the case of fault reconstruction, which is of high importance in scenarios where fault-tolerant control algorithms are desired.

References

[1] Y. Tian, K. Zhang, B. Jiang, and X.-G. Yan, "Interval observer and unknown input observer-based sensor fault estimation for high-speed railway traction motor," *Journal of the Franklin Institute*, vol. 357, no. 2, pp. 1137–1154, 2020.

[2] V. Puig, "Fault diagnosis and fault tolerant control using set-membership approaches: Application to real case studies," *International Journal of Applied Mathematics and Computer Science*, vol. 20, p. 619–635, Dec. 2010.

[3] J. C. Castillo, G. O. Gordillo, V. Puig, R. V. Méndez, G. G. Ramírez, J. R. Reyes, and C. A. Zaragoza, "Sensor fault detection for lpv systems using interval observers," *IEEE Access*, 2024.

[4] J. Meseguer, V. Puig, T. Escobet, and J. Saludes, "Observer gain effect in linear interval observer-based fault detection," *Journal of process control*, vol. 20, no. 8, pp. 944–956, 2010.

[5] V. Puig, J. Quevedo, T. Escobet, and A. Stancu, "Robust fault detection using linear interval observers," *IFAC Proceedings Volumes*, vol. 36, no. 5, pp. 579–584, 2003.

[6] J. Meseguer, V. Puig, T. Escobet, and R. Sarrate, "Observer gain effect in linear interval observer-based fault detection," in *2007 46th IEEE Conference on Decision and Control*, pp. 995–1002, IEEE, 2007.

[7] F. Nejjari, V. Puig, S. M. de Oca, and A. Sadeghzadeh, "Robust fault detection for lpv systems using interval observers and zonotopes," in *Proceedings of the 48h IEEE Conference on Decision and Control (CDC) held jointly with 2009 28th Chinese Control Conference*, pp. 1002–1007, IEEE, 2009.

[8] D. Rotondo, A. Cristofaro, T. A. Johansen, F. Nejjari, and V. Puig, "Robust fault and icing diagnosis in unmanned aerial vehicles using lpv interval observers," *International Journal of Robust and Nonlinear Control*, vol. 29, no. 16, pp. 5456–5480, 2019.

[9] S. Montes de Oca, V. Puig, and J. Blesa, "Robust fault detection based on adaptive threshold generation using interval lpv observers," *International Journal of Adaptive Control and Signal Processing*, vol. 26, no. 3, pp. 258–283, 2012.

[10] D. Efimov, T. Raïssi, S. Chebotarev, and A. Zolghadri, "Interval state observer for nonlinear time varying systems," *Automatica*, vol. 49, no. 1, pp. 200–205, 2013.

[11] D. Efimov and T. Raïssi, "Design of interval observers for uncertain dynamical systems," *Automation and Remote Control*, vol. 77, pp. 191–225, 2016.

[12] L. Cao, X. Yang, G. Wang, Y. Liu, and Y. Hu, "Fault detection based on extended state observer and interval observer for uavs," *Aircraft Engineering and Aerospace Technology*, vol. 94, no. 10, pp. 1759–1771, 2022.

[13] S. A. Nugroho, V. Hoang, M. Radosz, S. Wang, and A. F. Taha, "New insights on one-sided lipschitz and quadratically-inner bounded nonlinear dynamic systems," in *2020 American control conference (ACC)*, pp. 4558–4563, IEEE, 2020.

[14] J. P. Arango, L. Etienne, E. Duviella, K. Langueh, P. Segovia, and V. Puig, "Unknown input observer for quadratically inner bounded-one sided lipschitz perturbed nonlinear systems," *International Journal of Systems Science*, pp. 1–14, 2024.

[15] W. Zhang, H. Su, F. Zhu, and G. Azar, "Unknown input observer design for one-sided lipschitz nonlinear systems," *Nonlinear Dynamics*, vol. 79, pp. 1469–1479, 07 2014.

[16] J. D. Avilés and J. A. Moreno, "Interval observer design for nonlinear systems: Stability radii approach," *IEEE Access*, vol. 6, pp. 52801–52813, 2018.

[17] H. K. Khalil, *Nonlinear Control*. Upper Saddle River, NJ 07458.: Prentice Hall, 2015.

[18] Z. Zhang and J. Shen, "A survey on interval observer design using positive system approach," *Franklin Open*, vol. 4, p. 100031, 2023.

[19] S. Guo and F. Zhu, "Actuator fault detection and interval reconstruction based on interval observers," *IFAC-PapersOnLine*, vol. 50, no. 1, pp. 5061–5066, 2017.

[20] N. Meslem, N. Ramdani, and Y. Candau, "Interval observers for uncertain nonlinear systems. application to bioreactors.," *IFAC Proceedings Volumes*, vol. 41, no. 2, pp. 9667–9672, 2008.

[21] A. Zulfiqar, M. Rehan, and M. Abid, "Observer design for one-sided lipschitz descriptor systems," *Applied Mathematical Modelling*, vol. 40, no. 3, pp. 2301–2311, 2016.