Reactive Stepping to Prevent Falling for Humanoids

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Abstract— **This paper addresses a reactive motion of a humanoid robot to maintain a balance against disturbance. Although a small disturbance can be absorbed without stepping, it is necessary for reactive stepping to prevent a humanoid robot from falling by a large disturbance. On demand a stepping, it is expected to modify a position of a swing foot as soon as possible to keep its stability. To realize such reactive stepping, not only the COG (Center of Gravity) and the ZMP (Zero-Moment Point), but also the foot placement are determined by a real-time gait planning method based on analytical approach according to trunk position and velocity error. The effectiveness of proposed method is confirmed by simulation.**

I. Introduction

An improvement of stability is necessary for a humanoid robot to operate in human environment. A humanoid robot can prevent from falling over by a reactive stepping when a large force is affected, although it can absorb a small disturbance by motion of a body. Therefore humanoid robots have a potential to recover balance against an unexpected large disturbance. For this reactive stepping, how to update motion satisfying a dynamic stability according to sensor information as soon as possible is one of the most important techniques.

However the dynamics of humanoid robot becomes non-linear and high dimension. Therefore, to calculate in online, most of reactive stepping in previous researches are based on a linear inverted pendulum model and are realized by its state feedback[1]-[7]. Miura discretized an linear inverted pendulum model by a step cycle under instantanenous changing of a support leg, and determined a landing position by feedback of the COG position and velocity[1]. Komura presented a modification of a foot position to counteract strong disturbance using the angular momentum inducing the inverted pendulum[2]. Nishiwaki proposes the dynamically stable pattern method which updates the COG trajectory through the ZMP tracking control in a short cycle using the preview control[3]. Platt presented *the capture point* which investigates a stable region by the inverted pendulum with flywheel model[4]. Hyon proposes a passivity based full-body force control method and presented a compensatory motion for a by stepping[5]. Diedam proposes adaptive foot positioning under several constraints using linear model predictive control[6]. Tajima demonstrated a humanoid robot that can recover from human pushing disturbance during running in place[7].

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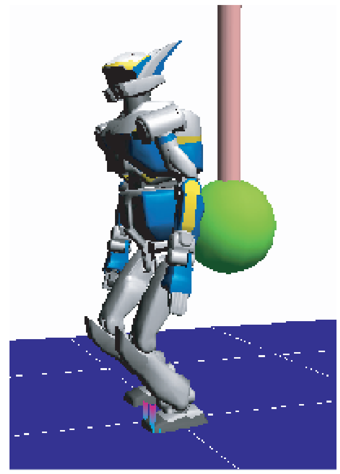


Fig. 1. Large disturbance affects on humanoid robot

The aim of our goal is to improve a response against disturbance and to realize keeping a balance against any disturbance by reactive stepping as necessary (Fig.1). At first, a fast real-time gait planning method based on analytical solution is formulated as shown in Section II. Then we extend a real-time gait planning method and not only the COG and the ZMP but also the foot placement are determined in Section III. The swing motion which can be modified immediately for a request is shown in Section IV. The effectiveness of proposed reactive motion by simulation is shown in Section V. Finally we conclude in Section VI

II. Simultaneous Planning of the COG and the ZMP

We previously proposed online simultaneous planning of the COG and the ZMP which allows immediate modification of foot placement[10],[11] based on analytical approach[9]. In this method, to modify a foot placement immediately, the 3rd and 4th order polynomials are applied to the ZMP trajectory with a fluctuation. In a similar way, the faster calculation method for the unknown coefficients of analytical solution was appeared by Hong[12]. This paper also derives fast COG and ZMP simultaneous planning using above formulation.

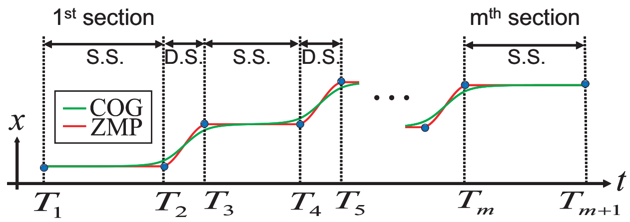


Fig. 2. The COG and The ZMP trajectories

Let us focus on the COG motion on sagittal plane, and the COG dynamics of a humanoid robot in x-axis can be approximated by an inverted pendulum which is given as

$$\begin{aligned} \ddot{x}(t) &= \omega^2(x(t) - p_x(t)), \\ \omega &\equiv \sqrt{\frac{g + \ddot{z}(t)}{z(t) - p_z(t)}}, \end{aligned} \tag{1}$$

where x and z are horizontal and vertical position of the COG. p_x is the ZMP position. g means gravity constant. The COG motion on lateral plane can be also formulated as the same equation.

At first, a biped gait is divided into several sections every single support phase (S.S.) and double support phase (D.S.) shown in Fig.2 and each section is applied to an analytical solution. Where T_1 is current time, and m is number of sections. If N steps are planned in advance, m becomes $2N + 1$.

At j-th section, let us assume that the ZMP $p_x^{(j)}$ can be represented by N_j-th order polynomial, that is

$$\begin{aligned} p_x^{(j)}(t) &= \sum_{i=0}^{N} P_i^{(j)}(\Delta t_j)^i, \\ \Delta t_j &\equiv t - T_j, \end{aligned} \tag{2}$$

where $P_i^{(j)}$ is coefficient of polynomials which is determined by boundary conditions and connectivity of the trajectory of the COG and the ZMP. We assume that the COG height is constant and is calculated at an average of initial and terminal value in each section.

$$\begin{aligned} \omega_j &= \sqrt{\frac{g}{\tilde{z}_j}}, \\ \tilde{z}_j &= \frac{(z(T_{j+1}) - p_z(T_{j+1})) - (z(T_j) - p_z(T_j))}{2} \end{aligned} \tag{3}$$

From (1)-(3), the analytical solution with 4th order polynomial of the ZMP trajectory can be obtained.

$$\begin{aligned} x^{(j)}(t) &= V^{(j)}\cosh(\omega_j \Delta t_j) + W^{(j)}\sinh(\omega_j \Delta t_j) \\ &+ \left(P_0^{(j)} + \frac{2}{\omega_j^2}P_2^{(j)} + \frac{24}{\omega_j^4}P_4^{(j)}\right) + \left(P_1^{(j)} + \frac{6}{\omega_j^2}P_3^{(j)}\right)\Delta t_j \\ &+ \left(P_2^{(j)} + \frac{12}{\omega_j^2}P_4^{(j)}\right)(\Delta t_j)^2 + P_3^{(j)}(\Delta t_j)^3 + P_4^{(j)}(\Delta t_j)^4 \end{aligned} \tag{4}$$

Substituting the initial position and velocity of the COG $x^{(j)}(T_j)$, $\dot{x}^{(j)}(T_j)$ and the ZMP $p_x^{(j)}(T_j)$, $\dot{p}_x^{(j)}(T_j)$ at j-th section into (2), (4) and its derivatives, some coefficients can be calculated as

$$P_0^{(j)} = p_x^{(j)}(T_j), \tag{5}$$

$$P_1^{(j)} = \dot{p}_x^{(j)}(T_j), \tag{6}$$

$$V^{(j)} = x^{(j)}(T_j) - p_x^{(j)}(T_j) - \frac{2}{\omega_j^2}P_2^{(j)} - \frac{24}{\omega_j^4}P_4^{(j)}, \tag{7}$$

$$W^{(j)} = \frac{1}{\omega_j}\left(\dot{x}^{(j)}(T_j) - \dot{p}_x^{(j)}(T_j) - \frac{6}{\omega_j^2}P_3^{(j)}\right). \tag{8}$$

In term of continuity, the relation between the sections of the COG and the ZMP can be represented.

$$x^{(j)}(T_{j+1}) = x^{(j+1)}(T_j) \tag{9}$$

$$\dot{x}^{(j)}(T_{j+1}) = \dot{x}^{(j+1)}(T_j) \tag{10}$$

$$p_x^{(j)}(T_{j+1}) = p_x^{(j+1)}(T_j) \tag{11}$$

$$\dot{p}_x^{(j)}(T_{j+1}) = \dot{p}_x^{(j+1)}(T_j) \tag{12}$$

From (2) - (12), the terminal position and velocity of the COG and the ZMP at j-th section $(t = T_{j+1})$ can be expressed as

$$\begin{aligned} \begin{bmatrix} \boldsymbol{X}(T_{j+1}) \\ \boldsymbol{p}(T_{j+1}) \end{bmatrix} &= \begin{bmatrix} \boldsymbol{A}_{11}^{(j)} & \boldsymbol{A}_{12}^{(j)} \\ \boldsymbol{0} & \boldsymbol{A}_{22}^{(j)} \end{bmatrix} \begin{bmatrix} \boldsymbol{X}(T_j) \\ \boldsymbol{p}(T_j) \end{bmatrix} \\ &+ \begin{bmatrix} \boldsymbol{B}_{11}^{(j)} & \boldsymbol{B}_{12}^{(j)} \\ \boldsymbol{B}_{21}^{(j)} & \boldsymbol{B}_{22}^{(j)} \end{bmatrix} \begin{bmatrix} P_2^{(j)} \\ P_3^{(j)} \\ P_4^{(j)} \end{bmatrix}, \end{aligned} \tag{13}$$

where

$$\boldsymbol{X}(T_j) = \begin{cases} \left[x^{(j)}(T_j)\ \dot{x}^{(j)}(T_j)\right]^T & \text{if j = 1 \ldots m} \\ \left[x^{(m)}(T_{m+1})\ \dot{x}^{(m)}(T_{m+1})\right]^T & \text{if j = m + 1} \end{cases},$$

$$\boldsymbol{p}(T_j) = \begin{cases} \left[p_x^{(j)}(T_j)\ \dot{p}_x^{(j)}(T_j)\right]^T & \text{if j = 1 \ldots m} \\ \left[p_x^{(m)}(T_{m+1})\ \dot{p}_x^{(m)}(T_{m+1})\right]^T & \text{if j = m + 1} \end{cases},$$

$$\boldsymbol{A}_{11}^{(j)} = \begin{bmatrix} c_j & \frac{s_j}{\omega_j} \\ \omega_j s_j & c_j \end{bmatrix}, \boldsymbol{A}_{22}^{(j)} = \begin{bmatrix} 1 & \Delta T_j \\ 0 & 1 \end{bmatrix},$$

$$\boldsymbol{A}_{12}^{(j)} = \begin{bmatrix} 1 - c_j & -\frac{s_j}{\omega_j} + \Delta T_j \\ -\omega_j s_j & 1 - c_j \end{bmatrix},$$

$$\boldsymbol{B}_{11}^{(j)} = \begin{bmatrix} \frac{2}{\omega_j^2}(1 - c_j) + (\Delta T_j)^2 & \frac{6}{\omega_j^2}(1 - \frac{s_j}{\omega_j}) + (\Delta T_j)^3 \\ -\frac{2}{\omega_j}s_j + 2\Delta T_j & \frac{6}{\omega_j^2}(1 - c_j) + 3(\Delta T_j)^2 \end{bmatrix},$$

$$\boldsymbol{B}_{12}^{(j)} = \begin{bmatrix} \frac{24}{\omega_j}(1 - c_j) + \frac{12}{\omega_j^2}(\Delta T_j)^2 + (\Delta T_j)^4 \\ \frac{24}{\omega_j}(1 - \frac{s_j}{\omega_j}) + 4(\Delta T_j)^3 \end{bmatrix},$$

$$\boldsymbol{B}_{21}^{(j)} = \begin{bmatrix} (\Delta T_j)^2 & (\Delta T_j)^3 \\ 2\Delta T_j & 3(\Delta T_j)^2 \end{bmatrix}, \boldsymbol{B}_{22}^{(j)} = \begin{bmatrix} (\Delta T_j)^4 \\ 4(\Delta T_j)^3 \end{bmatrix},$$

$$\Delta T_j \equiv T_{j+1} - T_j.$$

Two unknowns can be obtained from the lower part of (13).

$$\begin{aligned} \begin{bmatrix} P_2^{(j)} \\ P_3^{(j)} \end{bmatrix} &= (\boldsymbol{B}_{21}^{(j)})^{-1} \\ &\times \left(\boldsymbol{p}^{(j)}(T_{j+1}) - \boldsymbol{A}_{22}^{(j)}\boldsymbol{p}^{(j)}(T_j) - \boldsymbol{B}_{22}^{(j)}P_4^{(j)}\right) \end{aligned} \tag{14}$$

Subsituting (14) into the upper part of (13), the terminal position and velocity of the COG can be expressed from the initial position and velocity of the COG and the boundary condition of the ZMP and a scalar unknown $P_4^{(j)}$.

$$\boldsymbol{X}(T_{j+1}) = \bar{\boldsymbol{A}}^{(j)}\boldsymbol{X}(T_j) + \begin{bmatrix} \bar{\boldsymbol{B}}_1^{(j)} & \bar{\boldsymbol{B}}_2^{(j)} \end{bmatrix} \begin{bmatrix} \boldsymbol{p}(T_j) \\ \boldsymbol{p}(T_{j+1}) \end{bmatrix} + \bar{\boldsymbol{C}}^{(j)} P_4^{(j)} \quad (15)$$

Where

$$\begin{aligned} \bar{\boldsymbol{A}}^{(j)} &= \boldsymbol{A}_{11}^{(j)}, \\ \bar{\boldsymbol{B}}_1^{(j)} &= \boldsymbol{A}_{12}^{(j)} + \boldsymbol{B}_{11}^{(j)}(\boldsymbol{B}_{21}^{(j)})^{-1}\boldsymbol{A}_{22}^{(j)}, \\ \bar{\boldsymbol{B}}_2^{(j)} &= \boldsymbol{B}_{11}^{(j)}(\boldsymbol{B}_{21}^{(j)})^{-1}, \\ \bar{\boldsymbol{C}}^{(j)} &= \boldsymbol{B}_{12}^{(j)} + \boldsymbol{B}_{11}^{(j)}(\boldsymbol{B}_{21}^{(j)})^{-1}\boldsymbol{B}_{22}^{(j)}. \end{aligned} \quad (16)$$

By the initial position and velocity of the COG and all of the pass points of the ZMP $\hat{\boldsymbol{p}}$, the COG position and velocity at the end of m-th section $\boldsymbol{X}(T_{m+1})$ can be expressed as

$$\boldsymbol{X}(T_{m+1}) = \hat{\boldsymbol{A}}^{(m)}\boldsymbol{X}(T_1) + \hat{\boldsymbol{B}}^{(m)}\hat{\boldsymbol{p}} + \hat{\boldsymbol{C}}^{(m)}\hat{\boldsymbol{P}}_4, \quad (17)$$

where

$$\hat{\boldsymbol{p}} = \begin{bmatrix} \boldsymbol{p}^T(T_1) & \cdots & \boldsymbol{p}^T(T_m) & \boldsymbol{p}^T(T_{m+1}) \end{bmatrix}^T, \quad (18)$$

$$\hat{\boldsymbol{P}}_4 = \begin{bmatrix} P_4^{(1)} & \cdots & P_4^{(m-1)} & P_4^{(m)} \end{bmatrix}^T, \quad (19)$$

$$\hat{\boldsymbol{A}}^{(m)} = \prod_{i=m}^{1} \bar{\boldsymbol{A}}^{(i)}, \quad (20)$$

$$\hat{\boldsymbol{B}}^{(m)} = \Big[\left(\prod_{i=m}^{2} \bar{\boldsymbol{A}}^{(i)}\right) \bar{\boldsymbol{B}}_1^{(1)} \quad \left(\prod_{i=m}^{3} \bar{\boldsymbol{A}}^{(i)}\right) \left(\bar{\boldsymbol{A}}^{(2)}\bar{\boldsymbol{B}}_2^{(1)} + \bar{\boldsymbol{B}}_1^{(2)}\right) \quad \cdots \quad \bar{\boldsymbol{A}}^{(m)}\bar{\boldsymbol{B}}_2^{(m-1)} + \bar{\boldsymbol{B}}_1^{(m)} \quad \bar{\boldsymbol{B}}_2^{(m)} \Big], \quad (21)$$

$$\hat{\boldsymbol{C}}^{(m)} = \Big[\left(\prod_{i=m}^{2} \bar{\boldsymbol{A}}^{(i)}\right) \bar{\boldsymbol{C}}^{(1)} \quad \left(\prod_{i=m}^{3} \bar{\boldsymbol{A}}^{(i)}\right) \bar{\boldsymbol{C}}^{(2)} \quad \cdots \quad \bar{\boldsymbol{A}}^{(m)}\bar{\boldsymbol{C}}^{(m-1)} \quad \bar{\boldsymbol{C}}^{(m)} \Big]. \quad (22)$$

Then, unknown coefficients can be obtained.

$$\hat{\boldsymbol{P}}_4 = \left(\hat{\boldsymbol{C}}^{(m)}\right)^{+} \left(\boldsymbol{X}(T_{m+1}) - \hat{\boldsymbol{A}}^{(m)}\boldsymbol{X}(T_1) - \hat{\boldsymbol{B}}^{(m)}\hat{\boldsymbol{p}}\right) \quad (23)$$

The value of an element of $\hat{\boldsymbol{P}}_4$ is equivallent to the ZMP fluctuation. In fact, it is enough to allow the ZMP fluctuation only at the first and the last sections[11].

$$\begin{bmatrix} P_4^{(1)} \\ P_4^{(m)} \end{bmatrix} = \begin{bmatrix} \left(\prod_{i=m}^{2} \bar{\boldsymbol{A}}^{(i)}\right) \bar{\boldsymbol{C}}^{(1)} & \bar{\boldsymbol{C}}^{(m)} \end{bmatrix}^{-1} \times \left(\boldsymbol{X}(T_{m+1}) - \hat{\boldsymbol{A}}^{(m)}\boldsymbol{X}(T_1) - \hat{\boldsymbol{B}}^{(m)}\hat{\boldsymbol{p}}\right) \quad (24)$$

In (23) and (24), only $2 \times m$ pseudo inverse matrix and 2×2 inverse matrix are required respectively. Therefore it is suitable to calculate in real-time. Equation (23) or (24) can be connected any trajectories of the COG and the ZMP in principle, although the ZMP fluctuation may be caused. Thus it can be generated sequentially as shown in Fig.3.

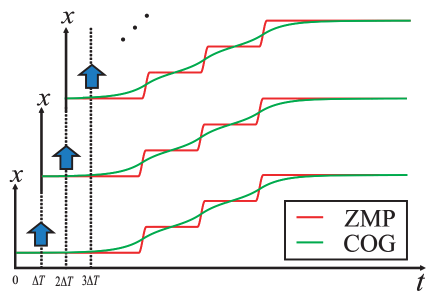


Fig. 3. Sequential gait planning

III. Reactive Stepping

A. Extention of Simultaneous Planning of the COG and the ZMP

In previous section, the desired ZMP position which is given from the desired foot placement is always satisfied in exchange for the ZMP fluctuation. In this section, the ZMP position is determined so that the desired foot placement can be satisfied as much as possible.

The desired future ZMP position $\boldsymbol{p}_{zmp} = [p_x \ p_y \ p_z]^T$ can be calculated as

$$\begin{aligned} \boldsymbol{p}_{zmp}(T_{2k-1}) &= \boldsymbol{u}^{(k)} + \boldsymbol{R}^{(k)}\boldsymbol{v}_i \equiv \boldsymbol{u}^{(k)} + \bar{\boldsymbol{v}}_i^{(k)}, \quad (25) \\ \boldsymbol{p}_{zmp}(T_{2k}) &= \boldsymbol{u}^{(k)} + \boldsymbol{R}^{(k)}\boldsymbol{v}_f \equiv \boldsymbol{u}^{(k)} + \bar{\boldsymbol{v}}_f^{(k)}, \quad (26) \\ k &= 2 \ldots [(m+1)/2] \end{aligned}$$

where $\boldsymbol{u}^{(k)} = [u_x^{(k)} \ u_y^{(k)} \ u_z^{(k)}]^T$ is vector of k-th desired foot placement. $\boldsymbol{R}^{(k)}$ is k-th desired sole attitude. $\boldsymbol{v}_i$ and $\boldsymbol{v}_f$ are the ZMP offsets from origin of either sole. The elements of x-axis in (25) and (26) can be expressed as

$$\begin{aligned} p_x^{(2k-1)}(T_{2k-1}) &= u_x^{(k)} + \bar{v}_{x,i}^{(k)}, \quad (27) \\ p_x^{(2k)}(T_{2k}) &= u_x^{(k)} + \bar{v}_{x,f}^{(k)}. \quad (28) \end{aligned}$$

The sequence of the desired foot placement from (27) and (28) can be written as

$$\boldsymbol{p}_{pos} = \boldsymbol{K}\boldsymbol{u} + \bar{\boldsymbol{v}}, \quad (29)$$

where

$$\boldsymbol{p}_{pos} = \begin{bmatrix} p_x^{(3)}(T_3) & \cdots & p_x^{(m)}(T_m) & p_x^{(m)}(T_{m+1}) \end{bmatrix}^T$$

$$\boldsymbol{u} = \begin{bmatrix} u_x^{(3)} & \cdots & u_x^{([(m+1)/2])} \end{bmatrix}^T$$

$$\bar{\boldsymbol{v}} = \begin{bmatrix} \bar{v}_{x,i}^{(3)} & \bar{v}_{x,f}^{(3)} & \cdots & \bar{v}_{x,i}^{([(m+1)/2])} & \bar{v}_{x,f}^{([(m+1)/2])} \end{bmatrix}^T$$

$$\boldsymbol{K} = \begin{bmatrix} 1 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 1 & 0 & \vdots \\ \vdots & & 0 & \ddots & \ddots & 0 \\ 0 & \cdots & & 0 & 1 & 1 \end{bmatrix}^T$$

To avoid a large fluctuation of the ZMP by immediate modification of the preplanned foot placement, vector of the ZMP pass points in (17) is also regarded as variable. $\hat{\boldsymbol{B}}^{(m)}$

is divided into three terms with respect to the current and the future ZMP position $\boldsymbol{p}_{pos}$ and velocity $\boldsymbol{p}_{vel}$.

$$\begin{bmatrix} \hat{\boldsymbol{B}}_i^{(m)} & \hat{\boldsymbol{B}}_p^{(m)} & \hat{\boldsymbol{B}}_v^{(m)} \end{bmatrix} \begin{bmatrix} \boldsymbol{p}_{ini} \\ \boldsymbol{p}_{pos} \\ \boldsymbol{p}_{vel} \end{bmatrix} \equiv \hat{\boldsymbol{B}}^{(m)} \hat{\boldsymbol{p}}$$

Then (17) can be rewritten as

$$\begin{aligned} &\hat{\boldsymbol{B}}_p^{(m)} \boldsymbol{p}_{pos} + \hat{\boldsymbol{C}}^{(m)} \hat{\boldsymbol{P}}_4 \\ &= \boldsymbol{X}(T_{m+1}) - \hat{\boldsymbol{A}}^{(m)} \boldsymbol{X}(T_1) - \hat{\boldsymbol{B}}_i^{(m)} \boldsymbol{p}_{ini} - \hat{\boldsymbol{B}}_v^{(m)} \boldsymbol{p}_{vel}, \end{aligned} \tag{30}$$

where

$$\boldsymbol{p}_{ini} = \begin{bmatrix} p_x^{(1)}(T_1) & p_x^{(2)}(T_2) \end{bmatrix}^T$$

$$\boldsymbol{p}_{vel} = \begin{bmatrix} \dot{p}_x^{(1)}(T_1) & \cdots & \dot{p}_x^{(m)}(T_m) & \dot{p}_x^{(m)}(T_{m+1}) \end{bmatrix}^T$$

$\boldsymbol{p}_{ini}$ is the current foot placement. $\boldsymbol{p}_{pos}$ and $\hat{\boldsymbol{P}}_4$ are the future foot placement and the ZMP fluctuation respectively. Substituting (29) into (30), (30) can be formulated as a quadratic programming problem.

$$\min \tfrac{1}{2} \left\{ (\boldsymbol{u} - \boldsymbol{u}^{ref})^T \boldsymbol{Q}_1 (\boldsymbol{u} - \boldsymbol{u}^{ref}) + \hat{\boldsymbol{P}}_4^T \boldsymbol{Q}_2 \hat{\boldsymbol{P}}_4 \right\} \tag{31}$$

$$\begin{aligned} \text{subject to} \quad & (30), \\ & \boldsymbol{u}_{lower} \le \boldsymbol{u} \le \boldsymbol{u}_{upper}, \\ & \hat{\boldsymbol{P}}_{4,lower} \le \hat{\boldsymbol{P}}_4 \le \hat{\boldsymbol{P}}_{4,upper} \end{aligned}$$

Where, $\boldsymbol{u}^{ref}$ is the desired foot placement. $\boldsymbol{u}_{lower}$ and $\boldsymbol{u}_{upper}$ are set as feasible foot placement. $\hat{\boldsymbol{P}}_{4,lower}$ and $\hat{\boldsymbol{P}}_{4,upper}$ are set as admissible ZMP fluctuation. $\boldsymbol{Q}_1$ and $\boldsymbol{Q}_2$ are defined as positive definite symmetric matrix. Solution of (31) without inequality conditions can be analytically calculated as

$$\begin{aligned} \boldsymbol{s} &= \boldsymbol{s}^{ref} \\ &- \boldsymbol{Q}^{-1} \boldsymbol{M}^T (\boldsymbol{M} \boldsymbol{Q}^{-1} \boldsymbol{M}^T)^{-1} (\boldsymbol{M} \boldsymbol{s}^{ref} - \boldsymbol{N}), \end{aligned} \tag{32}$$

where

$$\boldsymbol{s} = \begin{bmatrix} \boldsymbol{u} \\ \hat{\boldsymbol{P}}_4 \end{bmatrix}, \quad \boldsymbol{s}^{ref} = \begin{bmatrix} \boldsymbol{u}^{ref} \\ \boldsymbol{0} \end{bmatrix}, \tag{33}$$

$$\boldsymbol{M} = \begin{bmatrix} \hat{\boldsymbol{B}}_p^{(m)} \boldsymbol{K} & \hat{\boldsymbol{C}}^{(m)} \end{bmatrix}, \tag{34}$$

$$\begin{aligned} \boldsymbol{N} &= \boldsymbol{X}(T_{m+1}) - \hat{\boldsymbol{A}}^{(m)} \boldsymbol{X}(T_1) \\ &\quad - \hat{\boldsymbol{B}}_p^{(m)} \bar{\boldsymbol{v}} - \hat{\boldsymbol{B}}_i^{(m)} \boldsymbol{p}_{ini} - \hat{\boldsymbol{B}}_v^{(m)} \boldsymbol{p}_{vel}, \end{aligned} \tag{35}$$

$$\boldsymbol{Q} = \begin{bmatrix} \boldsymbol{Q}_1 & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{Q}_2 \end{bmatrix}. \tag{36}$$

B. Reflect Foot Placement to Disturbance

We suppose that a humanoid robot keeps a balance by trunk motion. The compensation values of the COG position and velocity are reflected to the pattern generator as the initial values which is shown in Fig.4.

$$x^{(1)}(T_1) = x^{cmd} + x^{cmp} \tag{37}$$

$$\dot{x}^{(1)}(T_1) = \dot{x}^{cmd} + \dot{x}^{cmp} \tag{38}$$

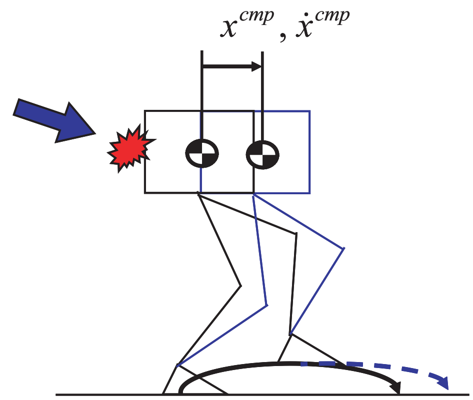


Fig. 4. The COG errors corresponding to disturbance

x^{cmd} and $\dot{x}^{cmd}$ means the COG position and velocity which is calculated from the pattern generator at a previous time. x^{cmp} and $\dot{x}^{cmp}$ means the compensation values of the COG position and velocity from a stabilization controller. The compensation value which is defined as a difference between the desired and the current trunk position includes not only an external disturbance but also an internal modeling error, an attitude estimation error, a sensor measurement error and so on. Therefore we prepare a threshold value $\tilde{x}^{cmp}_{threshold}$ for the compensation value.

$$\begin{cases} \text{if } \tilde{x}^{cmp} > \tilde{x}^{cmp}_{threshold}: & x^{cmp} = \tilde{x}^{cmp} - \tilde{x}^{cmp}_{threshold} \\ \text{if } \tilde{x}^{cmp} < -\tilde{x}^{cmp}_{threshold}: & x^{cmp} = \tilde{x}^{cmp} + \tilde{x}^{cmp}_{threshold} \\ \text{otherwise}: & x^{cmp} = 0 \end{cases}$$

IV. Swing Leg Trajectory

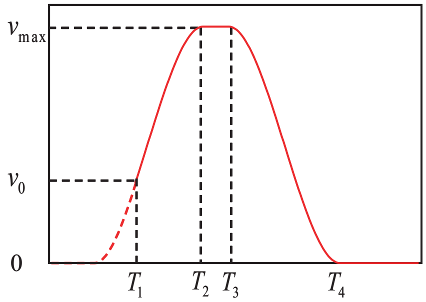
To modify a foot placement immediately, swing leg motion is desirable to generate sequentially as well as the COG motion. The trajectory of swing leg should also be satisfied the initial and the terminal conditions, and the continuity. 4-1-4th order polynomial is applied to the horizontal motion in x and y axes. The vertical motion in z axis is generated by two 4-1-4th order polynomials shown in Fig.5.

This trajectory is composed of 3 parts, that is the acceleration period $[T_1 : T_2]$, the constant velocity $[T_2 : T_3]$ and the deceleration period $[T_3 : T_4]$.

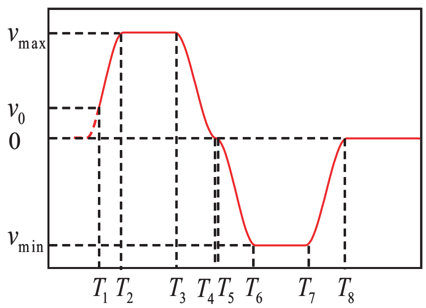
$$\begin{aligned} &T_1 \le t \le T_2 : \\ &\quad x_1(t) = a_{10} + a_{11}(t - T_1) + a_{12}(t - T_1)^2 \\ &\qquad\qquad + a_{13}(t - T_1)^3 + a_{14}(t - T_1)^4 \end{aligned} \tag{39}$$

$$\begin{aligned} &T_2 \le t \le T_3 : \\ &\quad x_2(t) = a_{20} + a_{21}(t - T_2) \end{aligned} \tag{40}$$

$$\begin{aligned} &T_3 \le t \le T_4 : \\ &\quad x_3(t) = a_{30} + a_{31}(t - T_4) + a_{32}(t - T_4)^2 \\ &\qquad\qquad + a_{33}(t - T_4)^3 + a_{34}(t - T_4)^4 \end{aligned} \tag{41}$$

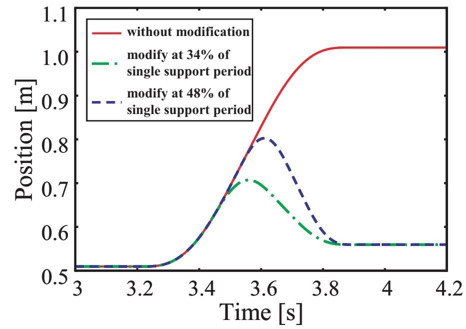


(a) horizontal axis

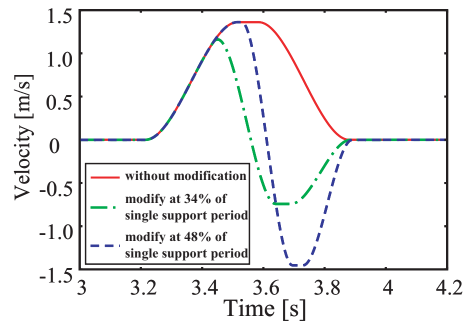


(b) vertical axis

Fig. 5. Velocity profiles of swing leg



(a) Position



(b) Velocity

Fig. 6. Swing leg trajectory with immediate modification

We set the boundary condition as

Initial conditions:	$x_1(T_1), \dot{x}_1(T_1), \ddot{x}_1(T_1),$
Terminal conditions:	$x_3(T_4), \dot{x}_3(T_4), \ddot{x}_3(T_4),$
Condinuity:	$x_1(T_2) = x_2(T_2), x_2(T_3) = x_3(T_3),$
	$\dot{x}_1(T_2) = \dot{x}_2(T_2), \dot{x}_2(T_3) = \dot{x}_3(T_3),$
	$\ddot{x}_1(T_2) = 0, \ddot{x}_3(T_3) = 0$

From these conditions, the coefficients of polynomials in (39)-(41) can be obtained analytically. Second velocity profile of vertical motion at $[T_5 : T_8]$ can be generated to move an opposite direction but same calculation way as the first velocity profile. The maximum velocity provides a feasibility indication of a swing motion and it can be calculated as follows.

$$v_{\max} = \frac{x(T_4) - x(T_0)}{T_4 - T_2} \qquad (42)$$

Examples of the position and the velocity of the foot trajectories with immediate modification are shown in Fig.6 (a) and (b). The preplanned foot motion which moves from approximately 0.5[m] to 1[m] is shown in solid line. The step lengths are changed from 0.25[m] to -0.2[m] at 34% and 48% of single support phase respectively (dots and dashed line). Time parameters $T_i (i = 1 \dots 4)$ of a horizontal foot motion in case of modification of foot placement are also distributed according to the remaining of traveling time of swing leg.

V. Simulation

To confirm the effectiveness of proposed method, simulation results will be shown using HRP-2[13]. HRP-2 was hit by pendulum whose mass is 7.5[kg] and it moves 4.3[m/s]. although robot have felled over in case of without reactive stepping (Fig.7), it could keep a balance with reactive stepping by proposed method (Fig.8).

VI. Conclusion and Future Works

This paper addressed a reactive motion of a humanoid robot to maintain a balance against disturbance. To realize reactive stepping, not only the COG and the ZMP but also the foot placement were determined by a real-time gait planning method based on analytical approach according to trunk position and velocity error. The effectiveness of proposed method was confirmed by simulation.

We are preparing for experimentation about reactive stepping and it will be shown in future.

VII. Acknowledgments

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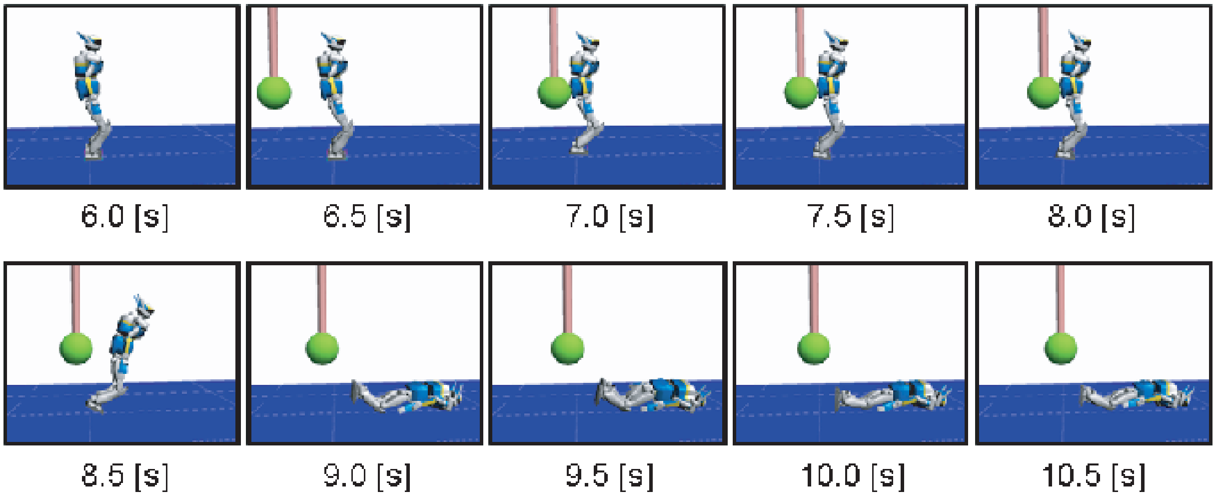

Fig. 7. Without modification of foot place

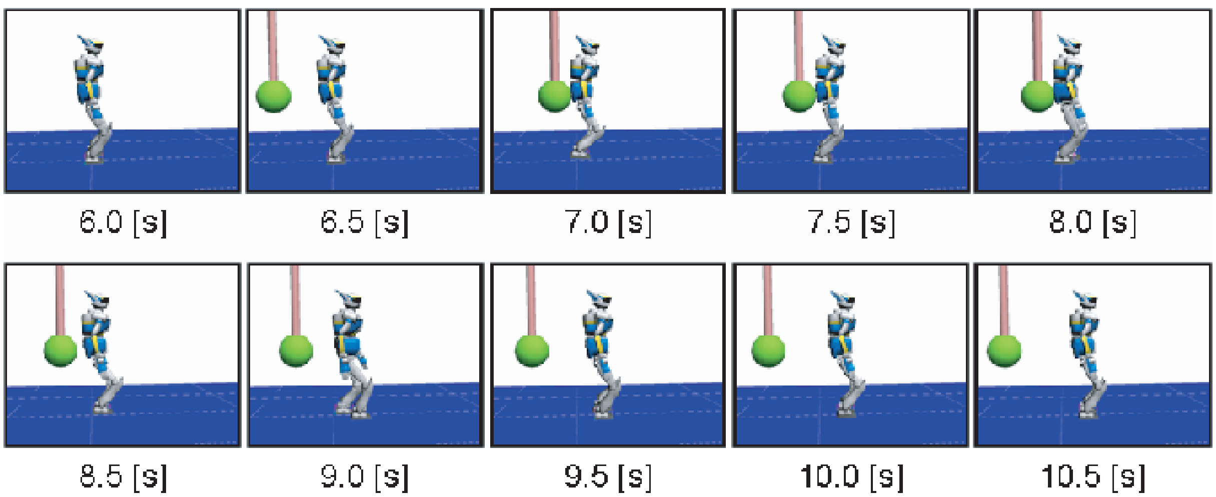

Fig. 8. With modification of foot place

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