Tensegrity frameworks: Dynamic analysis review and open problems

Josep M. Mirats Tur [1]

Institut de Robòtica i Informàtica Industrial (IRI), CSIC-UPC, Barcelona, Spain

Sergi Hernàndez Juan [2]

Institut de Robòtica i Informàtica Industrial (IRI), CSIC-UPC, Barcelona, Spain

Abstract

The main objective of this paper is twofold. First, to conclude the overview about tensegrity frameworks, started by the same authors in a previous work, covering the most important dynamic aspects of such structures. Here, the most common approaches to tensegrity dynamic modeling used so far are presented, giving the most important results about their dynamic behavior under external action.

Also, the main underlying problems are identified which allow the authors to give a clear picture of the main research lines currently open, as well as the most relevant contributions in each of them, which is in fact the second main objective of this paper. From the extensive literature available on the subject, four main areas have been identified: design and form-finding methods which deal with the problem of finding stable configurations, shape changing algorithms which deal with the problem of finding stable trajectories between them and, also control algorithms which take into account the dynamic model of the tensegrity structure and possible external perturbations to achieve the desired goal and performance.

Finally, some applications of such structures are presented emphasizing the increasing interest of the scientific community on tensegrity structures.

Key words: tensegrity frameworks, dynamic analysis, open problems
PACS: 05.45.-a, 45.40.Ln, 45.50.j
1991 MSC: 70C20, 70B15

Email addresses: jmirats@iri.upc.edu (Josep M. Mirats Tur),
shernand@iri.upc.edu (Sergi Hernàndez Juan).

[1] Josep M. Mirats Tur is a full time researcher.
[2] Sergi Hernàndez is currently a PhD student.

1 Introduction

The word tensegrity is an abbreviation for *tensile integrity* which was coined by Buckminister Fuller in the early 60's [26]. Tensegrity were created by people coming from the art community (Snelson [81]) being rapidly applied to other disciplines such as in the architectural context, for structures such as geodesic domes (Fu [25]) or later in space engineering to develop deployable antennas (Tibert [94]). A general definition for a tensegrity was given by Pugh [70]:

> *A tensegrity system is established when a set of discontinuous compressive components interacts with a set of continuous tensile components to define a stable volume in space.*

Here, the compressive elements (struts) can not decrease their length while the tensile elements (cables) can not increase it. In fact, there may exist a third kind of element, namely a bar, which can not vary its lenght. Some examples of tensegrity structures are given in Fig. 1.

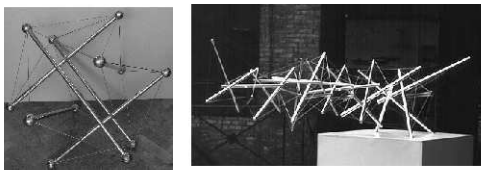


Fig. 1. Examples of tensegrity structures.

In a recently published effort, Hernàndez and Mirats-Tur [31] presented different existing definitions for tensegrity structures, as well as their main properties and a deep static analysis review. Tensegrity structures have already been shown to have superior features than traditional approaches in areas like architecture or civil engineering. Some of their properties, such as high energetic efficiency (energy is stored in the structure's members as tension and compression forces), deployability, deformability and redundancy, as well as their biological inspiration (cell and protein interaction models, see Ingber [32]), make this kind of structures good candidates to design both mobile robots and manipulators.

Up to now, tensegrity structures have been mainly used for static applications where the length of all members is kept constant and actuation is performed only to compensate for external perturbations. In the last decades, the tensegrity framework has been also used to build deployable structures, although the tensegrity paradigm has not been fully exploited either. It is not since very recent years that we find some relevant works towards this goal: for instance, Aldrich [2] put together several simple tensegrity structures to build a redundant manipulator robot. Paul et al. [64] and Masic and Skelton [47] proposed different self-propelled tensegrity architectures to build mobile robots, but only Paul et al. [64] managed to build a working prototype.

In this paper authors give a comprehensive review to tensegrity dynamic issues and explore the performed research on this area so as to enumerate the currently open problems. First of all, to fully understand this kind of structures, it is necessary to complete the static analysis presented by Hernàndez and Mirats-Tur [31]. Section 2 presents an overview of the most common approaches to tensegrity dynamic modeling used in the available literature, as well as obtained results about their dynamic behavior under external action derived from those models.

After a full theoretical overview of tensegrity structures, it will be possible to identify the underlying problems of such structures, which shall be presented in section 3. Following, sections 4 through 6, present the most relevant literature available for each of the identified open problems, paying special attention to how the open problems in section 3 are being currently interpreted and tackled. Next, different interesting applications of tensegrity structures are given in section 7 and finally, the most important conclusions of this work are outlined in section 8.

2 Dynamic analysis

Most important research on tensegrity dynamics has been mainly focused on the study of the force-displacement relationship, that is, how the structure changes its shape under the action of external forces, studying, for instance, oscillation damping and frequency or geometric deformations. Section 2.1 presents the different approaches used to characterize the dynamic behavior of tensegrity structures. Next, some important general results on the dynamics of such structures are given in section 2.2.

Please note that this section does not intend to be a complete review of all research efforts produced during the last decades in this area. It only contains the more relevant approaches, always from authors' point of view, so as to maintain the paper self-contained.

2.1 Force-displacement relationship

The study of the force-displacement relationship is essential to understand how the structure behaves when subjected to external perturbations as well as to find out the new stable configuration under such conditions. This study has been carried out from two different points of view. First, considering the structure is at an equilibrium configuration and can only experience small perturbations around it. Second, and more general, considering the structure is at an arbitrary position and may experience large deformations.

Obviously, the first approach is simpler. It does not take into account any dynamic issue, instead, only the geometry of the tensegrity structure around an equilibrium position is used to find out the force-displacement relationship. These kind of methods are used to find the final stable configuration of a tensegrity under external perturbation, but not the way the structure changes its shape to reach that configuration.

For instance, Pellegrino [66] use the equilibrium condition $\underline{\underline{R}}(\underline{p})^T\delta\underline{\omega} = \delta\underline{F}_{ext}$ and the compatibility condition $\underline{\underline{R}}(\underline{p})\delta\underline{p}' = \delta\underline{d}$ for small perturbations around an equilibrium configuration to find out how the structure behaves under such perturbations. Here, $\underline{\underline{R}}(\underline{p})$ is the rigidity matrix of the structure, $\underline{\omega}$ the stresses present in the edges, $\underline{p}'$ the nodal displacements, $\underline{d}$ the edge's elongations and $\underline{F}_{ext}$ the external forces on the nodes. This approach works fine for structures either kinematically or statically indeterminate but it does not work for tensegrity structures which are, in general, both kinematically and statically indeterminate. The reason for this can be seen from the general (particular + homogeneous) solution of the equilibrium equation,

$$\underline{\omega} = \overbrace{\underline{\omega}_i(\delta \underline{F}_{ext})}^{particular} + \overbrace{\underline{\underline{SS}}\,\underline{\alpha}(\delta \underline{F}_{ext})}^{homogeneous}, \tag{1}$$

and the compatibility condition or nodal displacements,

$$\underline{p}' = \overbrace{\underline{p}'_i(\delta \underline{d})}^{particular} + \overbrace{\underline{\underline{M}}\,\underline{\beta}(\delta \underline{d})}^{homogeneous}, \tag{2}$$

where the columns of the matrix $\underline{\underline{SS}}$ are base vectors for the space of all possible self-stress states (self-stress states are the solution of the linear equations $\underline{\underline{R}}(\underline{p})^T \delta\underline{\omega} = 0$) and the columns of $\underline{\underline{M}}$ are base vectors of mechanisms (a mechanism is a deformation of the framework, which may be finite if there is a perceptible change in the position of the nodes, or infinitesimal if there is a first or second order change in the nodes position). $\underline{\alpha}(\delta \underline{F}_{ext})$, $\underline{\beta}(\delta \underline{d})$ are the coefficients for the linear combination in terms of $\delta \underline{F}_{ext}$ and $\delta \underline{d}$ respectively, and $\underline{\omega}_i(\delta \underline{F}_{ext})$, $\underline{p}'_i(\delta \underline{d})$ are the particular solutions for the given external forces and edge lengthening.

From Eq. 1 and Eq. 2 it is easy to see that both the linear combination coefficients and the particular solutions depend on the external perturbations. Hence, without additional information, it is not possible to exactly know how the tensegrity structure will behave under such perturbations. In this direction, Oppenheim and Williams [60] proposed a method which takes into account the fact that all equilibrium configurations have minimum energy (Connelly [13]). Therefore, by minimizing an energy function associated to the structure in terms of the nodal configuration and edge stresses, it is possible to find out the new stable configuration under a given set of external perturbations. Oppenheim and Williams [60] worked with a symmetric structure, shown in Fig. 2, which allowed them to obtain the force-displacement relationship using geometrical considerations in terms of a unique parameter θ, which is the relative rotation between the top (ABC) and bottom (abc) platforms. In this way, they avoided the use of the equilibrium matrix.

More general methods use either the Newtonian or Lagrangian formulation to get force-displacement relationships which are valid in any configuration. The simpler of the two formulations, and also the first one to appear, is the Newtonian formulation which is mainly based on Newton's second law of motion:

$$\underline{F} = \underline{\underline{M}}\,\underline{\ddot{p}},$$

where $\underline{F}$ is the net force and torque applied to the rigid bodies, $\underline{\underline{M}}$ is the generalized mass matrix and $\underline{\ddot{p}}$ is the acceleration of the generalized coordinates. This equation connects the domain of forces and energies with that of

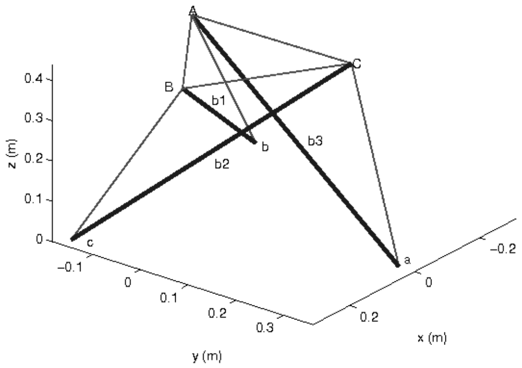


Fig. 2. Tensegrity structure. Black thick lines represent bars and thin gray lines represent cables.

positions and accelerations. Usually, energy and, linear and angular momentum conservation laws are used to avoid dealing directly with the rigid bodies forces and accelerations.

In the particular case of tensegrity structures the system of rigid bodies is formed by the rigid bars. They may be completely characterized by the Euclidean position of a given point (i.e. the center of mass) and its orientation ($\underline{p}_i = (x_{cm_i}, y_{cm_i}, z_{cm_i}, \theta_{x_i}, \theta_{y_i}, \theta_{z_i})^T$), or, alternatively, by the Euclidean position of each vertex ($\underline{p}_{ij} = (x_i, y_i, z_i, x_j, y_j, z_j)^T$) assuming the mass of each bar to be concentrated in its vertices. Both approaches have some numerical problems associated to the inversion of the generalized mass matrix. While the first one may provide more accurate results, the second approach avoid the ill definition of rotations and angular velocities of the bar.

Whatever the approximation used, forces acting on each bar are the tension of the cables attached to them, and any possible external force applied to the nodes of the structure. Regarding cable tension, it may be modeled in different ways, for instance as a simple linear spring ($F^{cable} = k(l - l_0)$), or as a mass-spring system ($F^{cable} = k(l - l_0) + D\dot{l}$, where k is the spring stiffness constant, l its actual length, l_0 is its rest length and D its viscous damping coefficient).

In order to simplify as much as possible the non-linear dynamic model of a tensegrity structure some hypothesis are commonly used: cables are assumed massless and linear elastic; bars are assumed rigid and rod like. Some shape symmetries (equal length members for example) and constraints (coplanar points for example), are also used to reduce the number of variables. Also, in general, no gravitational fields are considered.

Several authors have used the Newtonian formulation to model the dynamic behavior of a tensegrity structure. As a matter of fact, Kanchanasaratool and Williamson [37] studied a 6-bar tensegrity platform. They developed a simplified particle system model with unitary point masses at the nodes which are subject to geometric constraints (no three points on the top are collinear and the lower bar nodes are fixed). In order to guarantee that the model constraints are satisfied, a constraint force, $\underline{F}^{const}$, is added to each node. In this case, the formulation for each vertex is

$$\ddot{\underline{p}}_k = \sum_j \underline{F}_{jk}^{cable}(\underline{p}, \dot{\underline{p}}) + \underline{F}_k^{const}(\underline{p}),$$

where the summation is over all adjacent vertices.

Skelton et al. [79] also found a simplified version of the Newtonian formulation for a class-1 (at most one bar is attached at each node) tensegrity shell. They used the first and second order constraints on the bar lengths,

$$\begin{aligned} \tfrac{dL_{bar}}{dt} &= 0 \\ \tfrac{d^2L_{bar}}{dt^2} &= 0, \end{aligned}$$

to formulate a non-linear system of equations in terms of $\ddot{\underline{p}}$, and then found an analytical expression for it by solving the system of equations. Later on, Skelton [74] extended this work to be applicable to any tensegrity structure with n rigid bars. More recently, Skelton [75] presented a similar work but using matrix differential equations instead of vector differential equations. In this work, a matrix with the coordinates of each node in each column instead of a unique column vector is used.

The research based on Newtonian formulation presented so far use the complete non-linear dynamic model of the tensegrity structure. However, for more practical applications it is enough to use the linearized version of the model around an equilibrium configuration. For example, de Jager and Skelton [16], still using a Newtonian formulation, gave the tensegrity dynamic model in the state space notation,

$$\begin{aligned} \dot{\underline{x}} &= f(\underline{x}, \underline{v}, \underline{u}) & & & \dot{\underline{x}} &= \underline{\underline{A}}\,\underline{x} + \underline{\underline{B}}_v\,\underline{v} + \underline{\underline{B}}_u\,\underline{u} \\ \underline{z} &= g(\underline{x}, \underline{v}, \underline{u}) & & \xrightarrow[\frac{d}{dt}]{linearization} & \underline{z} &= \underline{\underline{C}}_z\,\underline{x} + \underline{\underline{D}}_{zv}\,\underline{v} + \underline{\underline{D}}_{zu}\,\underline{u}\,, \\ \underline{y} &= h(\underline{x}, \underline{v}, \underline{u}) & & & \underline{y} &= \underline{\underline{C}}_y\,\underline{x} + \underline{\underline{D}}_{yv}\,\underline{v} + \underline{\underline{D}}_{yu}\,\underline{u} \end{aligned} \tag{3}$$

where $\underline{x}$ is the state vector, $\underline{v}$ the perturbation vector, $\underline{u}$ the control vector and $\underline{z}$ and $\underline{y}$ the output and measure vectors (measures from sensors on the structure) respectively. The linear model on the right hand side of Eq. 3 was derived by linearizing the non-linear one in the left hand side.

However, most of the dynamic models found in the literature obtained for tensegrity structures are based on Lagrangian formulation. Joseph Louis Lagrange worked during several years into a reformulation of Newton's classical mechanics, which was finally introduced in 1788 [39]. With this new formulation, the movement of a mechanical system can be described as the solution of a system of second order differential equations, called the *Euler-Lagrange* equations. These are derived from an scalar function, the so called *Lagrangian* of the system, see Eq. 4, which, for classical mechanics, is the difference between the kinetic and potential energies. This formulation considerably simplifies some physical issues: there are forces actuating on the system, sometimes unknown a priory (as for instance non conservative forces), which don't have to be taken into account, as they appear *mathematically*.

$$\mathcal{L} = \mathcal{T} - \mathcal{U} \tag{4}$$

Now, using the minimum action principle which intuitively states that the evolution of any physical system will follow the path of minimal action, Eq. 4 can be formulated as

$$\frac{d}{dt}\left(\frac{\delta \mathcal{T}}{\delta \underline{\dot{p}}}\right) - \frac{\delta \mathcal{T}}{\delta \underline{p}} + \frac{\delta \mathcal{U}}{\delta \underline{p}} + \frac{\delta \mathcal{D}}{\delta \underline{\dot{p}}} = \underline{F}, \tag{5}$$

where $\underline{p}$ is the vector of generalized coordinates, $\mathcal{D}$ is the factor for dissipative energies and $\underline{F}$ is the vector of forces and torques applied to the structure. These equations are valid under the assumption of independent generalized coordinates, although other sets of dependant coordinates are possible with the use of Lagrange multipliers.

As, in general, cables are assumed massless, the kinetic energy only takes into account the translational and rotational energy of the b structure bars:

$$\mathcal{T} = \frac{1}{2}\sum_{i=1}^{b}\left(\underline{\dot{q}}_i^T m_i \underline{\dot{q}}_i + \underline{\dot{\theta}}_i^T \underline{\underline{J}}_i \underline{\dot{\theta}}_i\right) \tag{6}$$

Here we assume a general coordinate vector $\underline{p}_i^T = (\underline{q}_i^T, \underline{\theta}_i^T)$, $\underline{q}_i^T$ containing the exact position of the bar's center of mass, and $\underline{\theta}_i^T$ it's orientation. $\underline{\underline{J}}_i$ is the inertia matrix referenced to the $\underline{q}_i^T$ coordinates.

The potential energy term accounts for the elastic energy stored in the cables or struts as well as the structures' potential energy due to its position in a gravitational field applied to the n nodes:

$$\mathcal{U} = \frac{1}{2}\sum_{i=1}^{e} k_i \Delta l_i(\underline{p})^2 + \sum_{i=1}^{n} m_i g q_z. \tag{7}$$

where e is the number of edges, k the stiffness coefficient and Δl the length increment for each edge. As a matter of fact, in most of the reviewed literature a gravitational field has not been taken into consideration thus only accounting for the elastic energy stored in the cables or struts.

Finally, the dissipative energy factor collects all energy losses due to kinetic damping on cables and struts, kinetic friction of the bar joints, etc.,

$$\mathcal{D} = \frac{1}{2}\sum_{i=1}^{e} \underline{\dot{p}}_i^T \underline{\underline{C}}_i \underline{\dot{p}}_i + \sum_{i=1}^{n} \mu_i F_{N_i} + \cdots. \tag{8}$$

In this last expression, $\underline{\underline{C}}_i$ is the matrix containing the damping coefficients for each edge and μ_i the friction coefficient for each of the nodes.

Hence, using Eqs. 5 to 8 it is possible to obtain the most general statement of the Lagrange formulation for a tensegrity structure, which is shown in Eq. 9. Please, refer to Sultan [83] for a detailed derivation of these equations and the associated matrices.

$$\underline{\underline{M}}(\underline{p})\underline{\ddot{p}} + \underline{\underline{V}}(\underline{p}, \underline{\dot{p}})\underline{\dot{p}} + \underline{\underline{K}}(\underline{p})\underline{p} = \underline{F}. \tag{9}$$

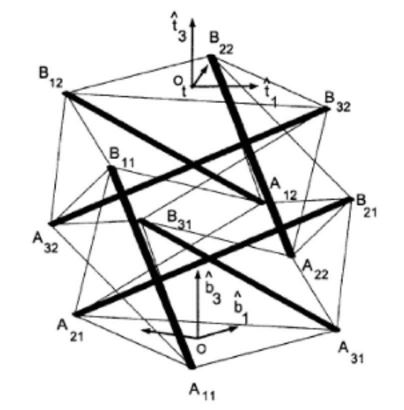
Motro et al. [52, 53], performed the first studies on how tensegrity structures dynamically behave when external loads are applied on the structure nodes. Their main objective was to determine the transfer function between an input excitation and the structure oscillations. This was achieved by using a second order dynamic model, such as the one in Eq. 9, around an equilibrium position, hence assuming constant matrices.

Sultan [83] and Sultan et al. [86, 87] used Lagrange formulation for a particular class of tensegrity known as SVD (Saddle, Vertical and Diagonal cables) shown in table 1. They exploited the high degree of symmetry of these structures in order to simplify the dynamic model, and also found the linearized version of the dynamic model around an equilibrium configuration $\underline{p}_e$.

Murakami [55, 56], Murakami and Nishimura [58, 57] also used Lagrangian formulation to model the dynamic behavior of a tensegrity structure taking into account additional non-linear effects, such as the deformability of the structure edges and the non-linear elasticity of the tensional members. In fact, they used both Newtonian and Lagrangian formulations and reported, as expected, equivalent results in the structure's dynamic response.

All the works presented so far find a dynamic model of the tensegrity structure to predict its behavior. From another point of view, when empirical data is available it is possible to use model estimation techniques. In this direction, Bossens et al. [9] developed a method to identify a linearized dynamic model of a tensegrity structure around a stable configuration. They used a known

Table 1
Two stage SVD tensegrity structure used by Sultan [83] on the left. It can be parameterized by the position (x, y, z) and orientation (ϕ, ψ, θ) of the top platform with respect to the bottom platform and the declination (δ) and azimuth (α) angles of all bars. The dynamic model of that structure at an equilibrium configuration $(\underline{p}_e)$ is shown on the right



$$\underline{\underline{M}}\,\ddot{\tilde{\underline{p}}} + \underline{\underline{V}}\,\dot{\tilde{\underline{p}}} + \underline{\underline{K}}\,\tilde{\underline{p}} = \tilde{\underline{F}}$$
$$\tilde{\underline{p}} = \underline{p} - \underline{p}_e$$
$$\tilde{\underline{F}} = \underline{F} - \underline{F}_e$$

perturbation signal, acceleration or displacement y_0, on the structure to measure its response, also in terms of acceleration or displacement, at the top platform points, y_1, y_2, y_3, and a SIMO (Single Input Multiple Output) curve fitting method such that

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} B_1(\underline{\theta}) \\ B_2(\underline{\theta}) \\ B_3(\underline{\theta}) \end{bmatrix} \frac{1}{A(\underline{\theta})} y_0, \qquad (10)$$

where $B_i(\underline{\theta})$ are the transfer function numerator polynomials for each output, $A(\underline{\theta})$ is the common polynomial denominator and $\underline{\theta}$ are the polynomial coefficients.

Finally, some authors have simulated the behavior of the tensegrity structure under perturbations, not requiring an explicit solution of the dynamic model. Analytical methods for the simulation of such structures were classified by Barnes [6] into incremental, iterative and minimization methods. Incremental and iterative methods use the matrix formulation of finite elements (Reddy [72]). For instance, Furuya [27] presented a work consisting on dividing the structure in a large set of small, simpler, linked elements, and then apply the problem constraints to all of them. On the minimization side, a widely used method is the dynamic relaxation method (Belkacem [8]). Domer et al. [20] studied the tensegrity geometric non linearities by using neural networks to improve the accuracy of the dynamic relaxation method and hence obtain a better fit of real measured data.

Perhaps one of the most interesting results, presented by Sultan [83] and Sultan et al. [87], is that the difference between the complete non-linear dynamic model and its linearized version at an equilibrium configuration decreases as the pre-stress of the cables increase, demonstrated for SVD tensegrity structures, see table 1.

Regarding the oscillatory behavior of tensegrity structures, several research groups, such as Furuya [27], Oppenheim and Williams [61] for a simple highly symmetric tensegrity, Moussa et al. [54] for a kind of modular tensegrities or Sultan et al. [87] for SVD tensegrity structures, have shown that the frequency of the oscillation modes of a tensegrity structure increase when the pre-tension of its cables also increase.

Murakami [55] also studied this issue for simple prismatic tensegrities, and found out that the increase is proportional to the square root of the prestress. This result oppens the possibility to adjust the resonant frequency of the structure to meet some requirements. Another important result regarding the oscillation modes of a tensegrity structure, shown by Motro et al. [53] and Oppenheim and Williams [61], is that the geometric flexibility of the structure leads to a slower decay of vibrations than the expected exponential decay.

An important characteristic of tensegrity structures is their geometric stiffening, that is, its stiffness increases when external resolvable forces are applied to it, or equivalently, when infinitesimal mechanisms are activated. For instance, Motro et al. [52] and Oppenheim and Williams [60, 59] showed that the nodal displacement was not proportional to the applied force.

Motro et al. [53] and Oppenheim and Williams [60] also showed that the stiffness of a tensegrity structure increases when the pre-tension of its members also increase, that is, for a given applied force, the nodal displacement decreases with the increase of the member pre-tensions.

3 Open Problems

In order for a tensegrity to exist, it must comply with a set of geometrical and statical constraints. First, distances, d_{ij}, between nodes which are linked by a cable, strut or bar, pertaining to the set of edges E, are constrained to lower, l_{ij}, and upper, u_{ij}, bounds, (i.e. the physical bounds of the actuated members) while all the other distances remain unconstrained.

$$\begin{aligned} l_{ij}^2 \leqslant d_{ij}^2 \leqslant u_{ij}^2 \quad \{ij\} \in E \\ \texttt{unconstrained} \; \{ij\} \notin E \end{aligned} \tag{11}$$

It is important to note that not all the possible sets of distances will be geometrically compatible for a given dimension of the working space. The set of squared distances between n points can be arranged in matrix form by means of an Euclidean Distance Matrix (EDM from now on). The set of EDM's of n points define a convex cone which in turn becomes a convex set when the constraints in Eq. 11, which are also convex, are taken into account. Thus, any point belonging to this convex set will correspond to a geometrically compatible set of distances for a given dimension, $\flat$, of the working space $\mathbb{R}^\flat$.

It is mathematically possible to consider $\flat > 3$ although the resulting structure can not be built. When the dimension of the working space is constrained

$$\flat \leqslant 3, \tag{12}$$

the set of geometrically compatible distances becomes concave. This may be a problem if we want to randomly generate combinations of distances between nodes since not all of them will be correct. A simple example of that is shown in Fig. 3. In this example, a set of $\{d_{ij}\}$ distances between 4 points is randomly generated, but the structure is only realizable in $\mathbb{R}^3$ since, in $\mathbb{R}^2$, the distance d_{24} is too short.

Regarding the statical constraints, the structure must be at equilibrium with the external forces at any time, so the net forces and torques at each node must be null. This constraint can be stated as

$$\underline{\underline{R}}(\underline{p})^T \underline{\omega} = \underline{F}, \tag{13}$$

where $\underline{F}$ are the external equilibrium forces applied to each node.

Since cables can not withstand compression and struts can not withstand tension, the stress in a cable must be non-negative and the stress in a strut must be non-positive. Note there are no stress constraints for the bars since they can withstand both tension and compression forces. Additional constraints are

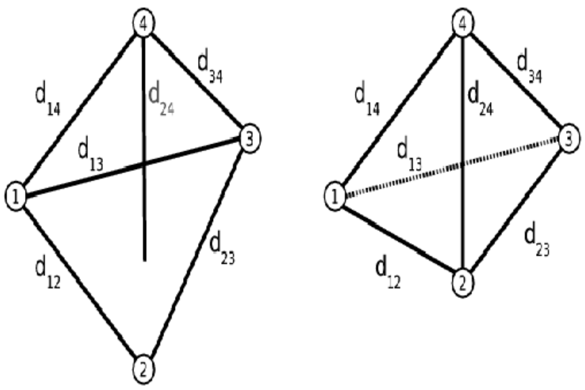


Fig. 3. A set of randomly generated distances which are geometrically compatible in $\mathbb{R}^3$ (on the right) but not in $\mathbb{R}^2$ (on the left).

possible in order, for instance, to avoid rigid motions or to force symmetries in the structure (as equal length edges or coplanar nodes).

$$\begin{aligned} \omega_{ij} \geqslant 0 \ \{ij\} \in C \\ \omega_{ij} \leqslant 0 \ \{ij\} \in S \end{aligned} . \tag{14}$$

So, any given tensegrity structure must comply at least, and at any time, with the constraints presented in Eqs. 11, 12, 13 and 14. In a general case, when no symmetry of any kind is taken into account, a tensegrity framework is completely defined by the spatial position of its nodes and the stressess (or equivalently the rest length) of its edges, or alternatively, with the azimuth, declination and length of each strut as well as the stresses of each edge. Whatever the generalized coordinates set used, a tensegrity structure is completely characterized by $n\flat + e$ parameters, being n the number of nodes, $\flat$ the dimension of the working space and e the number of edges. The dimension of the space $\mathcal{X}$ defined by these parameters is considerably high even for the most simple tensegrity in $\mathbb{R}^2$ (14 for Snelson's X) or $\mathbb{R}^3$ (30 for the triangular base regular prism).

Inequality constraints, such as lenght and stress constraints, intersect the whole parameterization space $\mathcal{X}$ with halfspaces which, if there exist solution, define a non empty subset with the same dimension as the original parameterization space. However, equality constraints, such as the ones in Eq. 13, define a subset of $\mathcal{X}$ with lower dimension.

This means that in general will be difficult to find configurations (a set of nodal coordinates and edge stresses) inside the feasible region, since it is defined by a set of non-linear equations which does not have a general solution. In other words, the probability of randomly generating a feasible configuration goes to 0. More intuitively, the different parameters of the structure are tightly coupled to each other and therefore can not be freely chosen. Furthermore, since the static equilibrium constraints in Eq. 13 depend on the external forces applied to the nodes, this feasible region will change whenever the external loads change.

For highly symmetric structures, fully parametrizable with just a few parameters, it is possible to find the dependencies between the different parameters of the structure. In a general case with arbitrary structures, and up to the knowledge of the authors, there are yet no methods to successfully handle the full feasible region. Having in mind the application of tensegrity structures to shape-shifting robots, this problem appear in four main areas which are sketched in Fig. 4.

The most fundamental problem is the design of such structures, that is, given the desired number of nodes and a given set of constraints, find out all stable topologies and configurations. Since the structure is allowed to be actuated, design also deals with the selection and placement of sensors and actuators. Current research on design of tensegrity structures is presented in section 4.

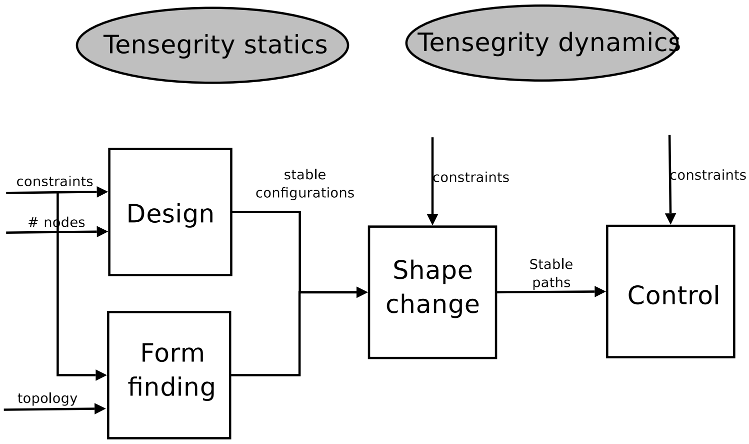


Fig. 4. Visual representation of the open problems for shape-shifting tensegrity structures: design and form-finding to find stable configurations, shape changing algorithms to find stable trajectories and control methods to compensate for external perturbations.

A subproblem of the more general issue of design, known as form-finding, is: given an initial topology for the tensegrity structure and a set of constraints to find out all the stable configurations. Form-finding has been extensively studied in the literature. The solution to any of these problems give stable configurations for a tensegrity structure. Now, to go from one stable configuration to any other, therefore effectively changing the shape of the structure, it would be necessary to find a stable path that complies with all the given constraints at any point. Current research on the problem of shape-change is presented in section 5.

Finally, even with a stable trajectory between two stable configurations, and in order to reduce the path following error, it is necessary to apply some kind of control method to compensate for both the dynamic effects of the structure and unexpected external perturbations that may appear. Regarding the control issue, new problems arise since the dynamics of any tensegrity structure can be modeled using second order non-linear differential equations with constraints (as shown in section 2):

$$\underline{\underline{M}}(\underline{q})\ddot{\underline{q}} + \frac{1}{2}\left(\frac{\delta\underline{\underline{M}}^T(\underline{q})}{\delta\underline{q}}\dot{\underline{q}}\right)\dot{\underline{q}} + \underline{f}_g^T = \underline{\underline{H}}(\underline{q})\underline{f}, \tag{15}$$

where $\underline{q}$ are the generalized coordinates, $\underline{\underline{M}}(\underline{q})$ is the mass matrix, $\left(\frac{\delta\underline{\underline{M}}^T(\underline{q})}{\delta\underline{q}}\dot{\underline{q}}\right)$ is the damping matrix, $\underline{f}_g$ is the gravity force, $\underline{\underline{H}}(\underline{q})$ is the constraint matrix and $\underline{f}$ are the external forces.

Literature available on tensegrity structures control is extensive and deals with the more general problem of actively controlling the tensegtity structure. Current research on these problems is presented in section 6.

4 Design for tensegrity structures

Design, when applied to tensegrity structures, means the generation of new topologies; that is, which nodes are connected to each other, which kind of edge is used in these connections, which is the spatial nodal configuration and which are the edge stresses. The new topology is designed to achieve a desired performance criterion, such as the level of stiffness, stiffness to mass ratio or deformation under external forces, while complying with the geometrical and statical constraints seen before.

Until recently, most of the proposed approaches start from a given topology, based in either basic geometrical shapes or people's intuition, to find out the feasible tensegrity configuration subject to the problem constraints and performance criterion. This is known as *form-finding* and has already been presented by Hernàndez and Mirats-Tur [31], who extended a previous survey by Tibert and Pellegrino [95] with the most current approaches. These methods mainly use iterative procedures to sequentially converge the initial structure into the feasible region, although there are other successful approaches based, for instance, on genetic algorithms or analytical solutions.

If we also want to design the topology of the structure, the original problem of form-finding becomes harder since the feasible region, if it exists, changes as nodes are connected or disconnected. An example of this fact is provided in Fig. 5, where two different feasible regions for the Snelson's X are shown. However, the proposed methods to deal with this problem are quite similar to those used for the form-finding issue, being iterative methods the most commonly used. It is important to note that, while the literature available on form-finding methods is extensive, very few approaches exist dealing with the problem of topology design.

Such approaches, based on non-linear programming, use the desired performance criterion as the cost function to be optimized. For instance, de Jager et al. [15] and Masic et al. [50] used the compliance energy of the structure under external forces, $\underline{f}^T\underline{u}$, where $\underline{u}$ are the nodal displacements and $\underline{f}$ are the external forces applied to the nodes. Here, the compliance is used as a measure of stiffness in order to achieve the best mass-to-stiffness ratio for the new structure.

These methods start from a given maximum allowed connectivity between all the nodes, being the type of every edge already defined. So, the proposed methods try to minimize the number of edges to achieve the desired performance. To this end, de Jager et al. [15] and Masic et al. [50] introduced a new variable into the problem: the maximum allowed volume ($\sum_{ij\in E} v_{ij} = V_{max}$), or equivalently, the maximum allowed mass for the structure. Then, the volume

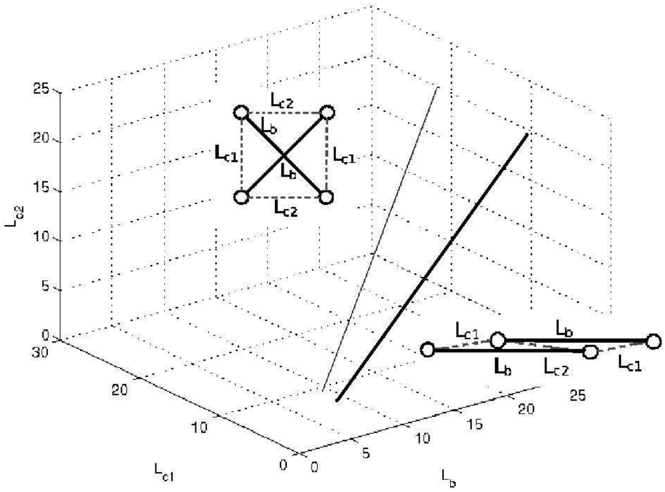


Fig. 5. Two feasible regions for Snelson's X. The thick line is the feasible region for the complete structure and the thin line is the feasible region when the upper cable L_{c2} is removed. The corresponding configuration is depicted next to its feasible region.

associated to each edge is determined by their stresses and two additional constraints: the strength constraint for all edges:

$$|\omega_{ij}|d_{ij}^2 - v_{ij}\sigma_{ij} \leqslant 0 \; ij \in C, \tag{16}$$

and the buckling constraint for all bars:

$$|\omega_{ij}|d_{ij}^5 - \frac{\pi}{4}E_{ij}v_{ij}^2 \leqslant 0 \; ij \in B, \tag{17}$$

where d_{ij} is the length of member ij, σ_{ij} is the maximum yield stress and E_{ij} is the modulus of elasticity.

Additionally, edge length, equilibrium and stress sign constraints were used. So, by eliminating those edges with negligible volume in the optimization process, the topology is effectively modified mantaining only those members which are necessary for the stability of the structure. A comparison between an initial configuration and an optimized one can be found in the work by Masic et al. [50].

A completely different approach is the one proposed by Paul et al. [63] using genetic algorithms. In this case, they evolve an initial arbitrary topology, in terms of both the connectivity between nodes and the type of each edge, into a stable one in the given work space. They use a genotype built from the spatial coordinates of each node, a list of pairs of struts that must swap edges and a list of pairs of cables that must swap edges too.

However, the fitness criterion used in the evolution procedure only takes into account the volume of the generated structure, which is maximized. Other structural performance criteria, such as the stiffness or mass-to-stiffness ratio, were not used. One of the main contributions of this method is that, for the first time, new irregular tensegrity topologies are generated, which are neither based on people's intuition nor on simple geometric shapes.

Up to now, we have dealt with *passive* tensegrity structures. When the tensegrity structure can be actuated and/or sensed, several other problems appear which fall into the category of design. These problems are not unique to tensegrities, but common to all truss like structures, hence already existing methods are adapted to take into account the particular challenges associated to tensegrity. Two of these problems that have received a lot of attention from the tensegrity community are:

- **input/output selection:** given the topology of a tensegrity structure, choose the optimum number and distribution of sensors and actuators to achieve a given performance criterion.
- **integrated structure/control design:** given a tensegrity structure, find out the best topology and/or nodal configuration and edge stresses, as well as the number and distribution of sensors and actuators, not only to achieve stability, but also to improve the performance of the desired controller.

Finding the best input/output selection, that is, the best configuration of sensors and actuators, has been an area of intensive research in civil engineering for many years. A good review of input/output selection methods was given by van de Wal and de Jager [97]. Two main issues were identified in this review: the candidate selection and the feasibility test for each candidate.

The first issue deals with the problem of selecting combinations of sensors and actuators to be checked for feasibility. The naive approach of checking all possible combinations is not possible due to the combinatorial explosion it experiences when the number of sensors (N_s) and/or actuators (N_a) increase. To overcome this problem, de Jager and Skelton [16] use a method based on finding all minimal dependant sets which represent all the feasible combinations with a minimum number of sensors and actuators. This is equivalent to find all maximal independent sets which represent all the non-feasible combinations with a maximum number of sensors and actuators. The main drawback of this method is its computational complexity. The problem is still $\mathcal{NP}$-hard, although authors shown that for $(N_s + N_a)$ not too large, the practical computing time is polynomial.

Another possible way of avoiding exhaustive checking of all candidates is using an optimization method. For example Skelton and Li [78] and Li and Skelton [43] started from a configuration with sensors and actuators in all the edges of the structure, and then successively eliminated those whose contribution to the desired closed loop performance is smaller.

de Jager [14] also used iterative procedures based on Mixed Integer Linear Programming (MILP) to search for candidates to be actuated as well as to achieve an on-line implementation. In this case, however, a lot of approximations are made in order to simplify the problem, such as that the bars can only vary its length and move without modifying its orientation, or assuming a fixed increment in the bar's length

$$\Delta l_{ij} = \pm s_{ij} \Delta \bar{l}_{ij} \; \{ij\} \in B,$$

where the parameter $s = 1$ if the bar is actuated or $s = 0$ if not, and $\Delta \bar{l}$ defines the minimum variation in length. Also, the stress feasibility is not checked until the spatial configuration is found.

Regarding the feasibility test for each candidate, there exist several possible criteria. For example, de Jager and Skelton [16] used a robust control criterion which consists in checking the existence conditions of a H_∞ controller achieving the specified performance level. This method is applied to improve the dynamic stiffness of the structure and to reduce its vibrations.

Another possible feasibility test is the minimization of the control energy. In this case, the actuator and measurement noises would deteriorate the output performance no matter how large the control effort was (Skelton and Li [78]), so it is necessary to use high precision components which implies a higher overall cost. Skelton and Li [78] and Li and Skelton [43] reformulated the problem, calling it *economic design*, to find the minimum number of sensors and actuators with the minimum necessary precision to achieve the desired closed loop performance. Finally, the criterion used by de Jager [14] was to achieve a given closed loop shape change performance in order to go from one initial configuration to another while keeping the stability of the structure.

The design of the structure and its controller have traditionally been two independent problems, however integrated design of the structure and its controller may provide additional benefits as it has been recently tackled in the literature. For instance, a given placement of an edge may provide the best static performance, but, when actuated, the overall closed loop performance may not be as good as possible.

Most of the methods proposed to deal with this problem use some kind of iterative technique. For example, de Jager et al. [17] extended the non-linear optimization method used by de Jager et al. [15] and Masic et al. [50] with additional constraints to limit the control effort of the members of the structure. That is, if the control effort of each member is assumed to be proportional to its change in length ($\Delta l_{ij} = l_{ij} - l_{ij0}$), or equivalently, to the variation in its elasticity coefficient (e_{ij}), the maximum control effort of the structure can be limited by

$$\sum_{ij \in E} e_{ij} \leqslant S_e.$$

During the optimization process, every edge needing a control effort smaller than a given threshold is ignored for control purposes, although it can still be used to achieve stability. Therefore, they simultaneously design the topology, spatial configuration and stresses of the structure and the distribution and number of the actuated and sensed members.

Masic et al. [49] and Masic and Skelton [48] approach, although integrating structure and control design, only optimizes the prestress of the structure but not its spatial configuration. It uses a special property of the stresses: it is well known that the set of pre-stresses of a tensegrity structure define a cone, that is, any pre-stress multiplied by a positive scalar is also a valid pre-stress. So, this method, while keeping the spatial configuration ($\underline{p}$) of the structure to preserve the stress matrix $\underline{\underline{\Omega}}(\underline{p})$, searches the proper stress vector ($\underline{\omega}$) inside the pre-stress cone

$$\underline{\omega} = \underline{\underline{\Lambda}}(\underline{p})\underline{\alpha},$$

where $\underline{\underline{\Lambda}}(\underline{p})$ are the extreme directions of the cone and $\underline{\alpha}$ the coefficients of the linear combination.

In this case, a gradient optimization method is used to find the optimal pre-stress ($\underline{\omega}^*$) inside the pre-stress cone in terms of the dynamic behavior of the structure under external perturbations and in terms of the optimal controller (Linear Quadratic Regulator - LQR).

To conclude this section, and regarding the design of more complex structures, Masic and Skelton [46] proposed a set of rules to build complex structures from simple units so that, if the individual modules are stable, then the whole structure is also stable.

5 Shape control for tensegrity structures

Shape control of a structure is usually carried out by a path-planning process that finds a feasible trajectory between two points, either in the real or in the parameterization space, which in fact may or may not exist due to environmental and internal constraints. In robotics, the initial and final poses of the robot as well as the path between them, are constrained by environmental and internal obstacles, i.e. bars cannot intersect, and the robot kinematic and dynamic models.

As presented before in section 3 all possible feasible configurations of a tensegrity structure are embedded in a subspace of lower dimension than the parameterization space. This problem is similar to that of closed kinematic chains [41], for instance two manipulators handling a single object, and make it impractical to use common path-planning techniques as those reported by Kavraki et al. [38] or LaValle [40].

First approaches to shape control of tensegrity structures were mainly focused on the deployment of simple and highly symmetric tensegrity structures (Furuya [27]), such as masts or planes. Those approaches used a priori knowledge of the structure topology and analytic methods. A possible approach, which we have called *passive deployment*, require the action of external forces in order to change the shape of the structure.

For example, Stern [82] and Duffy et al. [22] studied a family of n-strut prisms where the vertical cables were replaced by elastic ties. In this way, when external forces are applied to the structure, its volume can be considerably reduced and, then, by the principle of minimal potential energy and the action of the elastic ties, the structure returns to its equilibrium configuration when the forces are removed.

Smaili and Motro [80] proposed a completely different approach to the passive deployment of tensegrity structures based on the creation of finite mechanisms. They identified two ways to create finite mechanisms in a tensegrity structure:

- Activation of finite mechanisms by self-stress removal.
- Activation of finite mechanisms independent from the self-stress. In this case the self-stress still actuates when the mechanism is activated.

In the first case, by removing the self-stress in some of the cables, some of the constraints on the spatial position of the affected nodes are eliminated. In this case, applying external forces to these nodes it is possible to fold the structure. Also, restoring the state of self-stress to the affected cables is enough to deploy the structure again. Note that, here, path-planning is not involved since the folding of the structure is the result of an external agent and no trajectory is planned at all.

Another way of deploying a tensegrity structure is by actuating some or all of its members, which we have called *active deployment*. Motro [51] studied three different ways to actively fold and unfold tensegrity structures: strut actuation, cable actuation and mixed mode where both struts and cables may be actuated. In the case of active deployment of a symmetric tensegrity, which may completely be parameterized with just a few variables, the problem of path-planning may be simplified due to the highly symmetric nature of the needed trajectory. In this case the feasible region is considerably simplified and it is possible to use analytic methods to find closed expressions for the length of the actuated members as a function of the structures' parameters.

Sultan et al. [88] and Masic and Skelton [46] studied the symmetrical motions, i.e. all actuated members behave equally, which always keep the equilibrium conditions for simple tensegrity structures. In these cases, authors manage to give closed form expressions for the length of the actuated members, $L = f(p_1, p_2, \ldots, p_n)$, but do not explicitly find the relationships between the different parameters which are imposed by the feasible region. Therefore, not all the possible trajectories in the real world are feasible since no real solution may exist for a given length of the edges.

For instance, Sultan et al. [88] worked with 2 stage SVD structures, see table 1. Such structure, considering symmetrical configurations, can be parameterized by only two variables: declination, α and azimuth, δ, angles of the bars, so the length of the actuated members can be stated as a function of these variables. However, the time variation of these angles to achieve the desired trajectory in the real world, i.e. the deployment of the structure, are independently chosen.

Pinaud et al. [69] proposed a very similar work, however, they explicitly found the relationship between the different parameters of the structure, assuring the final trajectory in the parameterization space is completely embedded into the simplified feasible region.

More recently, iterative procedures have appeared which search the feasible trajectory inside the feasible region. These methods start from an initial trajectory, most probably wrong, and iteratively adjust it in order to be completely embedded inside either the simplified or the complete feasible region. For example, Sultan and Skelton [91] parameterized the initial trajectory in the parameterization space as a polynomial of a certain degree, s, for the same SVD structure used by Sultan et al. [88]. Then, the coefficients of that polynomials are adjusted so that the desired trajectory, and its first derivative, fits into the feasible region.

However, due to the impossibility of finding such trajectory for continuous time, they sampled the time interval, simultaneously imposing the constraints of the optimization problem on only those sampled points. Inbetween the sampled points there are no constraints, so depending on the degree of the polynomials and the sampling resolution, the trajectory will be kept more or less close to the feasible region. A symbolic draw of this method is provided in Fig. 6.

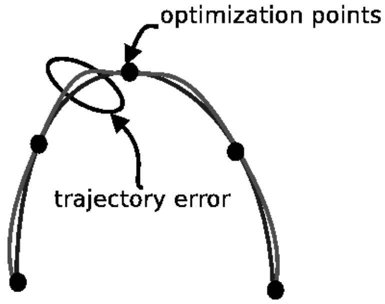


Fig. 6. An hypothetical feasible region in black and the computed feasible trajectory (gray). Note that the position and the first derivative of the trajectory coincide with the feasible region at the sampled points, but an error appears elsewhere.

The method proposed by Sultan and Skelton [91] still exploits the symmetry of the structure, thus working with the simplified feasible region, but the iterative approach presented is quite general and several authors have used it to deal with more general path-planning tasks, i.e. asymmetrical reconfiguration, for more complex asymmetric structures.

To our knowledge, Pinaud et al. [68] and van de Wijdeven and de Jager [98] were the first to deal with this issue. They defined the desired trajectory for the structures' center of mass ([68]) or for the structures' nodes ([98]) in the real world and, then, divided it in a number of small segments, n_c. For each point, a non-linear optimization problem is solved in order to find the required spatial configuration of the nodes $\underline{p}_i$ and the edge stresses $\underline{\omega}_i$ so as to keep the structure at an equilibrium configuration and also achieve the desired position. The number and placement of the actuators is fixed a priori, so it may still be impossible to fulfill the desired trajectory. A sketch of this procedure is depicted in Fig. 7.

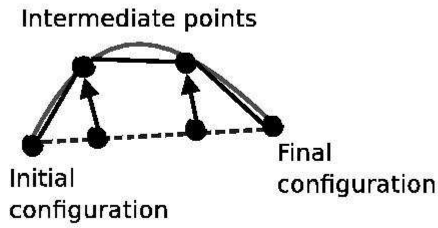


Fig. 7. Initial straight trajectory between initial and final configuration (dotted gray line) and valid trajectory belonging to the feasible region (solid gray line). In black, the linearized approximation for the final trajectory after the method proposed by Pinaud et al. [68] or van de Wijdeven and de Jager [98] is applied.

Both methods minimize the actuated tendon forces thus minimizing the overall energy of the structure. However, while Pinaud et al. [68] simultaneously optimize both the nodal configuration and the edges stresses, van de Wijdeven and de Jager [98] iteratively find, first, the optimum stresses and, then, the corresponding nodal configurations until the whole solution, i.e. nodal configuration and edge stresses, is feasible.

As a concluding remark, note that all the path-planning methods presented in this section, both passive and active, use cables as a mean to change the shape of the tensegrity structure while keeping the bars at constant length.

6 Control of tensegrity structures

The main issues regarding the control of tensegrity structures are associated to its dynamic model, presented before in section 3, and the possible external perturbations applied to the structure. In this case, the shape and characteristics of the feasible region will not directly affect the structure performance since the desired set point should have already been found using a form-finding, design (static) or path-planning (dynamic) procedure.

So, due to inaccuracies in the dynamic model of the structure, or the completely lack of such model, and also to unknown external perturbations, the performance of the structure will degrade (i.e. it would oscillate when it is supposed to remain still or it will not follow the desired path). In order to eliminate, or at least minimize these undesired effects it is necessary to use some kind of control technique, either passive, using the knowledge about the structure dynamics and material properties, or active, placing sensors and actuators connected through a feedback control ([11]).

Very few works exist which deal with the passive control of tensegrity structures, to our knowledge only Skelton et al. [79] suggested the use of such techniques. On the other hand, the active control of tensegrity structures have been an area of intensive research in the last few years. Actually, Skelton et al. [77] concluded that since only small amounts of energy are needed to change the shape of tensegrity structures, they are advantageous for active control. Up until recently, tensegrity structures have been mainly used in civil engineering (domes) and space applications (antennas) to build structures which, once deployed, have to keep its original configuration under external perturbations. This focused the research in the development of vibration control techniques.

Figure 8 shows a sketch of a generic closed loop control system. $G(s)$ represents the transfer function of the tensegrity structure, $K(s)$ the transfer function of the controller, $w(s)$ are the set points of the system, $u(s)$ is the actuator input, $z(s)$ is the output of the structure at which the performance is evaluated and $y(s)$ are the sensed parameters of the structure. All these functions are stated in the Laplace domain.

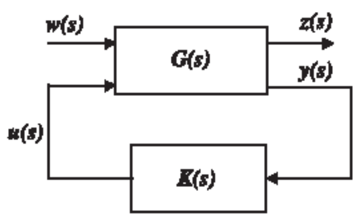


Fig. 8. A sketch of a generic closed loop control system.

There exist several possibilities for the selection of the feedback and actuation parameters. Regarding the feedback parameters, Djouadi et al. [19] and Averseng and Crosnier [4], Averseng et al. [5] used the nodal displacements, Chan et al. [11] used the tension force of the members and Raja and Narayanan [71] used the full or partial state of the structure (i.e. nodal position and velocities); Chan et al. [11] and Averseng et al. [5] also used the acceleration of some parts of the structure. For the actuation parameters, the most common approach is to actuate the members of the structure themselves, being the length of the members the most widely used (Djouadi et al. [19],Chan et al. [11] and Raja and Narayanan [71]). However, Averseng and Crosnier [4], Averseng et al. [5] used a completely different approach, placing the actuators outside the structure in order to do not interfer with its integrity.

Most of the proposed methods to actively control a tensegrity structure use the minimization of the vibrations as performance measurement, however, the control techniques used to achieve that objective are quite different. For example, Djouadi et al. [19] proposed a method based on instantaneous optimal control, intended for structures undergoing large displacements, which use a linearized version of the structure dynamics at each iteration. In this case it is shown that the member forces introduced by the actuators lead to additional damping and stiffness effects, which introduce error in the estimation of the structure's response due to the use of a linearized dynamic model.

Chan et al. [11] proposed two different schemes both based on removing energy from the structure (active damping). The first method (called *local integral force feedback*) used the transfer function shown in eq. 18 to achieve a negative energy flow in each actuated edge. In this equation g_c is the gain of the controller, $T_i(s)$ is the tension of the i-th member and u_i the required displacement for each actuator. So, this control law is decentralized (*local*) in the sense that each actuated member is controlled independently from the others subject only to its own measured forces.

$$u_i(s) = \frac{g_c}{s} T_i(s). \tag{18}$$

The second method proposed by Chan et al. [11] (called *acceleration feedback control*) use global information of the structure. In this case the authors also suggest the use of an estimated model of the tensegrity structure using the method proposed by Bossens et al. [9] (see eq. 19 for the transfer function of this controller). The first method, while simpler to implement, does not have the same effectiveness in damping the undesired oscillations as the second one.

$$u(s) = g_c \frac{s^2 + 80s + (22\pi)^2}{s(s + 26\pi)} y(s) \tag{19}$$

Averseng and Crosnier [4], Averseng et al. [5] proposed another active control strategy for tensegrity plane grids. They divided the control law into two parts:

- Static control: This stage filters out the high frequency displacements due to the vibration of the structure and uses a simple PI controller.
- Dynamic control: In this case a robust control technique (H_∞) is used which minimizes the transfer function between the external perturbations ($w(s)$) and the output ($y(s)$), and also minimizes the influence of unmodeled dynamics, identification errors, etc..

Both static and dynamic controllers simultaneously contribute to the overall behavior of the structure. Another method to actively control a tensegrity structure is the one proposed by Raja and Narayanan [71] based on optimal control.

All the methods presented so far have a solid theoretical background and achieve good results in simulation, however only Chan et al. [11] and Averseng et al. [5] actually tested their respective algorithms in real structures. Also note that the placement of sensors and actuators is chosen a priori.

In some other approaches, instead of feeding back the information of the sensors in order to compute a suitable control action, a search for the best action among all possible actions is performed. So, given the difference between the desired and current configuration of the structure, the problem is to find the best combination of actions to reduce the error. However, an exhaustive search is not possible due to the combinatorial explosion it experience when the number of sensors and actuators increase.

Shea et al. [73] proposed the general framework for intelligent control shown in Fig. 9. The main objective of this approach is to improve the performance of the structure by learning from experience over the whole live of the structure.

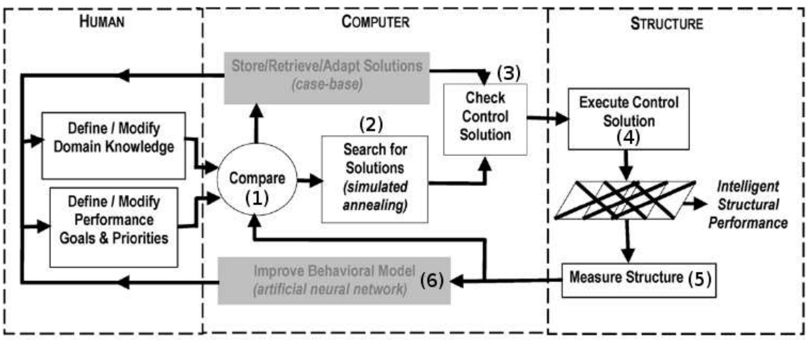


Fig. 9. Framework for intelligent computational control of complex structural systems as proposed by Shea et al. [73].

In this framework, appart from the search for the best action (bloc number 2), learning techniques are used to both improve the needed computational methods by using previously valid actions to guide the search for a good solution in similar conditions (bloc number 3), and to generalize past events to improve the static and dynamic models of the structure (i.e. the domain knowledge base) (bloc number 6). The input to such framework is the desired performance of the structure in terms of deformation ranges, maintaining stress or slope, etc., and the priorities between them.

Shea et al. [73], and Fest et al. [23], [24] presented a simplified implementation of such framework, using only the comparison and the search stages, for a double layer tensegrity grid. In these cases, the best action is defined as the set of struts to be actuated and the magnitude of their change. In order to simplify the search problem, the lengthening of the struts is limited to discrete steps, and therefore, assuming P possible strut lengths and a total of n possible struts to actuate, the dimension of the search space is $N = P^n$, hence increasing exponentially with the number of actuators.

As it is not feasible to test all the possible solutions, stochastic search methods are used. A simulated annealing algorithm is used by Shea et al. [73] and Fest et al. [23], while Fest et al. [24] use the Probabilistic Global Search Lausanne (PGSL) method. All of these methods use the dynamic relaxation method to predict the behavior of the structure for each of the possible candidate solutions generated by the stochastic search. One of the main problems of this approach is the computational needs of the search algorithm which make the structures adjustable instead of active (i.e. it is not possible to execute in real time the needed actions). Only the method proposed by [24] can be executed in real time.

In order to speed up the previous method, Domer and Smith [21] introduce the use of case-based reasoning methods which was already suggested in the general framework of Fig. 9. In this case, previously successful task-action pairs are used to reduce the search time and improve performance over time, as the system solves new situations. They also use an artificial neural network, trained to compensate for effects that are not included in the model of the structure. This improves the prediction of the behavior of the structure for each possible action. More recently, Adam and Smith [1] have used additional performance criterions, such as maximizing the stiffness of the structure, minimizing the stress, etc., as a complement to the main performance goal in order to reduce the number of possible actions and therefore speeding up the search process.

Extensive research have also been done regarding the path following control of tensegrity structures. Both open and closed loop control laws have been proposed. In the first category, Masic and Skelton [46], Pinaud et al. [68], Masic and Skelton [47] and Pinaud et al. [69] avoided the effects caused by the dynamic model of tensegrity structures by assuming a quasi-static motion, that is, by slowly varying the actuated members. All these methods have been applied to highly symmetrical tensegrity structures in order to be able to find a closed form expression for the actuated members, and the only objective of the open loop control law is to keep the structure always into the feasible region.

Other open loop control methods proposed by Sultan et al. [88], Sultan and Skelton [91] and Aldrich and Skelton [3] take into account the dynamic model of tensegrity structures in order to design the control laws. Sultan and Skelton [91] and Aldrich and Skelton [3], add the dynamic model of the structure as a constraint to the optimization procedure used. Also, for all the possible trajectories from one feasible point to the other, they search for the one with minimal execution time ([91]) and minimal required energy ([3]). Sultan et al. [88] substitute the equilibrium closed form expressions for the actuated member lengths into the dynamic model, and find the necessary conditions a feasible trajectory must verify.

Another different approach to the open control of tensegrity structures has been proposed by Kanchanasaratool and Williamson [36]. Since the dynamic model of a tensegrity structure is in general not invertible, and even if the inverse is known to exist, it would be unlikely to find a closed form expression, they proposed to use a neural network to approximate it. In order to train the neural network they used several steady state input (desired pose for the center of mass) output (necessary length for the actuated members) pairs obtained from simulation.

In order to be able to compensate for external perturbations and also for unmodeled effects of the dynamic model, closed loop control methods have also been studied. van de Wijdeven and de Jager [98] and Aldrich [2] both proposed closed loop control methods designed to improve the suppression of vibrations during the motion of the structure. van de Wijdeven and de Jager [98] use a linearized version of the dynamic model to design an $\mathcal{H}_2$ controller while Aldrich [2] use a computed-torque controller, a kind of feedback linearization technique.

An interesting fact about the control of tensegrity structures, and in general of any hyper-actuated mechanical system, have been pointed out by Paul and Lipson [62], Paul et al. [65]. They show that, due to the highly coupled mechanical nature of tensegrity structures, there exist redundancy in their control, that is, several control laws can lead to the same behavior. In this case Paul and Lipson [62], Paul et al. [65] used genetic algorithms to evolve the activation sequence of the actuators and the duration and magnitude of its motion to achieve the desired behavior of the structure, and find out that multiple solutions were possible.

7 Applications

Different applications can be found in the literature where tensegrity structures have been used. The first field where tensegrity structures were applied was civil engineering. In this field, tensegrity principles have been applied to the design of domes, Pellegrino [67], Hanaor [28] and Burkhardt [10], or grids Hanaor [30].

Perhaps the widest area of application is that of deployable structures, where a number of authors have contributed. For instance, Hanaor [29] worked with deployable structures in general; Skelton and He [76] proposed the use of deployable tensegrity for the $NESTOR$ project (Neutrinos form Supernovae and TeV sources Ocean Range); Sultan and Skelton [90] presented a structure such that the deployment time is minimized and the deployment trajectory found is close to the equilibrium manifold; Duffy et al. [22] presented a review on self-deployable tensegrity structures; Masic and Skelton [45] considered deployment as a tracking control problem and Tibert [94] fully characterized deployable tensegrity structures for space applications.

Regarding space applications, Tibert and Pellegrino [93] also studied the use of tensegrity structures to be used as on-board reflectors for small satellites, and Sultan et al. [84] proposed the design of a space telescope in which the structure itself was based on tensegrity principles.

More recently, there have been some contributions in the fields of both manipulators and mobile robots based on tensegrity structures. Tran et al. [96] and Marshall and Crane [44] used a tensegrity structure to obtain a parallel manipulator. Both approaches combined rigid actuated members together with passive members which placed the platform at its lowest energy configuration.

Regarding mobile robotics, Masic and Skelton [47] introduced a self-propelled tensegrity worm which was actuated by the interaction of a longitudinal wave propagating through the structure with the environment. Paul et al. [64, 63, 65] proposed the use of some simple tensegrity structures to obtain a mobile device by adequately shortening/lengthening some of its cables.

Perhaps the most interesting application can be found in biology related fields. It seems that a surprising number of natural systems, as carbon atoms, water molecules, proteins, viruses or tissues may be explained using tensegrity models. This has been first suggested by Ingber [34, 35, 32, 33] and, later, other authors also studied the matter, see for instance Beer et al. [7], Chen and Ingber [12], Wang et al. [100] or Sultan et al. [92]. Tensegrity structures are also very similar to muscle-skeleton structures of highly efficient land animals that can reach speeds up to 60 mph. See for instance Vogel [99] or Levin [42]. Such animals incorporate tensional elements in their muscle-skeleton system such that they maintain the structure integrity, acting it, and storing and distributing energy.

Other applications exist that use the tensegrity paradigm. Sultan et al. [85] developed a tensegrity based flight simulator. Also, Sultan and Skelton [89] designed a 3 axis force and torque sensor based on a two stage SVD tensegrity structure as an example of a smart sensor. They take advantage of the large number of possible sensing elements which provide more accurate, fault tolerant and redundant measures. Also, Defossez [18], proposed the use of tensegrity structures as shape memory actuators, using the fact that there may exist multiple stable states for a given tensegrity structure, each with a different level of potential energy, and, when perturbated, the structure have the tendency to go back to the more stable state (minimum energy).

8 Conclusions

From an engineering point of view, tensegrity are a special class of structures whose elements may simultaneously perform the purposes of structural force, actuation, sense and feedback control. They have a very high resistance/weight coefficient and are easily deformable. In such kind of structures, theoretically, pulleys or other kind of actuators may stretch/shorten some of the constituting elements in order to substantially change their form with a little variation of the structure's energy.

So a big challenge exists for roboticians to obtain new kind of deformable robots, either manipulator or mobile robots, using tensegrity structures with some or all of its elements being actuated. In the case that more elements than degrees of freedom in the space are actuated we obtain a hyper-actuated structure which may be, appart of moved, shape controlled. Whichever the aim, it must be done after a deep study and understanding about the tensegrity static and dynamic behavior is performed. In a previous publication Hernàndez and Mirats-Tur [31] presented a deep static analysis of tensegrity structures, together with an up to date literature review on that matter. The objective of this paper has been to deal with the dynamic issue as well as to identify the open research problems on tensegrity structures and how these have been tackled up to now in the literature by different research groups.

We presented in section 2 the different used approaches to study the force-displacement relationship, or in other words, how the structure changes its shape under the action of external forces, as well as the main results in this area. We then identified some challenges that still require attention by the tensegrity research community to be fully solved. Perhaps the most important issue when dealing with actuated tensegrity structures is that the set of all the possible feasible configurations of the structure are contained in a sub-space which dimension is lower than the dimension of the parameterization space. This is due to the constraints the tensegrity must hold in a feasible realization. From here, we identified three basic problems, named design of tensegrity structures, shape change, involving path-planning issues, and control of the structure in order to compensate for dynamic effects and external perturbations in the task of following a specified trajectory. A full section has been devoted to each one of these problems analyzing how they have been interpreted and tackled in the existing literature.

Finally we would like to remark that, despite their huge potential of applicability, only a few structures of this kind have been built at the present time. We think that a good field of application for such structures is to allow them to move by the use of adequate actuators and sensors, and expect that in the next years research on tensegrity structures will focus on their dynamics and control, and, in our specific interest, obtaining new deformable and totally environmental adaptable robots.

9 Acknowledgments

This work has been partially supported by the projects CICyT DPI2006-14001 and PROFIT CIT-020400-2007-78 both financed by the Education and Science Ministry of the Spanish Government.

References

[1] B. Adam and I.F.C. Smith. Tensegrity active control: Multiobjective approach. *Journal of Computing in Civil Engineering*, 21:3, 2007.

[2] J.B. Aldrich. *Control synthesis for a class of light and agile robotic tensegrity structures.* PhD thesis, University of California, 2004.

[3] J.B. Aldrich and R.E. Skelton. Time-energy optimal control of hyper-actuated mechanical systems with geometric path constraints. In *European Control Conference*, 2005.

[4] J. Averseng and B. Crosnier. Static and dynamic robust control of tensegrity systems. *IASS 2004 Symposium on Shell and Spatial Structures - From Models to Realization*, 45(3):169–174, 2004.

[5] J. Averseng, J.F. Dube, B. Crosnier, and R. Motro. Active control of a tensegrity plane grid. In *44th IEEE Conference on Decision and Control and European Control Conference*, pages 6830–6834, 2005.

[6] M.R. Barnes. Form finding and analysis of tension structures by dynamic relaxation. *Journal of Space Structures*, 14(2):89–104, 1999.

[7] R.D. Beer, H.J. Chiel, R.D. Quinn, and R.E. Ritzmann. Biorobotic approaches to the study of motor systems. *Current Opinion in Neurobiology*, 8(6):777–782, 1998.

[8] S. Belkacem. *Recherche de forme par relaxation dynamique des structures reticulees spatiales autoconstraints.* PhD thesis, L'Universite Paul Sabatier de Toulouse, 1987.

[9] F. Bossens, R.A. de Callafon, and R.E. Skelton. Experimental modeling and modal analysis of a tensegrity structure. *International Journal of Solids and Structures*, 2003.

[10] R. Burkhardt. *A practical guide to tensegrity design.* Cambridge University Press, 2005.

[11] W.L. Chan, D. Arbelaez, F. bossens, and R.E. Skelton. Active vibration control of a three-stage tensegrity structure. In *SPIE 11th Annual International Symposium on Smart Structures and Materials*, San Diego, March 2004.

[12] C.S. Chen and D.E. Ingber. Tensegrity and mechanoregulation: from skeleton to cytoskeleton. *Osteoarthritis Cartilage*, 7(1):81–94, 1999.

[13] R. Connelly. Rigidity and energy. *Inventiones Mathematicae*, 66:11–33, 1982.

[14] A.G. de Jager. Design for shape control of tensegrities. In *Proceedings of the American Control Conference*, pages 2528–2533, 2006.

[15] B. de Jager, M. Masic, and R.E. Skelton. Optimal topology and geometry for controllable tensegrity systems. In *15th Triennial World Congress of the International Federation of Automatic Control*, 2002.

[16] B. de Jager and R.E. Skelton. Input-output selection for planar tensegrity models. *IEEE Transactions on Control Systems Technology*, 13(5), 2005.

[17] B. de Jager, R.E. Skelton, and M. Masic. Integrated control/structure design for planar tensegrity models. In *Proceedings of the 2002 International Conference on Control Applications*, volume 2, pages 862–867.

[18] M. Defossez. Shape memory effect in tensegrity structures. *Journal of Mechanics Research Communications*, 30(4):311–316, 2003.

[19] S. Djouadi, R. Motro, J.C. Pons, and B. Crosnier. Active control of tensegrity systems. *Journal of Aerospace Engineering*, 11(2):37–44, 1998.

[20] B. Domer, E. Fest, and V. Lalit. Combining dynamic relaxation method with artificial neural networks to enhance simulation of tensegrity structures. *Journal of Structural Engineering*, 129(5):672, 2003.

[21] B. Domer and F. Smith. An active structure that learns. *Journal of Computing in Civil Engineering*, 19:16, 2004.

[22] J. Duffy, J. Rooney, B. Knight, and C.D. Crane III. A review of a family of self-deploying tensegrity structures with elastic ties. *The Shock and Vibration Digest*, 32(2):100, 2000.

[23] E. Fest, K. Shea, and B. Domer. Adjustable tensegrity structures. *Journal of Structural Engineering*, 129:515–526, 2003.

[24] E. Fest, K. Shea, and Ian F. C. Smith. Active tensegrity structure. *Journal of Structural Engineering*, 130(10):1454–1465, 2004.

[25] F. Fu. Structural behavior and design methods of tensegrity domes. *Journal of Constructional Steel Research*, 61(1):23–35, 2005.

[26] R.B. Fuller. Tensile-integrity structures. United States Patent 3063521, November 1962.

[27] H. Furuya. Concept of deployable tensegrity structures in space applications. *Journal of Space Structures*, 7(2):143–151, 1992.

[28] A. Hanaor. Aspects of design of double layer tensegrity domes. *Journal of Space Structures*, 7(2):101–113, 1992.

[29] A. Hanaor. Double layer tensegrity grids as deployable structures. *Journal of Space Structures*, 8:135–143, 1993.

[30] A. Hanaor. Geometrically rigid double-layer tensegrity grids. *Journal of Space Structures*, 9(4):227–238, 1994.

[31] S. Hernàndez and J.M. Mirats-Tur. Tensegrity frameworks: Static analysis review. *Journal of Mechanism and Machine Theory*. Available on

line August 2007.
[32] D.E. Ingber. Cellular tensegrity: defining new rules for biological design that govern the cytoskeleton. *Journal of Cell Science*, 104:613–627, 1993.
[33] D.E. Ingber. Architecture of life. *Scientific American*, 52:48–57, 1998.
[34] D.E. Ingber. Tensegrity I. Cell structure and hierarchical systems biology. *Journal of Cell Science*, 116(7):1157–1173, 2003.
[35] D.E. Ingber. Tensegrity II. How structural networks influence cellular information processing networks. *Journal of Cell Science*, 116:1397–1408, 2003.
[36] M. Kanchanasaratool and D. Williamson. Motion control of a tensegrity platform. *Communications in Information and Systems*, 2(3):299–324, 2002.
[37] N. Kanchanasaratool and D. Williamson. Modelling And Control of class NSP Tensegrity Structures. *International Journal of Control*, 75: 123–139, 2002.
[38] L. Kavraki, P. Svestka, J.C. Latombe, and M. Overmars. Probabilistic roadmaps for path planning in high dimensional configuration spaces. *IEEE Transactions on Robotics and Automation*, 12, 1996.
[39] J.L. Lagrange et al. *Mecanique analytique.* Mallet-Bachelier, 1853.
[40] S. LaValle. *Rapidly-exploring Random Trees: A new tool for pathplanning.* On-line, 1998.
[41] S.M. LaValle, J.H. Yakey, and L.E. Amato. A probabilistic roadmap approach for systems with closed kinematic chains. In *Proceedings of the 1999 International Conference on Robotics and Automation*, pages 1671–1676, 1999.
[42] S.M. Levin. The tensegrity-truss as a model for spinal mechanics: Biotensegrity. *Journal of Mechanics in Medicine and Biology*, 2(3), 2002.
[43] F. Li and R.E. Skelton. Sensor/actuator selection for tensegrity structures. In *45th IEEE Conference on Decision and Control*, pages 2332–2337, 2006.
[44] M. Marshall and C. D. Crane. Design and analysis of a hybrid parallel platform that incorporates tensegrity. In *Proc. of the ASME 2004 Design Engineering Technical Conferences and Computers and Information in Engineering Conference*, pages 535–540, 2004.
[45] M. Masic and R.E. Skelton. Deployable plates made from stable-element class 1 tensegrity. In *Proceedings of 2002 SPIE Smart Structures and Materials*, pages 220–230, 2002.
[46] M. Masic and R.E. Skelton. Open-loop shape control of stable unit tensegrity structures. In *Proceedings of the Third World Conference on Structural Control*, pages 439–447, 2003.
[47] M. Masic and R.E. Skelton. Open-loop control of class-2 tensegrity towers. In *Proceedings of SPIE Modeling, Signal Processing, and Control*, volume 5383, pages 298–308, 2004.
[48] M. Masic and R.E. Skelton. Selection of prestress for optimal dynamic/control performance of tensegrity structures. *International Jour-*

nal of Solids and Structures, 43(7-8):2110–2125, 2006.

[49] M. Masic, R.E. Skelton, and M. Carvalho de Oliveira. Integrated structure and control design of modular tensegrities. In *44th IEEE Conference on Decision and Control, European Control Conference*, pages 8240–8245, 2005.

[50] M. Masic, R.E. Skelton, and P.E. Gill. Optimization of tensegrity structures. *International Journal of Solids and Structures*, 43:4687–4703, 2006.

[51] R. Motro. *Tensegrity: Structural systems for the future.* Kogan Page, 2003.

[52] R. Motro, S. Najari, and P. Jouanna. Static and dynamic analysis of tensegrity systems. In *Proceedings of the ASCE International Symposium on Shell and Spatial Structures: Computational aspects*, pages 270–279. Springer, 1986.

[53] R. Motro, S. Najari, and P. Jouanna. Tensegrity systems. from design to realization. In *Proceedings of the First International conference on Lightweight structures in architecture*, 1986.

[54] B. Moussa, N. Ben Kahla, and J.C. Pons. Evolution of natural frequencies in tensegrity systems: A case study. *Journal of Space Structures*, 16 (1):57–73, 2001.

[55] H. Murakami. Static and dynamic analysis of tensegrity structures. Part 1: Nonlinear equations of motion. *International Journal of Solids and Structures*, 38:3599–3613, 2001.

[56] H. Murakami. Static and dynamic analysis of tensegrity structures. Part 2: Quasi-static analysis. *International Journal of Solids and Structures*, 38:3615–3629, 2001.

[57] H. Murakami and Y. Nishimura. Static and dynamic characterization of regular truncated icosahedral and dodecahedral tensegrity modules. *International Journal of Solids and Structures*, 38(50-51):9359–9381, 2001.

[58] H. Murakami and Y. Nishimura. Static and dynamic characterization of some tensegrity modules. *Journal of Applied Mechanics*, 68:19–27, 2001.

[59] I.J. Oppenheim and W.O. Williams. Tensegrity prisms as adaptive structures. *ASME Annual meeting*, 1997.

[60] I.J. Oppenheim and W.O. Williams. Geometric effects in an elastic tensegrity structure. *Journal of Elasticity*, 59:51–65, 2000.

[61] I.J. Oppenheim and W.O. Williams. Vibration of an elastic tensegrity structure. *European Journal of Mechanics and solids*, 20(6):1023–1031, 2001.

[62] C. Paul and H. Lipson. Redundancy in the control of robots with highly coupled mechanical structures. In *2005 IEEE/RSJ International Conference on Intelligent Robots and Systems*, pages 3585 – 3591.

[63] C. Paul, H. Lipson, and F.J. Cuevas. Evolutionary form-finding of tensegrity structures. In *Proceedings of the Conference on Genetic and Evolutionary Computation*, 2005.

[64] C. Paul, J.W. Roberts, H. Lipson, and F.V. Cuevas. Gait production in a tensegrity based robot. In *Proceedings of the International Conference on Advanced Robotics*, 2005.

[65] C. Paul, F.J. Valero-Cuevas, and H. Lipson. Design and control of tensegrity robots for locomotion. *IEEE Transactions on Robotics*, 22 (5):944–957.

[66] S. Pellegrino. Analysis of prestressed mechanisms. *International Journal of Solids and Structures*, 26(12):1329–1350, 1989.

[67] S. Pellegrino. A class of tensegrity domes. *Journal of Space Structures*, 7(2):127–142, 1992.

[68] J.P. Pinaud, M. Masic, and R.E. Skelton. Path planning for the deployment of tensegrity structures. *SPIE 10th Annual International Symposium on Smart Structures and Materials*, 2003.

[69] J.P. Pinaud, S. Solari, and R.E. Skelton. Deployment of a class 2 tensegrity boom. In *Proceedings of SPIE Smart Structures and Materials*, volume 5390, pages 155–162, 2004.

[70] A. Pugh. *An introduction to tensegrity.* University of California Press, 1976.

[71] M.G. Raja and S. Narayanan. Active control of tensegrity structures under random excitation. *Journal of Smart Materials and Structures*, 16(3):809–817, 2007.

[72] J. N. Reddy. *Introduction to the Finite Element Method*. McGraw-Hill Science/Engineering/Math; 2 edition, 1993.

[73] K. Shea, E. Fest, and I.F.C. Smith. Developing intelligent tensegrity structures with stochastic search. *Journal of Advanced Engineering Informatics*, 16(1):21–40, 2002.

[74] R. Skelton. Dynamics and control of tensegrity systems. In *IUTAM Symposium on Vibration Control of Nonlinear Mechanisms and Structures*, pages 309–318. Springer, 2005.

[75] R. Skelton. Dynamics of tensegrity systems: Compact forms. In *45th IEEE Conference on Decision and Control*, pages 2276–2281, 2006.

[76] R.E. Skelton and M. He. Smart tensegrity structure for nestor. *Smart Structures and Materials*, pages 780–787, 1997.

[77] R.E. Skelton, J.W. Helton, R.Adhikari, J.P. Pinaud, and W. Chan. *An introduction to the mechanics of tensegrity structures*, chapter 17. CRC Press, 2002.

[78] R.E. Skelton and F. Li. Economic sensor/actuator selection and its application to flexible structure control. *Smart Structures and Materials*, 5383:194–201, 2004.

[79] R.E. Skelton, J.P. Pinaud, and D.L. Mingori. Dynamics of the shell class of tensegrity structures. *Journal of the Franklin Institute*, 338:255–320, 2001.

[80] A. Smaili and R. Motro. Folding/unfolding of tensegrity systems by removal of self-stress. In *International symposium on shell and spatial structures*, pages 595–602, 2005.

[81] K.D. Snelson. Continuous tension, discontinuous compression structures. United States Patent 3169611, February 1965.

[82] I. Stern. *Development of Design Equations for Self-Deployable N-Strut Tensegrity systems*. PhD thesis, University of Florida, 1999.

[83] C. Sultan. *Modeling, design and control of tensegrity structures with applications*. PhD thesis, Purdue Univeristy, 1999.

[84] C. Sultan, M. Corless, and R.E. Skelton. Peak-to-peak control of an adaptive tensegrity space telescope. In *Proceedings of the SPIE Smart Structures and Materials*, volume 3667, pages 190–201, 1999.

[85] C. Sultan, M. Corless, and R.E. Skelton. Tensegrity flight simulator. *Journal of Guidance, Control, and Dynamics*, 23(6):1055–1064, 2000.

[86] C. Sultan, M. Corless, and R.E. Skelton. The prestressability problem of tensegrity structures: some analytical solutions. *International Journal of Solids and Structures*, 38:5223–5252, 2001.

[87] C. Sultan, M. Corless, and R.E. Skelton. Linear dynamics of tensegrity structures. *Journal of Engineering structures*, 24:671–685, 2002.

[88] C. Sultan, M. Corless, and R.E. Skelton. Symmetrical reconfiguration of tensegrity structures. *International Journal of Solids and Structures*, 39(8):2215–2234, 2002.

[89] C. Sultan and R.E. Skelton. Force and torque smart tensegrity sensor. In *Proceedings of the SPIE Smart Structures and Materials*, volume 3323, pages 357–368, 1998.

[90] C. Sultan and R.E. Skelton. Tendon control deployment of tensegrity structures. In *Proceedings of the SPIE Smart Structures and Materials*, volume 3323, pages 455–466, 1998.

[91] C. Sultan and R.E. Skelton. Deployment of tensegrity structures. *International Journal of Solids and Structures*, 40(18):4637–4657, 2003.

[92] C. Sultan, D. Stamenovic, and D.E. Ingber. A computational tensegrity model predicts dynamic rheological behaviors in living cells. *Annals of Biomedical Engineering*, 32(4):520–530, 2004.

[93] A. G. Tibert and S. Pellegrino. Deployable tensegrity reflectors for small satellites. *Journal of Spacecraft and Rockets*, 39(5):701–709, 2002.

[94] A.G. Tibert. *Deployable tensegrity structures for space applications*. PhD thesis, Royal institute of technology, 2003.

[95] A.G. Tibert and S. Pellegrino. Review of form-finding methods for tensegrity structures. *International Journal of Solids and Structures*, 18(4):209–223, 2003.

[96] T.M. Tran, C.D. Crane, and J. Duffy. The reverse displacement analysis of a tensegrity based parallel mechanism. In *Proceedings of the 5ht Biannual world Automation Congress*, 2002.

[97] M. van de Wal and B. de Jager. A review of methods for input/output selection. *Automatica*, 37(4):487–510, 2001.

[98] J.J.M. van de Wijdeven and A.G. de Jager. Shape change of tensegrity structures: design and control. In *Proceedings of the American Control Conference*, 2005.

[99] S. Vogel. *Cats'Paws and Catapults: Mechanical worlds of nature and people.* WW Norton & Company, 1998.

[100] N. Wang, K. Naruse, D. Stamenovic, J.J Fredberg, S.M. Mijailovich, I.M. Tolic-Norrelykke, T. Polte, R. Mannix, and D.E. Ingber. Mechanical behavior in living cells consistent with the tensegrity model. In *Proceedings of the National Academy of Sciences*, volume 98, pages 7765–7770. National Academy of Sciences, 2001.