NUMERICAL COMPUTATION OF MANIPULATOR SINGULARITIES

Oriol Bohigas, Dimiter Zlatanov, Lluís Ros, Montserrat Manubens and Josep M. Porta

May 2012

NUMERICAL COMPUTATION OF MANIPULATOR SINGULARITIES

MANIPULATOR SINGULARITIES

$$\boldsymbol{\Phi}(\boldsymbol{q}) = \mathbf{0} \quad \boldsymbol{L} \cdot \boldsymbol{m} = \mathbf{0}$$

C-SPACE
INPUT SINGULARITIES
OUTPUT

SIX TYPES

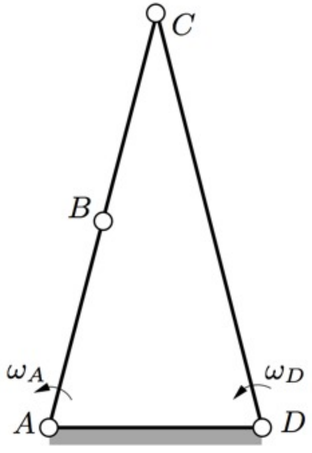
RI RO IO II RPM IIM

REDUNDANT INPUT

$$\left.\begin{array}{c} \boldsymbol{\Phi}(\boldsymbol{q}) = \mathbf{0} \\ \boldsymbol{L}_O \boldsymbol{\xi} = \mathbf{0} \\ \|\boldsymbol{\xi}\|^2 = 1 \end{array}\right\}$$

$$\boldsymbol{\xi} = \left[\boldsymbol{\Omega}^{a\mathsf{T}}, \boldsymbol{\Omega}^{p\mathsf{T}}\right]^{\mathsf{T}}$$

$$\boldsymbol{\Omega}^a \neq \mathbf{0}$$

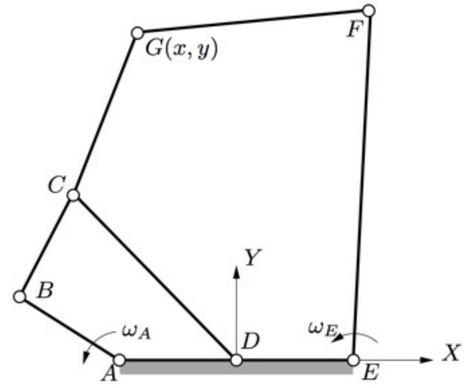


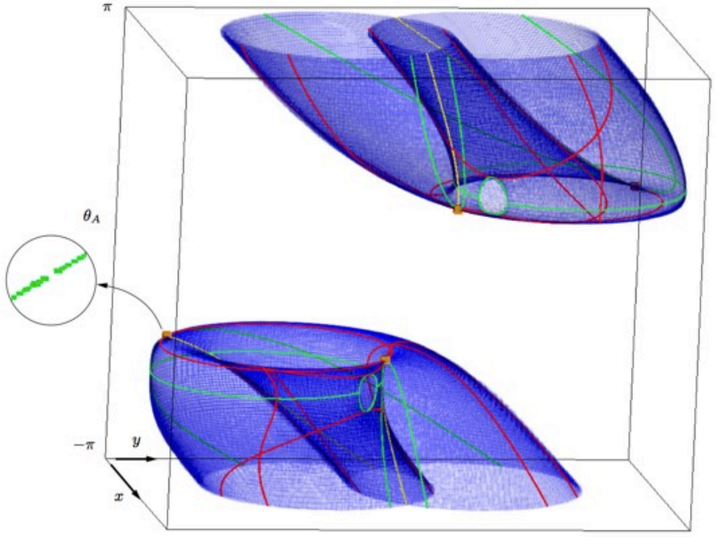
NUMERICAL COMPUTATION

BRANCH AND PRUNE BASED ON LINEAR RELAXATIONS

EXAMPLE

2 DOF MANIPULATOR





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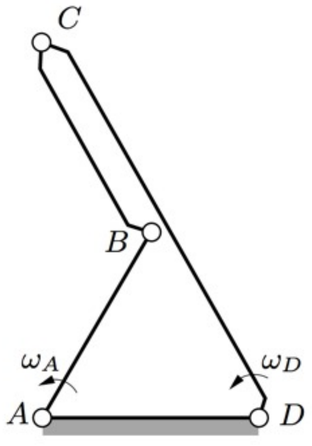

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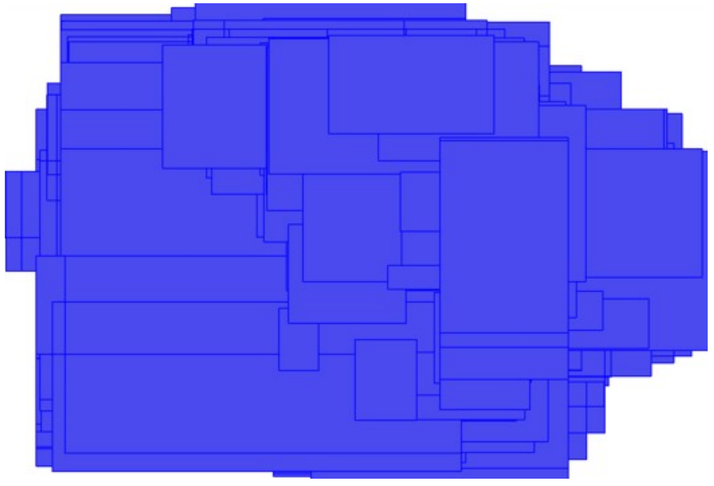

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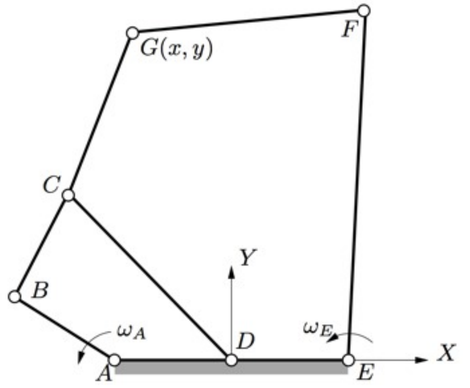

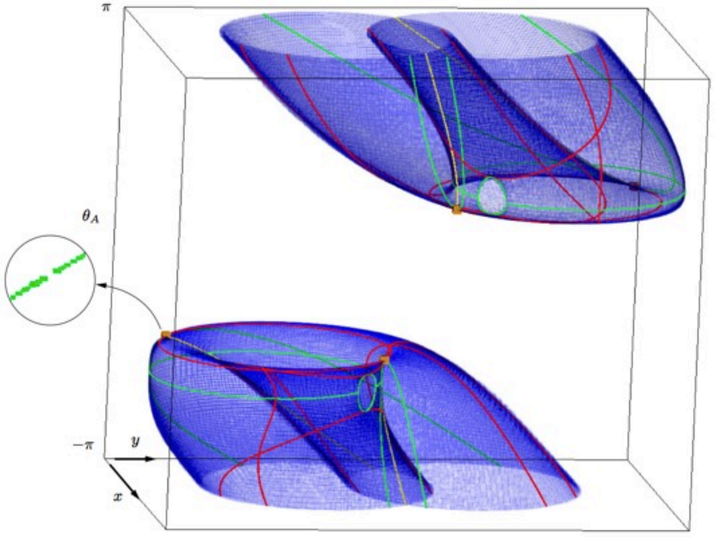

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IMPOSSIBLE OUTPUT

$$\left.\begin{array}{c} \boldsymbol{\Phi}(\boldsymbol{q}) = \mathbf{0} \\ \boldsymbol{L}_O^{\mathsf{T}} \boldsymbol{\xi} = \mathbf{0} \\ \|\boldsymbol{\xi}\|^2 = 1 \end{array}\right\}$$

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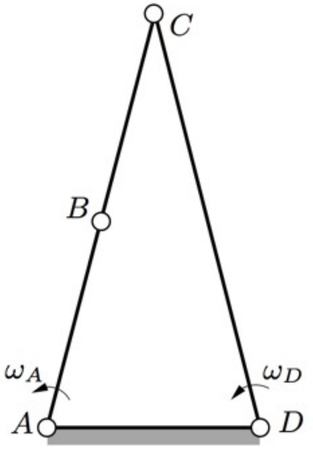

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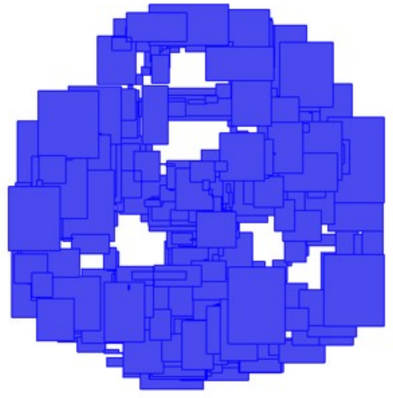

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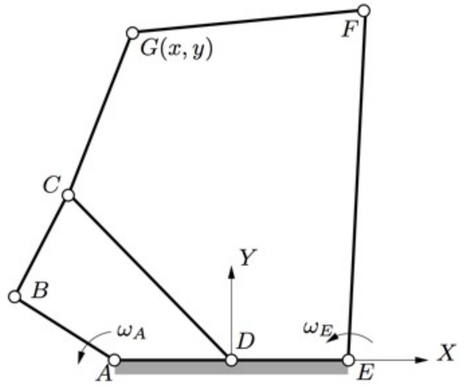

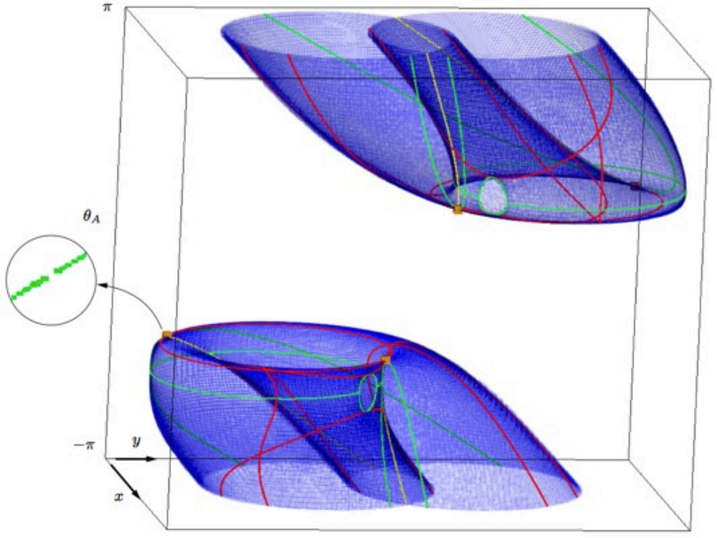

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RI RO IO **II** RPM IIM

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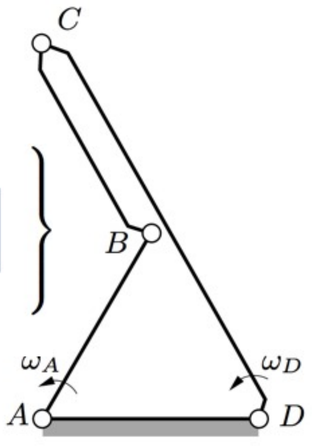
$$\begin{bmatrix} \boldsymbol{D}_a^{\mathsf{T}} \\ \boldsymbol{D}_p^{\mathsf{T}} \end{bmatrix} \boldsymbol{\psi}_2 = \begin{bmatrix} \boldsymbol{\Omega}^{a*} \\ \mathbf{0} \end{bmatrix}$$

$$\|\boldsymbol{\psi}_2\|^2 = 1$$

$$\boldsymbol{\Omega}^{a*} \neq \mathbf{0}$$



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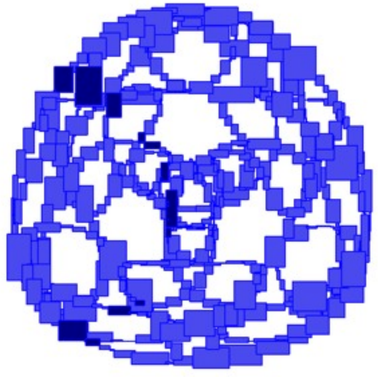

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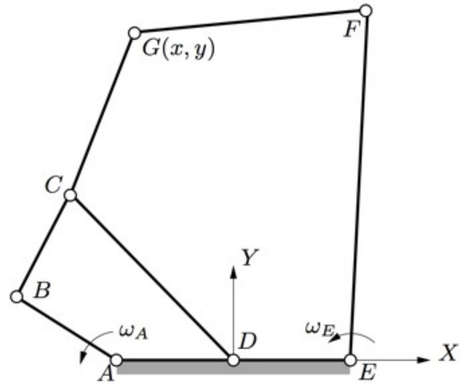

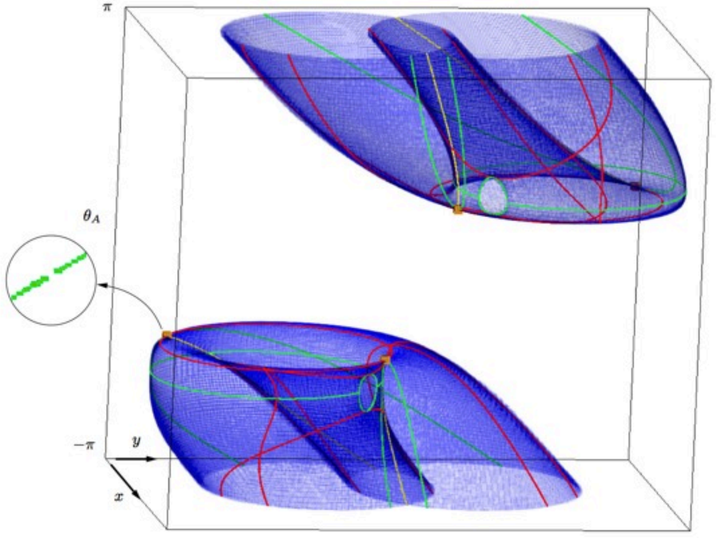

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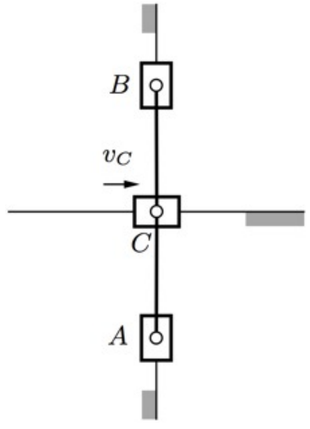
SIX TYPES

RI RO IO II **RPM** IIM

REDUNDANT PASSIVE MOTION

$$\left.\begin{array}{l} \boldsymbol{\Phi}(\boldsymbol{q}) = \mathbf{0} \\ \boldsymbol{L}_P \boldsymbol{\Omega}^p = \mathbf{0} \\ \|\boldsymbol{\Omega}^p\|^2 = 1 \end{array}\right\}$$



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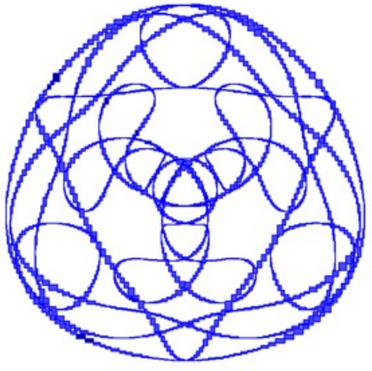

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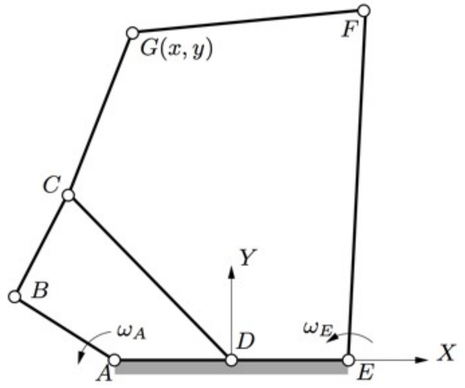

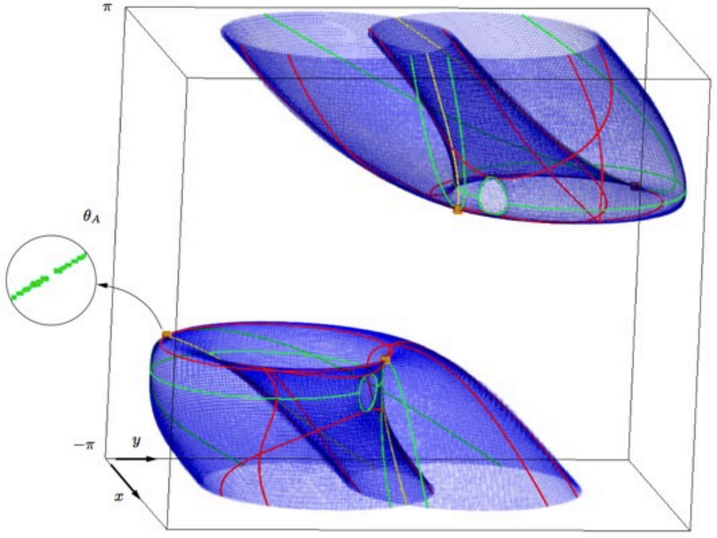

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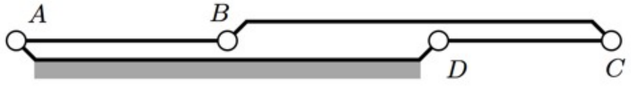
INCREASED INSTANTANEOUS MOBILITY

$$\left.\begin{aligned} \boldsymbol{\Phi}(\boldsymbol{q}) &= \mathbf{0} \\ \boldsymbol{L}^{\mathsf{T}}\boldsymbol{\xi} &= \mathbf{0} \\ \|\boldsymbol{\xi}\|^2 &= 1 \end{aligned}\right\}$$



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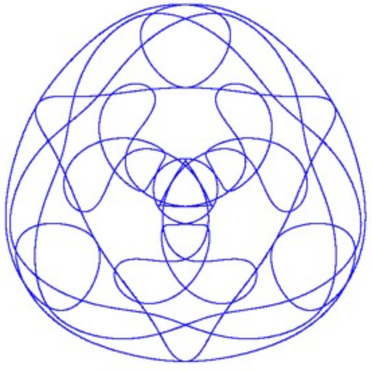

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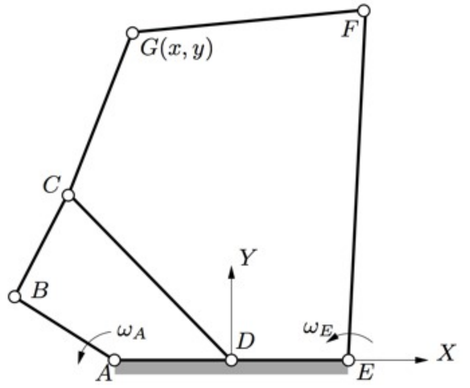

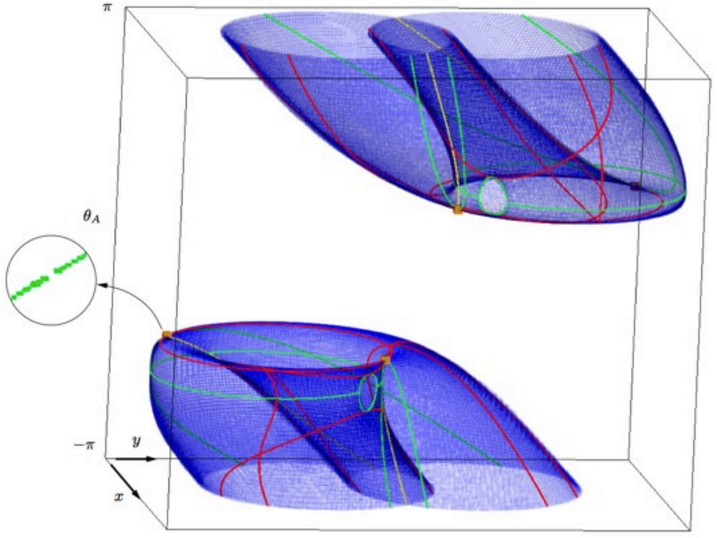

MANIPULATOR SINGULARITIES

CONFIGURATION SPACE $\boldsymbol{\Phi}(\boldsymbol{q}) = \mathbf{0}$

VELOCITY EQUATION $\boldsymbol{L} \cdot \boldsymbol{m} = \mathbf{0}$

$$\begin{bmatrix} -\boldsymbol{I} & \boldsymbol{A}_a & \boldsymbol{A}_p \\ \mathbf{0} & \boldsymbol{D}_a & \boldsymbol{D}_p \end{bmatrix} \begin{bmatrix} \boldsymbol{\Omega}^o \\ \boldsymbol{\Omega}^a \\ \boldsymbol{\Omega}^p \end{bmatrix} = \mathbf{0}$$

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→ SINGULARITIES

C-SPACE:	~~FIKP~~	~~IIKP~~
INPUT:	~~FIKP~~	
OUTPUT:	~~IIKP~~	

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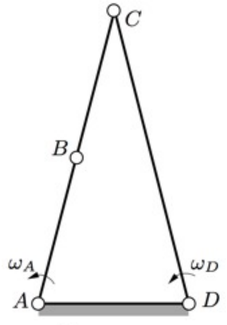

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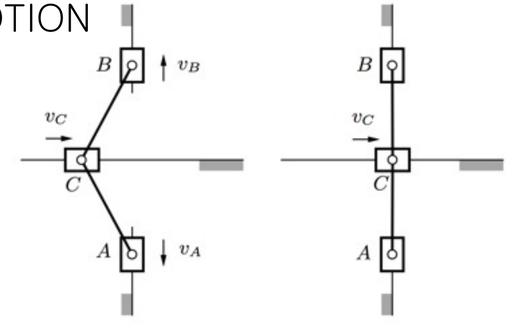

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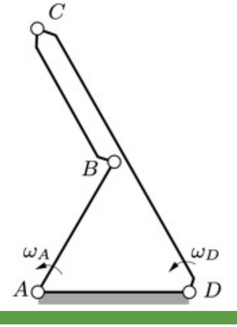

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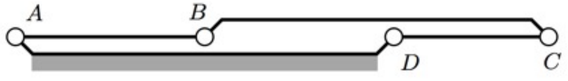

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linear equations in $\boldsymbol{x}$: $\boldsymbol{\Lambda}(\boldsymbol{x}) = 0$

quadratic equations $x_k = x_i^2$ or $x_k = x_i x_j$: $\boldsymbol{\Gamma}(\boldsymbol{x}) = 0$

} + BOUNDING BOX

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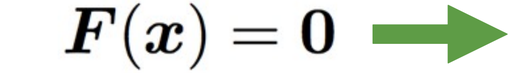

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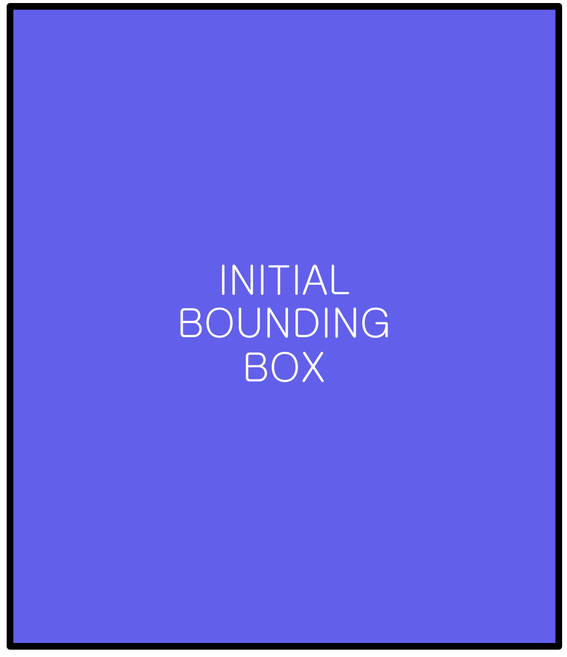

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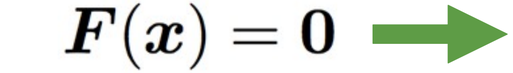

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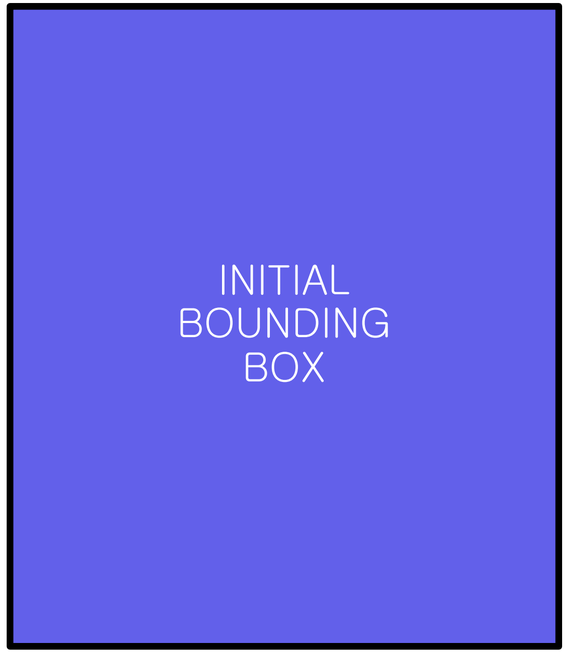

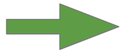

BOX SHRINKING:

BOX SPLITTING:

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INITIAL BOUNDING BOX

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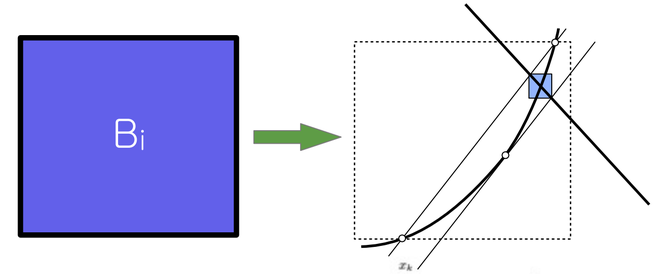

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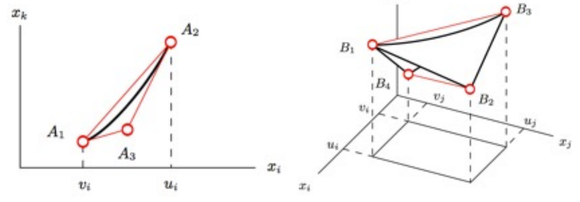

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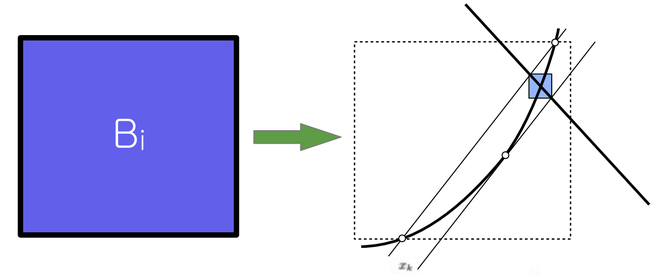
quadratic equations $x_k = x_i^2$ or $x_k = x_i x_j$: $\boldsymbol{\Gamma}(\boldsymbol{x}) = 0$

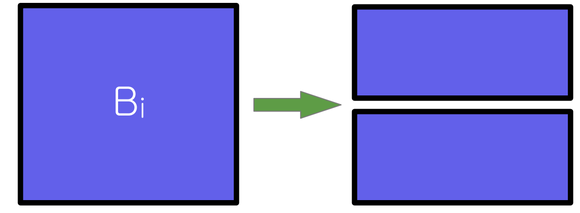
} + BOUNDING BOX

INITIAL BOUNDING BOX

BOX SHRINKING:



BOX SPLITTING:



NUMERICAL COMPUTATION OF MANIPULATOR SINGULARITIES

NUMERICAL COMPUTATION

BRANCH AND PRUNE BASED ON LINEAR RELAXATIONS

$$\boldsymbol{F}(\boldsymbol{x}) = \boldsymbol{0}$$

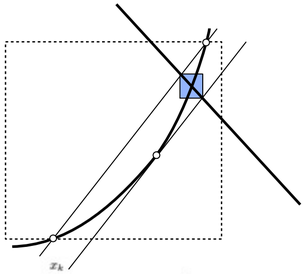
linear equations in $\boldsymbol{x}$: $\boldsymbol{\Lambda}(\boldsymbol{x}) = 0$

quadratic equations $x_k = x_i^2$ or $x_k = x_i x_j$: $\boldsymbol{\Gamma}(\boldsymbol{x}) = 0$

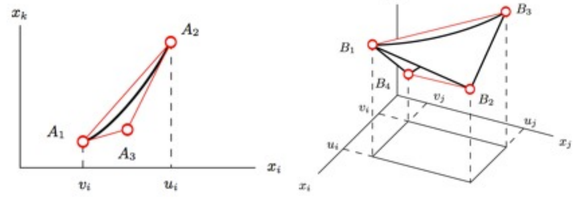
} + BOUNDING BOX

INITIAL BOUNDING BOX

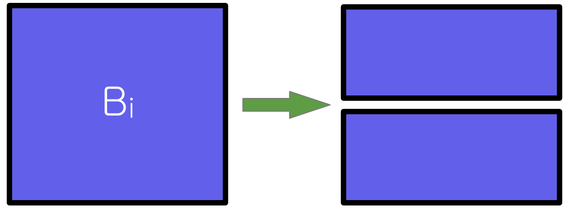
BOX SHRINKING: B_i



σ



BOX SPLITTING: B_i



NUMERICAL COMPUTATION OF MANIPULATOR SINGULARITIES

NUMERICAL COMPUTATION

BRANCH AND PRUNE BASED ON LINEAR RELAXATIONS

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linear equations in $\boldsymbol{x}$: $\boldsymbol{\Lambda}(\boldsymbol{x}) = 0$

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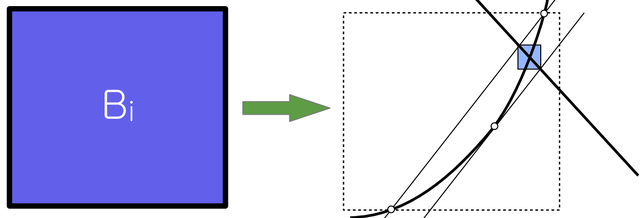
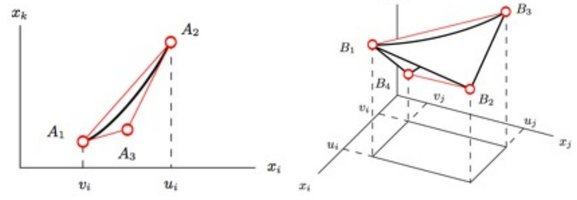
} + BOUNDING BOX

INITIAL BOUNDING BOX

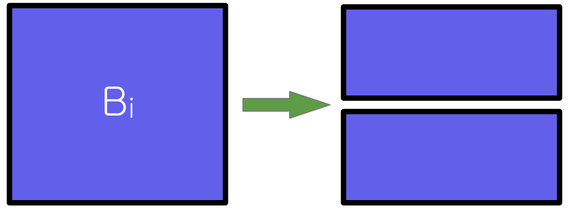
BOX SHRINKING:

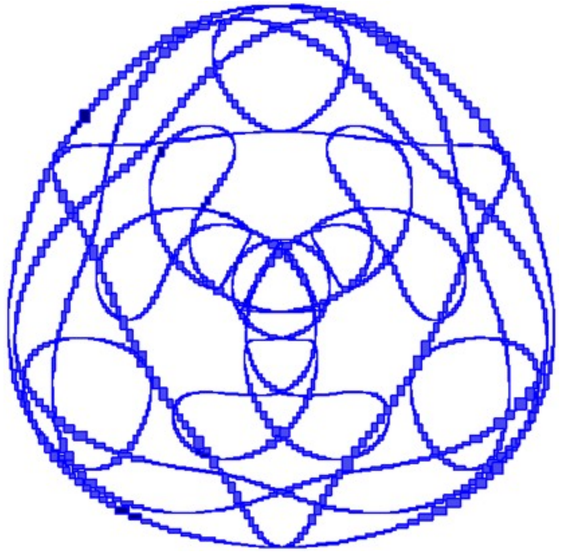


σ



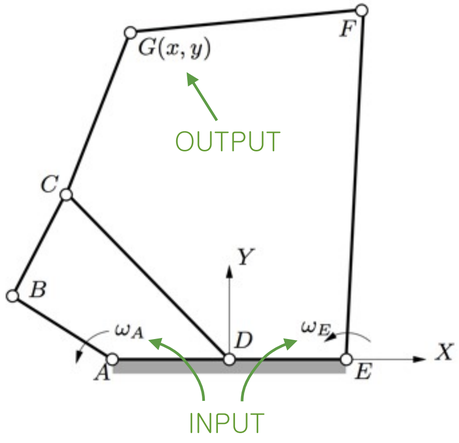
BOX SPLITTING:





NUMERICAL COMPUTATION OF MANIPULATOR SINGULARITIES

AN ILLUSTRATIVE EXAMPLE



$$
\begin{aligned}
0 &= -x + 2\cos\theta_D + \tfrac{3}{2}\cos\theta_C,\\
0 &= -y + 2\sin\theta_D + \tfrac{3}{2}\sin\theta_C,\\
0 &= \cos\theta_A + \cos\theta_B - 2\cos\theta_D - 1,\\
0 &= \sin\theta_A + \sin\theta_B - 2\sin\theta_D,\\
0 &= 2\cos\theta_D + \tfrac{3}{2}\cos\theta_C + 2\cos\theta_G - 3\cos\theta_E - 1,\\
0 &= 2\sin\theta_D + \tfrac{3}{2}\sin\theta_C + 2\sin\theta_G - 3\sin\theta_E,
\end{aligned}
$$

RI–IO

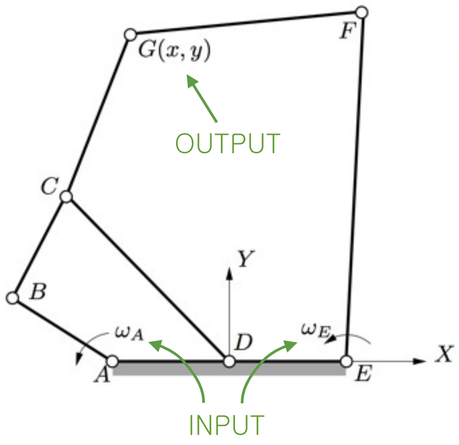
RO–II

RPM–II–IO

THERE ARE NO IIM

NUMERICAL COMPUTATION OF MANIPULATOR SINGULARITIES

AN ILLUSTRATIVE EXAMPLE



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\end{aligned}
$$

RI–IO

RO–II

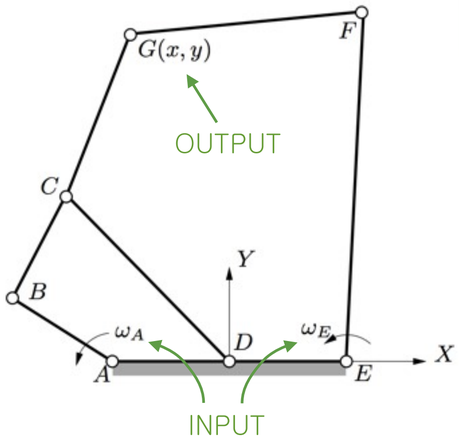
RPM–II–IO

THERE ARE NO IIM

PROJECTIONS OF THE CONFIGURATION SPACE AND ITS SINGULARITIES

NUMERICAL COMPUTATION OF MANIPULATOR SINGULARITIES

AN ILLUSTRATIVE EXAMPLE



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0 &= 2\sin\theta_D + \tfrac{3}{2}\sin\theta_C + 2\sin\theta_G - 3\sin\theta_E,
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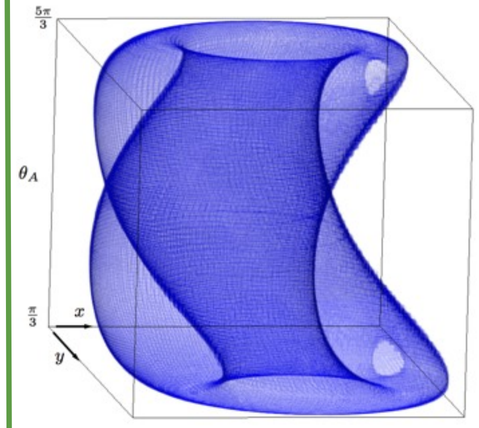
RI–IO

RO–II

RPM–II–IO

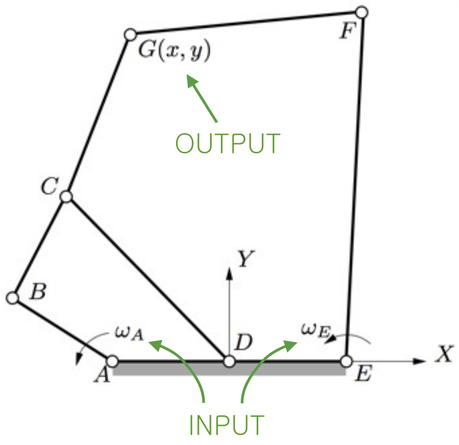
THERE ARE NO IIM

PROJECTIONS OF THE CONFIGURATION SPACE AND ITS SINGULARITIES



NUMERICAL COMPUTATION OF MANIPULATOR SINGULARITIES

AN ILLUSTRATIVE EXAMPLE



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\end{aligned}
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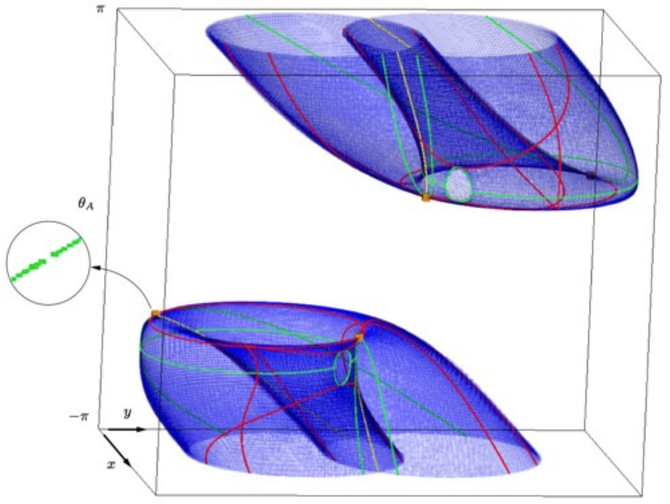
RI–IO

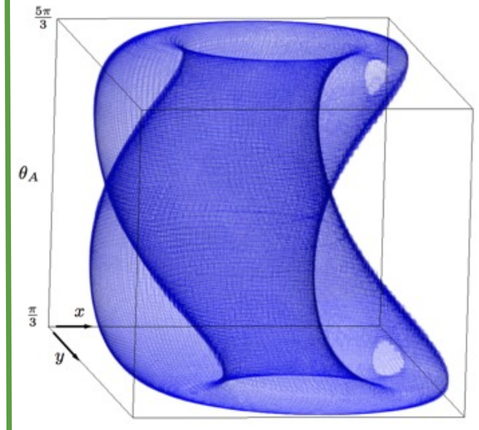
RO–II

RPM–II–IO

THERE ARE NO IIM

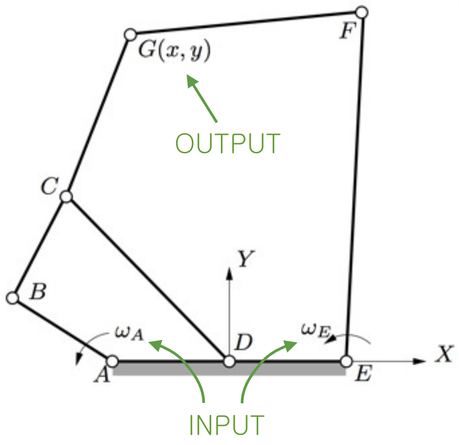
PROJECTIONS OF THE CONFIGURATION SPACE AND ITS SINGULARITIES





NUMERICAL COMPUTATION OF MANIPULATOR SINGULARITIES

AN ILLUSTRATIVE EXAMPLE



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RI–IO

RO–II

RPM–II–IO

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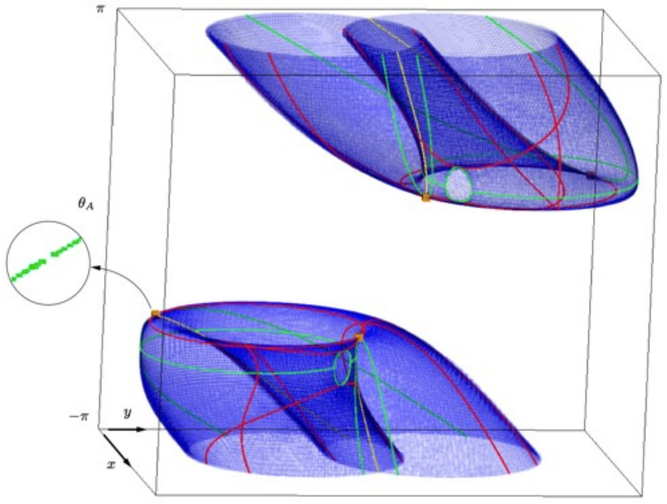
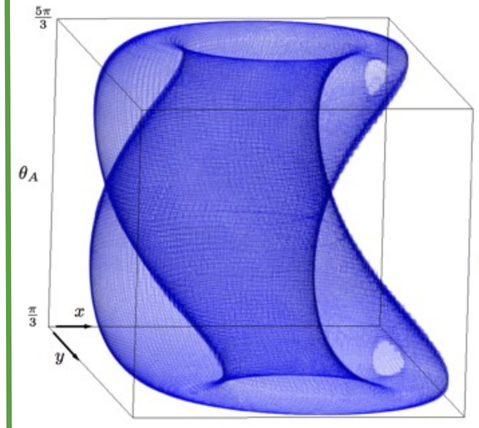
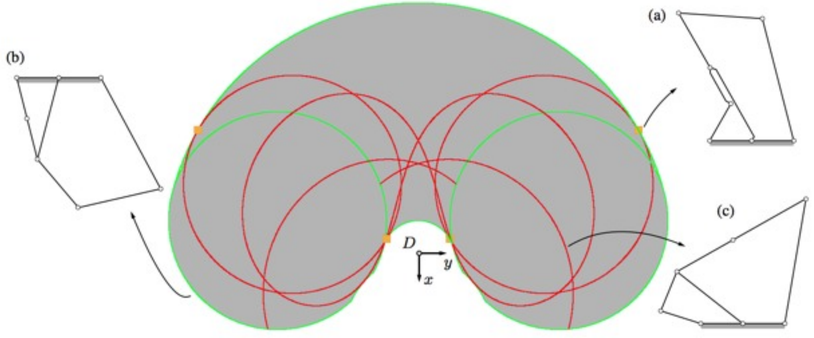
PROJECTIONS OF THE CONFIGURATION SPACE AND ITS SINGULARITIES



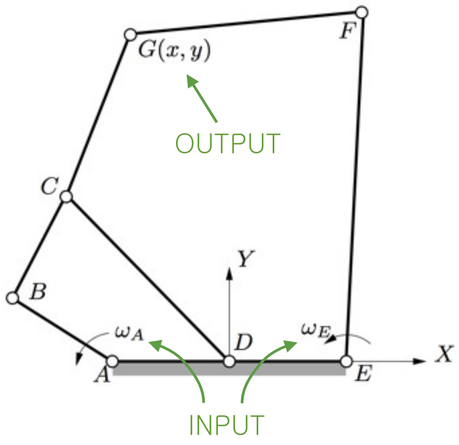




a) RPM–IO–II b) RI–IO c) RO–II

NUMERICAL COMPUTATION OF MANIPULATOR SINGULARITIES

AN ILLUSTRATIVE EXAMPLE



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RI–IO

RO–II

RPM–II–IO

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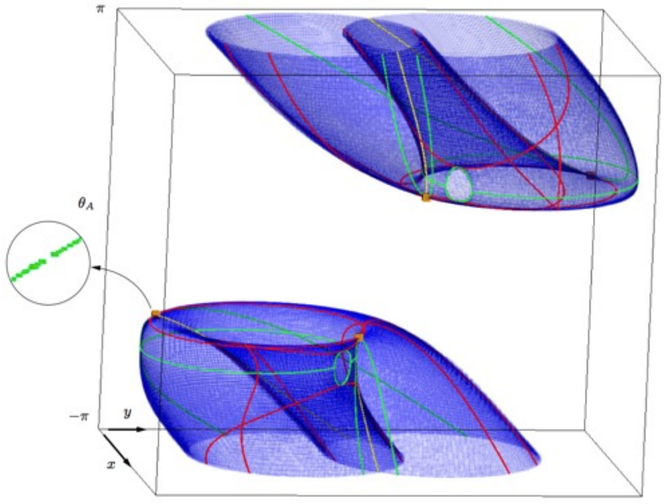
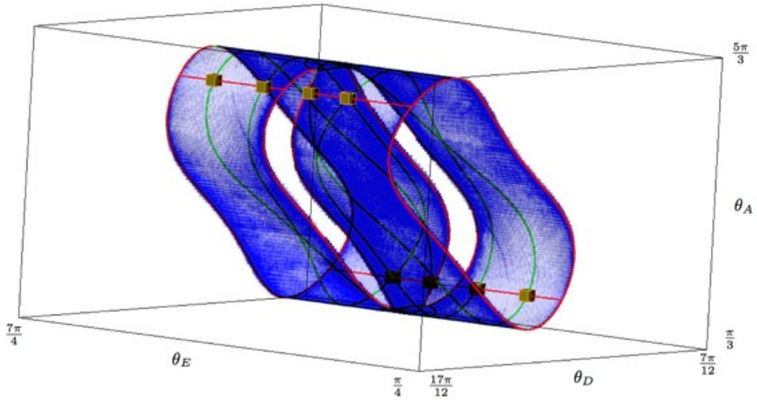
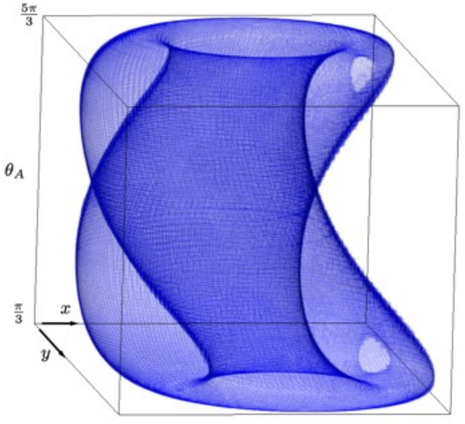
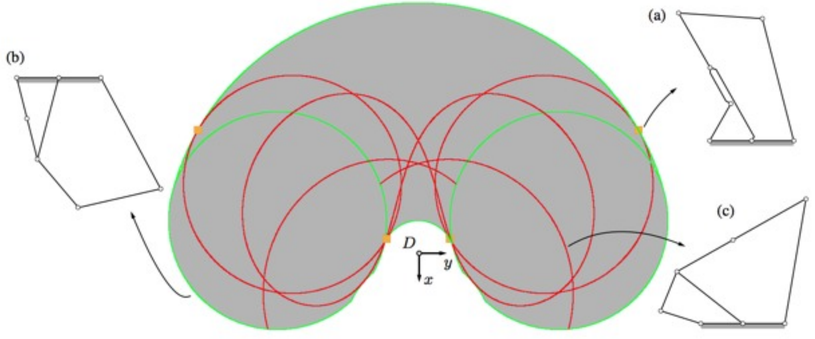
PROJECTIONS OF THE CONFIGURATION SPACE AND ITS SINGULARITIES





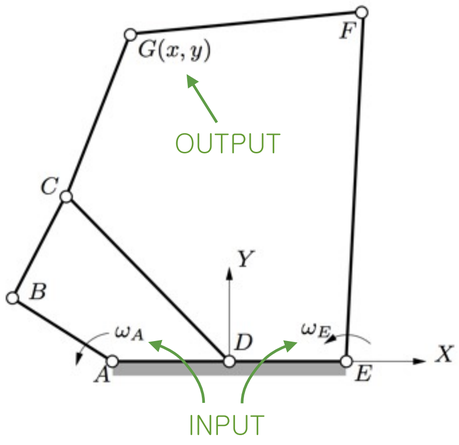




a) RPM–IO–II b) RI–IO c) RO–II

NUMERICAL COMPUTATION OF MANIPULATOR SINGULARITIES

AN ILLUSTRATIVE EXAMPLE



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\end{aligned}
$$

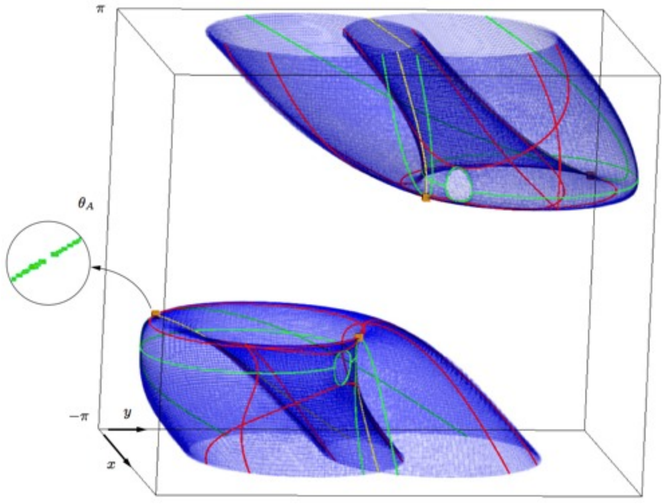
RI–IO

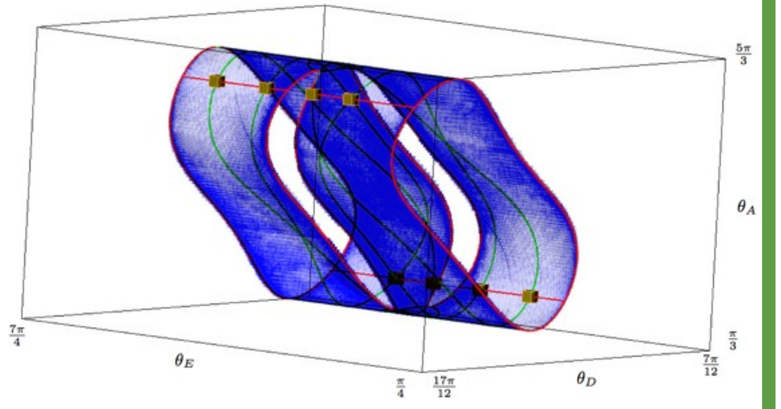
RO–II

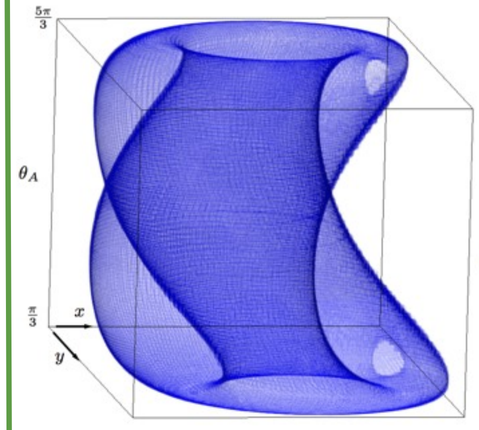
RPM–II–IO

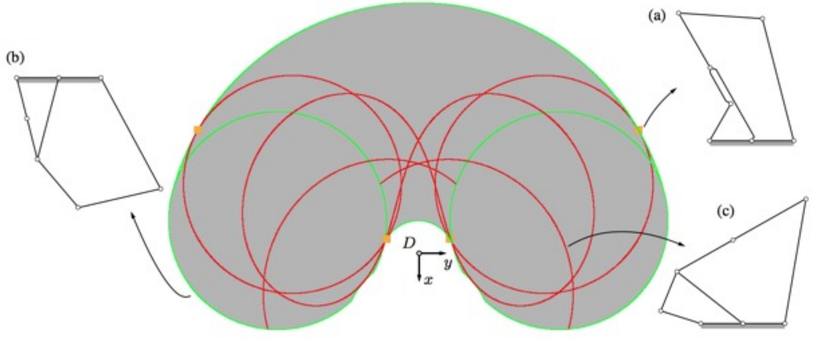
THERE ARE NO IIM

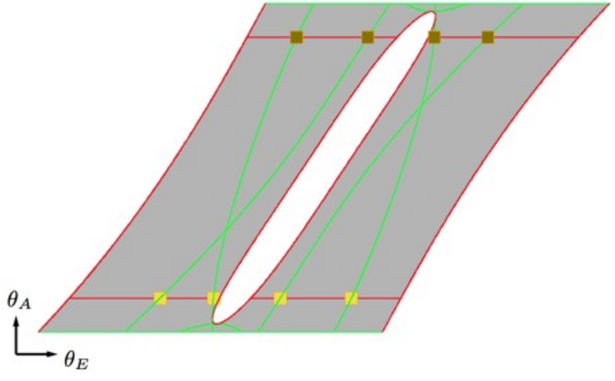
PROJECTIONS OF THE CONFIGURATION SPACE AND ITS SINGULARITIES











a) RPM–IO–II b) RI–IO c) RO–II