Optimal Design of a 6-DOF 4-4 Parallel Manipulator with Uncoupled Singularities

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Abstract
A 6-DOF 4-4 parallel manipulator is presented. Its forward kinematics can be solved by a sequence of three trilaterations and, as a consequence, its singularities can be described in geometric terms as the degeneration of three tetrahedra. Moreover, it is shown how the proposed manipulator belongs to the family of flagged parallel manipulators. This identification is useful because the topology of the singularity locus of flagged manipulators have been fully characterized and, what is more important, the singularities of flagged manipulators correspond to uncoupled translations and/or rotations in the workspace of the manipulator. An optimization of its workspace is carried out using Sequential Quadratic Programming and a virtual prototype of the optimal result have been implemented in SolidWorks™.

Keywords: parallel manipulators, robot kinematics, manipulator design, kinematics singularities.

1. Introduction

Parallel manipulators exhibit many advantages when compared to their serial counterparts, such as higher payload capacity, higher speed and acceleration, and higher accuracy and stiffness [1]. However, a general 6-6 parallel manipulator has some drawbacks such as difficult forward kinematics and singularity characterization, and small workspace so as to make motion planning and control difficult in most applications. Therefore, many efforts have been devoted to the study of parallel manipulators well-adapted to particular tasks that overcome some of the above drawbacks.

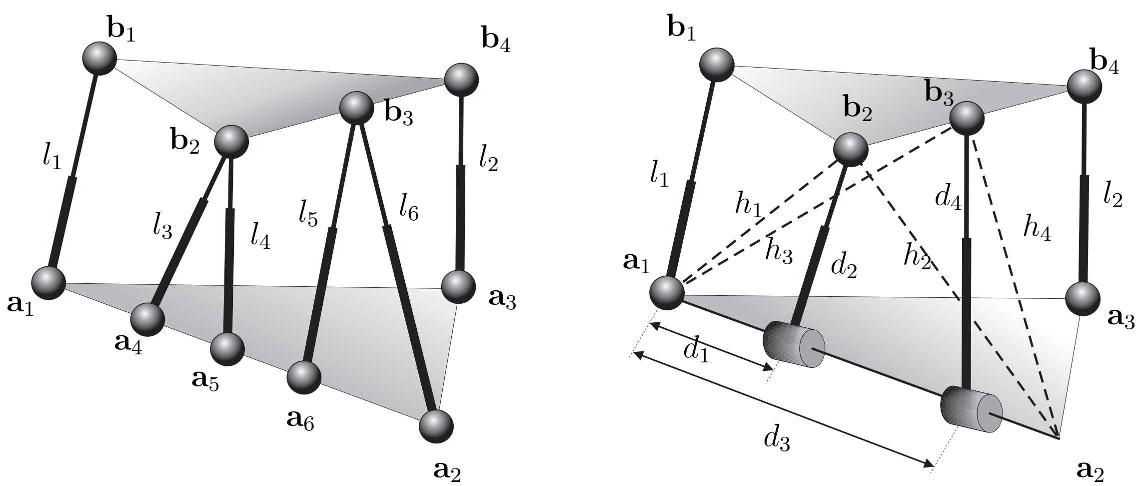
In this paper we present a 4-4 parallel manipulator, which possesses favorable kinematics and mechanical properties, when compared to other designs, because it belongs to a family of manipulators called *flagged* that have several interesting properties in terms of the simplicity of their singularity loci [2, 3].

The proposed manipulator is composed of two $\underline{P}R\underline{P}S$ serial chains and two $U\underline{P}S$ serial chains to attain 6 DOFs for the moving platform. To the best of our knowledge, the only 4-legged manipulators with 6-DOF proposed are the parallel manipulator in [4] and the haptic device proposed in [5], and on the contrary to our architecture, both they are actuated redundantly.

A dimensional optimal synthesis for the proposed manipulator is performed by taking into account design constraints such as leg interference and workspace limits. Using SolidWorks, a virtual prototype of the obtained optimal design is presented.

2. The proposed manipulator: geometric description and analysis

Let us consider the 6-4 parallel manipulator in Figure 1(a). A pair of legs sharing an endpoint can be replaced by a PRPS serial chain. After this substitution, the resulting parallel robot appears as in Figure 1(b). The set of actuated variables in both platforms are in a one-to-one correspondence, so both platforms are kinematically equivalent.



(a) (b)

Figure 1: *A 4-6 parallel manipulator (a), and its kinematically equivalent 4-4 manipulator (b).*

With this substitution, the original 6-4 parallel robot becomes a simpler 4-legged parallel robot with the same kinematic properties and a lower risk of leg interference. Therefore, we can deduce a solvable formulation with more efficient algorithms.

An algebraic closed-form solution for the forward kinematic of the proposed parallel robot can be found by solving a system of quadratic equations as described in [6]. However, a more straightforward solution is possible based on geometric arguments only. First, notice that we can obtain the distance from the spherical joint center of any of the two PRPS chains to any point on the axis of their first prismatic joints by applying the Pythagorean

Theorem. In other words, the distances h_1, h_2, h_3, and h_4 in Figure 1(b) can be readily obtained from d_1, d_2, d_3, and d_4.

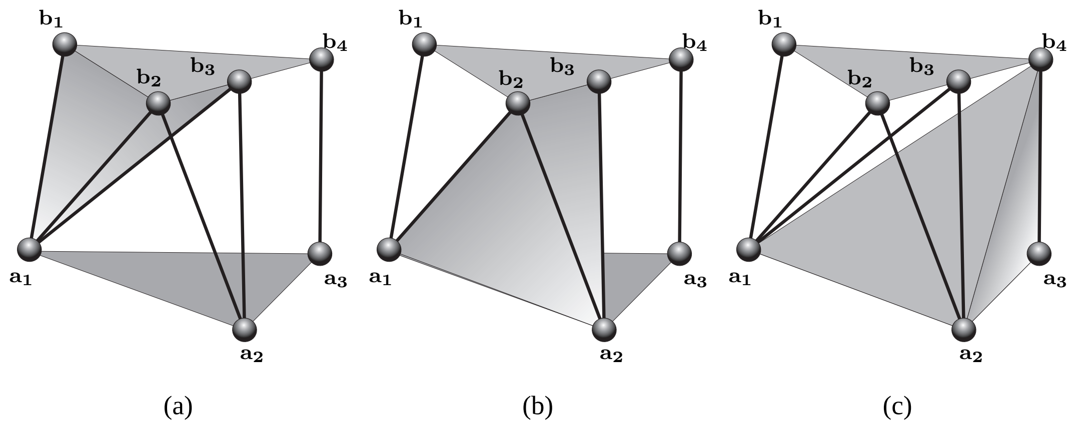


Figure 2: *The 3 trilaterations that solve the forward kinematics of the proposed manipulator.*

Next, it is easy to see that the location of points $\mathbf{a}_1$, $\mathbf{a}_2$, and $\mathbf{a}_3$, with respect to the platform reference frame, can be obtained by a sequence of the three trilaterations shown in Figure 2. Two mirror locations for points $\mathbf{a}_1$, $\mathbf{a}_2$ and $\mathbf{a}_3$ in each trilateration are possible, so that they give a total of 8 assembly modes [7].

Therefore, if, and only if, the points in the sets $\{\mathbf{b_1}, \mathbf{b_2}, \mathbf{b_3}, \mathbf{a_1}\}$, $\{\mathbf{a_1}, \mathbf{b_2}, \mathbf{b_3}, \mathbf{a_2}\}$ and $\{\mathbf{b_4}, \mathbf{a_1}, \mathbf{a_2}, \mathbf{a_4}\}$ form non-degenerate tetrahedra, there are eight possible configurations for the moving platform compatible with a given set of leg lengths. Otherwise, the parallel manipulator is in a singularity [8]. Alternatively, we can say that the manipulator is in a singularity if $\mathbf{a}_1$ is on the platform plane, the lines defined by $\mathbf{a}_1\mathbf{a}_2$ and $\mathbf{b}_2\mathbf{b}_3$ intersect, or $\mathbf{b}_4$ is on the base plane. This reinterpretation is important because it is not expressed in terms of leg locations but directly in terms of points locations and edges attached to either the base or the platform.

Now let us consider a plane, a line and a point so that the point is contained in the line, and the line in the plane. This geometric entity is called a flag. Therefore, if two flags are placed on the manipulator base and platform, then the manipulator singularities coincide with flag configurations in which either the vertex of one flag lies on the plane of the other flag or the two flag lines intersect. The flag attached to the base consists of point $\mathbf{a}_1$, the line defined by $\mathbf{a}_1$ and $\mathbf{a}_2$, and the plane defined by $\mathbf{a}_1$, $\mathbf{a}_2$, and $\mathbf{a}_3$. The flag attached to the platform consists of point $\mathbf{b}_4$, the line defined by $\mathbf{b}_4$ and $\mathbf{b}_2$, and the plane defined by

$\mathbf{b}_4$, $\mathbf{b}_2$, and $\mathbf{b}_1$.

A parallel manipulator whose singularities can be described in terms of incidences between two flags is called a flagged manipulator [2]. The relevance of these manipulators derives from the fact that their singularity analysis is quite simple because: (1) the topology of their singularity spaces is the same for all members of the family irrespective of changes in their kinematic parameters; (2) their singularity spaces can be easily decomposed into manifolds, or cells, forming what in algebraic geometry is called a *stratification*, derived from that of the flag manifold [3]; (3) each cell can be characterized using a single local chart whose coordinates directly correspond to uncoupled translations and/or rotations in the workspace of the manipulator; (4) any path connecting two assembly modes passes through a singularity (note that this assertion is not true in general [9]).

3. Optimal design and numerical results

The optimum design problem with dimensionless objective function $F(X)$ can be formulated as finding the optimal design parameters values to obtain the position workspace volume that is as close as possible to a prescribed one in the form $F(X) = \left|1 - \frac{\text{Vol}}{\text{Vol}_0}\right|$ where $|\cdot|$ is the absolute value; Vol is the position workspace volume, which is evaluated at each iteration, and Vol_0 refers to a prescribed value. The workspace optimization problem can be also subjected to design constraints such as $x_{min} \leq x \leq x_{max}$, $y_{min} \leq y \leq y_{max}$ and $z_{min} \leq z \leq z_{max}$. The optimum design algorithm takes into account interferences among legs as described next.

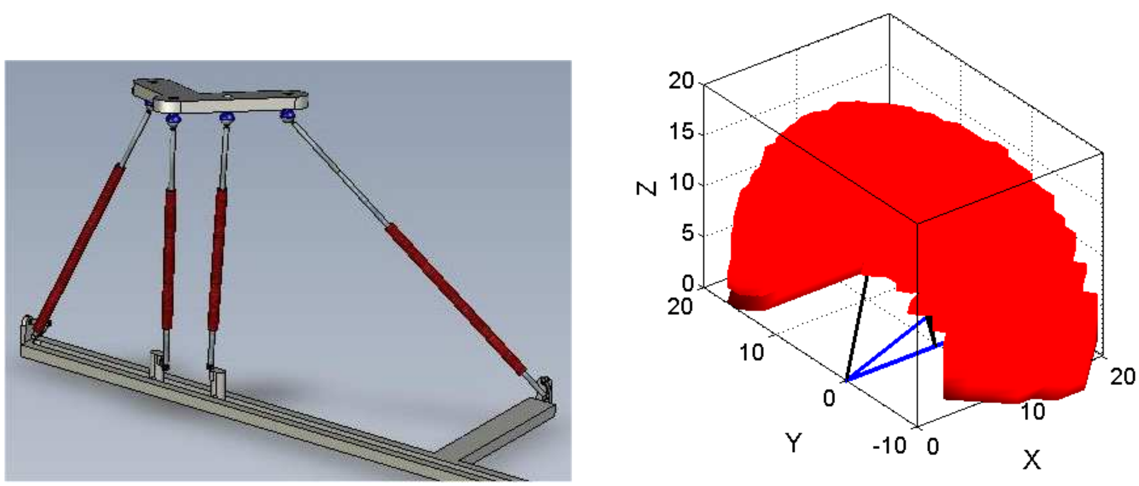
Considering that leg attachments motions are sufficiently small per sampling interval, the trajectory described by each leg can be assumed to be bounded by the tetrahedron described by the locations of the leg attachments before and after the motion. One way to guarantee that such leg crossings will not occur is by checking whether these tetrahedra do not intersect each other. That is, we must verify that for the four legs, the tetrahedra described by the four attaching points $\{\mathbf{a}_i(t), \mathbf{a}_i(t+\Delta t), \mathbf{b}_i(t), \mathbf{b}_i(t+\Delta t)\}$ do not intersect. A very fast test for tetrahedra intersection is based on the Separating Axis Theorem described in the computer graphics literature [10].

The optimization has been carried out for the position workspace of the proposed manipulator. An optimal design is obtained with a Sequential Quadratic Programming by means of a numerical algorithm proposed in [11]. Design variables are the locations of base and platform attachment points, namely $\mathbf{a}_3 = (a_{3x}, a_{3y}, 0)$, $\mathbf{b}_1 = (b_{1x}, b_{1y}, 0)$, $\mathbf{b}_2 = (b_2, 0, 0)$ and $\mathbf{b}_3 = (b_3, 0, 0)$, according to the kinematic sketch in Figure 1(b), where the base reference is located at $\mathbf{a}_1$ and platform reference at $\mathbf{b}_4$.

Table 1: *Design parameters and workspace limits for the optimal design*

Values	a_{3x}	a_{3y}	b_{1x}	b_{1y}	b_2	b_3	x_{max}	x_{min}	y_{max}	y_{min}	z_{max}	z_{min}
Initial Guess	10	4	4	-1	3	1.5	10.052	0	10.770	-5.026	9.334	0
Optimal	19.1829	7.0062	4.2892	-3.8858	2.6067	4.9094	20	0	22	-12	18	0

A prescribed workspace volume has been chosen equal to 8000. The proposed optimum design algorithm takes 220 iterations to converge to an optimal solution. Design parameters and characteristics for the optimal procedure are reported in table 1. The workspace volume of the optimal solution is 7992.



(a) (b)

Figure 3: *Numerical results for the optimal solution: the optimized architecture as described in Solidworks (a); representation of the position workspace (b).*

4. Conclusions

A new 6-DOF parallel manipulator has been proposed. It possesses favorable properties with respect to other designs, which can be summarized as follows: (1) It has a closed-form solution for its forward kinematics; (2) It is a flagged parallel robot. As a consequence, it has a well-behaved cell decomposition for its singularity locus; (3) It is simpler in structure (it posses four legs) and consequently cheaper than most other designs; (4) It possesses a large workspace, with great orientation capabilities; (5) The singularities can be characterized as uncoupled translations and/or rotations in the workspace of the manipulator.

We have also proposed an optimum design procedure for prescribed positioning capabilities. Preliminary practical design considerations have been shown and a mechanical design of the optimized structure has been also presented.

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