Unknown Input Observer design for LPV OSL-QIB nonlinear systems

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Abstract: This paper presents the design of an unknown input observer (UIO) for linear parameter-varying (LPV) one-sided Lipschitz quadratically inner-bounded (OSL-QIB) systems. This family of systems extends conventional LPV systems by adding nonlinear terms to improve observer performance and reduce errors due to the transformation of a nonlinear system to its LPV version (either because the nonlinearities are difficult to wrap as it is in the polytopic case, or because the Jacobian does not yield a good approximation). The UIO is initially designed in the absence of noise, allowing to establish LMI conditions to guarantee the convergence of the error dynamics, and then sensor noise is explicitly considered in the design so that its effect can be mitigated. Finally, a case study based on the Corning channel benchmark is used to test the performance of the UIO in the presence of unknown inputs (e.g., leakage, evaporation or rain).

Keywords: LPV, OSL-QIB, Unknown Input Observer, OCIS, Material and energy balances, fluid mechanics.

1. INTRODUCTION

State estimators, also known as observers, play a crucial role in the monitoring and control of dynamic systems, since these mathematical algorithms are able to estimate variables that are difficult or impossible to measure, and also allow to reduce the costs incurred in the purchase of sensors. Although observers are a tempting alternative for process monitoring, there are several factors that can negatively affect their performance, such as the existence of unknown inputs (unmodeled exogenous inputs). The design of unknown input observers (UIO) (i.e. an observer robust against unknown inputs) have extensively addressed in the literature, see as e.g. (Hou and Muller, 1992; Hui and Żak, 2005). There exist already many results concerning the design of UIO for linear systems. Although the design of estimation approaches for linear systems are well-established, it is well known that they only work in a region close to an equilibrium point in case of systems with nonlinear dynamics. This fact has motivated the formulation of UIO for different types of nonlinear systems. Lipschitz nonlinear systems are among the most studied classes of nonlinear systems, for which different types of UIOs have been formulated, and their convergence is guaranteed upon the fulfillment of linear matrix inequality (LMI) constraints (Abbaszadeh and Marquez, 2010; Witczak et al., 2007; Alenezi et al., 2020; Chen and Saif, 2006).

An alternative approach to design UIO algorithms for nonlinear systems is to consider that the nonlinearity is one-sided Lipschitz quadratically inner bounded (OSL-QIB). This alternative way of describing a nonlinear system is less restrictive for UIO design than the Lipschitz case. Although both descriptions can be considered qualitatively equivalent (Nugroho et al., 2020), the constants bounding the nonlinearity can be positive or negative in the case of OSL-QIB systems, which gives more versatility when designing an observer (Nugroho et al., 2020). There are also several different types of UIO design approaches for OSL-QIB systems. For instance, Zhang et al. (2014); Nguyen et al. (2018); Cuong Nguyen and Trinh (2016); Arango et al. (2024b) address situations where the OSL-QIB nonlinearity is considered together with delays, measurement noise, and parameter uncertainty. However, one important limitation in the previous references is that the UIO gains were calculated considering a single operating point, i.e., constant A matrix (Arango et al., 2024b), a fact that might lead to poor performance if the system moves far from the equilibrium point.

The applicability of the UIO for OSL-QIB systems at multiple operating points can be extended by including

⋆ This work has been co-financed by the Spanish State Research Agency (AEI) and the European Regional Development Fund (ERFD) through the projects SaCoAV (ref. PID2020-114244RB-I00) and L-BEST (PID2020-115905RB-C21).

the effect of a time-varying parameter $\theta(t)$ in the design matrices. The inclusion of $\theta(t)$ causes the linear part of the OSL-QIB system to become varying over time, which is usually termed as linear parameter-varying (LPV) systems. LPV systems are considered to be an intermediate point between a linear and a nonlinear system (Tóth et al., 2011), as the signal relations are considered linear, but the parameters within the state matrices are functions of a time-varying signal $\theta(t)$ known as *scheduling variables* (Tóth et al., 2011). Thanks to the explicit consideration of these signals, LPV systems can describe time-varying phenomena as well as non-linearities. As it is the case for both linear and nonlinear systems, UIO for LPV systems are designed including different features, such as unknown input reconstruction, robustness against parameter uncertainty, and discrete LPV systems (Ichalal and Mammar, 2015; de Oliveira and Pereira, 2020; Wang et al., 2015). Despite the advantages that LPV systems present for the design of UIO, it is possible to have approximation errors when LPV systems are transformed to their polytopic form so that the solution can be determined by a finite set of LMIs. These errors can be due to restrictions on the parametric grid or the linear interpolations that are used (Tóth, 2010; Briat, 2014).

Based on the above arguments, this paper presents the design of an observer for LPV OSL-QIB systems considering the impact of unknown inputs, with the aim of mitigating the effect of the possible approximation errors of the polytopic model by incorporating the nonlinearity in the framework of LPV systems, and in the structure of OSL-QIB systems. The addition of the LPV part in order to calculate the gains of the UIO in multiple regions of operation, and consequnetly improving its performance (which was not possible with the UIO OSL-QIB proposed by (Arango et al., 2024b)).

The structure of the paper is the following: In Section 2, the problem of UIO design for LPV OSL-QIB nonlinear systems is introduced. Section 3 introduces the UIO synthesis design procedure for the considered family of systems. Section 4 presents the obtained results applying the proposed approaches using an illustrative example. Finally, Section 5 draws the main conclusions and points several future research paths.

2. PROBLEM STATEMENT

2.1 Problem statement

The OSL-QIB framework provides greater flexibility when bounding the dynamics of a system (Zhang et al., 2014, 2016; Nugroho et al., 2020; Arango et al., 2024b) compared to Lipschitz case, since it can deal with a larger number of real-valued constants, not necessarily positive. This fact improves the convergence properties of the estimation algorithms when the OSL-QIB framework is used (Arango et al., 2024b). However, when a nonlinear system is expressed in OSL-QIB form in terms of the linearization error or the A matrix is constant, the observer gains are calculated at a fixed operating point, which would lead to poor state estimation performance under different types of inputs and transients. Considering the above arguments, we want to design an UIO such that the computation of its gains contemplates multiple operating points (adding the effect of the LPV term), and also include possible modeling errors that can arise in the polytopic representation of an LPV system.

2.2 Mathematical description of LPV OSL-QIB systems

Consider the following LPV OSL-QIB nonlinear system:

$$\begin{aligned} \dot{x} &= A(\theta(t))x + B(\theta(t))u + D_f f(Hx) + Dv, \\ y &= Cx + Rh, \end{aligned} \tag{1}$$

where $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}^m$ is the input, $y \in \mathbb{R}^q$ is the output, $v \in \mathbb{R}^p$ is an unknown input, $h \in \mathbb{R}^j$ is the measurement perturbation, and $A(\theta(t)) \in \mathbb{R}^{n\times n}$, $B(\theta(t)) \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{q\times n}$,$H \in \mathbb{R}^{n\times n}$,$D_f \in \mathbb{R}^{n\times n}$, $R \in \mathbb{R}^{q\times j}$, $D \in \mathbb{R}^{n\times q}$ are the system matrices. The nonlinearity $f(Hx)$ is assumed to fulfill the OSL-QIB conditions (Zhang et al., 2014).

Assumption 1. The parameter vector $\theta(t)$ consists of N time-varying parameters $\theta(t)^T = [\theta_1(t)\ \theta_2(t)....\theta_N(t)]$, which are smooth enough and bounded, implying that $\theta(t) \in \Theta$, where Θ is a polytopic set, being the simplest case a hyper-rectangular box defined by (Ichalal and Mammar, 2015)

$$\begin{aligned} \Theta = \big\{\theta(t) \in \mathbb{R}^N | \theta_1 \in [\theta_{1,min},\ \ \theta_{1,max}], .., \\ \theta_N \in [\theta_{N,min},\ \ \theta_{N,max}]\big\}, \end{aligned}$$

where $\theta_{i,min}$ and $\theta_{i,max}$ represents the lower and upper value of each scheduling parameter, respectively.

Condition 1: The non-linearity $f(Hx)$ satisfies the OSL condition if the following inequalities are fulfilled:

$$\Delta f^T H e \leq \rho e^T H^T H e, \tag{2}$$

where $\rho \in \mathbb{R}$ is the OSL constant, $e = \hat{x} - x$ is the error of estimation and $\Delta f = f(H\hat{x}) - f(Hx)$.

Condition 2: As in Condition 1, the following inequalities are formulated to state the QIB property:

$$\Delta f^T \Delta f \leq \beta e^T H^T H e + \gamma e^T H \Delta f, \tag{3}$$

and the scalars γ, β in (3) are the QIB constants.

Definition 1. (Continuous time robust observer)Arango et al. (2024b): An observer $\hat{x}$ of x with $\mathcal{L}_2$ gain μ is said to be robust with respect to perturbation v, if :

$$\int_0^\infty \left(e(s)^T e(s) - \mu^2 v(s)^T v(s)\right) ds < 0, \tag{4}$$

where $e(s)$ corresponds to the estimation error $(\hat{x}(s) - x(s))$.

3. UIO SYNTHESIS FOR LPV OSL-QIB SYSTEMS

Initially, let us consider the following UIO structure for system (1), following the work from (Zhang et al., 2014):

$$\begin{aligned} \dot{z} &= N(\theta)z + TB(\theta)u + G(\theta)y + TD_f f(H\hat{x}), \\ \hat{x} &= z - Ey, \end{aligned} \tag{5}$$

where z is the auxiliary variable, $\hat{x}$ is the estimation of the system state, and matrices $N(\theta) \in \mathbb{R}^{n\times n}, G(\theta) \in$

$\mathbb{R}^{n\times q}$, $E \in \mathbb{R}^{n\times q}$ and $T = I_n + EC \in \mathbb{R}^{n\times n}$ are the observer gains. The subtraction between the estimated state (5) and the real state of the system (1) allows to define the estimation error as

$$\begin{aligned} e = \hat{x} - x = z - ECx - x = z - Tx, \\ T = (I_n + EC). \end{aligned} \tag{6}$$

Taking the time derivative of (6), the error dynamics can be deduced:

$$\dot{e} = \dot{z} - T\dot{x}. \tag{7}$$

Considering that $R \neq 0$, it is possible to obtain the following error dynamics:

$$\begin{aligned} \dot{e} = N(\theta)e + (G(\theta)R + N(\theta)ER)h + \\ TD_f\Delta f - ER\dot{h} - TDv(t) \\ +TB(\theta)u + N(\theta)Tx - TAx - \\ TB(\theta)u + G(\theta)Cx \end{aligned} \tag{8}$$

From this equation it is easy to see that the dynamics of the estimation error is influenced by the sensor noise and its derivative (Arango et al., 2024b). Considering the error dynamics for the LPV OSL-QIB observer, it is possible to proceed with the UIO synthesis (Chen and Saif, 2006). In the following equations, the observer gains are stated as functions of the time-varying matrices:

$$\begin{aligned} E &= P^{-1}S, \\ K(\theta) &= P^{-1}Q(\theta), \\ N(\theta) &= TA(\theta) - K(\theta)C, \\ G(\theta) &= K(\theta) + K(\theta)CE - TA(\theta)E \end{aligned} \tag{9}$$

Theorem 1 presents the conditions to minimize the effect of noise on the system, and to try to guarantee convergence to the origin of the dynamics of the estimation error.

Theorem 1. If there exist matrices $P \succ 0$, S, $Q(\theta)$ and positive scalars τ_1, τ_2, τ_3, τ_4 and μ such that the following relations holds for all $\theta(t) \in \Theta$

$$\Omega_1 = \begin{bmatrix} \mathcal{H}(PA(\theta) + SCA(\theta) - Q(\theta)C) + a_1 & \Lambda^T \\ \Lambda & -2\tau_2 I_n \\ R^T Q(\theta)^T & 0_n \\ -R^T S^T & 0_n \end{bmatrix} \tag{10}$$

$$\Omega_2 = \begin{bmatrix} Q(\theta)R & -SR \\ 0_n & 0_n \\ -\tau_3\mu^2 I_n & 0_n \\ 0_n & -\tau_4 I_n \end{bmatrix} \tag{11}$$

$$\Omega = [\Omega_1 \ \Omega_2] \prec 0 \tag{12}$$

$$PD + SCD = 0 \tag{13}$$

$$\begin{aligned} a_1 = 2(\tau_1\rho + \tau_2\beta)H^T H + \tau_3 I_n, \\ \Lambda = D_f^T P + D_f^T C^T S^T - \tau_1 H + \tau_2\gamma H, \end{aligned} \tag{14}$$

then observer (5) is a robust observer of system (1) and its gains are given by (9).

Proof: By assuming $PD + SCD = 0$, and since P is invertible, it follows $D + P^{-1}SCD = 0$, and since $P^{-1}S = E$, $(I + EC)D = 0$, this leads to $TD = 0$. Furthermore:

$$\begin{aligned} N(\theta)T + G(\theta)C - TA(\theta) = \\ -K(\theta)C + N(\theta)EC + G(\theta)C = \\ -K(\theta)C + N(\theta)EC + K(\theta)C + K(\theta)CEC - TA(\theta)EC \\ = (N(\theta) + K(\theta)C - TA(\theta))\,EC = 0. \end{aligned} \tag{15}$$

Using the error dynamics (8) and the LME constraint $PD + SCD = 0$, the quadratic Lyapunov function (LF) $V = e^T Pe$ is proposed. Taking the LF derivative with respect to time, the expression (16) is obtained:

$$\begin{aligned} \dot{V} = e^T(N(\theta)^T P + PN(\theta))e + \mathcal{H}(h^T Q(\theta)Re) \\ +\mathcal{H}(\Delta f D_f^T T^T Pe). \end{aligned} \tag{16}$$

Pre-multiplying Equations (2) and (3) with the constants $2\tau_1$ and $2\tau_2 \in \mathbb{R}$, respectively, it is possible to find an upper bound for the LF derivative:

$$\Psi = \rho e^T H^T He - \Delta f He > 0, \tag{17}$$

$$\Gamma = \Delta f^T \Delta f + \gamma e^T H^T \Delta f + \beta e^T H^T He > 0, \tag{18}$$

$$\dot{V} \leq \dot{V} + \tau_1\mathcal{H}(\Psi) + \tau_2\mathcal{H}(\Gamma). \tag{19}$$

Considering Equations (17), (18), (19) and $\mathcal{L}_2$ gain definition, inequality (20) is defined for LF derivative as follows:

$$\begin{aligned} \dot{V} \leq \dot{V} + \tau_1\mathcal{H}(\Psi) + \tau_2\mathcal{H}(\Gamma)) + (\tau_3 + \tau_4)e^T e \\ -\tau_3\mu^2 h^T h - \tau_4\mu^2 \dot{h}^T \dot{h}. \end{aligned} \tag{20}$$

By introducing the variable $\zeta = [e \quad \Delta f \quad h \quad \dot{h}]$, Equation (20) can be rewritten as an LMI as indicated in Equation (12). According to Equation (20), it is possible to minimize the impact of the sensor perturbation on the system by replacing $\tau_3\mu^2$ by a new LMI variable ϵ such that $\frac{\epsilon}{\tau_3}$ is minimized, as the previous cases. With mathematical developments, the proof for Theorem 1 is concluded. Note that the previous semidefinite programming problem (SDP) subject to the constraints (12)-(13) is formulated for infinite values of the scheduling parameter $\theta(t)$.

Considering *Assumption 1*, since Θ is a convex set, the convexity property allows the conditions of Theorem 1 posed in terms of a θ parameter with infinite values to be reduced to a finite number of LMI conditions evaluated at each of the vertices of Θ (Bernal et al., 2022). Once the LMIs are solved in the vertices, the values of Q_i are obtained. Then, $K(\theta)$ is obtained from the polytopic interpolation of the values of $K_i = P^{-1}Q_i$. Finally, from the value of $K(\theta)$, the values of $N(\theta)$ and $G(\theta)$ in (9) can be determined.

4. APPLICATION EXAMPLE: AN OPEN-FLOW CHANNEL CASE STUDY

Most of the water used in agriculture is transported by networks of open-flow channel irrigation systems (OCIS) (Arango et al., 2024a). This kind of systems may suffer water losses during its operation due to, e.g., evaporation, leaks, theft, user demands, and other external factors (Bedjaoui et al., 2009; Bedjaoui and Weyer, 2011; Pillosu et al., 2012; Segovia et al., 2019; Conde et al., 2021b; Segovia et al., 2024). Moreover, level measurement instrumentation in long canals can increase the capital and operating costs of the process. Consequently, a robust observer against the possible adverse factors that may occur can be an attractive alternative to improve system performance (Conde et al., 2021b; Arango et al., 2024a).

There exist different types of models in the literature that describe the dynamics of water channels, such as the integrator with delay (ID) (Litrico and Fromion, 2004), inte-

grator with resonance (IR) (Horváth et al., 2015), orthogonal collocation (Pillosu et al., 2012), and models described by partial differential equations (PDE) (Conde et al., 2021a). While most of the reported models have certain advantages, most of them lack physical meaning and have implementation problems, either because they can only describe the plant behavior around an equilibrium point (e.g., linear models) or are highly nonlinear and thus costly for real-time computations (e.g., orthogonal placement and PDE models). The model reported by Conde et al. (2020) is chosen in this paper due to its phenomenological nature (it is built using balance equations and conservation laws) and exhibits very good performance when compared with data provided by EPA SWMM Conde et al. (2021a), i.e., a software tool that provides highly reliable data in open channel simulations.

4.1 Corning channel modeling

The selected open channel considered as case study in this paper is the Corning Channel in California, widely used in the OCIS literature as benchmark (see Figure 1). This benchmark was proposed by ASCE (American Society of Civil Engineers) task committee on canal automation algorithms as a standardized test case on canals with realistic properties (Clemmens et al., 1998). The Corning channel consists of 8 pools, each equipped with an undershoot gate in submerged flow. For testing purposes, only the first pool will be considered, as scalability problems may arise when considering the complete system due to the large number of vertices of the resulting LPV model.

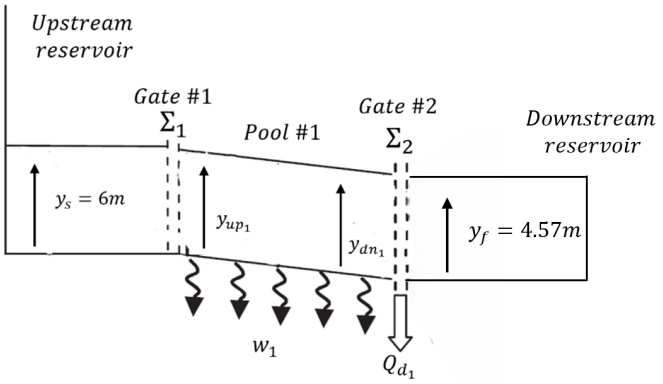

Fig. 1. First pool of the Corning channel (Clemmens et al., 1998)

According to the modeling methodology reported by Conde et al. (2020), the application of conservation laws (material balance and Bernoulli equation) over the first pool of the canal while considering the equations of the undershoot gates in submerged flow (Litrico and Fromion, 2004) leads to

$$A_{up_1}\dot{y}_{up_1} = Q_1(t) - Q_{tr_1}(t), \tag{21}$$
$$A_{dn_1}\dot{y}_{dn_1} = Q_{tr_1}(t) - Q_2(t), \tag{22}$$
$$Q_1 = c_1 w_1 u_1 \sqrt{2g}\sqrt{|y_s - y_{up_1}|}, \tag{23}$$
$$Q_2 = c_2 w_2 u_2 \sqrt{2g}\sqrt{|y_{dn_1} - y_f|}, \tag{24}$$
$$Q_{tr_1} = K_{tr_1} y_{up_1}\sqrt{|y_{up_1} - y_{dn_1} + \Delta z|}, \tag{25}$$

where $c_1 w_1\sqrt{2g} = c_2 w_2\sqrt{2g} = 20.14\dfrac{m^{3/2}}{s}$ are parameters related to the gate dimensions , $u_1, u_2 \in [0.3,\ 1]\ m$ are the gate openings, $K_{tr_1} = 7.47\frac{m^2}{s}$ is the transition constant, $A_{up_1} = 69887m^2$ is the upstream area, $A_{dn_1} = 70154m^2$ is the downstream area, $\Delta z = 1.11m$, and y_{up_1}, y_{dn_1} are the upstream and downstream levels, respectively.

The first step consists in determining an LPV representation of the nonlinear model given by Equations (21)–(25) using the nonlinear embedding methodology (Kwiatkowski et al., 2006). The resulting LPV model is given by matrices

$$A(\theta) = \begin{bmatrix} -\dfrac{1}{A_{up_1}}\theta_1 & 0 \\ \dfrac{1}{A_{dn_1}}\theta_1 & 0 \end{bmatrix} \tag{26}$$
$$B(\theta) = \begin{bmatrix} \dfrac{1}{A_{up_1}}\theta_2 & 0 \\ 0 & -\dfrac{1}{A_{dn_1}}\theta_3 \end{bmatrix},$$
$$D_f f(Hx) = \begin{bmatrix} \dot{y}_{up_1} \\ \dot{y}_{dn_1} \end{bmatrix} - A(\theta)x - B(\theta)u$$
$$D_f = H = I_2 \tag{27}$$

with

$$\theta_1 = K_{tr_1}\sqrt{y_{up_1} - y_{dn_1} + \Delta z}$$
$$\theta_2 = c_1 w_1\sqrt{2g}\sqrt{y_s - y_{up_1}}$$
$$\theta_3 = c_2 w_2\sqrt{2g}\sqrt{y_{dn_1} - y_f},$$

and the following bounds on the scheduling parameters are determined: $0.44 \le \theta_1 \le 1.2$, $6.48 \le \theta_2 \le 22.1$, $0.63 \le \theta_3 \le 22.7$. Moreover,

$$y = [0\ 1]\begin{bmatrix} y_{up_1} \\ y_{dn_1} \end{bmatrix}. \tag{29}$$

With the LPV system given by Equation (26) and the bounds on the scheduling parameters, a system with 2^3 vertices can be found, which results in 8 matrices with their respective weights (Bernal et al., 2022). The values of the OSL-QIB constants are reported in the work of (Arango et al., 2024a)

4.2 Observer implementation over the Corning channel

Once the system is expressed in the form (1), the observer design methodology presented in this paper can be implemented in simulation in the Corning channel. With this new approach based on LPV OSL-QIB modeling, the observer (5) can fully exploit the nonlinear characteristics because the LPV model is able to consider multiple operating points in the calculation of the gains. The polytopic LPV model can imply modelling errors in the transformation, so the nonlinear part is used to compensate for them.

To test the performance of the observer, three different scnearios are presented in order to test its robustness to unknown inputs, transient capture and parametric uncertainty together with measurement noise. To measure

the performance of the RUIO in each scenario, the Integral Squared Error (ISE), defined as

$$ISE = \frac{1}{t_f}\int_0^{t_f} \| e(t) \|^2 \, dt, \tag{30}$$

is used (Arango et al., 2024b), where t_f is the total simulation time in seconds.

Scenario 1: "Transient characteristics test". The first scenario aims to highlight the behavior of the observer in the presence of transients generated by changes in the inputs, and also its ability to filter sensor noise. Figure 2 shows the results obtained. Note that the observer captures the transients adequately, and its performance is not significantly affected by noise.

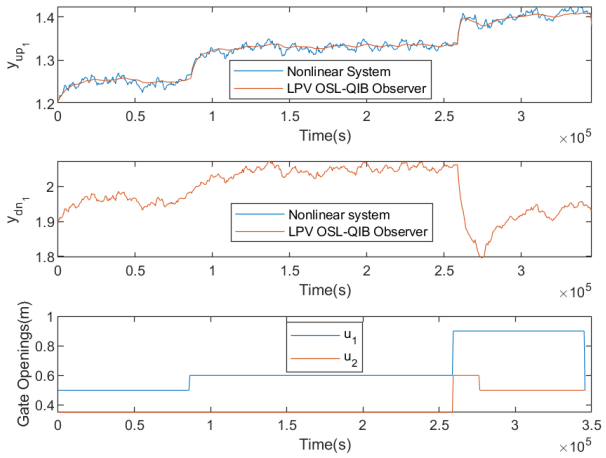

Fig. 2. LPV OSL-QIB Observer performance against inputs $ISE = 2.8 \times 10^{-4}$

Scenario 2: "Robustness against unknown inputs". This scenario considers the occurrence of leaks. It is then expected that the gates will be opened to take in a larger amount of water that compensates the leaks, preventing the canal from emptying and allowing to continue transporting water. Figure 3 shows the results obtained during the simulation. It can be observed that the observer is robust to unknown inputs and captures system transients in the presence of noise.

In the light of the results obtained so far, the UIO could be an interesting strategy to be implemented in OCIS since it is robust to various situations that may occur in practice.

Scenario 3: "Simultaneous unknown inputs and transients". The last scenario includes two possible uncertainty scenarios, where the effect of rainfall on the system (modeled as $Q_2 = Q_2 + \lambda_r Q_{tr_1}$) and leakage are considered simultaneously. In order to retain as much water as possible, the floodgates should remain closed when only water is present. When leaks occur, the flood gates should be opened to maintain the water supply in the network.

From the results presented in Figure 4, it can be concluded that the observer presents a good performance for this application case study since it is able to deal with the nonlinear dynamics of the system, while rejecting possible disturbances and unknown inputs.

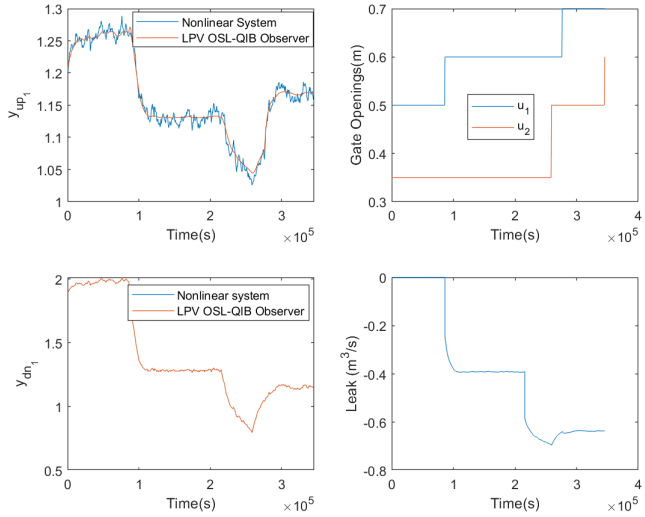

Fig. 3. LPV OSL-QIB Observer performance against unknown inputs and gate movements $ISE = 6.22 \times 10^{-4}$

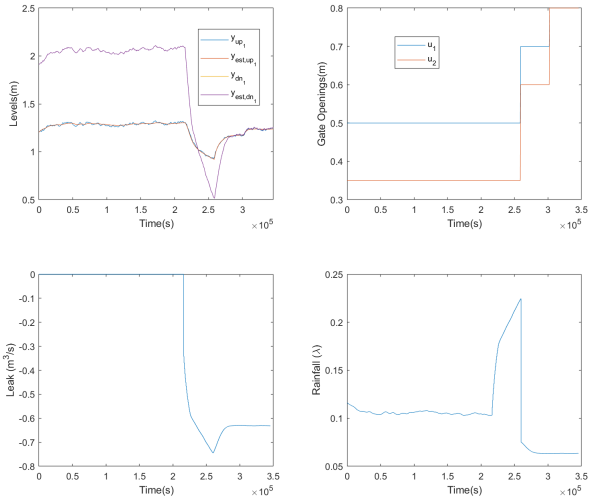

Fig. 4. LPV OSL-QIB Observer performance against leaks and rain $ISE = 8.4 \times 10^{-4}$

5. CONCLUSIONS

In this paper, an UIO was designed for a new family of nonlinear systems named LPV OSL-QIB, and its performance was tested in an open-flow channel application. The results obtained allowed to conclude that the OSL-QIB LPV UIO is an interesting alternative to perform state estimation and exhibits robustness against different situations that may arise in practice. Furthermore, it was observed that LPV systems can be coupled with nonlinear systems such as OSL-QIB for estimation purposes. The fact of applying the UIO to a case close to reality, and obtaining interesting results in terms of performance makes this algorithm interesting to test in other applications (bioprocesses, UAV, robotics, etc.).

As future work we intend to evaluate the scalability of the observer when state estimation is required for high dimensional systems (such as the complete Corning Channel).

Another interesting point would be to compare the results of the UIO LPV OSL-QIB obtained from the considered application (Pool 1 of the Corning channel) with real data or close to it.

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