Finite Element based Sequential Bayesian Non-Rigid Structure from Motion

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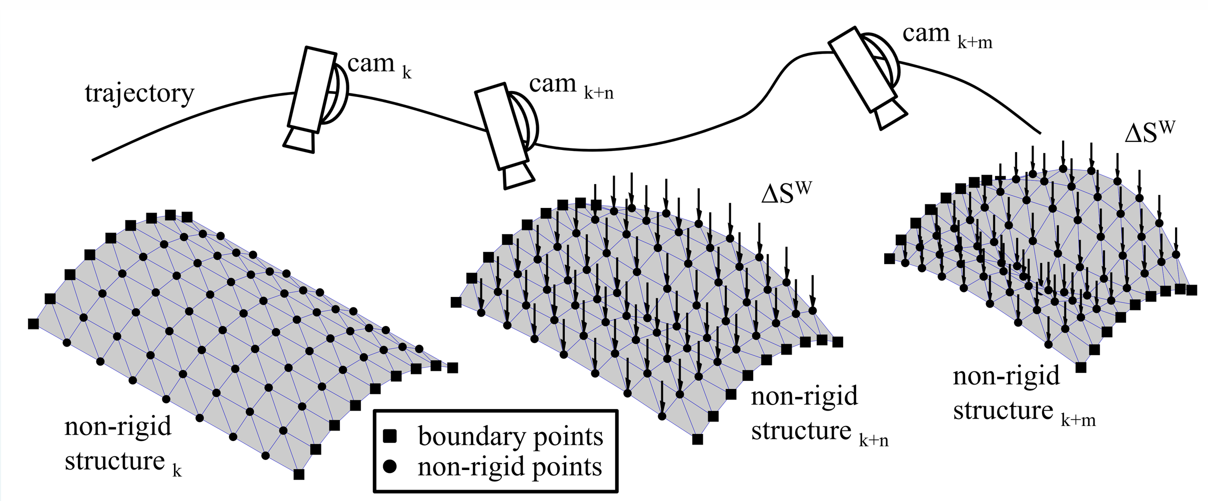
Contributions

Non-Rigid Structure from Motion embedding Navier's equations coded as FEM (Finite Element Method) within Bayesian sequential estimator coded as EKF (Extended Kalman Filter):

- DA (Data Association) is computed. Classically, DA assumed as prior (2D tracking)
- Potential 30 Hz real-time performance
- Full-perspective camera, no orthographic simplified camera
- Dealing with isometric and non-isometric scenes just by tuning
- Can cope with partial occlusions
- No boundary points have to be identified
- Able to perform even with small size maps

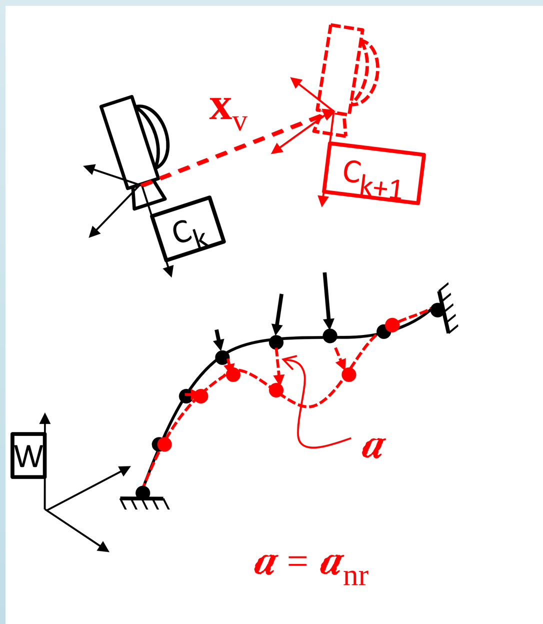
EKF+FEM Non-Rigid Structure Sequential Estimation

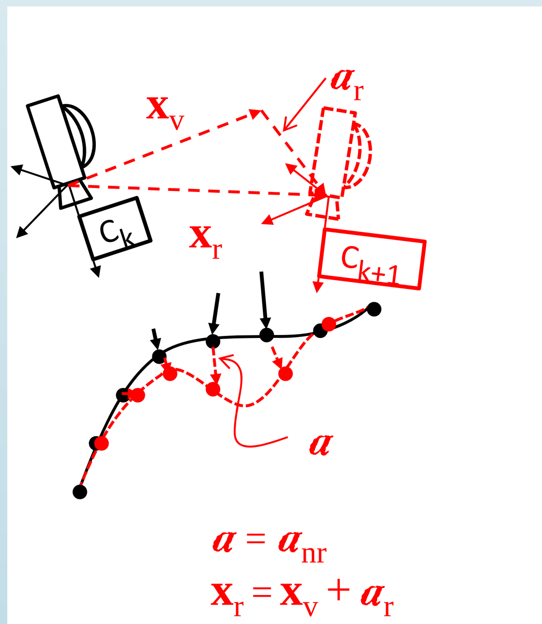


- Thin-plate FEM model. Tuning: Young's modulus E and Poisson's ratio ν
- FEM linear system:

$$\boldsymbol{K}\ \boldsymbol{a} = \boldsymbol{f}$$

- No prior boundary points: $\boldsymbol{K}^{+}$ matrix Moore-Penrose (MP) pseudoinverse





- Camera-centric pose prediction:

$$\mathbf{g}_r = \begin{pmatrix} \mathbf{r}^{C_k}_{k+1} \\ \mathbf{q}^{C_k}_{k+1} \\ \mathbf{v}^{C_k}_{k+1} \\ \omega^{C_k}_{k+1} \end{pmatrix} = \begin{pmatrix} \mathbf{r}^{C_k}_{k} + (\mathbf{v}^{C_k}_{k} + \mathbf{V}^C)\Delta t \\ \mathbf{q}^{C_k}_{k} \times \mathbf{q}\left((\omega^{C_k}_{k} + \Omega^C)\Delta t\right) \\ \mathbf{v}^{C_k}_{k} + \mathbf{V}^C \\ \omega^{C_k}_{k} + \Omega^C \end{pmatrix}$$

- Non-rigid FEM-based scene prediction:

$$\mathbf{y}^{C_k}_{k+1} = \mathbf{y}^{C_k}_{k} + \boldsymbol{K}^{+}_{k}\Delta\mathbf{S}^{C}$$

- Normalized forces:

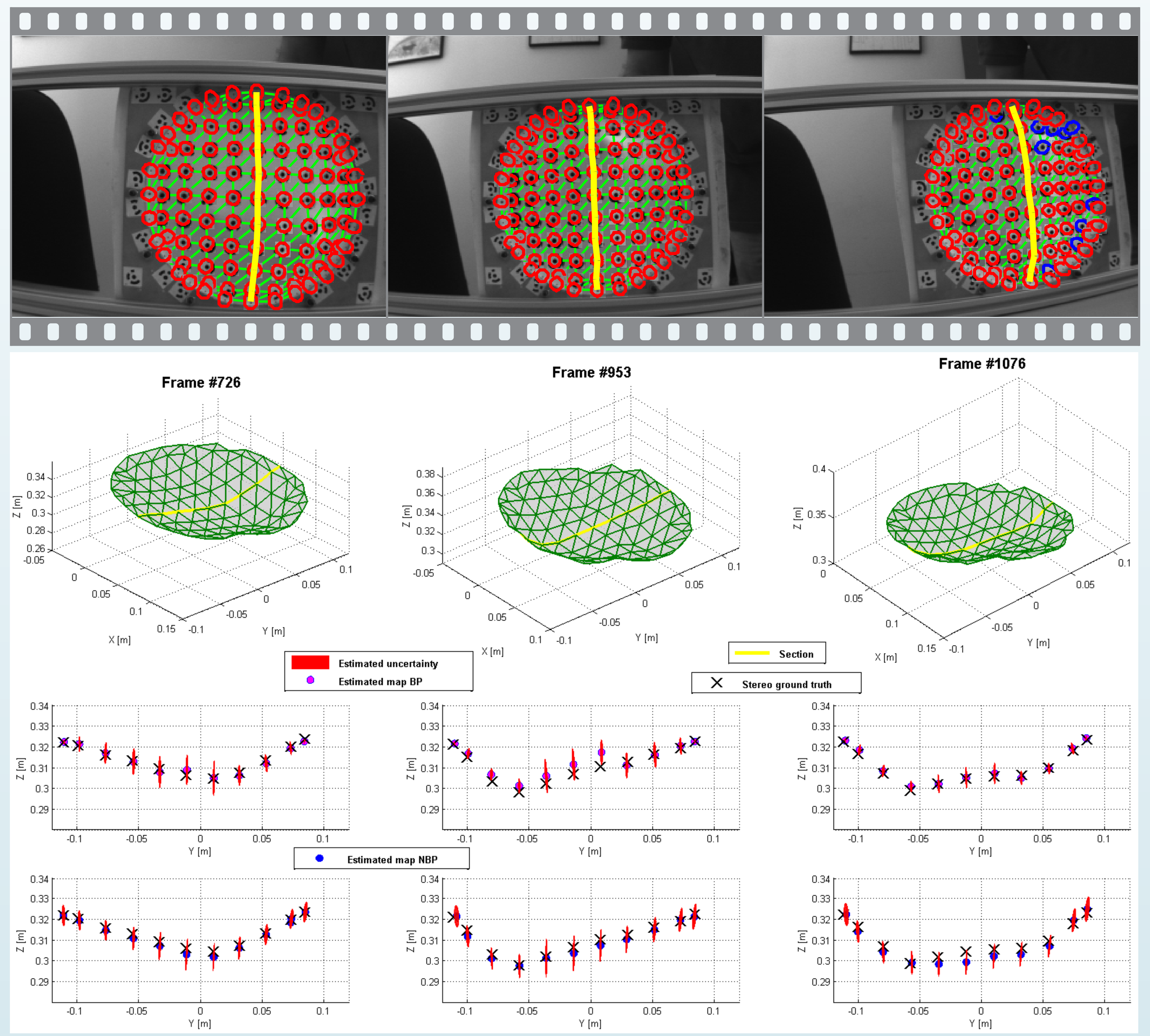
$$\Delta\mathbf{S}^{C}_{i} = \frac{1}{Eh}\left(\Delta f^{C}_{xi}, \Delta f^{C}_{yi}, \Delta f^{C}_{zi}\right)^{\top}$$

- $O\left(n^3\right)$ computational complexity:
  - EKF $O\left(n^3\right)$
  - MP pseudoinverse $O\left(12n^3\right)$
- DA EKF prediction elliptical 3σ search region

Experimental Results

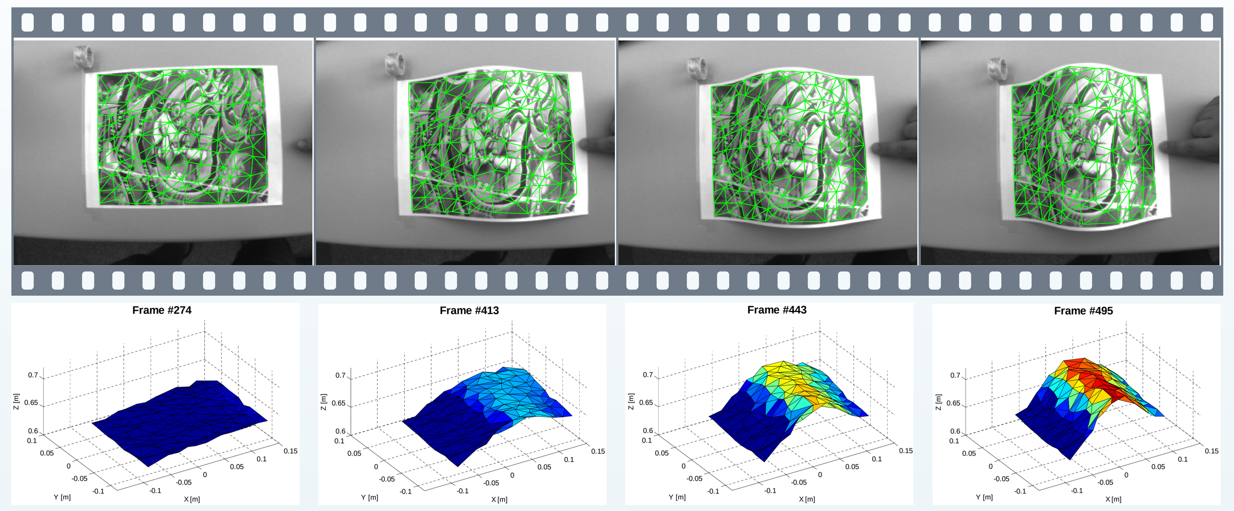
The proposed method is validated with real image sequences 320×240. The first sequence frames are used to estimate structure at rest by means of rigid EKF-SLAM

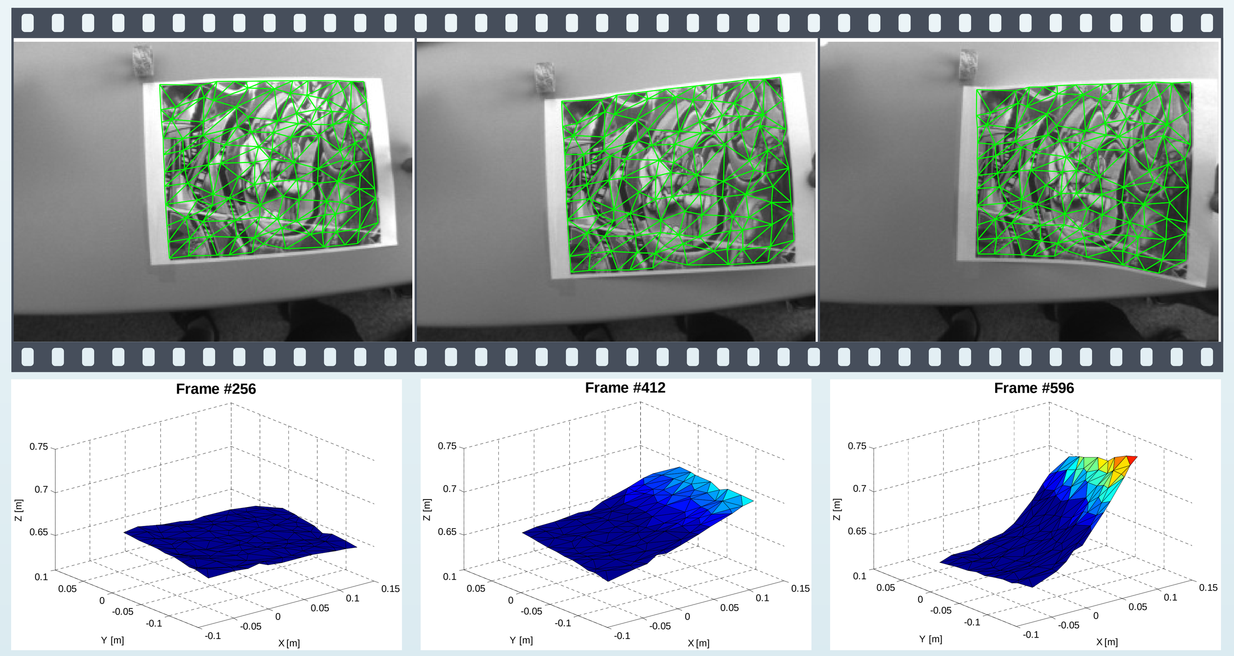
Multiply Deformed Silicone Sequence



- Non-Isometric deformation
- Accurate and consistent wrt ground truth
- Waving hand-held camera

Paper Sequences





- Isometric deformation
- Natural landmarks
- Irregular triangles
- Static or waving camera

Material Parameter Tuning

Parameter	Silicone	Paper
h (m)	$1.5 \cdot 10^{-3}$	$1.0 \cdot 10^{-4}$
ν	0.499	0.499
$\frac{\Delta f}{Eh}$ (m) (std.)	$1.5 \cdot 10^{-5}$	$2.0 \cdot 10^{-9}$

Computation Time

$\approx 1 \frac{\text{sec}}{\text{frame}}$ ≈ 100 map points Matlab

References

[1] Antonio Agudo, Begoña Calvo and J. M. M. Montiel. FEM models to code non-rigid EKF monocular SLAM. In *International Workshop 4DMOD (ICCV) 2011*

[2] J. A. Castellanos, J. Neira and J. D. Tardós. Limits to the consistency of EKF-based SLAM. In *IFAC Symposium on Intelligent Autonomous Vehicles 2004*

Conclusions

- EKF+FEM can solve NRSfM
- Low cost linear FEM is accurate enough
- Rigid boundary points removed
- Complexity depends on map size but not on number of frames

Future Work

- 30 Hz Real-time performance
- Comparison with respect to state-of-the-art NRSfM methods
- Determining extreme deformations limits
- Processing medical endoscope sequences

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