Undelayed landmarks initialization for monocular SLAM

APPENDIX
INVERSION OF THE RADIAL DISTORTION FUNCTION

In projective cameras, lens distortion can be modeled in the normalized image plane as a one-to-one mapping $[x_d, y_d]^\top = d([x, y]^\top)$ from the ideally projected point $[x, y]^\top$ to the distorted one $[x_d, y_d]^\top$. A simple yet efficient model arises when one considers perfectly round lenses parallel to the image plane. This gives distortions only in the radial direction. The *radial distortion function* $d()$ is often approximated via a polynomial of the ideal pixel radius $r = \sqrt{x^2 + y^2}$ by

$$\begin{bmatrix} x_d \\ y_d \end{bmatrix} = d\left(\mathbf{d}, \begin{bmatrix} x \\ y \end{bmatrix}\right) = (1 + d_2 \cdot r^2 + d_4 \cdot r^4 + \cdots) \begin{bmatrix} x \\ y \end{bmatrix}. \tag{1}$$

where $\mathbf{d} = \{d_2, d_4, \cdots\}$ is the set of radial distortion coefficients. Several methods are available, sometimes in the form of software packages, to determine the coefficients $\mathbf{d}$ via offline calibration procedures [1]–[3].

For dense or full image correction the inverse mapping is tabulated from $d()$. However, we are interested only in a reduced, sparse set of points. We can save computing resources by only calculating the undistorted coordinates of these points. This requires the inversion of the distortion function. For this, it is clarifying to re-write it as

$$r_d = d(\mathbf{d}, r) = (1 + d_2 \cdot r^2 + d_4 \cdot r^4 + \cdots) \cdot r \tag{2}$$

$$\begin{bmatrix} x_d \\ y_d \end{bmatrix} = \frac{r_d}{r} \begin{bmatrix} x \\ y \end{bmatrix} \tag{3}$$

where $r_d = \sqrt{x_d^2 + y_d^2}$ is the radius of the distorted pixel. This way, all the distortion is isolated in (2), which contains only odd powers of the radius r.

The analytical inversion of a polynomial is only feasible up to the fourth degree, and thus only one-parameter radial distortion functions are exactly invertible. For higher order models we propose an approximated inversion which is optimal in the least-squares sense. Being the kernel of $d()$ an arbitrary choice, we choose the correction one $c()$ to have the same structure, leading to the correction scheme:

$$r = c(\mathbf{c}, r_d) = (1 + c_2 \cdot r_d^2 + c_4 \cdot r_d^4 + \cdots) \cdot r_d \tag{4}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{r}{r_d} \begin{bmatrix} x_d \\ y_d \end{bmatrix} \tag{5}$$

where $\mathbf{c} = \{c_2, c_4, \cdots\}$ is the set of radial correction coefficients. The inverted model (4 – 5) can be rapidly computed online. Then we seek for the optimal set $\mathbf{c}$ so that $c(\mathbf{c}, d(\mathbf{d}, r)) \approx r$. We assume that a certain radial distortion model $r_d = d(\mathbf{d}, r)$ is available, because it has been obtained with one of the existing camera calibration procedures. This calibration also provided the camera intrinsic parameters $\mathbf{k} = \{u_0, v_0, \alpha_u, \alpha_v\}$. We proceed as follows: from the direct model $r_d = d(\mathbf{d}, r)$ (2) we generate a data set of N corresponding pairs $\{r_i, r_{d,i}\}, i \in [1, N]$. This data set must be representative of the whole range of possible image radius. In the normalized image plane of a camera with intrinsic parameters $\mathbf{k}$, the maximal radius (the semi-diagonal) is well approximated with $r_{max} = \sqrt{(u_0/\alpha_u)^2 + (v_0/\alpha_v)^2}$, so we simply take $r_i = i \cdot r_{max}/N$ and $r_{d,i} = d(\mathbf{d}, r_i)$. Then we find the set of parameters $\mathbf{c}$ that minimizes the error function

$$f(\mathbf{c}) = \sum_{i=1}^{N} \left[r_i - c(\mathbf{c}, r_{d,i})\right]^2. \tag{6}$$

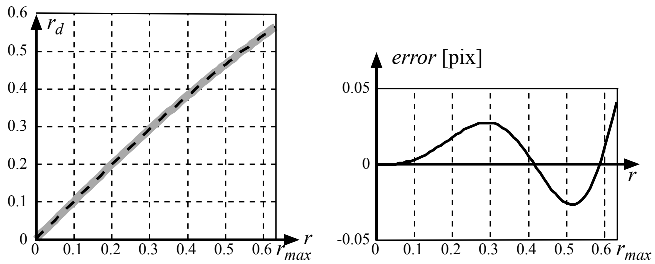

Fig. 1. Radial distortion correction for a real camera with calibration parameters $\mathbf{k} = [516.7, 355.1, 991.9, 995.3]$ and $\mathbf{d} = [-0.3017, 0.09632]$, giving $r_{max} = 0.631$. The correction parameters found with $N = 100$ are $\mathbf{c} = [0.2979, 0.2163]$. *Left:* distortion and correction mappings, overlapped. *Right:* Errors of the inverted mapping as a function of the pixel radius.

In the cases where $c(\cdot)$ is linear in the components of $\mathbf{c}$, as it is for the radial distortion model, (6) corresponds to a linear least-squares optimization problem. It is solved by means, for instance, of the pseudo-inverse method by writing the equations $r_i = (1 + c_2\, r_{d,i}^2 + c_4\, r_{d,i}^4 + \cdots)\, r_{d,i}$ for $1 \leq i \leq N$ as the linear system

$$\begin{bmatrix} (r_{d,1})^3 & (r_{d,1})^5 & \cdots \\ \vdots & & \\ (r_{d,N})^3 & (r_{d,N})^5 & \cdots \end{bmatrix} \begin{bmatrix} c_2 \\ c_4 \\ \vdots \end{bmatrix} = \begin{bmatrix} r_1 \\ \vdots \\ r_N \end{bmatrix} - \begin{bmatrix} r_{d,1} \\ \vdots \\ r_{d,N} \end{bmatrix} \tag{7}$$

which, re-written as $\mathbf{R}_d\, \mathbf{c} = (\mathbf{r} - \mathbf{r}_d)$, leads to the least-squares optimal solution

$$\mathbf{c} = \left[(\mathbf{R}_d^\top\, \mathbf{R}_d)^{-1} \mathbf{R}_d^\top \right] (\mathbf{r} - \mathbf{r}_d). \tag{8}$$

The accuracy of the result for a real inversion of distortion and correction functions of two parameters can be appreciated in Fig. 1 to be better than one twentieth of a pixel.

REFERENCES

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