Sparse Kalman Filter

Joan Solà

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1 Prediction-type operations

1.1 Robot motion

General function	$x = f(x, u, q)$	
Sparse function	$r = f(r, u, q)$	
In	r	robot
Invariant	m	mapped landmarks
Out	r	robot
Ignored in	i	
Used in	$x = r + m$	
Used out	$x = r + m$	
Ignored out	i	

Matrix partitions

$$x = \begin{bmatrix} r & m & i \end{bmatrix}$$

Jacobian

$$F = \begin{bmatrix} F_r & 0 & * \\ 0 & I & * \\ * & * & * \end{bmatrix}$$

Covariances

$$P = \begin{bmatrix} P_{rr} & P_{rm} & * \\ P_{mr} & P_{mm} & * \\ * & * & * \end{bmatrix}$$

Output covariances

$$FPF^\top + Q = \begin{bmatrix} F_r P_{rr} F_r^\top + Q_{rr} & F_r P_{rm} & * \\ P_{mr} F_r^\top & P_{mm} & * \\ * & * & * \end{bmatrix}$$

1.2 Landmark initialization

General function	$x = g(x, y, n)$	
Sparse function	$l = g(r, y, n)$	
In	r	robot+sensor
Invariant	$r + m$	robot+sensor+map
Out	l	new landmark
Ignored in	i	
Used in	$x = r + m$	
Used out	$x = r + m + l$	
Ignored out	$i - l$	

Matrix partitions

$$x = \begin{bmatrix} r & m & l & i \end{bmatrix}$$

Jacobian

$$G = \begin{bmatrix} I & 0 & * & * \\ 0 & I & * & * \\ G_r & 0 & * & * \\ * & * & * & * \end{bmatrix}$$

Covariances

$$P = \begin{bmatrix} P_{rr} & P_{rm} & * & * \\ P_{mr} & P_{mm} & * & * \\ * & * & * & * \\ * & * & * & * \end{bmatrix}$$

Output covariances

$$GPG^\top + (R) + (N) = \begin{bmatrix} P_{rr} & P_{rm} & P_{rr}G_r^\top & * \\ P_{mr} & P_{mm} & P_{mr}G_r^\top & * \\ G_rP_{rr} & G_rP_{rm} & G_rP_{rr}G_r^\top + G_yRG_y^\top + G_nNG_n^\top & * \\ * & * & * & * \end{bmatrix}$$

1.3 Landmark re-parametrization

General function	$x = j(x)$	
Sparse function	$l = j(k)$	
In	k	old landmark
Invariant	m	all map
Out	l	new landmark
Ignored in	i	
Used in	$x = k + m$	map with old lmk
Used out	$x = m + l$	map with new lmk
Ignored out	$i + k - l$	

Matrix partitions

$$x = \begin{bmatrix} k & m & l & i \end{bmatrix}$$

Jacobian

$$J = \begin{bmatrix} * & * & * & * \\ 0 & I & * & * \\ J_k & 0 & * & * \\ * & * & * & * \end{bmatrix}$$

Covariances

$$P = \begin{bmatrix} P_{kk} & P_{km} & * & * \\ P_{mk} & P_{mm} & * & * \\ * & * & * & * \\ * & * & * & * \end{bmatrix}$$

Output covariances

$$JPJ^\top = \begin{bmatrix} * & * & * & * \\ * & P_{mm} & P_{mk}J_k^\top & * \\ * & J_kP_{km} & J_kP_{kk}J_k^\top & * \\ * & * & * & * \end{bmatrix}$$

2 Correction-type operations

2.1 Individual landmark correction

General function	$y = h(x)$	
Sparse function	$y = h(r, l)$	
In 1	r	robot+sensor
In 2	l	observed landmark
Passive	m	other landmarks
Out	y	measurement
Updated	r+m+l	robot+sensor+landmarks
Ignored	i	

Matrix partitions

$$x = \begin{bmatrix} k & m & l & i \end{bmatrix}$$

Jacobian

$$H = \begin{bmatrix} H_r & 0 & H_l & * \end{bmatrix}$$

Covariances

$$P = \begin{bmatrix} P_{rr} & P_{rm} & P_{rl} & * \\ P_{mr} & P_{mm} & P_{ml} & * \\ P_{lr} & P_{lm} & P_{ll} & * \\ * & * & * & * \end{bmatrix}$$

Expectation matrix

$$E = H_r P_{rr} H_r^\top + H_r P_{rl} H_l^\top + H_l P_{lr} H_r^\top + H_l P_{ll} H_l^\top$$

Innovation matrix

$$Z = E + R$$

Band matrix

$$PH^\top = \begin{bmatrix} P_{xr} H_r^\top + P_{xl} H_l^\top \\ * \end{bmatrix}$$

Kalman gain

$$K = PH^\top Z^{-1}$$

Covariances update

$$P = P - K(PH^\top)^\top$$

2.2 Buffered landmarks correction

General function	$y = h(x)$	
Sparse function	$y = h(r, l_1, l_2)$	
In 1	r	robot+sensor
In 2	l_1	observed landmark
In 3	l_2	observed landmark
Passive	m	other landmarks
Out	$y1$	measurement
Out	$y2$	measurement
Updated	$r + m + l_1 + l_2$	robot + sensor + all landmarks
Ignored	i	

Matrix partitions

$$x = \begin{bmatrix} r & m & l_1 & l_2 & i \end{bmatrix}$$

Jacobian

$$H = \begin{bmatrix} H_{r1} & 0 & H_{l1} & 0 & * \\ H_{r2} & 0 & 0 & H_{l2} & * \end{bmatrix}$$

Covariances

$$P = \begin{bmatrix} P_{rr} & P_{rm} & P_{r1} & P_{r2} & * \\ P_{mr} & P_{mm} & P_{m1} & P_{m2} & * \\ P_{1r} & P_{1m} & P_{11} & P_{12} & * \\ P_{2r} & P_{2m} & P_{21} & P_{22} & * \\ * & * & * & * & * \end{bmatrix}$$

Expectation matrix

$$E = \begin{bmatrix} E_{11} & E_{12} \\ E_{21} & E_{22} \end{bmatrix}$$

with

$$E_{ij} = H_{ri} P_{rr} H_{rj}^\top + H_{ri} P_{rj} H_{lj}^\top + H_{li} P_{ij} H_{lj}^\top + H_{li} P_{ir} H_{rj}^\top$$

Innovation matrix

$$Z = \begin{bmatrix} E_{11} + R & E_{12} \\ E_{21} & E_{22} + R \end{bmatrix}$$

Band matrix

$$\begin{bmatrix} PH^\top \\ * \end{bmatrix} = \begin{bmatrix} (PH^\top)_1 & (PH^\top)_2 \\ * & * \end{bmatrix}$$

with

$$(PH^\top)_i = \begin{bmatrix} P_{xr} H_{ri}^\top + P_{xi} H_{li}^\top \end{bmatrix}$$

Kalman gain

$$\begin{bmatrix} K \\ * \end{bmatrix} = \begin{bmatrix} PH^\top \\ * \end{bmatrix} Z^{-1}$$

Covariances update

$$P = P - K (PH^\top)^\top$$