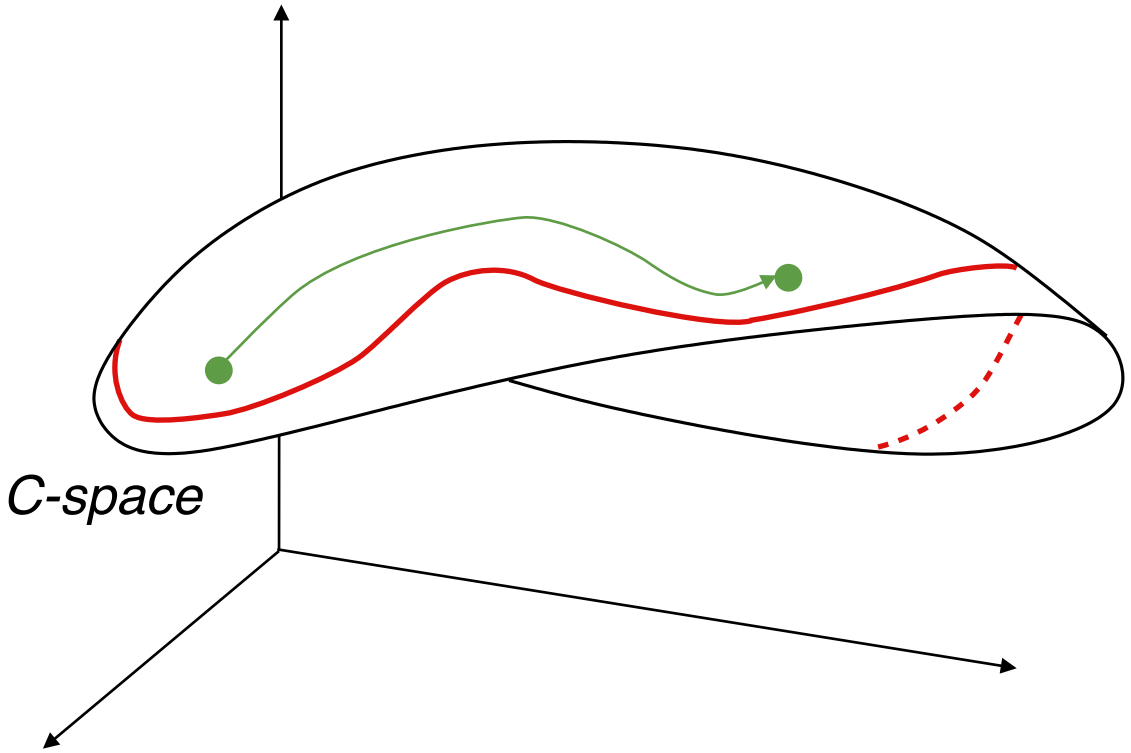
Planning Singularity-free Force-feasible Paths on the Stewart Platform

Oriol Bohigas, Montserrat Manubens, Lluís Ros

June 2012

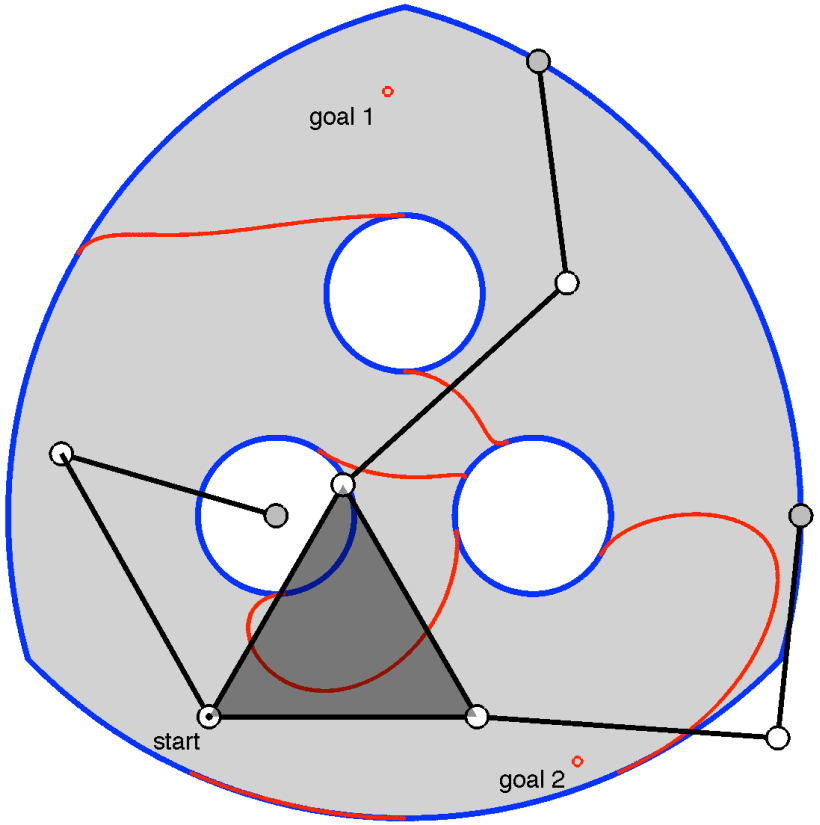
SINGULARITY-FREE PATH PLANNING



SINGULARITY-FREE PATH PLANNING

CLEARANCE GIVEN BY DET(J)

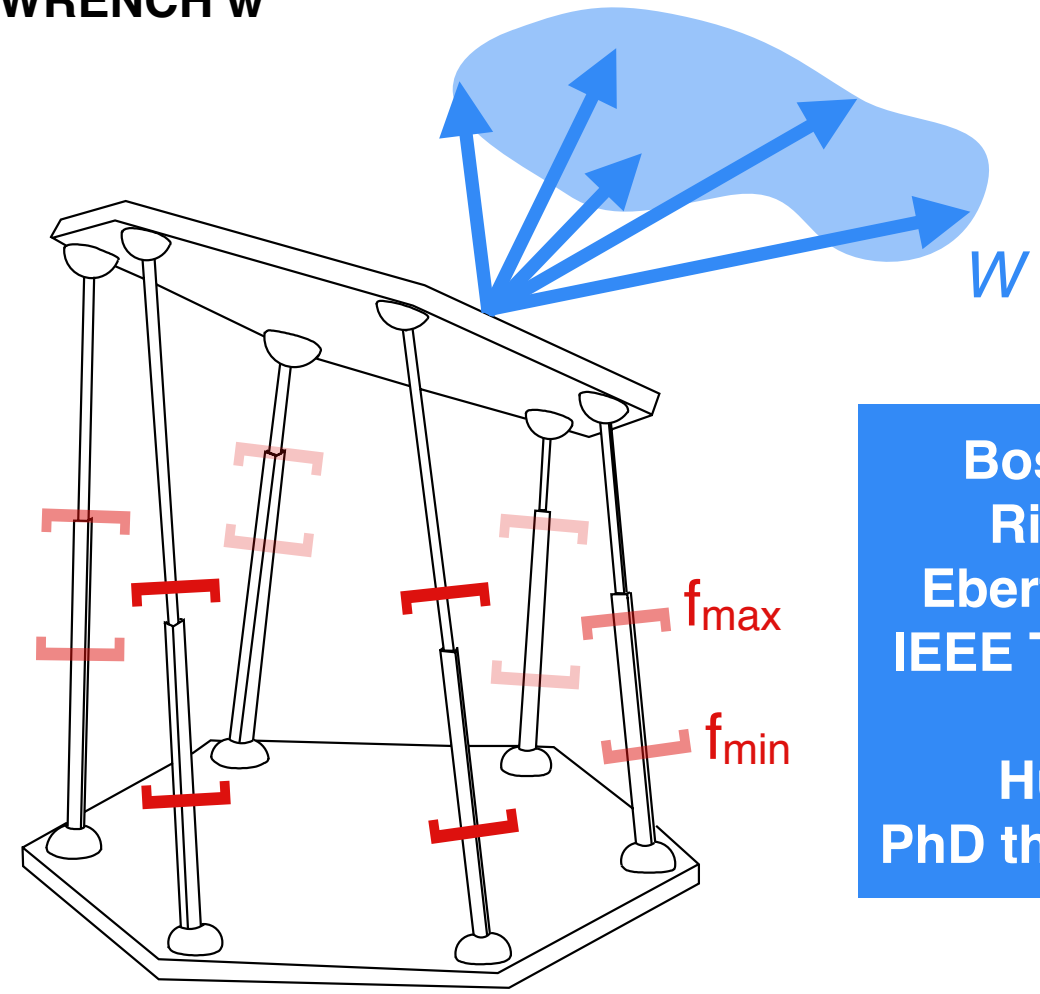
NOT PHYSICALLY MEANINGFUL



SINGULARITY-FREE PATH PLANNING

CLEARANCE GIVEN BY DET(J) — **NOT PHYSICALLY MEANINGFUL**

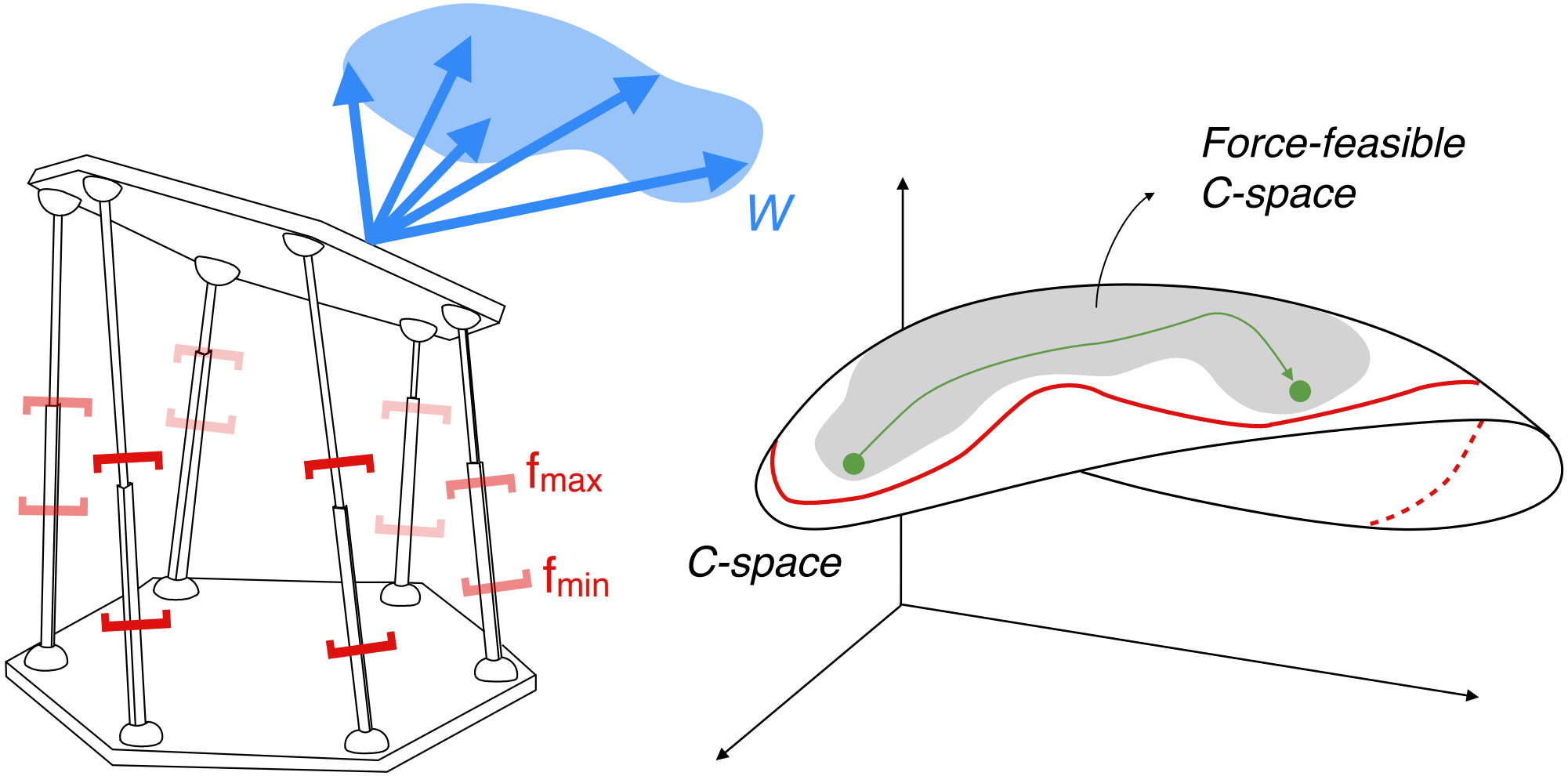
CLEARANCE GIVEN BY | **FORCE RANGES**
WRENCH w — **PHYSICALLY MEANINGFUL**



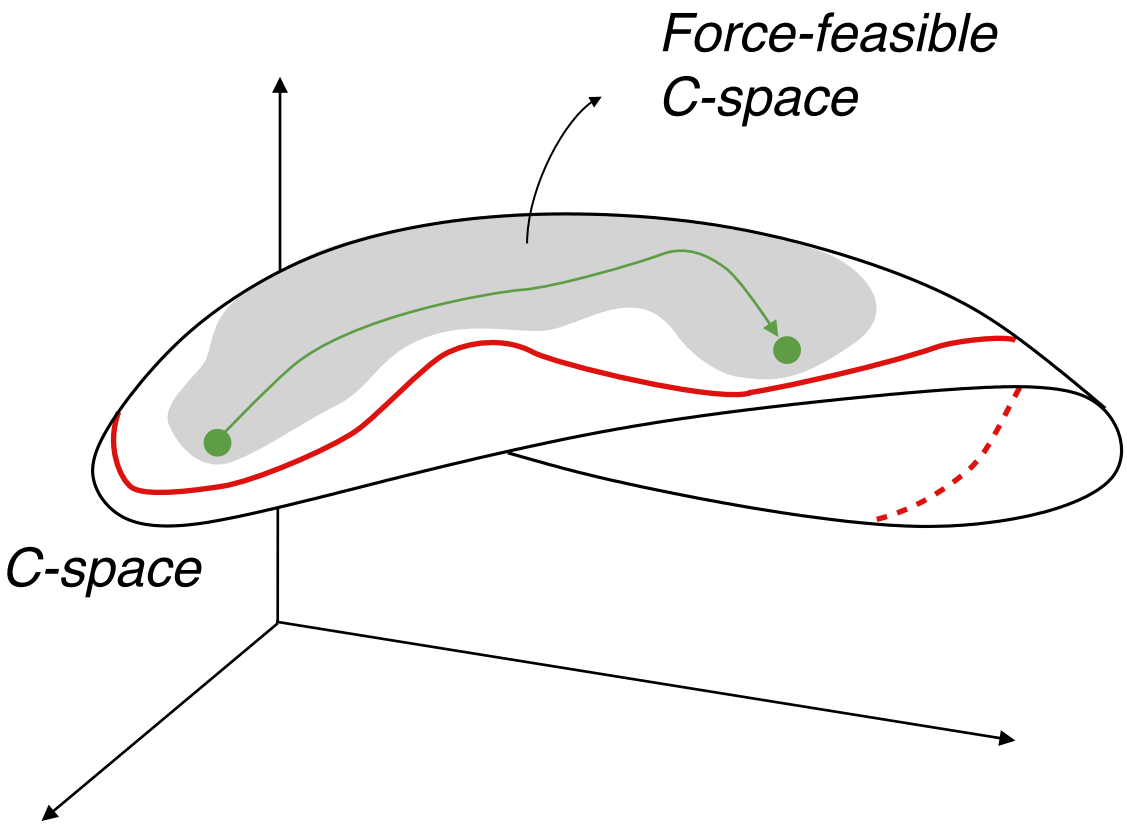
Bosscher
Riechel
Ebert-Uphoff
IEEE TRO 2006

Hubert
PhD thesis 2010

SINGULARITY-FREE FORCE-FEASIBLE PATH PLANNING



SINGULARITY-FREE FORCE-FEASIBLE PATH PLANNING



SYSTEM OF EQUATIONS FOR THE FORCE-FEASIBLE C-SPACE

HIGHER-DIMENSIONAL CONTINUATION FOR EXPLORATION

SYSTEM OF EQUATIONS FOR THE FORCE-FEASIBLE C-SPACE

HIGHER-DIMENSIONAL CONTINUATION FOR EXPLORATION

C-SPACE

$$\rho_i^2 = |\boldsymbol{p} + \boldsymbol{R}\boldsymbol{b}_i - \boldsymbol{a}_i|^2$$

$$\rho_i \in [\underline{\rho_i}, \overline{\rho_i}]$$

FORCES ON THE LEGS

$$\boldsymbol{J}(\boldsymbol{q}) \cdot \boldsymbol{f}_0 = \hat{\boldsymbol{w}}_0$$

$$\boldsymbol{B} = \boldsymbol{J}(\boldsymbol{q})^{\mathsf{T}} \boldsymbol{E}\, \boldsymbol{J}(\boldsymbol{q})$$

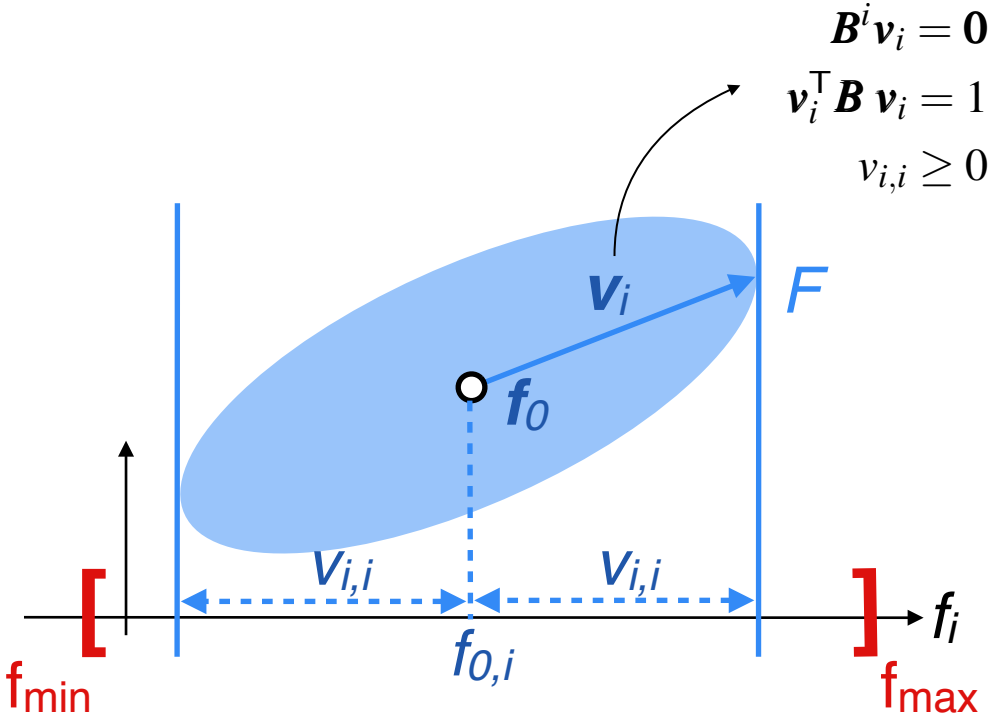
$$\boldsymbol{B}^i \boldsymbol{v}_i = \boldsymbol{0}$$

$$\boldsymbol{v}_i^{\mathsf{T}} \boldsymbol{B}\, \boldsymbol{v}_i = 1$$

$$v_{i,i} \geq 0$$

$$f_{0,i} - v_{i,i} \geq \underline{f_i}$$

$$f_{0,i} + v_{i,i} \leq \overline{f_i}$$



$$(\boldsymbol{f} - \boldsymbol{f}_0)^{\mathsf{T}} \boldsymbol{B}\, (\boldsymbol{f} - \boldsymbol{f}_0) \leq 1$$

$$\boldsymbol{B} = \boldsymbol{J}(\boldsymbol{q})^{\mathsf{T}} \boldsymbol{E}\, \boldsymbol{J}(\boldsymbol{q})$$

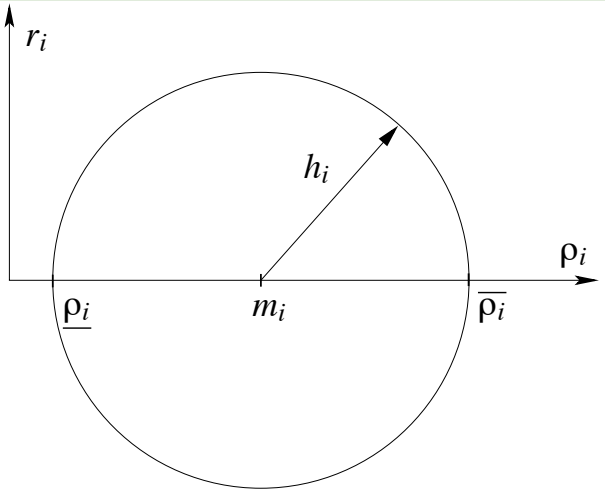
SYSTEM OF EQUATIONS FOR THE FORCE-FEASIBLE C-SPACE

HIGHER-DIMENSIONAL CONTINUATION FOR EXPLORATION

C-SPACE

$$\rho_i^2 = |\boldsymbol{p} + \boldsymbol{R}\boldsymbol{b}_i - \boldsymbol{a}_i|^2$$

$$\rho_i \in [\underline{\rho_i}, \overline{\rho_i}] \longrightarrow (\rho_i - m_i)^2 + r_i^2 = h_i^2$$



FORCES ON THE LEGS

$$\boldsymbol{J}(\boldsymbol{q}) \cdot \boldsymbol{f}_0 = \hat{\boldsymbol{w}}_0$$

$$\boldsymbol{B} = \boldsymbol{J}(\boldsymbol{q})^{\mathsf{T}} \boldsymbol{E}\, \boldsymbol{J}(\boldsymbol{q})$$

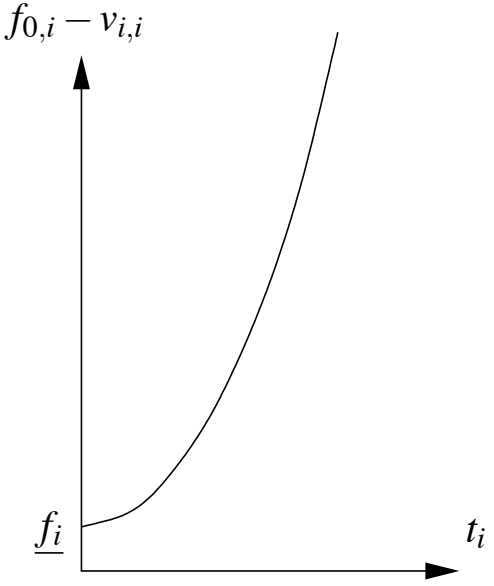
$$\boldsymbol{B}^i \boldsymbol{v}_i = \boldsymbol{0}$$

$$\boldsymbol{v}_i^{\mathsf{T}} \boldsymbol{B}\, \boldsymbol{v}_i = 1$$

$$v_{i,i} \geq 0 \longrightarrow v_{i,i} = s_i^2$$

$$f_{0,i} - v_{i,i} \geq \underline{f_i} \longrightarrow f_{0,i} - v_{i,i} = t_i^2 + \underline{f_i}$$

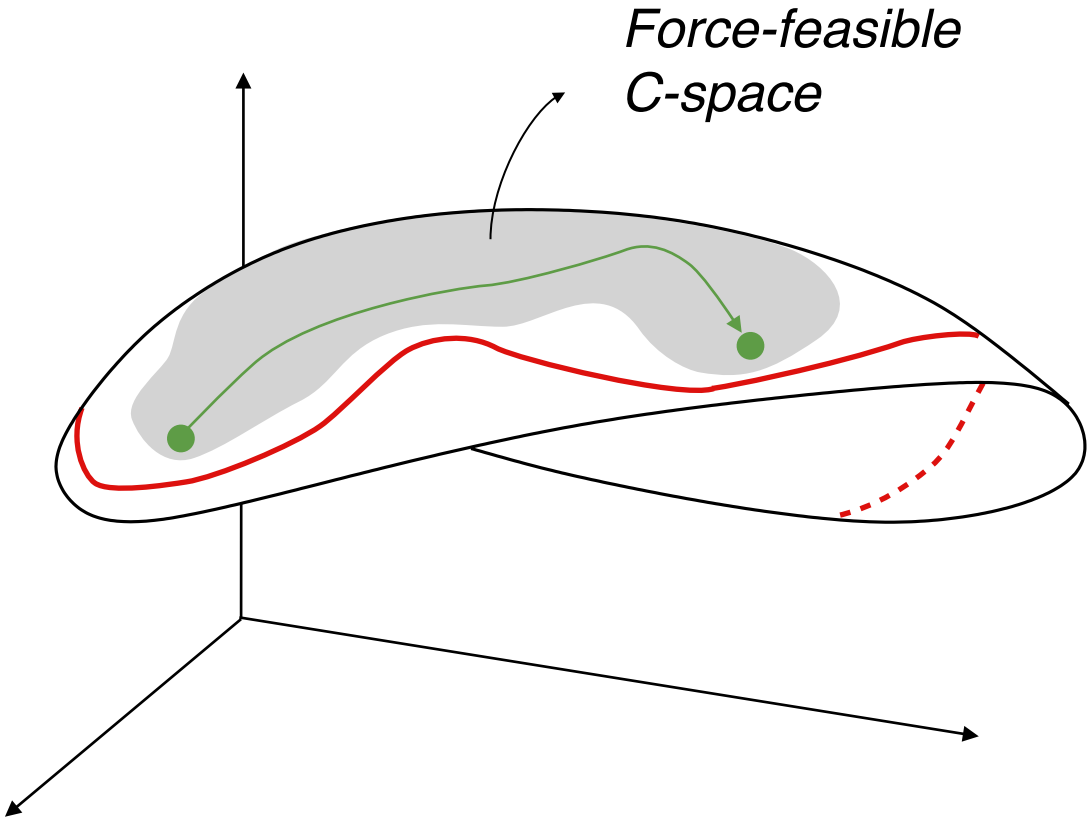
$$f_{0,i} + v_{i,i} \leq \overline{f_i} \longrightarrow f_{0,i} + v_{i,i} = -u_i^2 + \overline{f_i}$$



SYSTEM OF EQUATIONS FOR THE FORCE-FEASIBLE C-SPACE

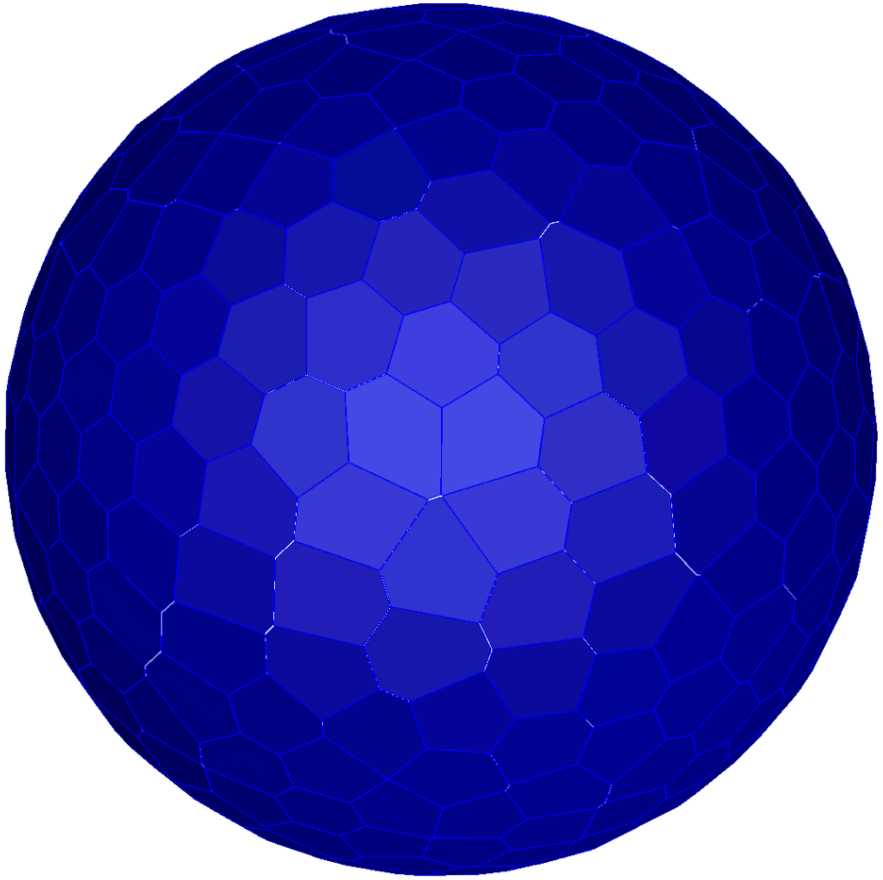
HIGHER-DIMENSIONAL CONTINUATION FOR EXPLORATION

$$
\left.
\begin{aligned}
&\rho_i^2 = |\boldsymbol{p} + \boldsymbol{R}\boldsymbol{b}_i - \boldsymbol{a}_i|^2 \\
&(\rho_i - m_i)^2 + r_i^2 = h_i^2 \\
&\boldsymbol{J}(\boldsymbol{q}) \cdot \boldsymbol{f}_0 = \hat{\boldsymbol{w}}_0 \\
&\boldsymbol{B} = \boldsymbol{J}(\boldsymbol{q})^{\mathsf{T}} \boldsymbol{E}\, \boldsymbol{J}(\boldsymbol{q}) \\
&\boldsymbol{B}^i \boldsymbol{v}_i = \boldsymbol{0} \\
&\boldsymbol{v}_i^{\mathsf{T}} \boldsymbol{B}\, \boldsymbol{v}_i = 1 \\
&v_{i,i} = s_i^2 \\
&f_{0,i} - v_{i,i} = t_i^2 + \underline{f_i} \\
&f_{0,i} + v_{i,i} = -u_i^2 + \overline{f_i}
\end{aligned}
\right\} \boldsymbol{F}(\boldsymbol{x}) = \boldsymbol{0}
$$



SYSTEM OF EQUATIONS FOR THE FORCE-FEASIBLE C-SPACE

HIGHER-DIMENSIONAL CONTINUATION FOR EXPLORATION

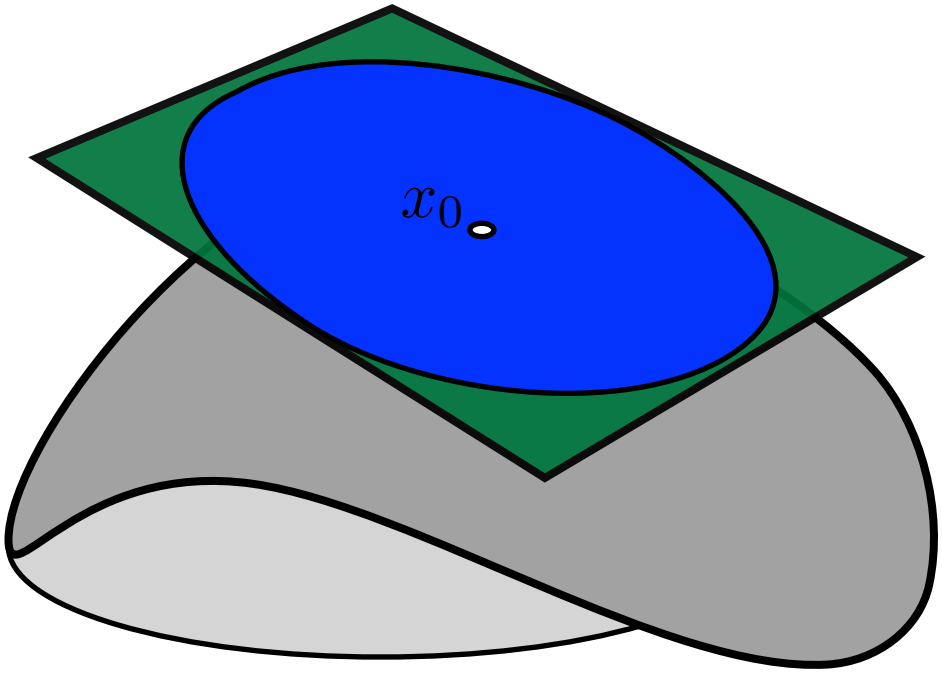


ITERATIVELY BUILDS THE **CHARTS** OF THE **ATLAS** FROM A STARTING POINT

SYSTEM OF EQUATIONS FOR THE FORCE-FEASIBLE C-SPACE

HIGHER-DIMENSIONAL CONTINUATION FOR EXPLORATION

INITIALIZE CHART

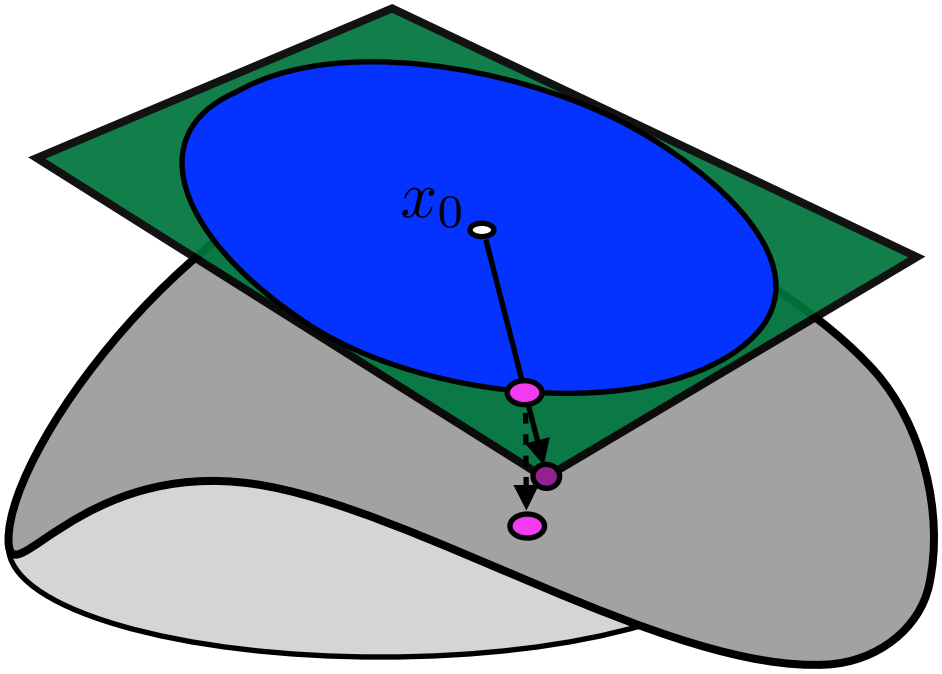


SYSTEM OF EQUATIONS FOR THE FORCE-FEASIBLE C-SPACE

HIGHER-DIMENSIONAL CONTINUATION FOR EXPLORATION

INITIALIZE CHART

SELECT POINT AND PROJECT



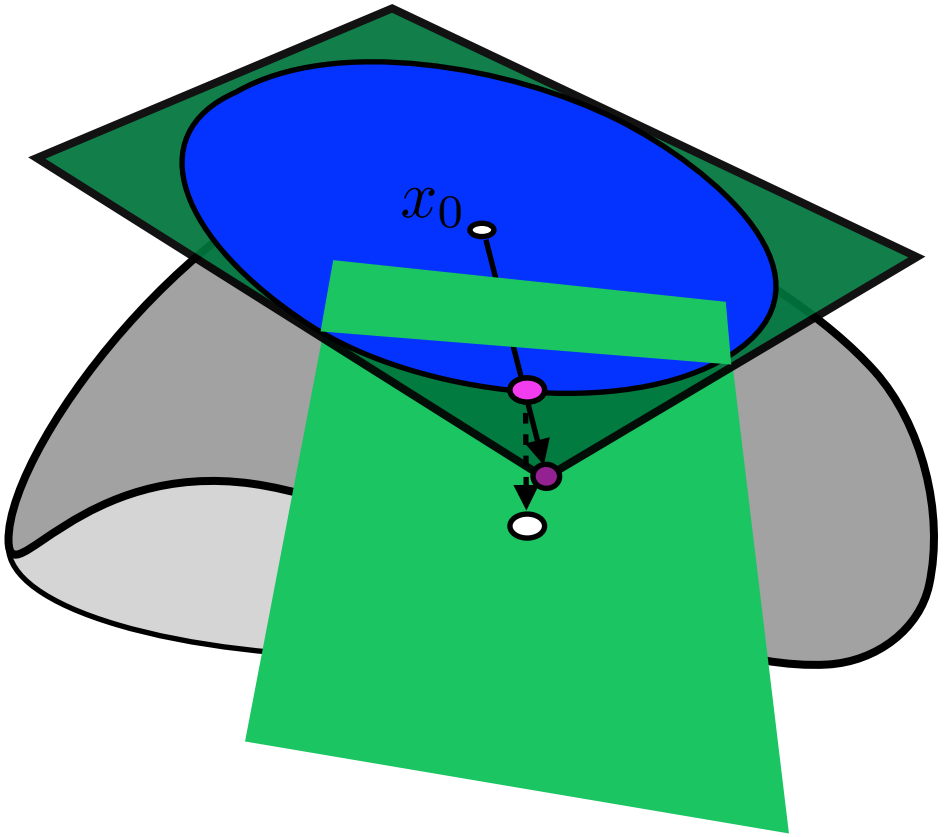
SYSTEM OF EQUATIONS FOR THE FORCE-FEASIBLE C-SPACE

HIGHER-DIMENSIONAL CONTINUATION FOR EXPLORATION

INITIALIZE CHART

SELECT POINT AND PROJECT

TEST VALIDITY OF NEW CHART



SYSTEM OF EQUATIONS FOR THE FORCE-FEASIBLE C-SPACE

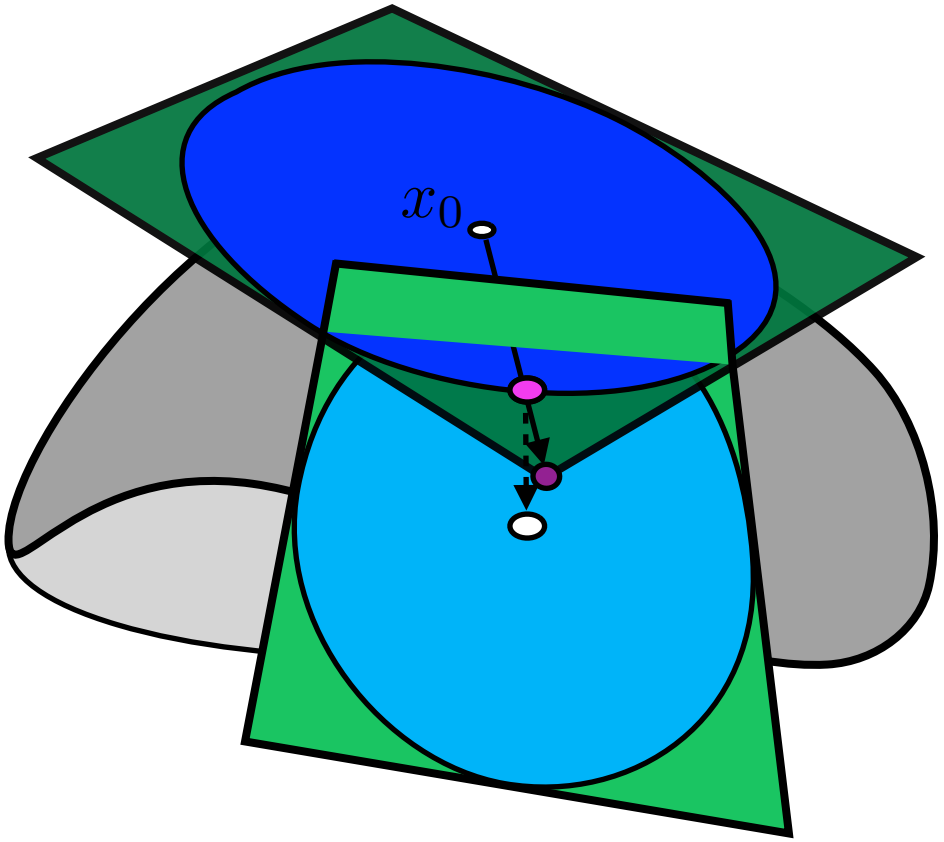
HIGHER-DIMENSIONAL CONTINUATION FOR EXPLORATION

INITIALIZE CHART

SELECT POINT AND PROJECT

TEST VALIDITY OF NEW CHART

INITIALIZE NEW CHART



SYSTEM OF EQUATIONS FOR THE FORCE-FEASIBLE C-SPACE

HIGHER-DIMENSIONAL CONTINUATION FOR EXPLORATION

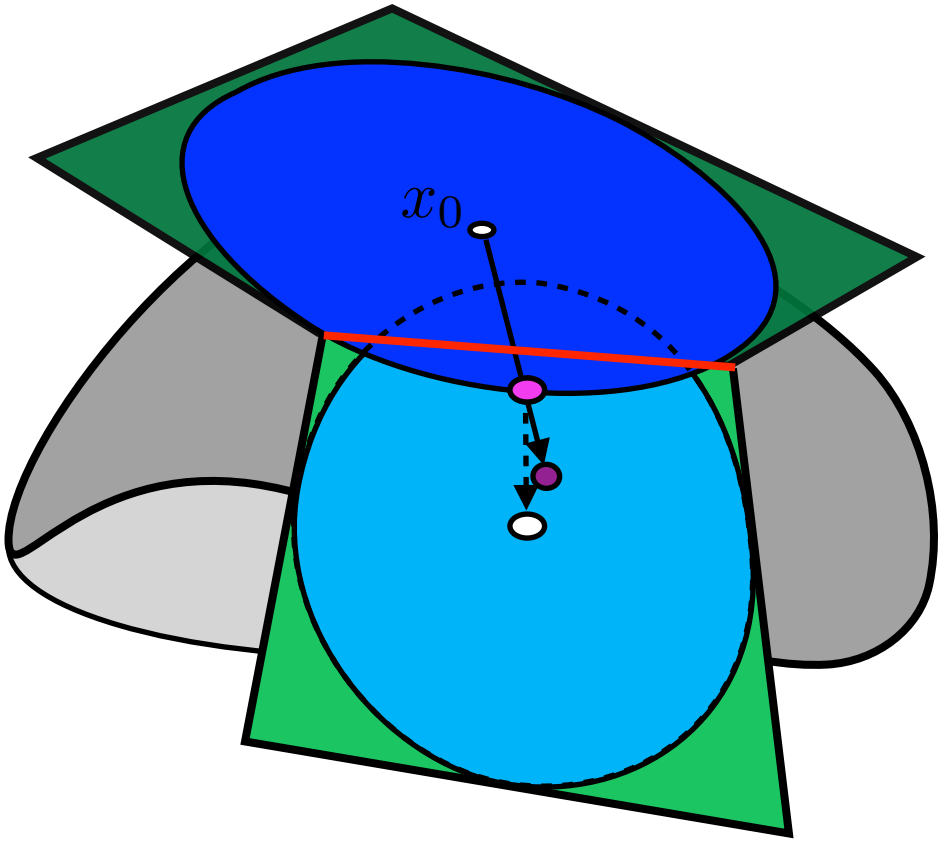
INITIALIZE CHART

SELECT POINT AND PROJECT

TEST VALIDITY OF NEW CHART

INITIALIZE NEW CHART

CROP THE CHARTS



SYSTEM OF EQUATIONS FOR THE FORCE-FEASIBLE C-SPACE

HIGHER-DIMENSIONAL CONTINUATION FOR EXPLORATION

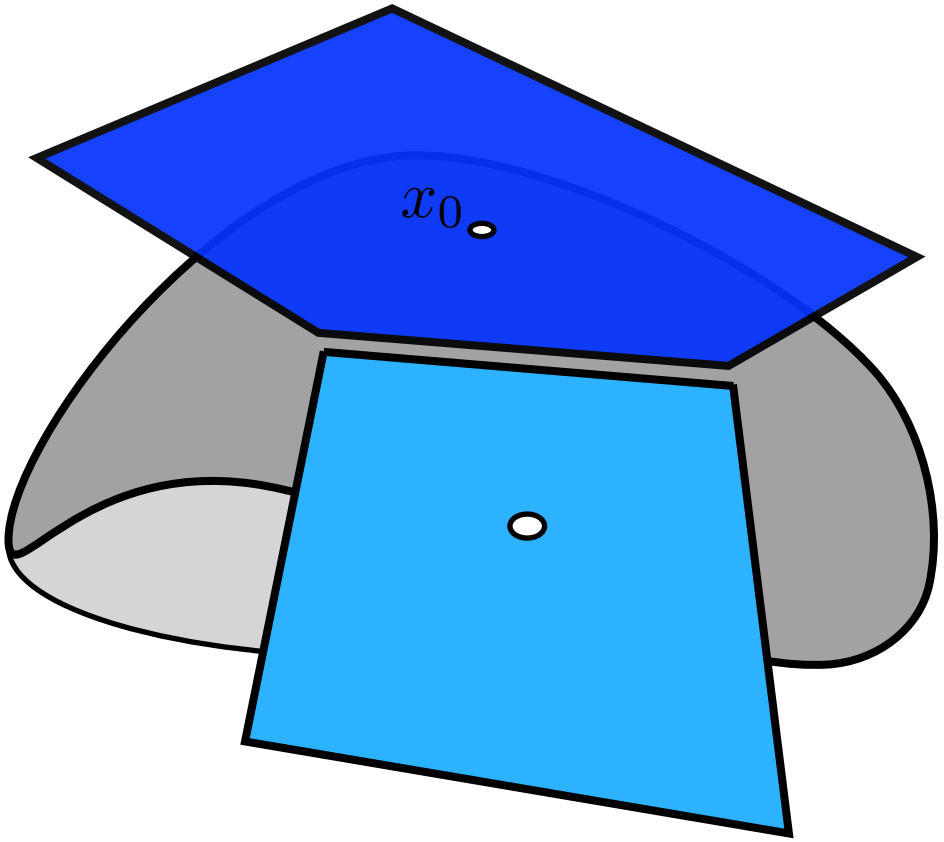
INITIALIZE CHART

SELECT POINT AND PROJECT

TEST VALIDITY OF NEW CHART

INITIALIZE NEW CHART

CROP THE CHARTS



SYSTEM OF EQUATIONS FOR THE FORCE-FEASIBLE C-SPACE

HIGHER-DIMENSIONAL CONTINUATION FOR EXPLORATION

INITIALIZE CHART

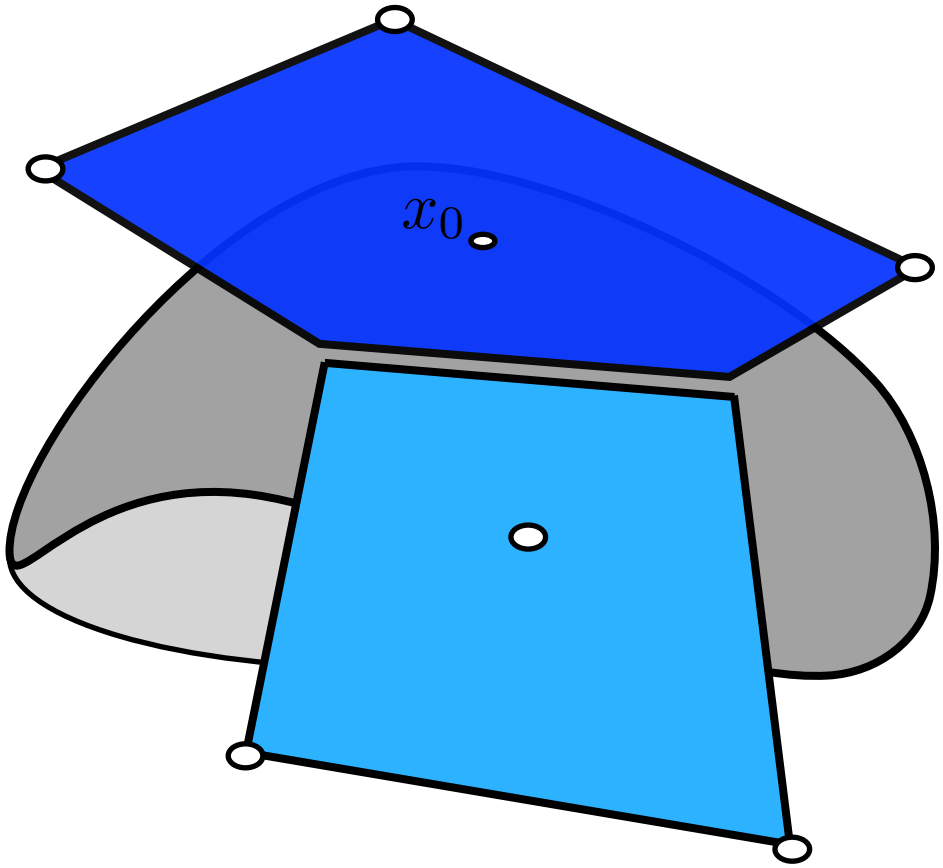
SELECT POINT AND PROJECT

TEST VALIDITY OF NEW CHART

INITIALIZE NEW CHART

CROP THE CHARTS

EXPAND THE ATLAS



SYSTEM OF EQUATIONS FOR THE FORCE-FEASIBLE C-SPACE

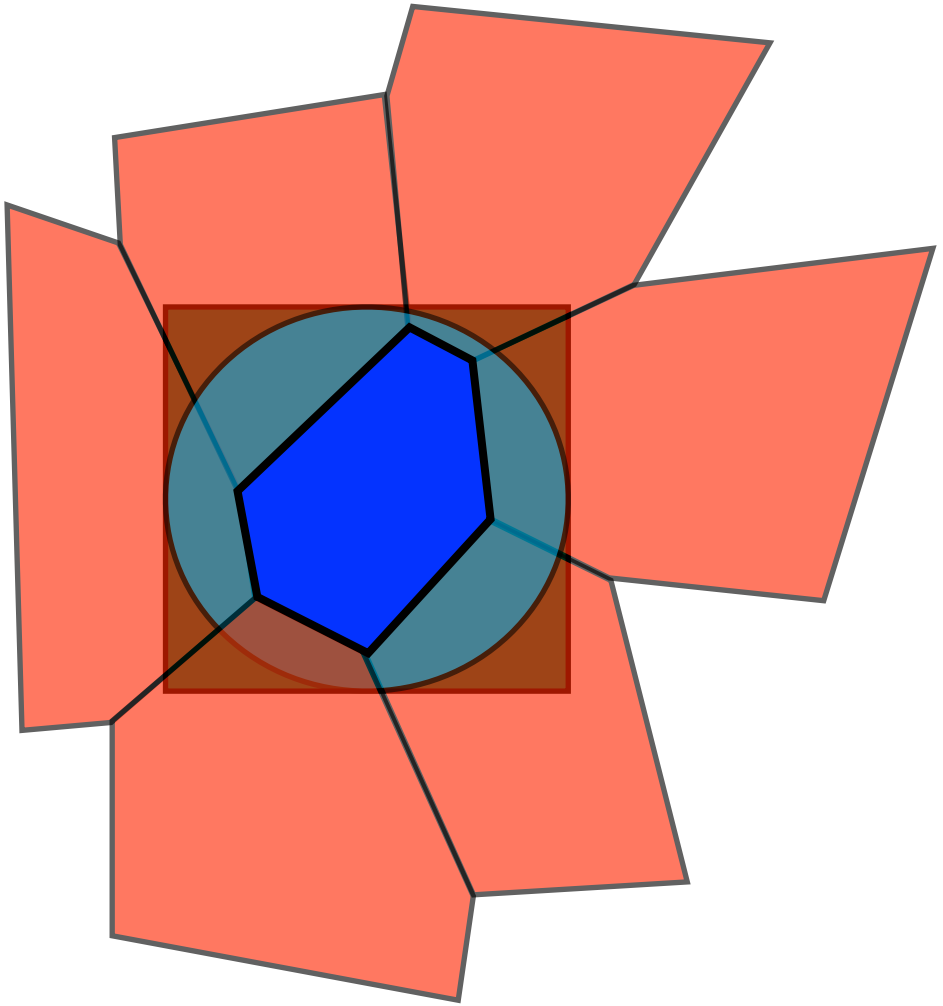
HIGHER-DIMENSIONAL CONTINUATION FOR EXPLORATION

NEIGHBOUR CHARTS CROP THE POLYTOPE

POLYTOPE INSIDE THE BALL

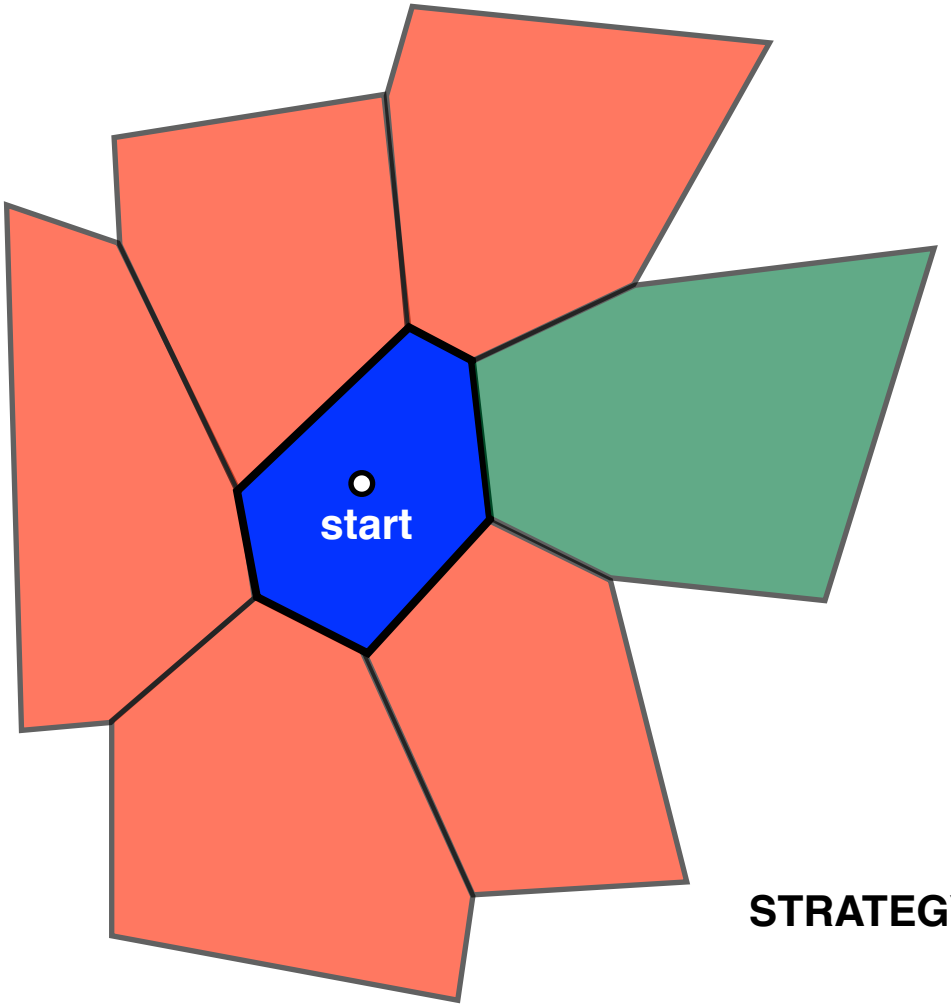
↓

CHART CLOSED



SYSTEM OF EQUATIONS FOR THE FORCE-FEASIBLE C-SPACE

HIGHER-DIMENSIONAL CONTINUATION FOR EXPLORATION

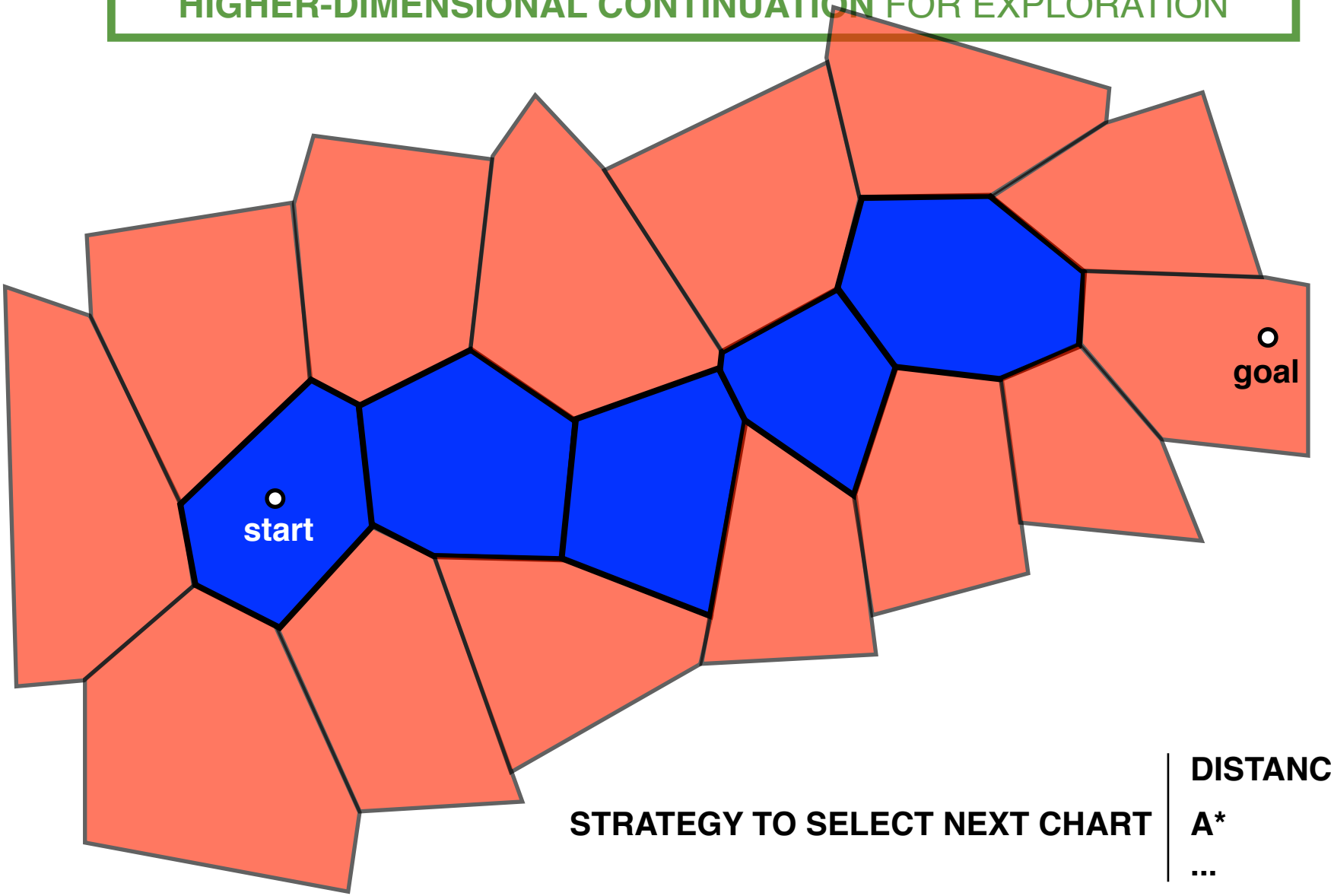


goal

STRATEGY TO SELECT NEXT CHART | **DISTANCE**

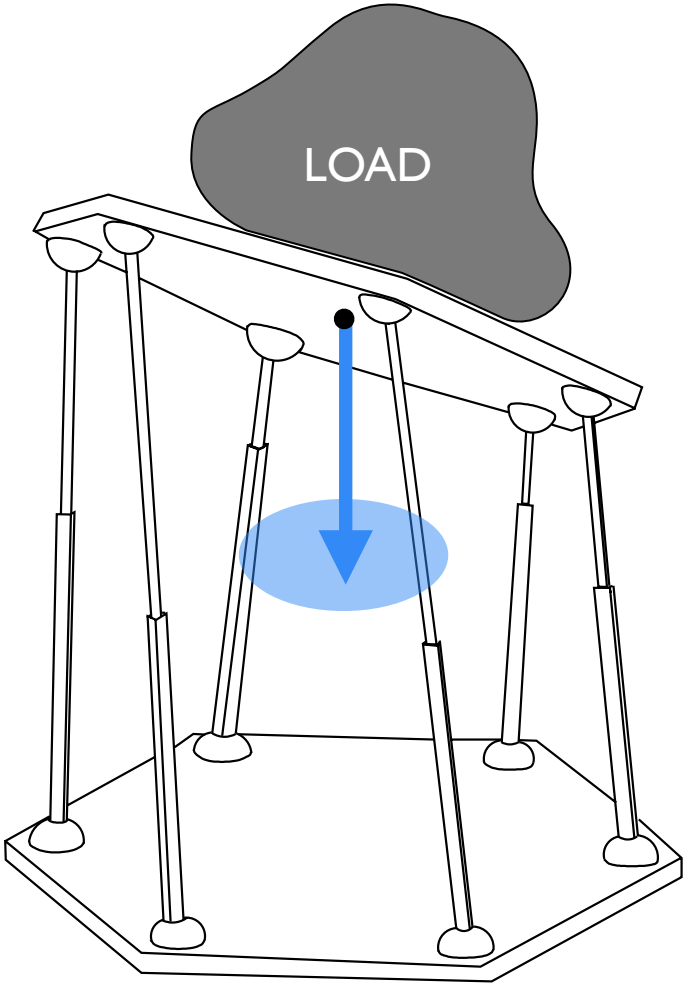
SYSTEM OF EQUATIONS FOR THE FORCE-FEASIBLE C-SPACE

HIGHER-DIMENSIONAL CONTINUATION FOR EXPLORATION



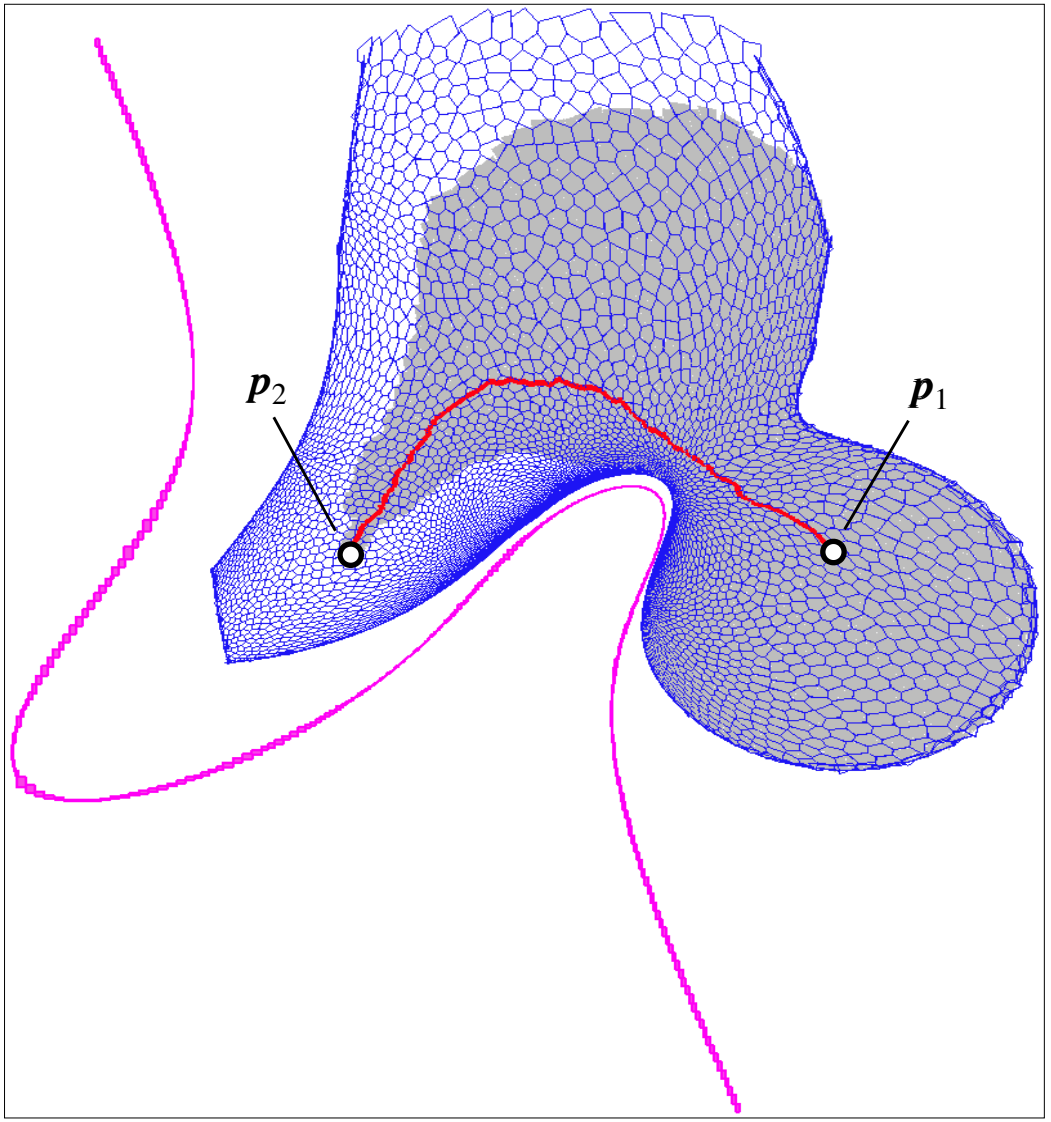
STRATEGY TO SELECT NEXT CHART	DISTANCE
	A*
	...

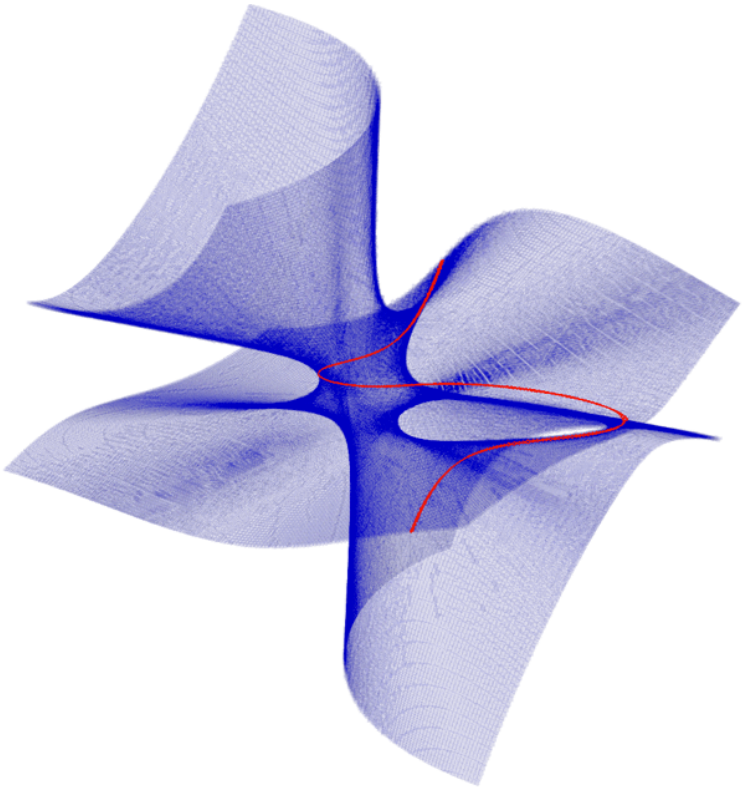
CONSTANT ORIENTATION



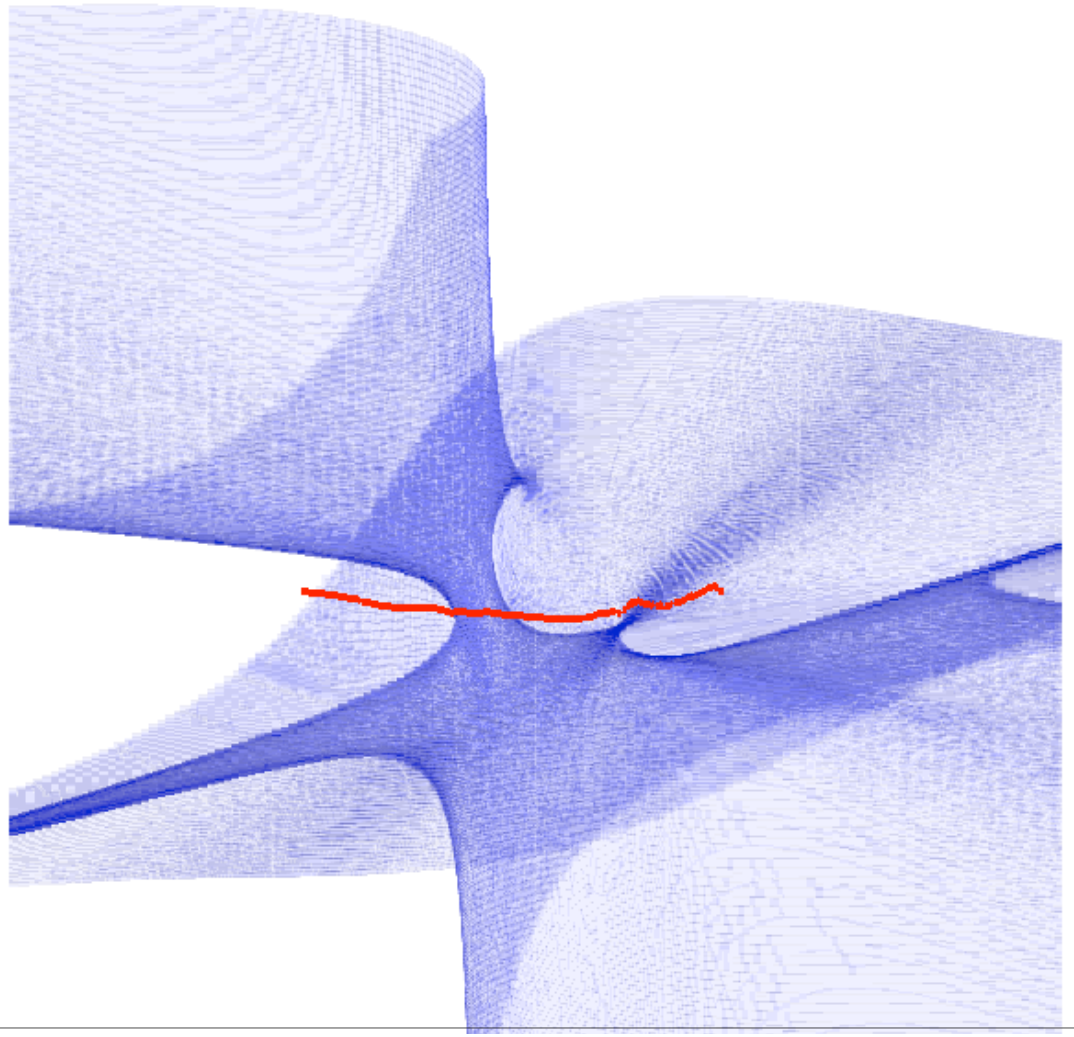
CONSTANT ORIENTATION

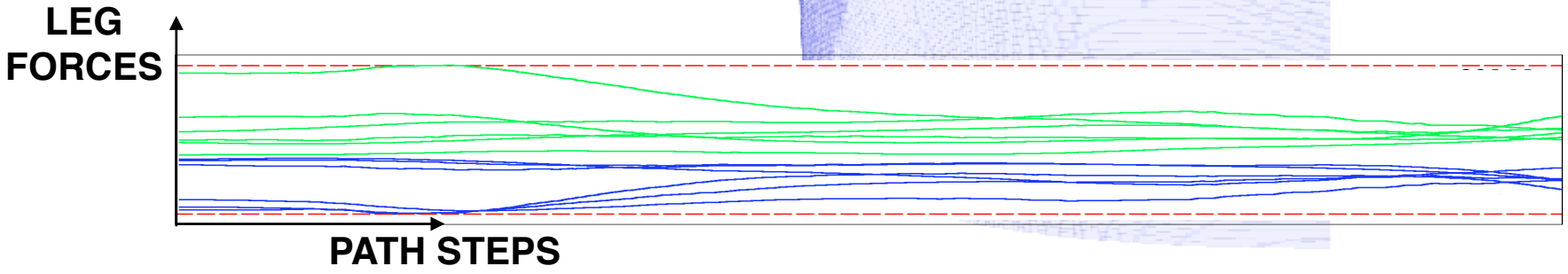
Z FIXED



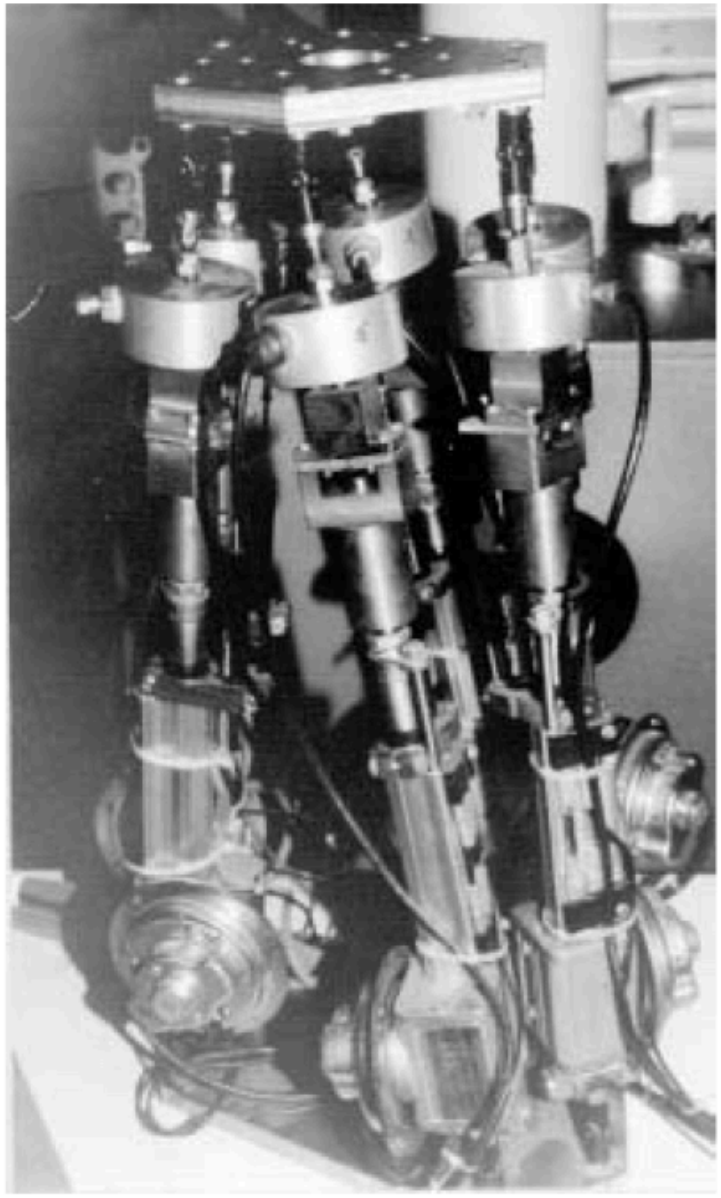


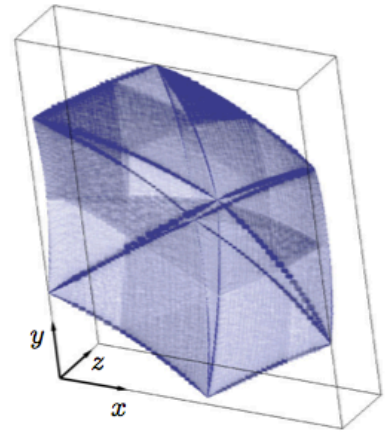
CONSTANT ORIENTATION

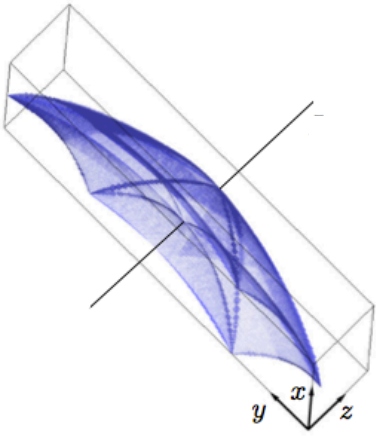


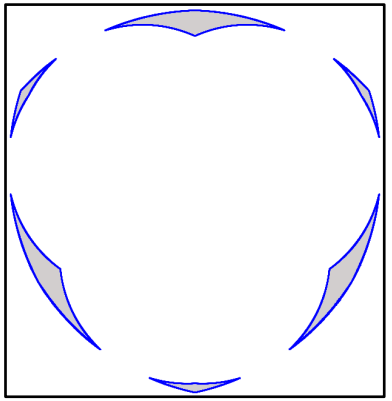


CONSTANT ORIENTATION

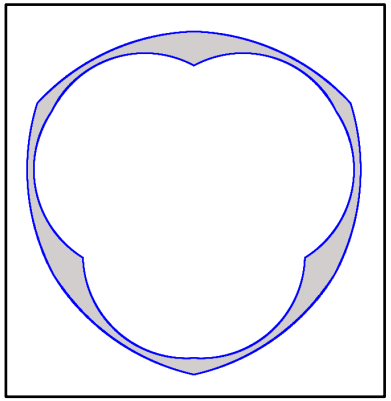




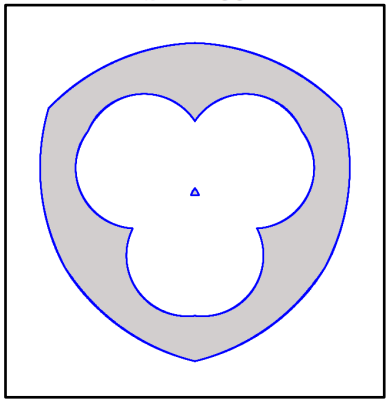




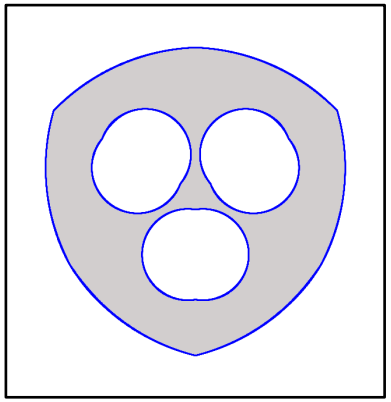
$z = 4.95$



$z = 5.00$



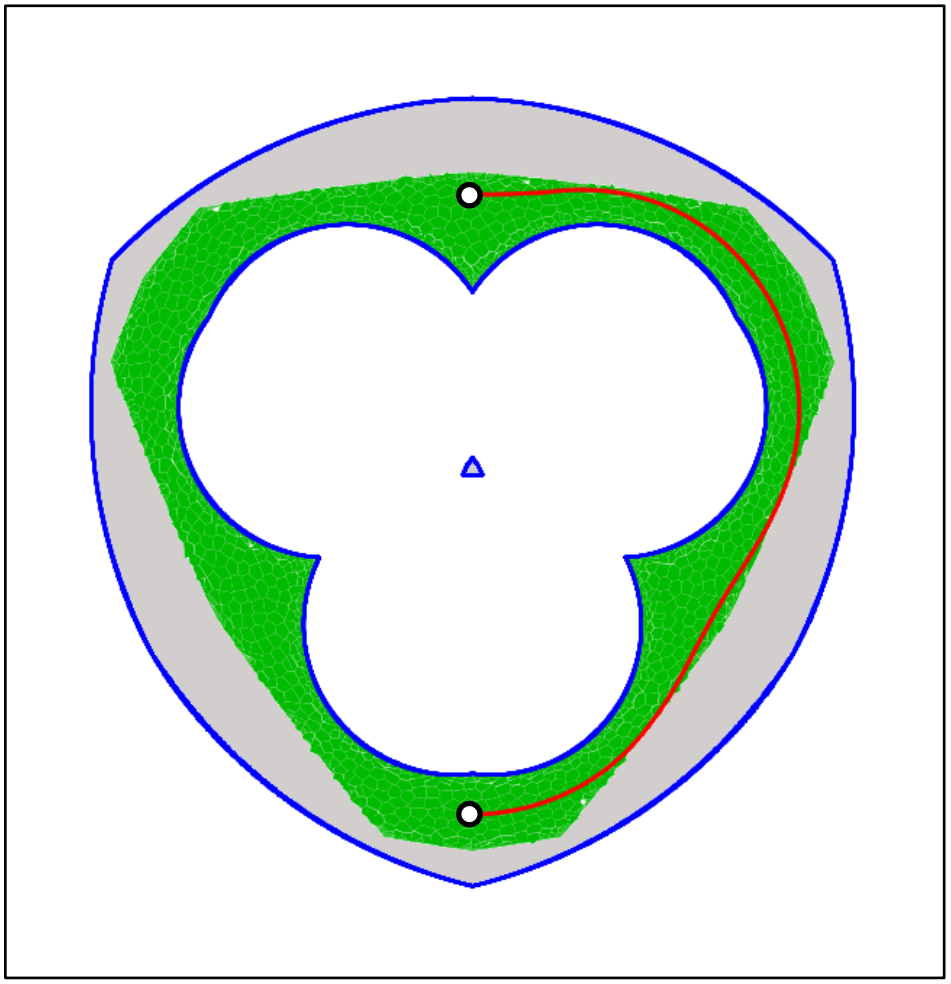
$z = 5.10$

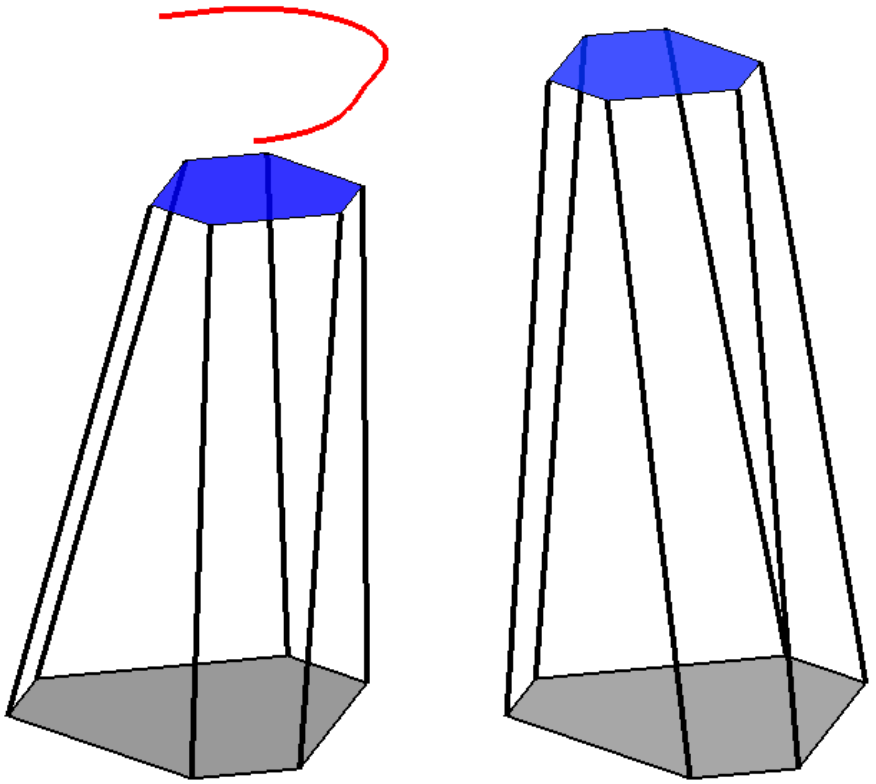


$z = 5.12$

CONSTANT ORIENTATION

6D





APPROACH TO COMPUTE SINGULARITY-FREE FORCE-FEASIBLE PATHS ON THE STEWART PLATFORM

- RESOLVABILITY OF A SET OF WRENCHES
- SYSTEM OF EQUATIONS FOR THE FORCE-FEASIBLE C-SPACE
- HIGHER-DIMENSIONAL CONTINUATION
- NO EXPLICIT REPRESENTATION OF SINGULARITY LOCUS
- ALLOWS COMPUTATION OF FORCE-FEASIBLE WORKSPACE
- TREATMENT OF COLLISIONS (RANDOMIZING)
- APPLICATION TO CABLE-DRIVEN MANIPULATORS