

Optimal control of urban drainage systems. A case study

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Abstract

This paper deals with the use of optimal control techniques in urban drainage systems containing gates and detention tanks as well as a telemetry/telecontrol system. Optimal control is used to provide control strategies which contribute to reducing the events of flooding and polluting discharges to the environment. The case of the Barcelona system is presented.

Keywords: Real-time control; Optimal control; Combined sewer networks; Telecontrol; Flood prevention; Nonlinear optimization; Modelling; Simulation

1. Introduction

Urban drainage systems are generally networks of sewers which carry urban wastewater and rainwater to one or more terminal points, where it is treated and/or discharged to the environment. Combined sewer systems carry rain- and wastewater together. In many cities where the conurbation has been growing fast and stormy rains are frequent, the existing combined sewer systems are unable to carry all the rain- and wastewater to the treatment plants when high-intensity rain occurs. This results in flooding of certain areas and combined sewer overflows (CSO) which release untreated water to the environment.

There is an increasing concern of society with environmental protection, including, among other crucial subjects, the issue of water management in urban areas, i.e., water supply, water distribution and effluent

disposal to the receiving environment (Krebs & Larsen, 1997; Price, 2000). Optimal control, to achieve conservation, quality improvement and/or energy savings has been successfully applied to water supply and distribution in several applications (e.g., Cembrano, Brdys, Quevedo, Coulbeck, & Orr, 1988; Brdys & Ulanicki, 1994; Cembrano & Quevedo, 1999). More recently, this type of control structure is being applied in the context of advanced urban drainage (e.g., Gelormino & Ricker, 1994; Pleau, Methot, Lebrun, & Colas, 1996).

Advanced urban drainage is an attempt to solve the problems of flooding and CSO. It involves the incorporation of detention tanks, with its associated inlet and outlet gates, and other flow-diversion gates, along with the installation of a telemetry and supervisory control system. The telemetry system contains rain-gauges distributed in several areas of the city, as well as flow or level meter (limnimeters) and quality meters in the main sewers, which periodically send information to a central dispatch. The supervisory control system allows operators to monitor the sewer network and command the gate operations.

Detention tanks are used to store water during high-intensity rain and gradually release it when the sewage network is not overloaded. Flow-diversion and detention-tank gates must be actuated so as to reduce flooding and polluting discharges to the environment.

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Optimal control in urban drainage networks deals with the problem of generating control strategies ahead of time, based on current and past readings of the telemetry system, to minimize flooding and CSO.

Real-time control of an urban drainage system may be *local* or *global*. When local control is applied, flow regulation devices use only measurements taken at its specific location. While this control structure is applicable in many simple cases, in a large city, with a strongly interconnected sewerage network and a complex network of actuators and sensors, it may not be the most efficient alternative. Conversely, global control, which computes control actions taking into account real-time measurements all through the network, is likely to make the best use of the infrastructure capacity and all the available sensor information.

So far, very few cities have actually implemented real-time control of urban drainage in North America and Europe. Most of these have implemented local reactive control rather than global control. (More information on the specific locations and their current status in urban drainage control is available from the Group on Real Time Control of Urban Drainage Systems at <http://web.tiscali.it/RTCUSD>.)

The Barcelona urban drainage network contains approximately 1600 km of sewers carrying a daily volume of some 300,000 m³ wastewater in dry weather as well as rainwater (an average precipitation of approx. 600 mm a year). It includes three detention tanks (other three are under construction, to be inaugurated by 2002) with a total of 273,000 m³ capacity and 16 gates. A telemetry network containing 24 rain gauges, more than 100 limnimeters (water level sensors in the sewers) and a few quality sensors connected to a Supervisory Control and Data Acquisition system (SCADA) have been in operation since 1994. At present, real-time local control is applied at the reservoirs and diversion gates.

In the context of a collaborative project, the authors have developed a global optimal control prototype for the Barcelona urban drainage system, which is presented in this paper. Section 2 deals with the concepts of optimal control in urban drainage, Section 3 describes the modelling of the Barcelona prototype network, Section 4 presents the statement of the optimal control problem and Sections 5 and 6 refer to implementation and results, respectively. The conclusions of the work and an outline of ongoing research are included in Section 7.

2. Optimal control in urban drainage

2.1. Operational model

An operational model of an urban drainage system is a set of equations which provide a fast approximate

evaluation of the hydraulic variables of the network and its response to control actions at the gates. This type of model is useful for the computation of optimal strategies, because it makes it possible to evaluate a large number of control actions in a short computation time.

This type of model is usually referred to as a conceptual, transfer-function model (Norreys & Cluckie, 1997; Cluckie, Lane, & Yuan, 1999; Nguyen, Loong, & Woo, 2001), or as virtual-reservoir model (Ballester, Martí, & Salamero, 1998). The sewage network is modelled with a set of main sewers connecting a set of virtual (and real) reservoirs. A virtual reservoir is an aggregation of a portion of the sewage network which approximates the hydraulics of rain and sewage water retention thereof. The hydraulics of virtual reservoirs is linearized so that, in discrete time:

$$\mathbf{x}^{k+1} = \mathbf{f}(\mathbf{x}^k, \mathbf{u}^k, \mathbf{w}^k), \quad (1)$$

where \mathbf{u} represents the vector of control variables related to gate positioning (e.g., flow through the gate), \mathbf{x} is the vector of observable states: stored volumes in reservoirs (real and virtual) and flows in main sewers, \mathbf{w} is the disturbance vector containing rainfall intensities in the different catchments, \mathbf{f} is the linear function expressing the mass balance of rain intake, sewer flow and reservoir volumes in the networks. Its structure depends on the topology of the network and its parameters must be estimated using real data from the sensors in the network. In the application described in this paper, adaptive, on-line parameter identification is used. Superindexes indicate time intervals.

2.2. Optimal control goals

The optimal control goals in urban drainage systems are generally concerned with flood prevention and minimization of combined sewer overflow (CSO) to the environment. The objective of applying optimal control is to compute, ahead of time, feasible strategies for the actuators in the network which produce the best admissible states of the network, in terms of these objectives, during a certain horizon. The control period must be defined taking into account the telemetry system sampling time and the time constants of the actuators in the network. The optimization horizon must be chosen adequately considering the hydraulic time constants of storm water evacuation, reservoir depletion and/or other factors affecting the urban drainage management.

The computed strategies are based on rain forecasts through the optimization horizon, so that the predictability horizon of the rain data, a measure of how long ahead in time forecasts have acceptable degrees of confidence, bounds the choice of the optimization horizon. Furthermore, the strategies must be

recomputed periodically, ideally at each control interval, in order to adjust the model parameters and the optimization according to the new sensor readings.

2.3. Interaction with the SCADA

The optimal control application interacts with the SCADA as follows: At each control interval, it must retrieve information on:

- states and observers: reservoir volumes and main sewer levels,
- actuator status,
- alarms, and
- SCADA variables, such as communication errors and time stamps.

Then, the variable parameters of the model are updated if appropriate, using the latest readings and the optimization computes a control strategy for a complete optimization horizon. The set points for the next control interval are then sent to the actuators.

3. Application to the Barcelona prototype network

3.1. Description of the network

Optimal control has been applied to a prototype network of the Barcelona urban drainage systems. The prototype area covers 12 of the catchments in the city (out of a total of 20). It contains three diversion gates and only one real detention tank, the smallest one currently in use, with 35,000 m³ capacity. It includes the main sewer carrying water to the treatment plant and the four main seafront CSO points. Five passive flow-diversion (overflow) devices exist and the network is metered by means of five rain-gauges and 14 limnimeters. Figs. 1 and 2 show a scheme of the prototype network and its situation on the city map, respectively.

3.2. Model identification in the prototype network

The equations of this prototype network were stated using a linear-dynamics model, as described in Section 2.1. The model parameters have been estimated with historic data of water level sensors and rain gauges, using the MATLAB System Identification Toolbox. A total of 22 parameters were estimated using the sensor data, collected every 5 min, of several rain events. Several identification methods were tested and the recursive least-squares (RLS) methods were retained. This identification method provided a good accuracy in all the parameters according to RMS error and parameter variance. As an example, Fig. 3 shows the comparison between the real data of limnimeter L₃₉ and the model prediction of water level downstream of

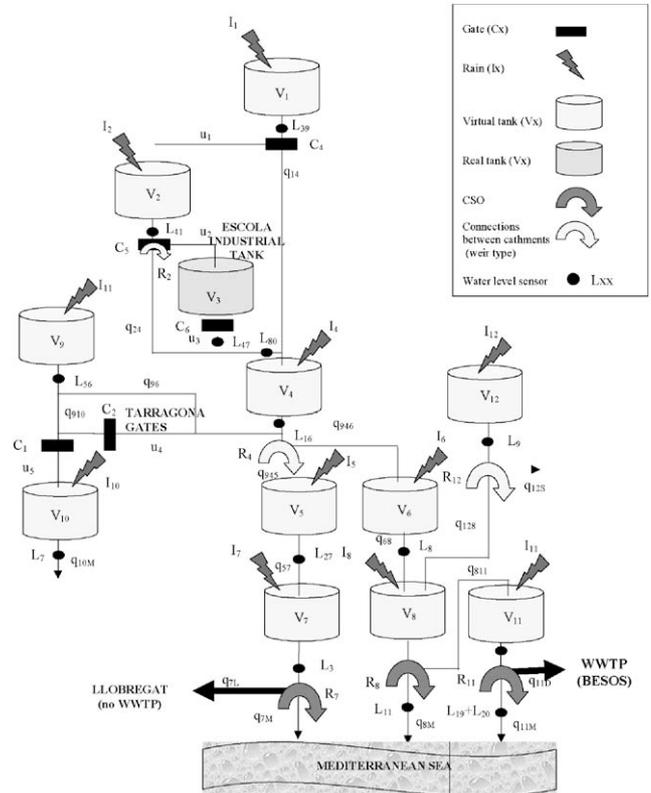


Fig. 1. Reservoir model of the Barcelona prototype network.

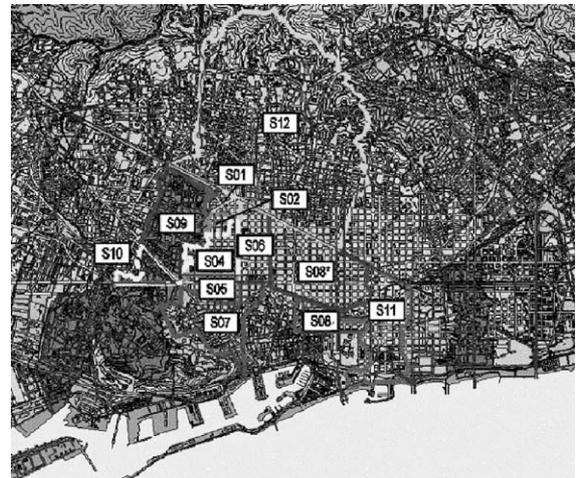


Fig. 2. Prototype network map in Barcelona.

virtual reservoir No.1, for a 30-min prediction horizon. The model prediction of the downstream water level shows a good adjustment to the real values measured by the limnimeter. The use of a recursive version of the parameter identification method makes possible to implement on-line model calibration, when the application is running on the real system, based on SCADA readings.

The optimal control application has been implemented off-line in this network, in order to assess the

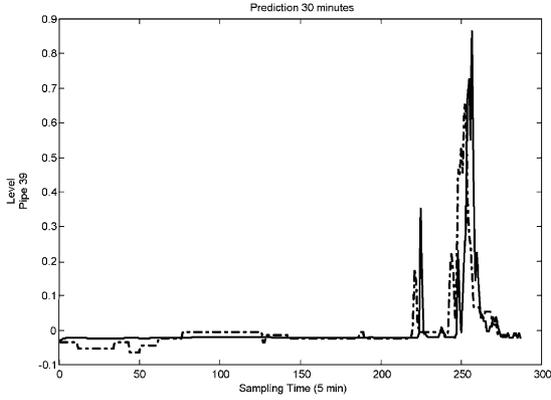


Fig. 3. Thirty-minutes prediction (solid) versus real measures (dashed) for limnimeter I₃₉.

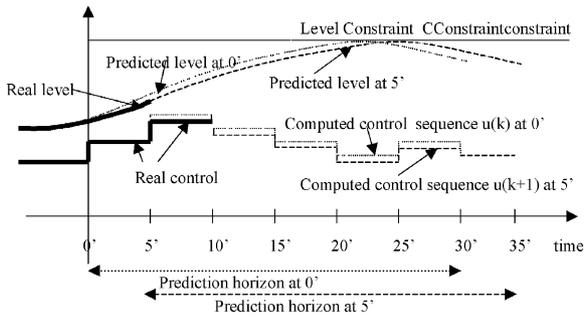


Fig. 4. Predictive control strategy.

feasibility of optimal control in the complete network. This constitutes a first step towards the implementation of real-time optimal control in the complete network.

The computation of the optimal control setpoints to be applied at the actuators is based on predictive control (Camacho & Bordons, 1999; Maciejowski, 1999; Mosca, 1995). More precisely it applies the receding horizon strategy. This consists of determining a virtual control input sequence of present and future values ($u^k, u^{k+1}, \dots, u^{k+N-1}$) that optimizes an open-loop performance function, according to a prediction of the system evolution over the horizon N . This prediction is performed assuming that disturbances (rain measures) and model parameters (no adaptation) will keep constant during the horizon. However, only the first control input of sequence (u^k) is actually applied to the system, until another sequence based on more recent data is computed as shown in Fig. 4. The same procedure is restarted at time $k + 1$, using the new measurements obtained from sensors and the new model parameters obtained from the recursive parameter estimation algorithm that is working in parallel. The resulting controller belongs to the class called *open-loop optimal-feedback control*. As the name suggests, it is assumed that feedback is used, but it is computed only on the basis of the information available at the present time.

In the Barcelona prototype, a sampling time (and control interval) of 5 min is used, based on actuator response times and on the scanning time of the telemetry system. An optimization horizon of 30 min is chosen, taking into account the time constants of reservoir depletion and the average transportation time from the inlets to the terminal points of the drainage system.

4. Statement of the optimal control problem

4.1. Cost function

The cost function is the mathematical expression of the urban drainage management goals. In this case, flood prevention is the first priority. Secondly, CSO reduction must be sought, albeit without compromising flood prevention and, finally, the network must drain as much water as possible, provided this does not interfere with any of the first two objectives.

A multi-objective cost function is then chosen as the summation over the optimization horizon of a weighted sum of the following terms:

- A sum of quadratic functions of the discrete-time instantaneous flow (one for each sewer in the prototype network) penalizing positive deviations from the sewers design flows.
- A sum of overflow volumes to the sea at each time interval.
- The instantaneous volume in the real reservoir (detention tank).

The expression of the cost function is as follows:

$$J = \sum_{k=0}^{N-1} \sum_j (q_j^k - q_j^*)^2 + \alpha \sum_l CSO_l^k + \beta \sum_i (V_i^k) + \beta \sum_i V_i^N, \quad (2)$$

where N is the optimization horizon in number of sampling periods, k indicates the time period, j is an identifier for each sewer in the prototype network, q_j^k is the flow through sewer j at sampling time k and superindex $*$ denotes the desirable value for this flow, according to its design criteria; l identifies the combined sewer overflow sites, CSO_l^k is the combined sewer overflow volume of site l at time k , i identifies the real reservoirs, V_i^k is the volume stored in reservoir i at time k , α and β are the relative weights associated to the second and third objective, respectively. The quadratic shape of the penalty terms, as well as their associated weighting factor, guarantee that the flood prevention goal takes priority over the other two. The relative weights of the second and third terms are chosen so as to minimize CSO preferentially over maximizing water evacuation.

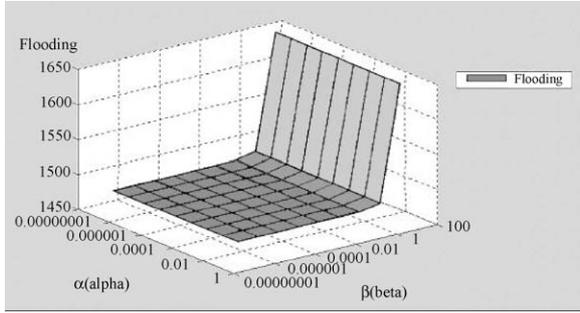


Fig. 5. Parameter influence on flooding.

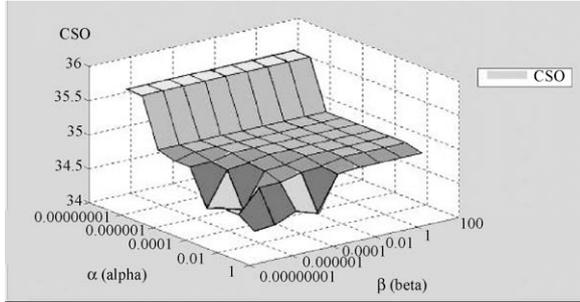


Fig. 6. Parameter influence on CSO.

Figs. 5 and 6 show the influence of the choice of the parameters on the fulfillment of the control objectives. Because of specific characteristics of this network, such as the flow limitation in the seafront interceptor sewer and the relatively low impact of the real reservoir on the lower areas of the network, the main improvement to be expected during heavy rain is flood reduction. Conversely, during light rain, which may be evacuated without significant flood volumes, optimal control will provide improvements on CSO. Fig. 5 shows the influence of alpha and beta on the optimal flooding volume during a heavy rain episode. Fig. 6 shows the influence of the choice on the optimal CSO during a light rain. Both figures use a logarithmic scale for the parameters.

The first plot shows that, as expected, the optimal value of flood volume is independent of the alpha value (in this parameter variation interval) and that there is an upper bound on beta (0.01) in order to keep the optimal flood value low. From the second plot, it may be concluded that, for a small rain, it is interesting to keep alpha greater than a lower bound (in this case, 0.001) and beta lower than an upper bound (in this case, 0.0001), in order to keep CSO as low as possible. The combination of these criteria makes it possible to choose the cost function parameters adequately.

4.2. Constraints

The performance index can only be minimized with respect to the control variables (flows through the gates), since these are the only variables which may be

acted upon. However, the value of the index also depends on variables that are not directly controlled, but must be evaluated for each control action. Similarly, the constraints apply both on directly controlled variables and on other variables related to elements which cannot be operated directly.

The optimization problem is constrained by:

- A large number of linear equality equations expressing the mass balance, including rain intake, in the reservoirs and the sewers.
- A reduced number of nonlinear equality flow equations related to the overflow devices.
- Bound constraints on the operative range of gates, on the allowable flows through the sewers and on maximum reservoir capacities,

which may be written as follows:

$$\begin{aligned}
 \mathbf{Ax} + \mathbf{Bu} + \mathbf{Cw} &= 0, \\
 \mathbf{g}(\mathbf{x}) &= 0, \\
 \mathbf{x}_{\min} &\leq \mathbf{x} \leq \mathbf{x}_{\max}, \\
 \mathbf{u}_{\min} &\leq \mathbf{u} \leq \mathbf{u}_{\max},
 \end{aligned} \tag{3}$$

where \mathbf{x} , \mathbf{u} and \mathbf{w} are the state, control and disturbance vectors, respectively, as defined in Section 2.1, \mathbf{A} , \mathbf{B} and \mathbf{C} are the coefficient matrices affecting \mathbf{x} , \mathbf{u} , and \mathbf{w} in the mass balance equations, \mathbf{g} is a nonlinear function of the flow and \mathbf{x}_{\min} , \mathbf{x}_{\max} , \mathbf{u}_{\min} and \mathbf{u}_{\max} are bounds on the states and the controls, respectively.

4.3. Initial conditions and disturbances

The initial conditions for the optimization are the current reservoir levels, to be obtained from the SCADA at each execution. The disturbances are rain forecasts for the optimization horizon at each catchment.

5. Optimal control in the prototype network

5.1. Individual optimizations

At each control interval, strategies are computed for the optimization horizon. In this application, the individual optimization problems are solved using a commercial library (GAMS, 1997).

The optimization method used to solve the problem is a generalized reduced gradient search, first suggested by Abadie and Carpentier (1969), implemented as part of the GAMS library, which can cater for the nonlinear performance index and constraints. It starts by finding a feasible solution; then, an iterative procedure follows, which consists of:

- finding a search direction, through the use of the Jacobian of the constraints, the selection of a set of

basic variables and the computation of the reduced gradient;

- performing a search in this direction, through a pseudo-Newton process;

until a convergence criterion is met. A description of the algorithm and its implementation may be found in GAMS (1997). A more detailed version thereof is reported in Drud (1985, 1992).

5.2. Long-horizon simulation

In order to test the efficiency of the optimal control application in real rain scenarios, a simulator of the real-time implementation was developed. This application runs a configurable number of individual optimizations, linking runs to each other by an iterative process of:

- Retrieving information about the state of the system. In real-time operation this is provided by the SCADA; in this off-line simulation, the states are computed using the system model.
- Retrieving current rain intensity information corresponding to each catchment. In real-time operation, this is also provided by the SCADA; in the off-line simulation, historic records of real rain scenarios are used.
- Running an optimization for a half-hour horizon assuming constant rain.
- Applying the first 5 min interval control action. Storing system state information after this control action.

The process may be run for user-defined horizons. In general, it is run for a horizon that covers a complete rain event, which may be of the order of a few hours.

6. Results

By using the above-described simulator, the optimal control system was tested in a number of real rain scenarios. The objective of the experiments was to compare the results of applying optimal control to those of using the original drainage system (with no active elements) and those of using the new detention reservoir passively, i.e. without commanding the inlet and outlet gates but keeping both of them open at all times.

The results are analyzed by studying, for each scenario, the evolution of: total flood volume, stored volume at the reservoir, volume sent to the treatment plant and CSO. The following tables and figures refer to two examples taken from historic records of 1999: a heavy rain, of 4-year average return period with peaks of 12-year return period and a light-rain of 0.7-year average return period with peaks of 10-year return period.

Figs. 7 and 8 show the evolution of the stored volume at the real reservoir, along with the rain intensity plot for the heavy and the light rain, respectively. The plots show the evolution of the reservoir used passively and with optimal control. In Fig. 7, it is clear how optimal control takes more advantage of the reservoir storage capacity than passive use, i.e., it keeps it fuller for a longer time.

Fig. 8 shows that the optimal reservoir evolution for light rain is close to the passive situation, because keeping the inlet and outlet gates open is actually very close to the optimal strategy for this rain condition.

Tables 1 and 2 summarize the results concerning flood volume, total volume of treated water and CSO for both rain scenarios. When analyzing these results, it is important to bear in mind that the prototype network includes just one detention reservoir (35,000 m³ storage capacity) out of six (273,000 m³ storage capacity) to be in operation shortly. Additionally, the geographical location of this reservoir relative to the CSO precludes attaining important CSO reductions in heavy-rain conditions, because several catchments with important rain intake volumes exist downstream of the reservoir, which drain water to the CSO sites.

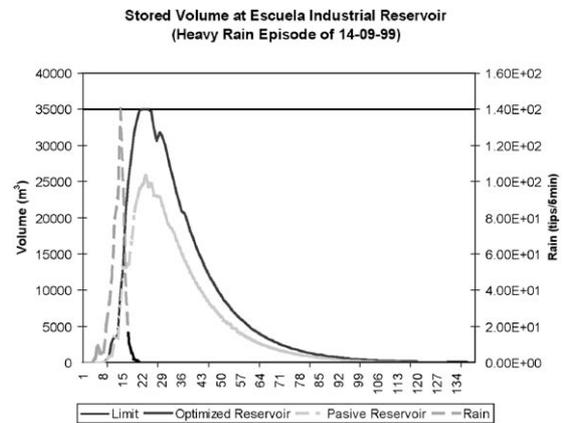


Fig. 7. Reservoir volume with heavy rain.

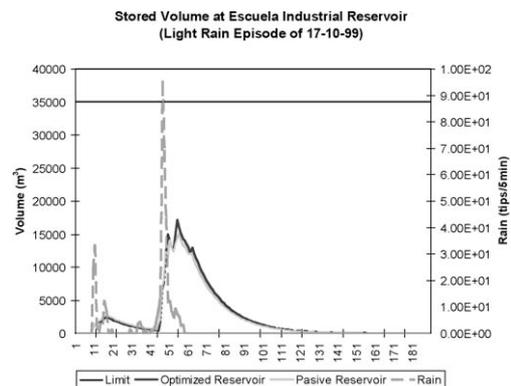


Fig. 8. Reservoir volume with moderate rain.

Table 1
Results with heavy-rain scenario

Heavy Rain Episode (14-09-99)			
	Passive no reservoir	Passive with reservoir	Optimized
CSO m ³	1,030,026	1,012,491	1,006,290
Improvement %		1.30	2.3
Treated Water m ³	117,243	129,234	133,062
Improvement %		10.22	13.49
Flooding m ³	184,859	137,295	128,478
Improvement %		25.65	30.42

Table 2
Results with light-rain scenario

Light Rain Episode (17-10-99)			
	Passive no reservoir	Passive with reservoir	Optimized
CSO m ³	465,489	444,204	443,043
Improvement %		2.77	3.02
Treated Water m ³	139,614	148,689	149,25
Improvement %		6.50	6.90

Nevertheless, some very encouraging results have been obtained. In the heavy-rain scenario (Table 1):

- An important flood reduction is achieved by using the reservoir (even passively) and a significant improvement is obtained by applying optimal control of the reservoir.
- A slight (for the above-explained reasons) improvement in CSO is obtained by using the reservoir passively and optimal control produces a further reduction.
- The use of the reservoir also increases the total amount of water sent to the treatment plant (otherwise released untreated to the environment) even in passive operation, but the use of optimal control improves this value significantly.

In the light rain scenario (Table 2), when flooding is not a problem:

- A moderate reduction of CSO is achieved by using the reservoir passively and a similar reduction is obtained with optimal control, because both strategies are very similar in this rain condition.
- A similar improvement is shown in the increase of treated water volumes.

The increase in the total water volumes sent to the treatment plant is achieved by delaying the release of a certain amount of water until after the rain has ceased, as shown in the reservoir evolutions of Figs. 7 and 8. This fact shows that optimal control also improves the usage of the infrastructure in that it takes fuller

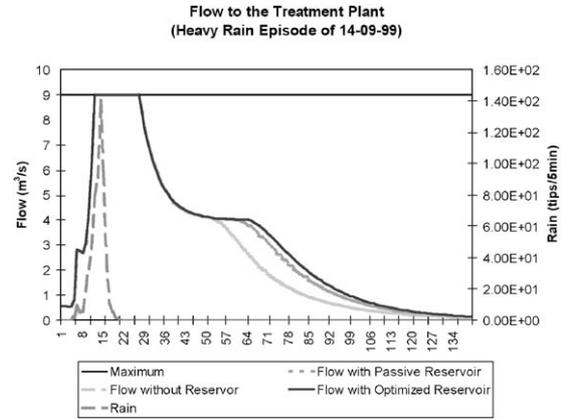


Fig. 9. Flow to the treatment plant in a heavy-rain scenario.

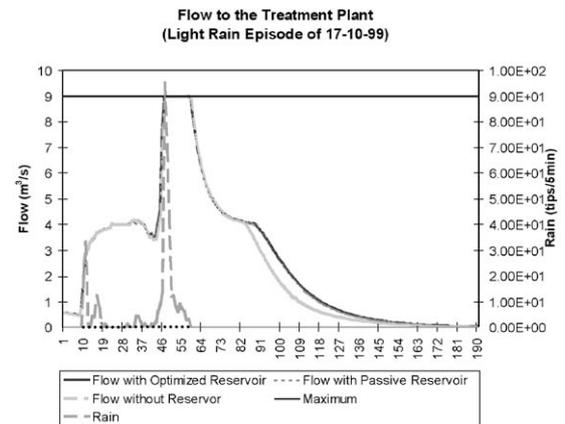


Fig. 10. Flow to the treatment plant in a light-rain scenario.

advantage of the water treatment plant capacity after a rain event. This may be seen more clearly in Figs. 9 and 10, which show the evolution of the flow sent to the treatment plant in both rain scenarios.

7. Current work: on-line implementation

After the validation of the on-line prototype object of this paper, the on-line implementation is currently underway. A computer-aided control software, named CORAL (Spanish acronym for Optimal Control of Sewer Networks), has been implemented. The features of this software include: model management; rain event reproduction; rain event simulation; mass balances computation, all of them running off-line, and a global control application, running on-line, in charge of: SCADA communications; parameter estimation; error detection and correction; optimal strategies determination and a real-time control interface. A scheme of the architecture is represented in Fig. 11.

The global control application flow-chart is represented in Fig. 12. Every 5 min, the application first reads

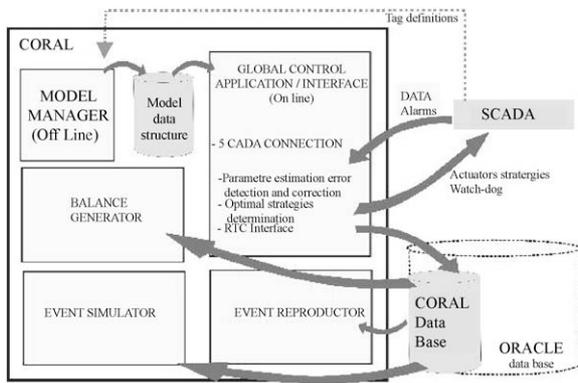


Fig. 11. CORAL architecture.

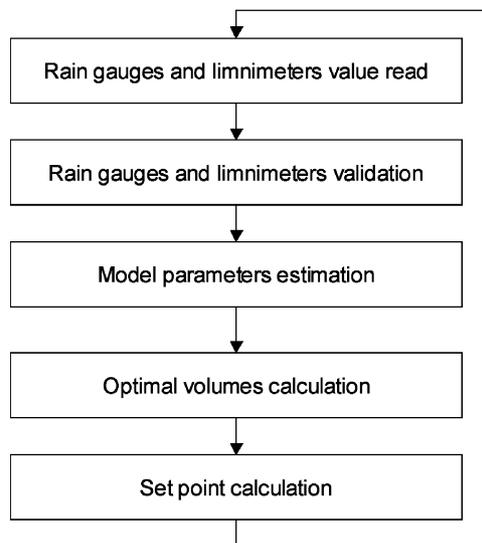


Fig. 12. Control application flow-chart.

all sensor data, provided by the SCADA system asynchronously. Then, it validates the sensor values in order to detect and replace or dismiss corrupt data. Afterwards, the model parameters are tuned and the optimization is run. Finally, the optimization gate-opening set points are computed for the next 5 min interval. This application starts automatically when a rain event occurs.

8. Conclusions

Optimal control in urban drainage systems is an efficient means of taking full advantage of a drainage network capacity in order to reduce flooding and CSO. The optimal control application described in this paper shows that significant reductions in flooding and CSO may be achieved, even in a prototype network with limited control possibilities (few controls, relatively small reservoir capacity, geographic distance between control and CSO points).

The predictive control set-up reduces the impact of the limited validity of rain forecasts on the computed strategy, by updating strategies as soon as new readings are available from the telemetry system.

The on-line implementation of the real-time optimal control in the area covered by the prototype is currently underway in the Barcelona urban drainage system. After this system has been validated in real operation, the extension to the complete network is envisaged.

Acknowledgements

This work has been partly funded by the CICYT (Project COO1999 AX072) Authors J. Quevedo and V. Puig are members of CERCA (Association for the Study and Research in Automatic control) and of LEA-SICA (Associated European Laboratory on Intelligent Systems and Advanced Control).

The contribution of the CLABSA staff in supplying data and validating the experiments is gratefully acknowledged.

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