

A Complete Method for Workspace Boundary Determination



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PROBLEM

GIVEN

A ROBOT AND A SET OF
OUTPUT VARIABLES

(pose of the end-effector)

FIND

THE SET OF POSSIBLE VALUES
FOR SUCH VARIABLES

(workspace)

OUTLINE

1. RELATED WORK

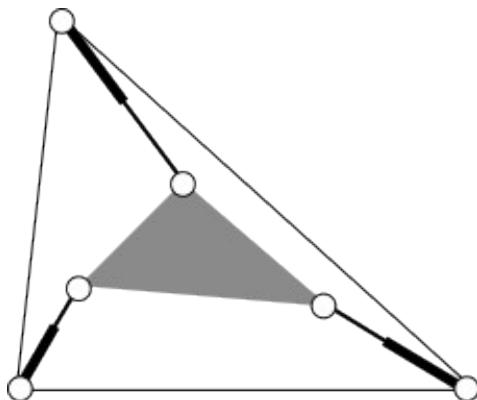
2. PROPOSED METHODOLOGY

3. EXAMPLES

4. CONCLUSIONS

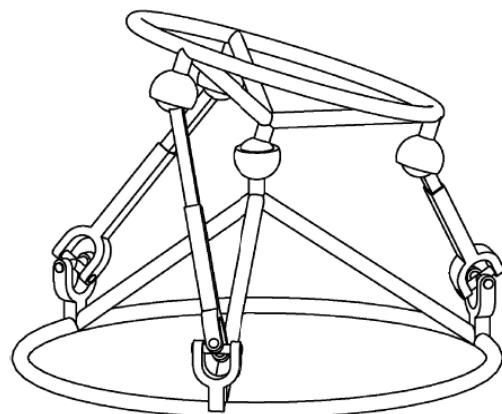
RELATED WORK

PLANAR PARALLEL MANIPULATORS



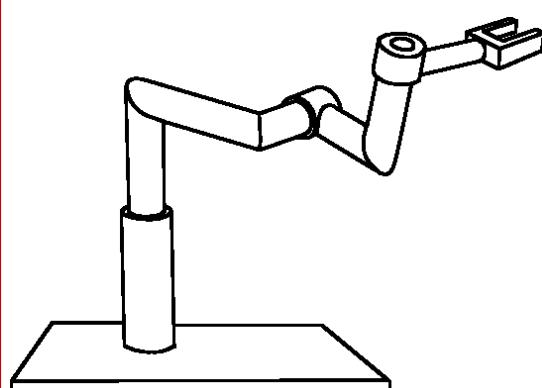
(Merlet et al. 1998)

SYMMETRIC SPHERICAL MANIPULATORS



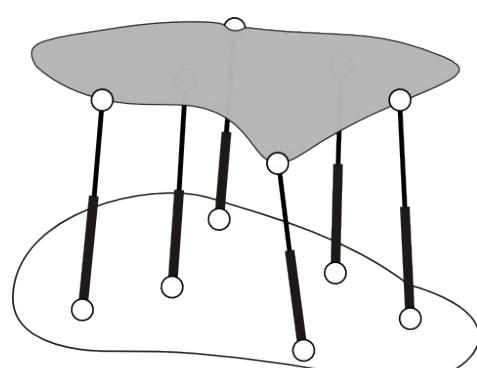
(Bonnev and Gosselin 2006)

SPATIAL 3R MANIPULATORS

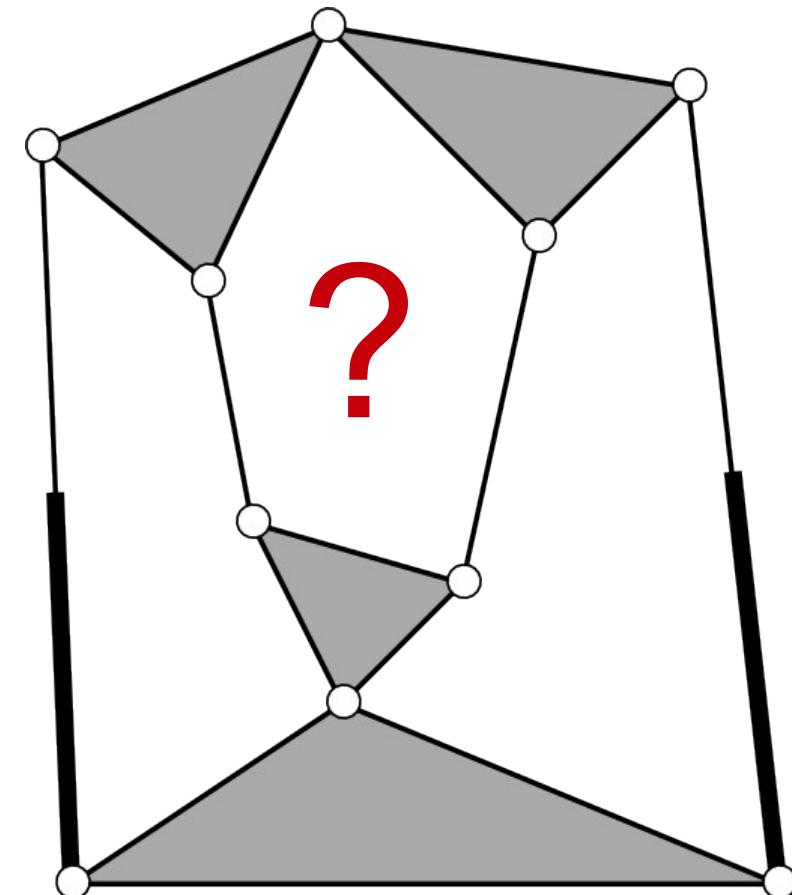


(Zein et al. 2006)
(Ottaviano et al. 2006)

GENERAL STEWART PLATFORM



(Gosselin 1990, Merlet 1995)
(Merlet et al. 1999)
(Snyman et al. 2000)



(Haug et al. 1996)

(Bohigas et al. 2010)

COMPARISON

HAUG ET AL.	PRESENTED APPROACH
COMPUTE WORKSPACE BOUNDARIES	
No particular formulation	Quadratic formulation
Continuation approach	Branch - and – Prune approach (using linear relaxations)
Seed points required for each connected component	Seed points not required
May fail to compute all connected components	Isolates all connected components
 COMPLETE	

OUTLINE

1. RELATED WORK

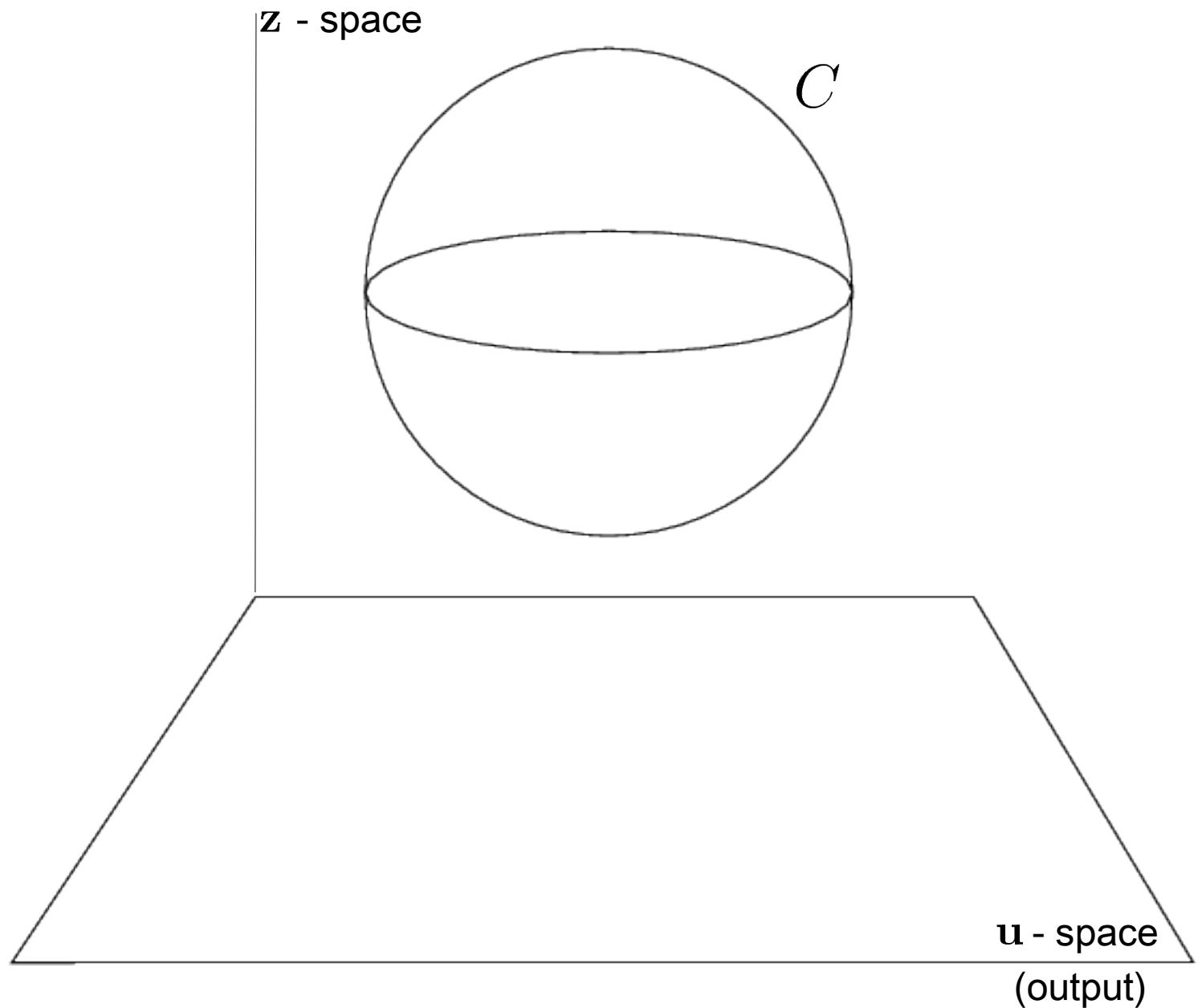
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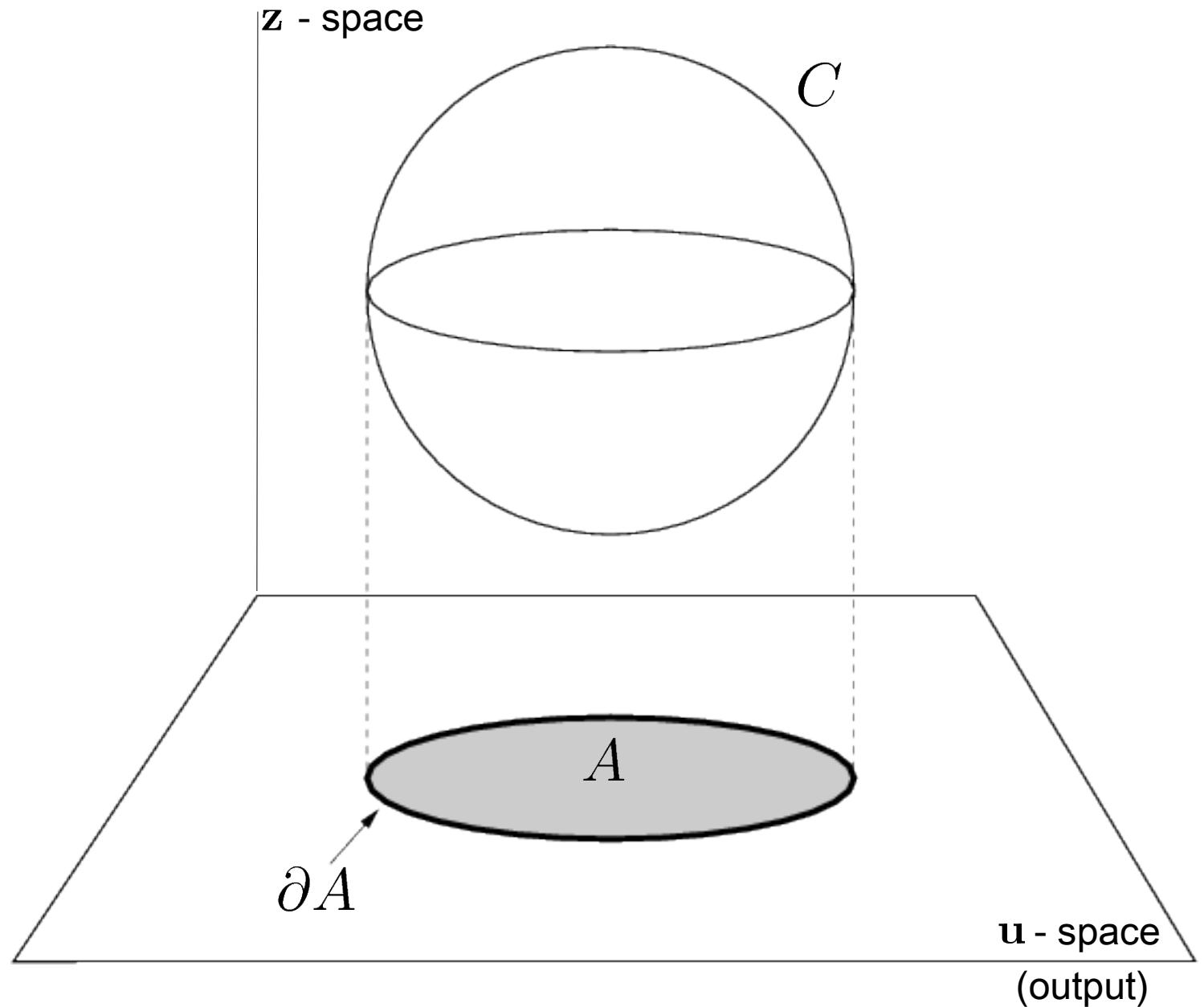
NECESSARY CONDITIONS

$$\underbrace{z, u}_{\Phi(\mathbf{q})} = 0$$



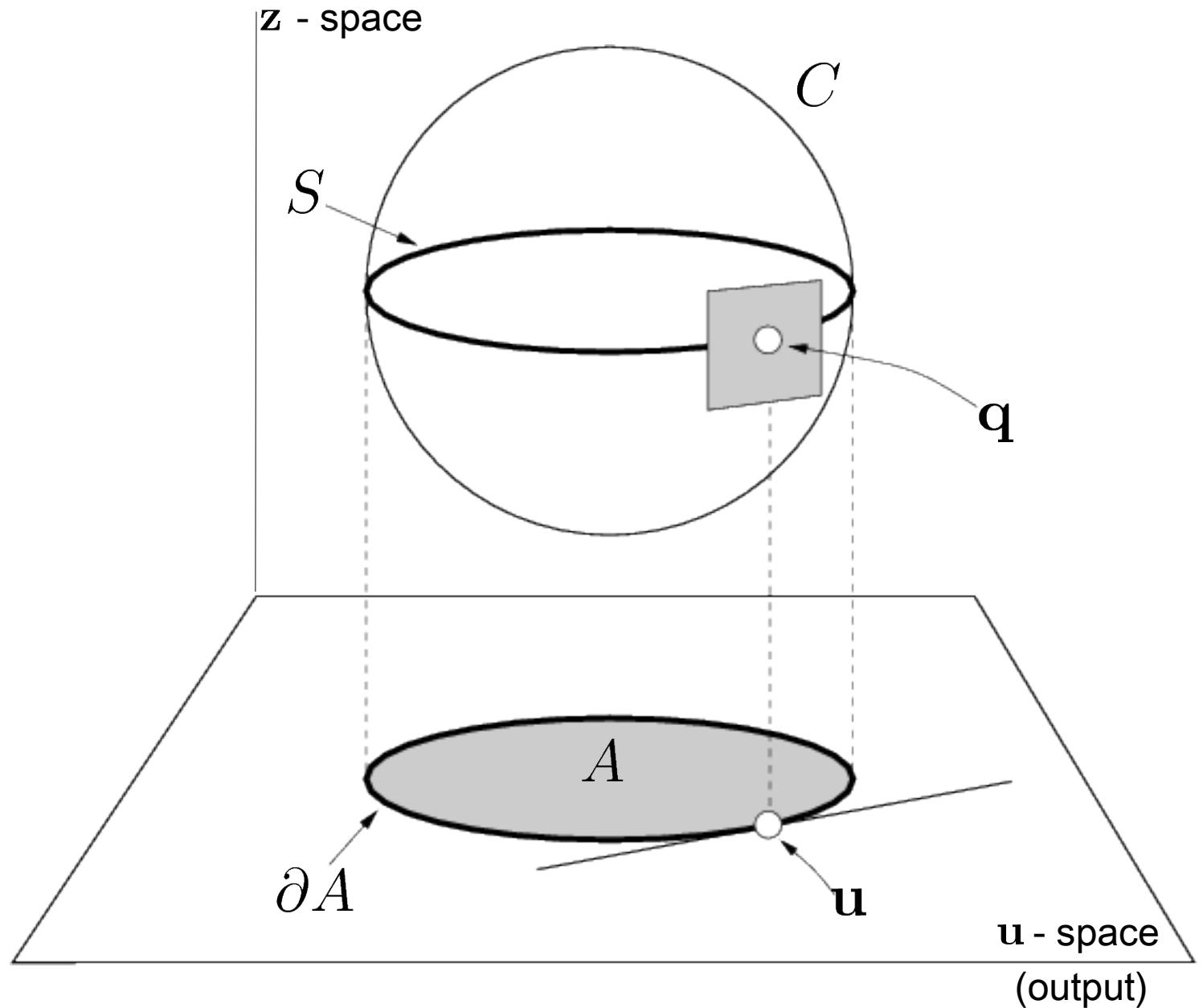
NECESSARY CONDITIONS

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NECESSARY CONDITIONS

$$\underbrace{z, u}_{\Phi(q) = 0}$$



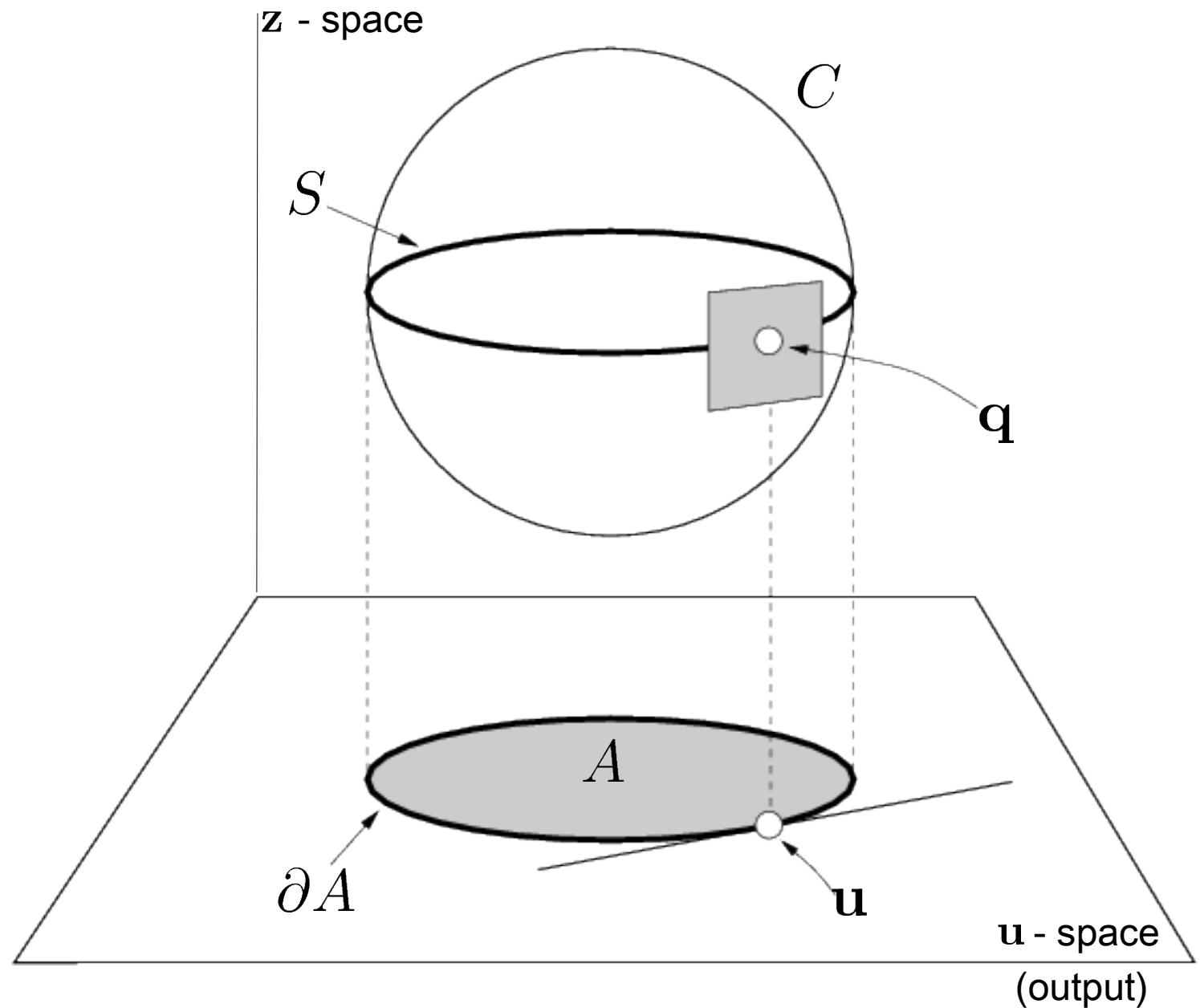
NECESSARY CONDITIONS

$\underbrace{\mathbf{z}, \mathbf{u}}$

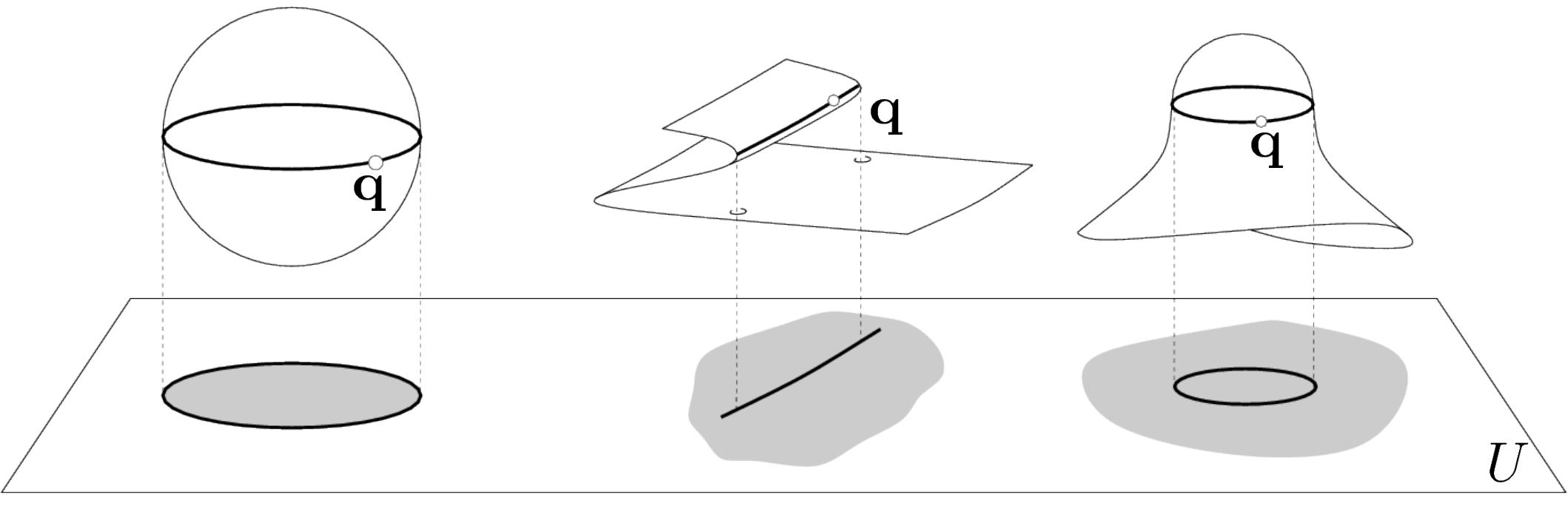
$$\Phi(\mathbf{q}) = 0$$

$$d\Phi_{\mathbf{z}}^T \xi = 0$$

$$\xi^T \xi = 1$$



SINGULARITY TYPES



BARRIER

NON-BARRIER

METHODOLOGY

SET KINEMATIC
CONSTRAINTS

$$\Phi(\mathbf{z}, \mathbf{u}) = 0$$

OBTAIN
SINGULARITY SET

$$\begin{aligned}\Phi(\mathbf{z}, \mathbf{u}) &= 0 \\ d\Phi_{\mathbf{z}}^T \xi &= 0 \\ \xi^T \xi &= 1\end{aligned}$$

CLASSIFY
SINGULARITY SET

BARRIERS
NON-BARRIERS

METHODOLOGY

SET KINEMATIC CONSTRAINTS

$$\Phi(\mathbf{z}, \mathbf{u}) = 0$$

OBTAIN SINGULARITY SET

$$\begin{aligned}\Phi(\mathbf{z}, \mathbf{u}) &= 0 \\ d\Phi_{\mathbf{z}}^T \xi &= 0 \\ \xi^T \xi &= 1\end{aligned}$$

CLASSIFY SINGULARITY SET

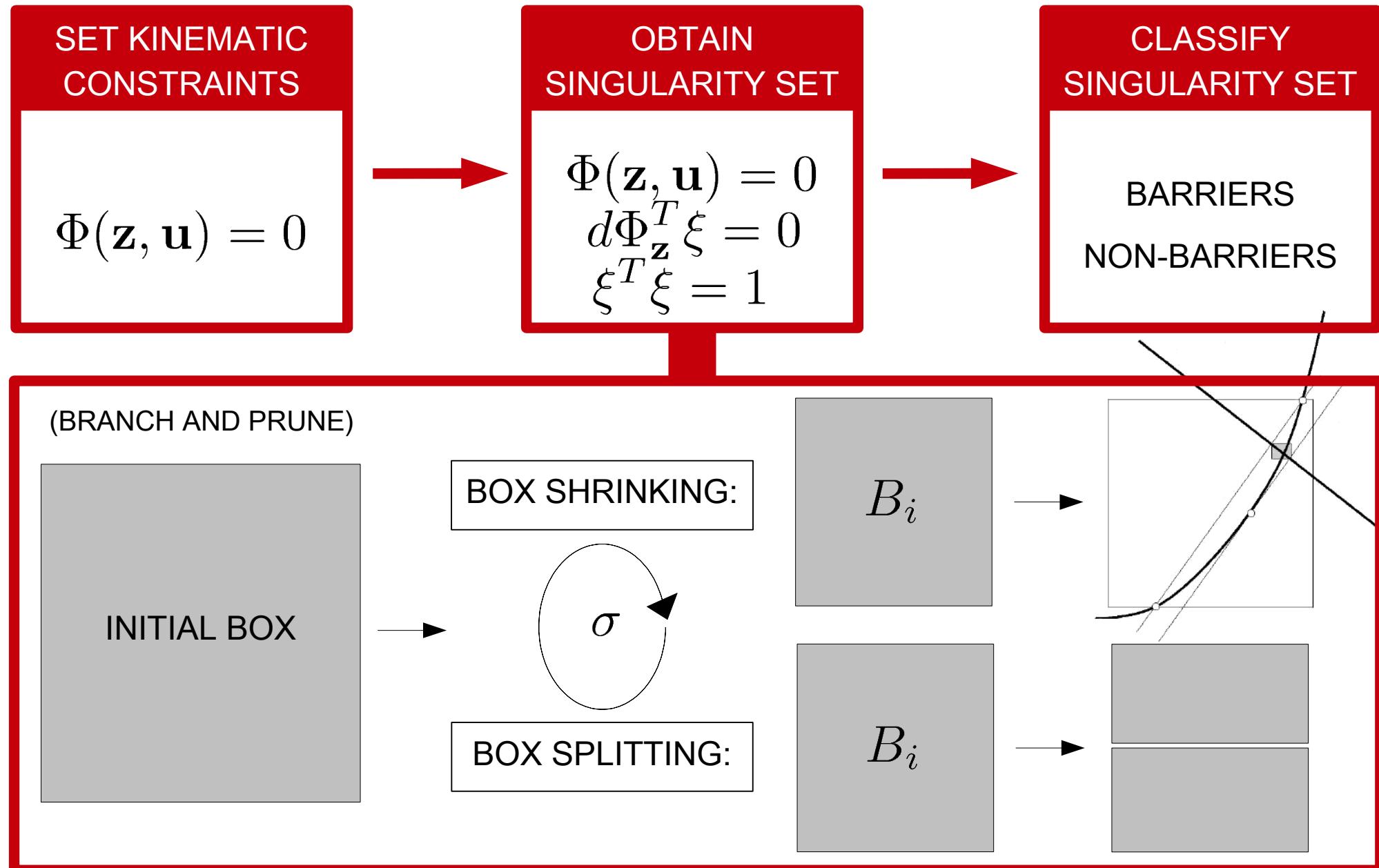
BARRIERS
NON-BARRIERS

$$\Phi(\mathbf{z}, \mathbf{u}) = 0 \rightarrow$$

CANONICAL FORM: QUADRATIC q_i $q_i q_j$ q_i^2

(Porta et al. 2009)

METHODOLOGY



METHODOLOGY

SET KINEMATIC CONSTRAINTS

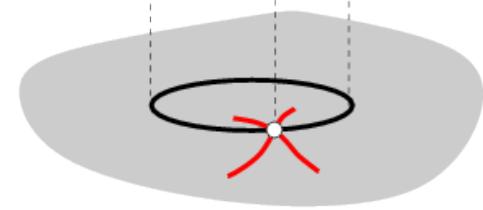
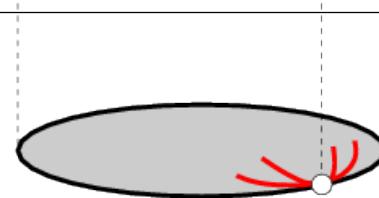
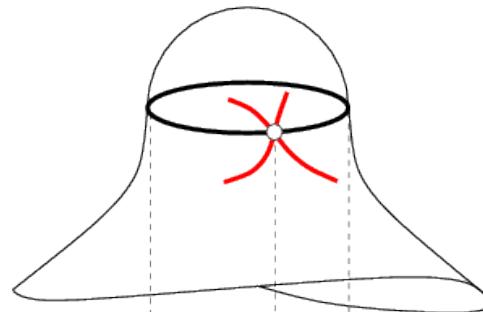
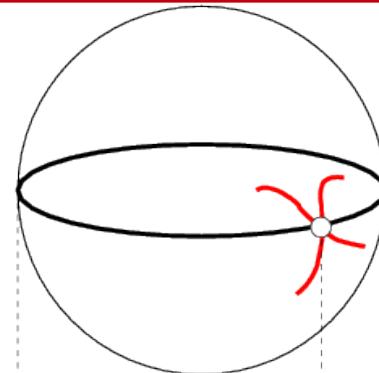
$$\Phi(\mathbf{z}, \mathbf{u}) = 0$$

OBTAIN SINGULARITY SET

$$\begin{aligned}\Phi(\mathbf{z}, \mathbf{u}) &= 0 \\ d\Phi_{\mathbf{z}}^T \xi &= 0 \\ \xi^T \xi &= 1\end{aligned}$$

CLASSIFY SINGULARITY SET

BARRIERS
NON-BARRIERS



DEFINITENESS OF A MATRIX

OUTLINE

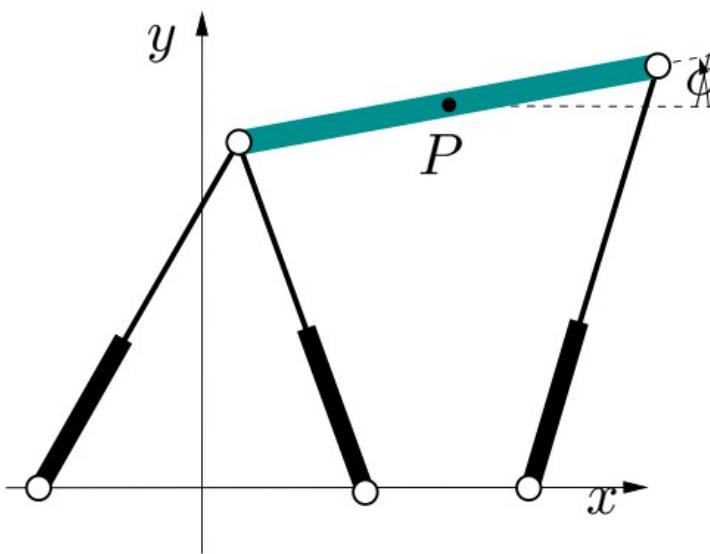
1. RELATED WORK

2. PROPOSED METHODOLOGY

3. EXAMPLES

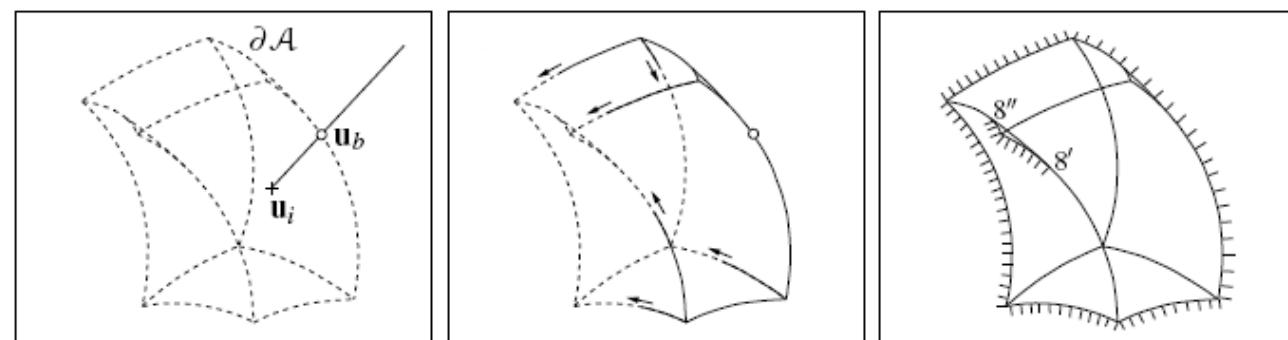
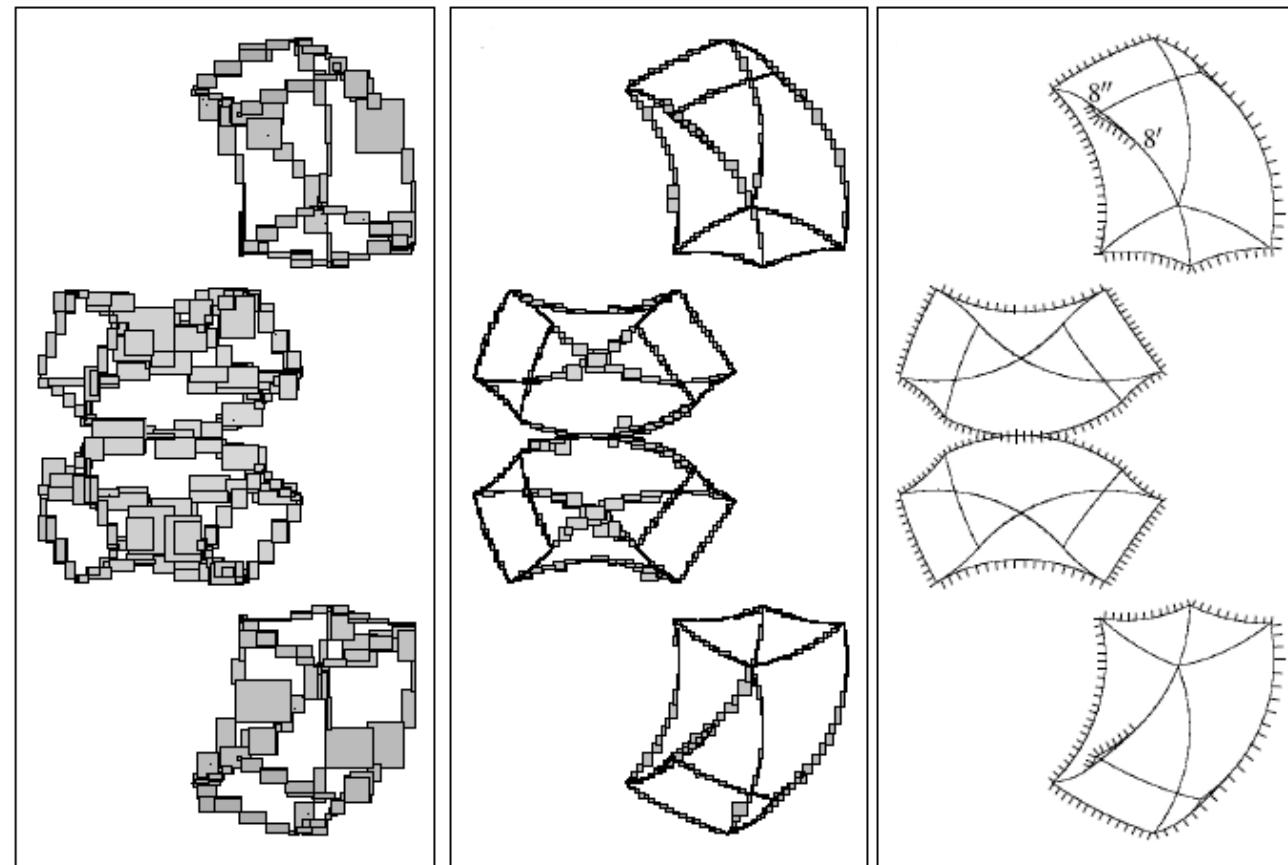
4. CONCLUSIONS

EXAMPLES - 3RPR PLANAR PARALLEL

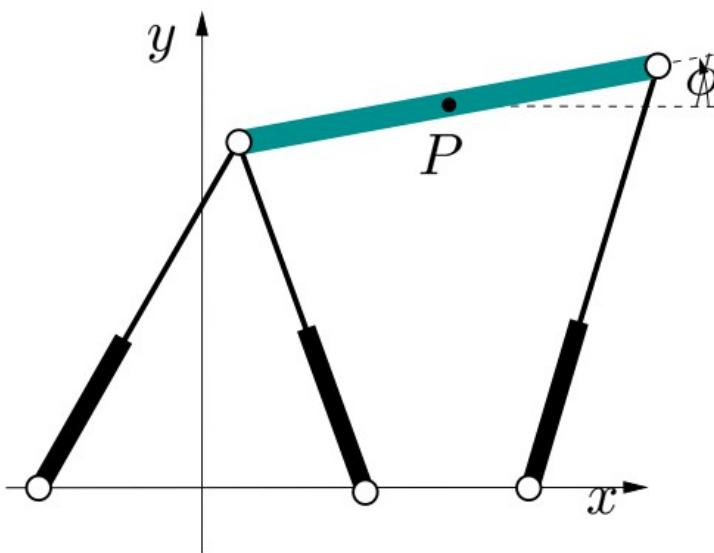


$$\mathbf{u} = [x, y]^T$$

23 variables
22 equations
43231 boxes
154 seconds

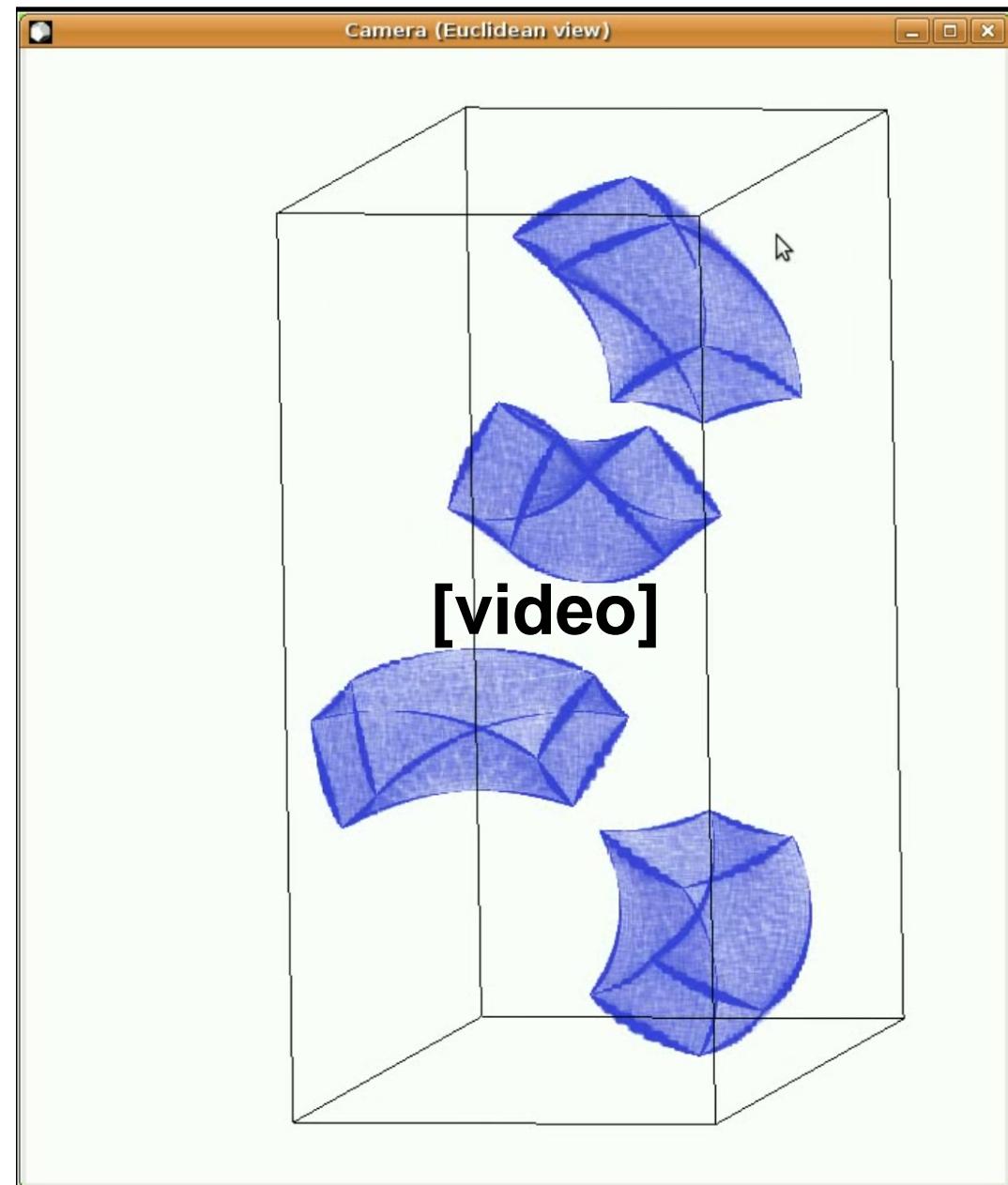


EXAMPLES - 3RPR PLANAR PARALLEL

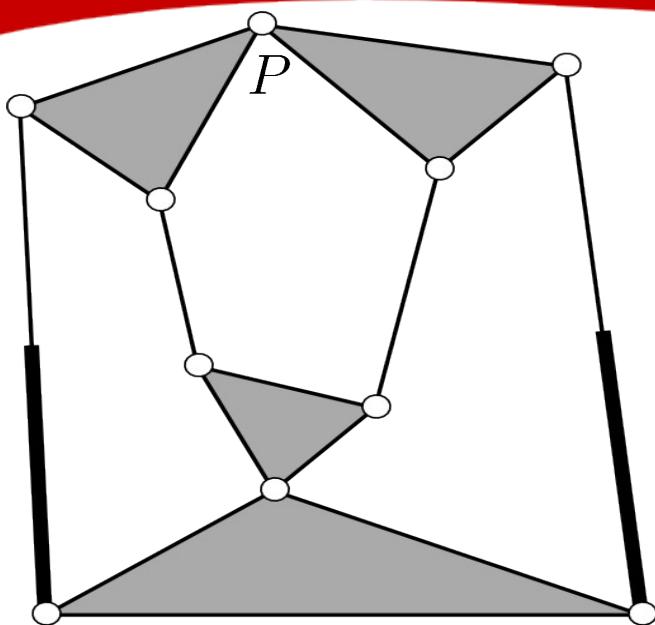


$$\mathbf{u} = [x, y, \phi]^T$$

23 variables
20 equations
105036 boxes
142 seconds



EXAMPLES – DOUBLE BUTTERFLY

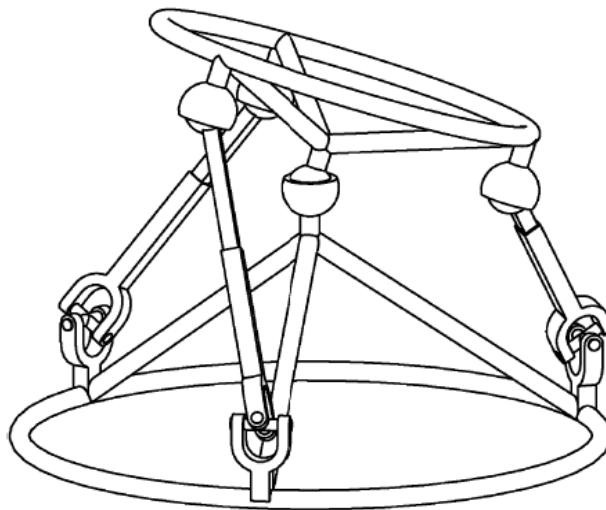


$$\mathbf{u} = [x, y]^T$$

37 variables
36 equations
143521 boxes
179 minutes

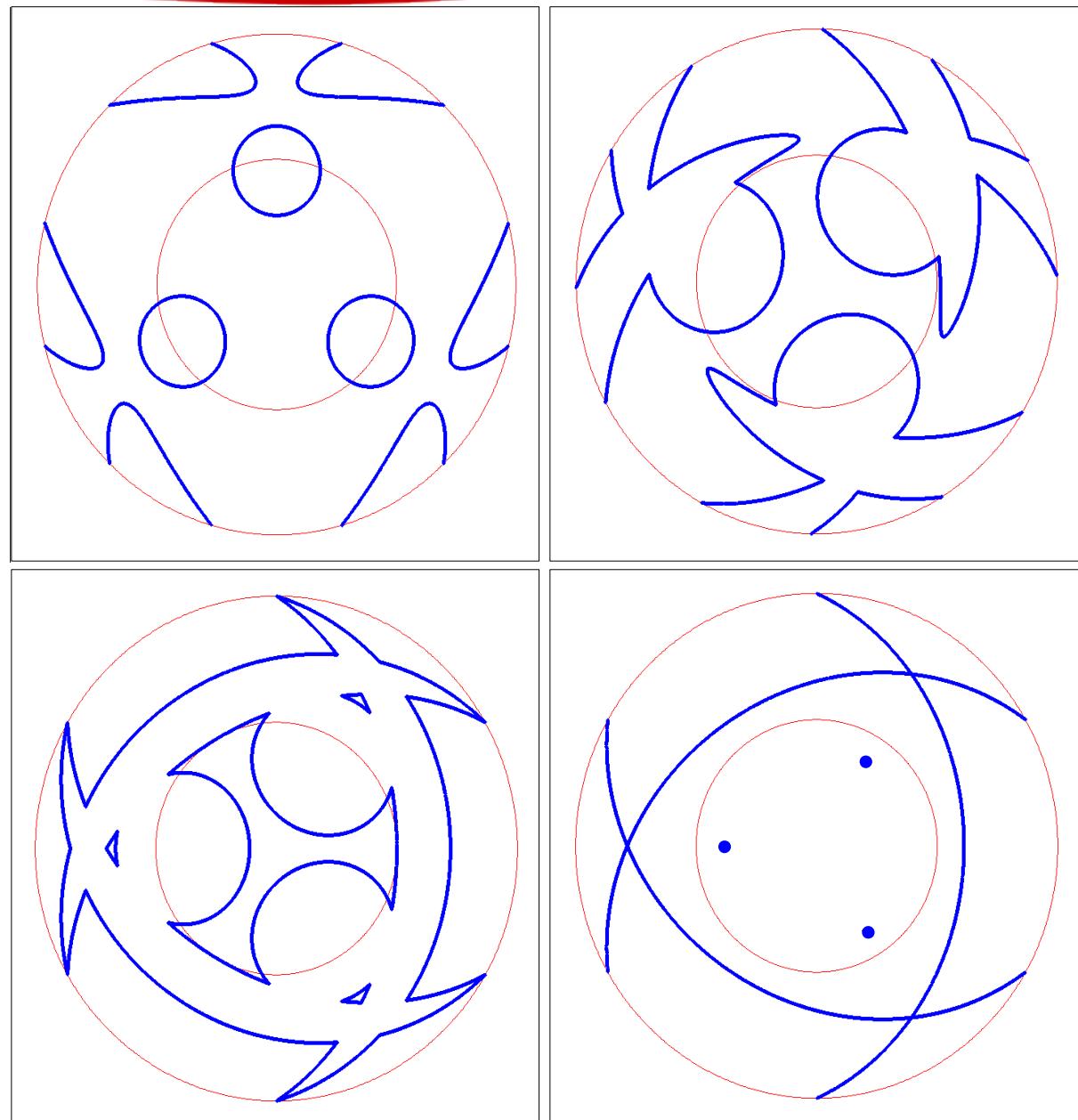


EXAMPLES – SYMMETRICAL SPHERICAL MECHANISM 3UPS

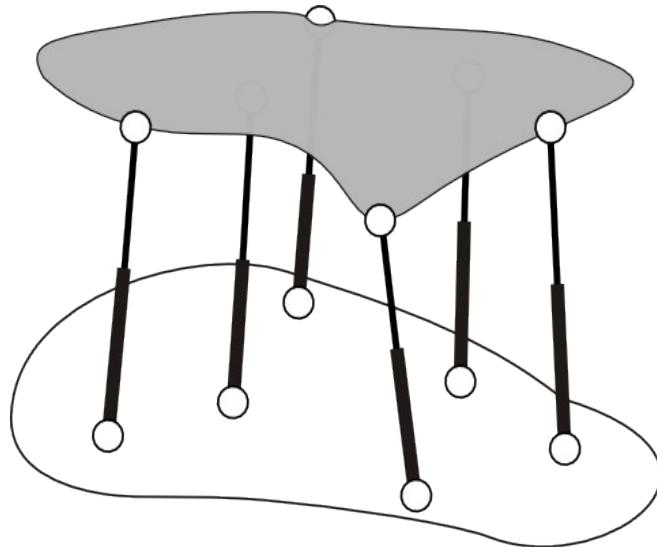


$$\mathbf{u} = [\phi, \theta]^T$$

66 variables
79 equations
1001 boxes
43 seconds

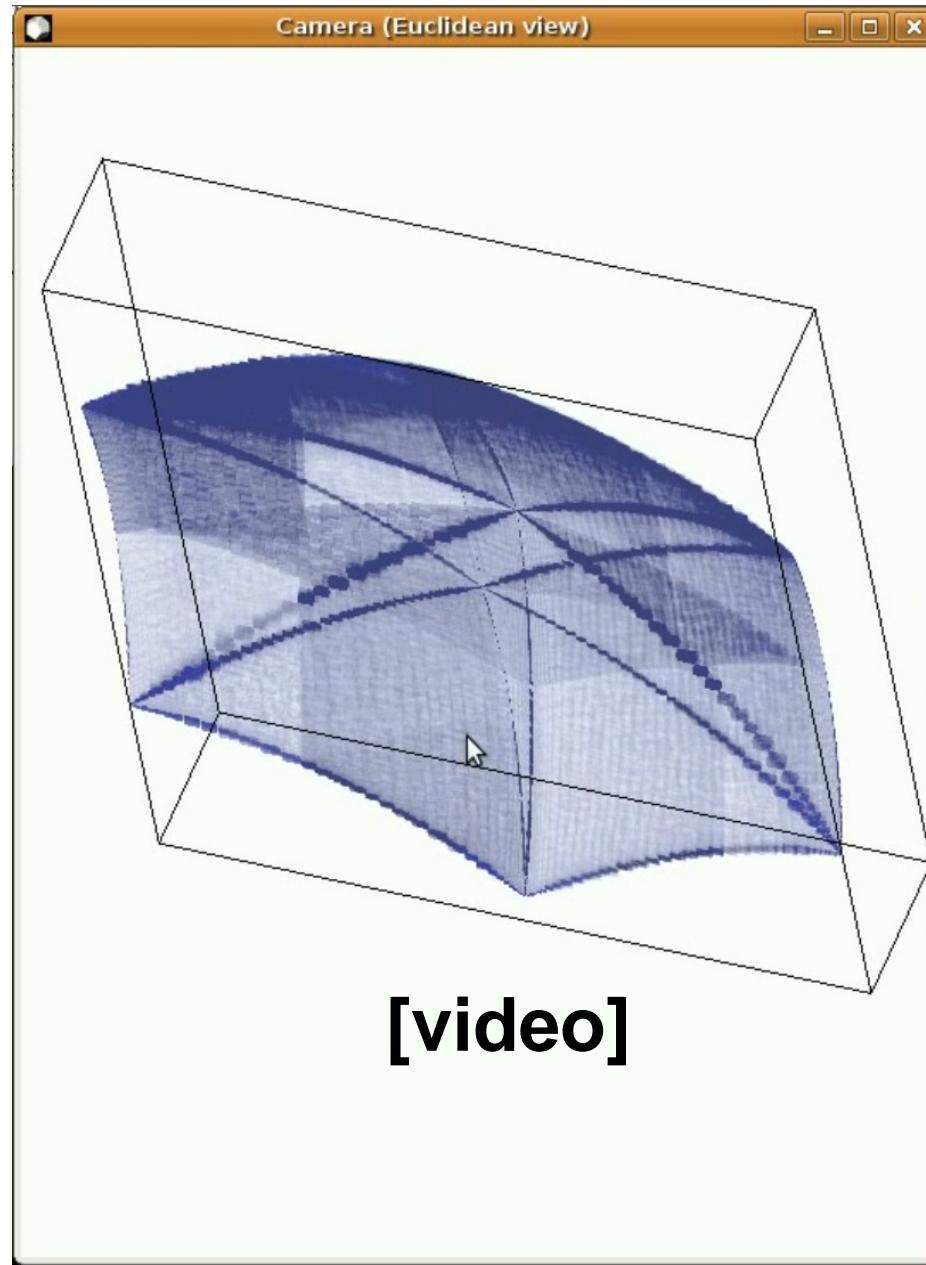


EXAMPLES – GENERAL STEWART PLATFORM

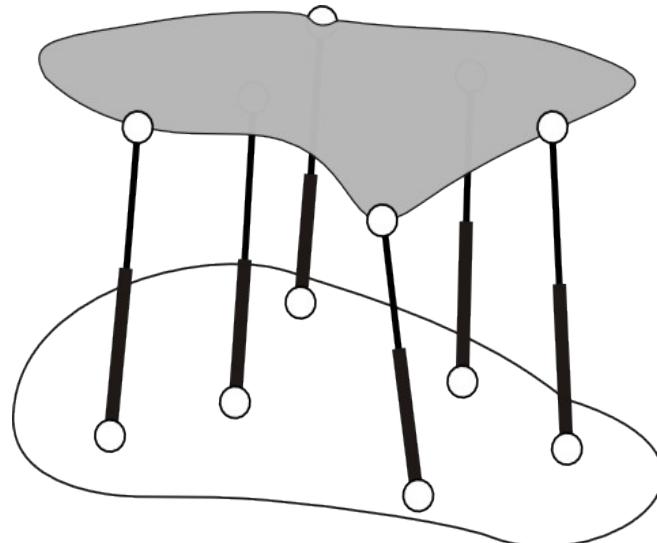


$$\mathbf{u} = [x, y, z]^T$$

75 variables
73 equations
372549 boxes
44.5 minutes

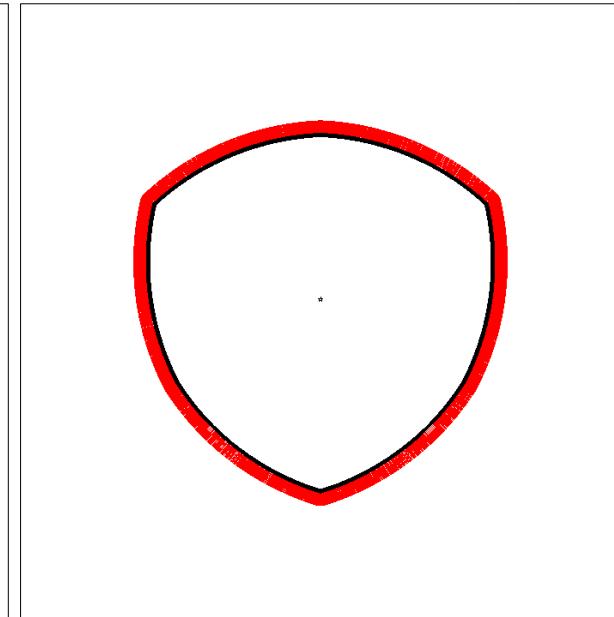
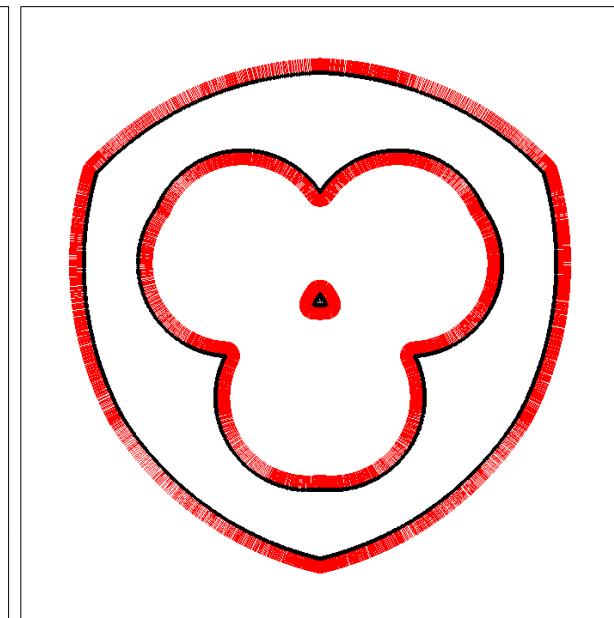
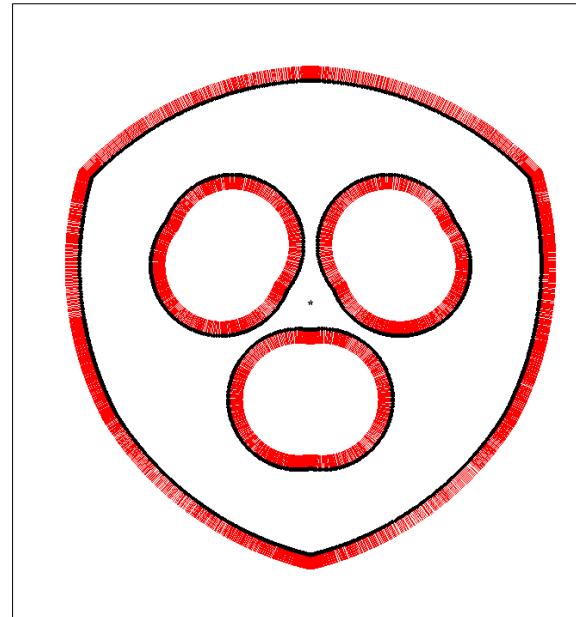
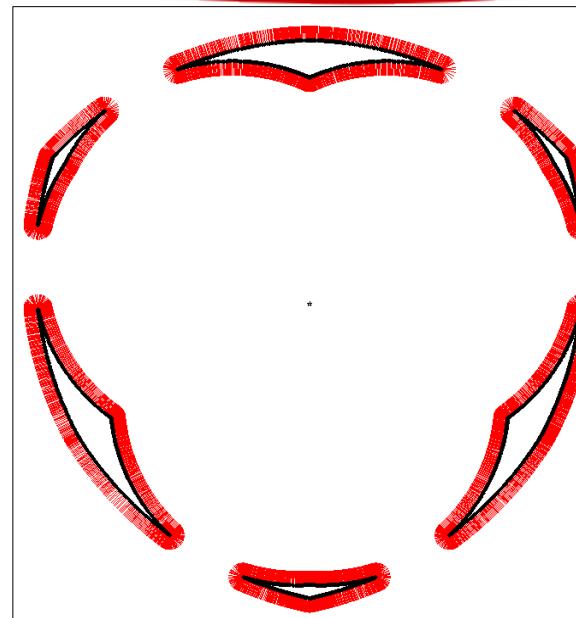


EXAMPLES – GENERAL STEWART PLATFORM



$$\mathbf{u} = [x, y]^T$$

74 variables
73 equations
12458 boxes
45 seconds



OUTLINE

1. RELATED WORK

2. PROPOSED METHODOLOGY

3. EXAMPLES

4. CONCLUSIONS

CONCLUSIONS

NEW METHOD FOR WORKSPACE BOUNDARY DETERMINATION

GENERAL

APPLICABLE TO ALL
LOWER-PAIR MECHANISMS
WITH JOINT LIMITS

COMPLETE

RETURNS ALL
CONNECTED COMPONENTS
EVEN IF LOWER-DIMENSIONAL

VERSATILE

FULL WORKSPACE
REACHABLE WORKSPACE
CONSTANT POSITION WORKSPACE
CONSTANT ORIENTATION WORKSPACE

ALSO GIVES

INTERIOR BARRIERS
END-EFFECTOR SINGULARITIES

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THANK YOU!