A unified method for computing position and orientation workspaces of general Stewart platforms

Oriol Bohigas, Lluís Ros and Montserrat Manubens

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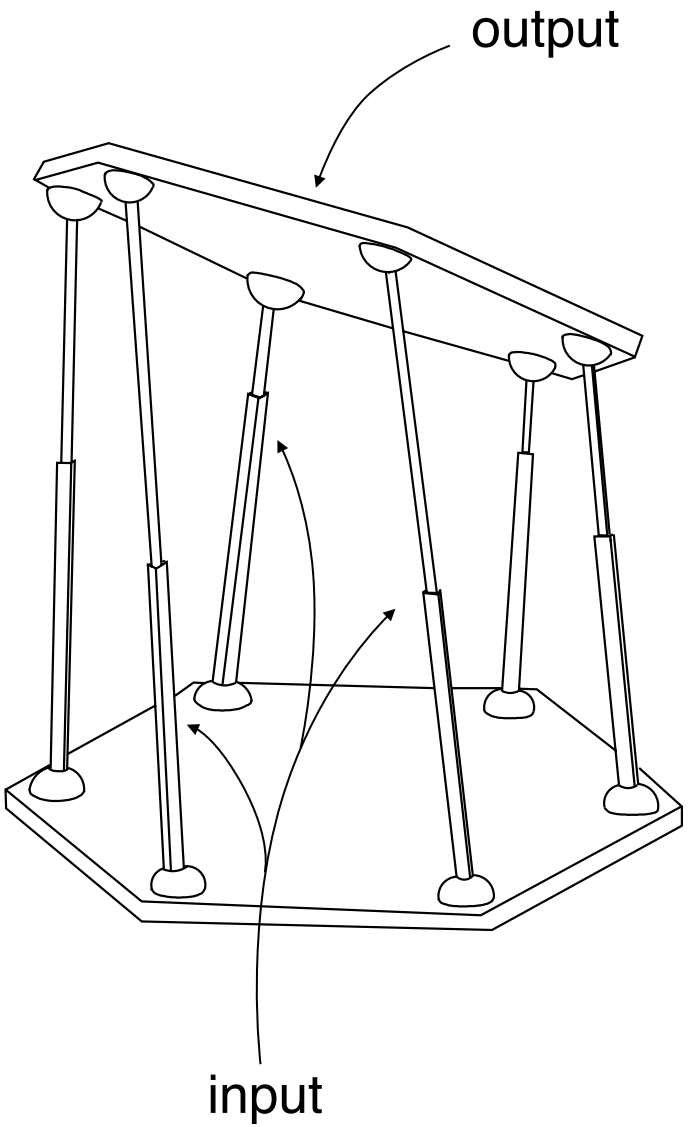


6 DIMENSIONAL WORKSPACE

OUTLINE

1. INTRODUCTION
2. FORMULATION
3. BOUNDARY ISOLATION
4. EXAMPLES
5. CONCLUSION

OUTLINE

1. INTRODUCTION

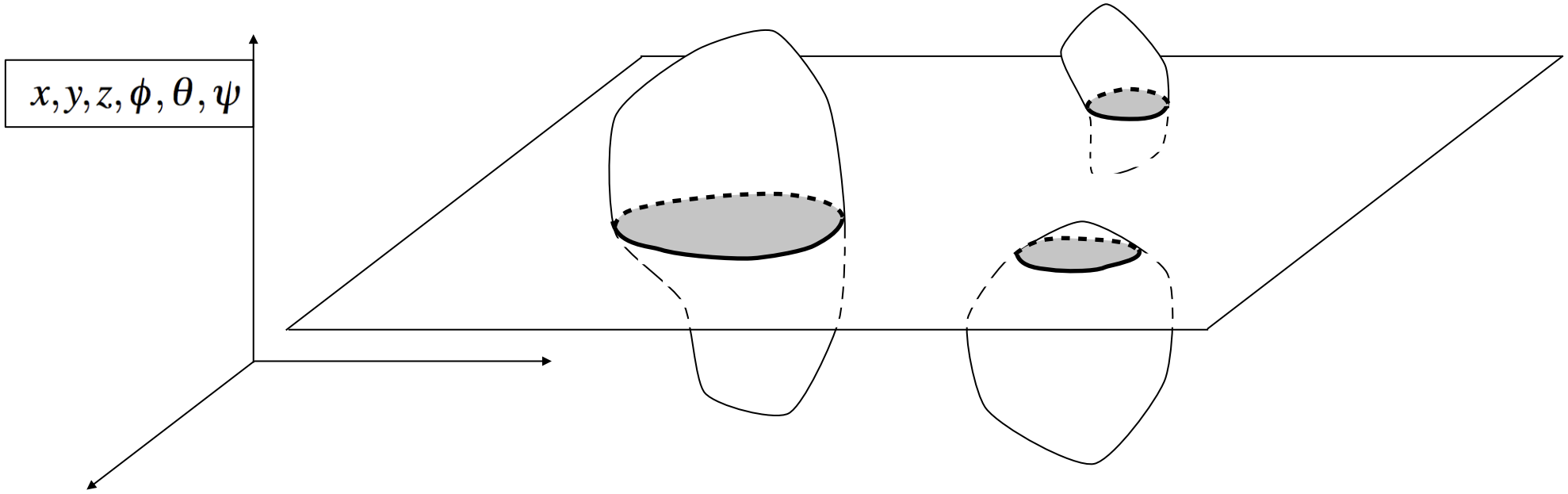
2. FORMULATION

3. BOUNDARY ISOLATION

4. EXAMPLES

5. CONCLUSION

INTRODUCTION



CONSTANT ORIENTATION

Gosselin, 1990

Merlet, 1992

Merlet et al., 1999

CONSTANT POSITION

Merlet, 1995

Bonev & Ryu, 2001

Pernkopf & Husty, 2006

Jiang & Gosselin, 2009

Haug et al., 1996

20 POSSIBLE SLICES !

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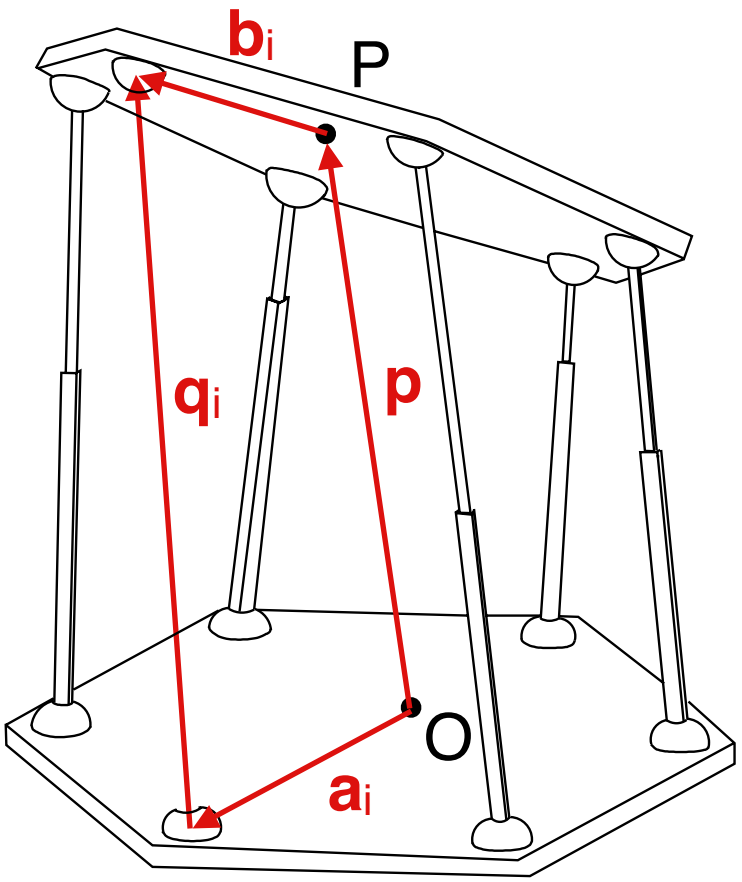
WORKSPACE EQUATIONS



$$L_i^2 = |\boldsymbol{q}_i|^2 = |\boldsymbol{p} + \boldsymbol{R}\boldsymbol{b}_i - \boldsymbol{a}_i|^2$$

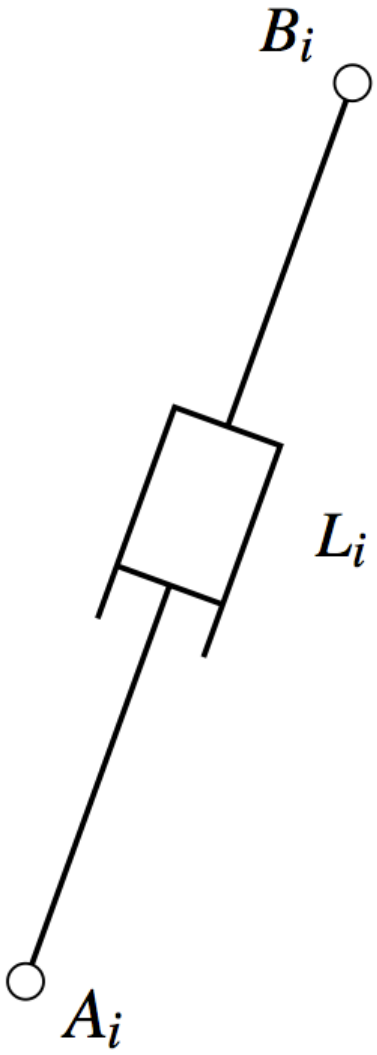
$$\boldsymbol{R} = \boldsymbol{R}_Z(\psi)\boldsymbol{R}_Y(\theta)\boldsymbol{R}_X(\phi)$$

Euler angles
Tilt - and - torsion
Euler parameters

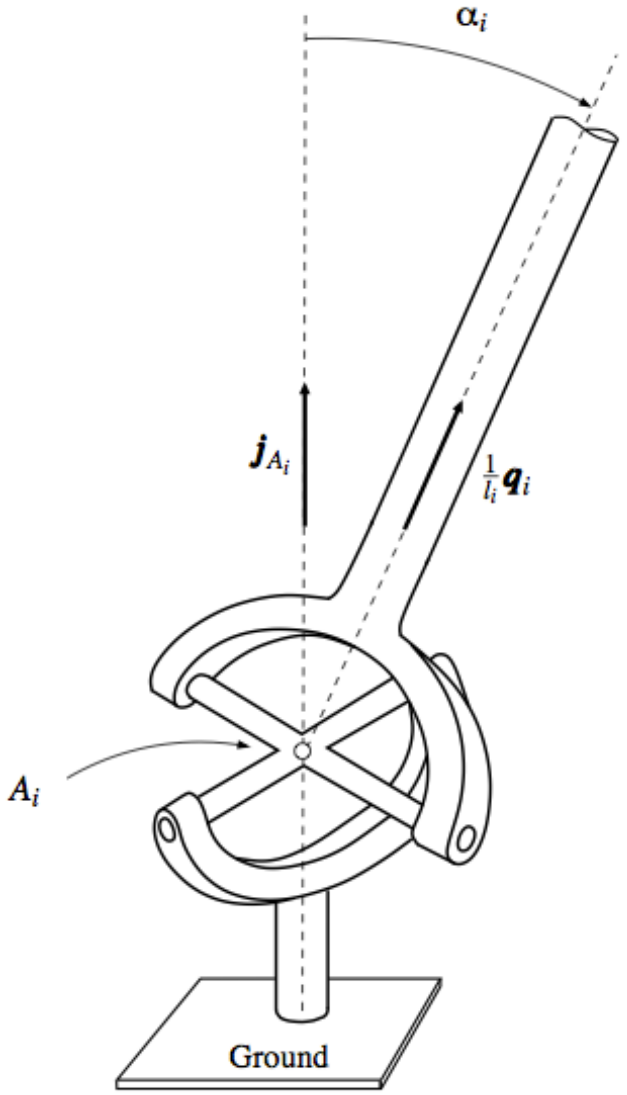
$$(x, y, z, \phi, \theta, \psi)$$

WORKSPACE EQUATIONS

ACTIVE JOINTS



PASSIVE JOINTS

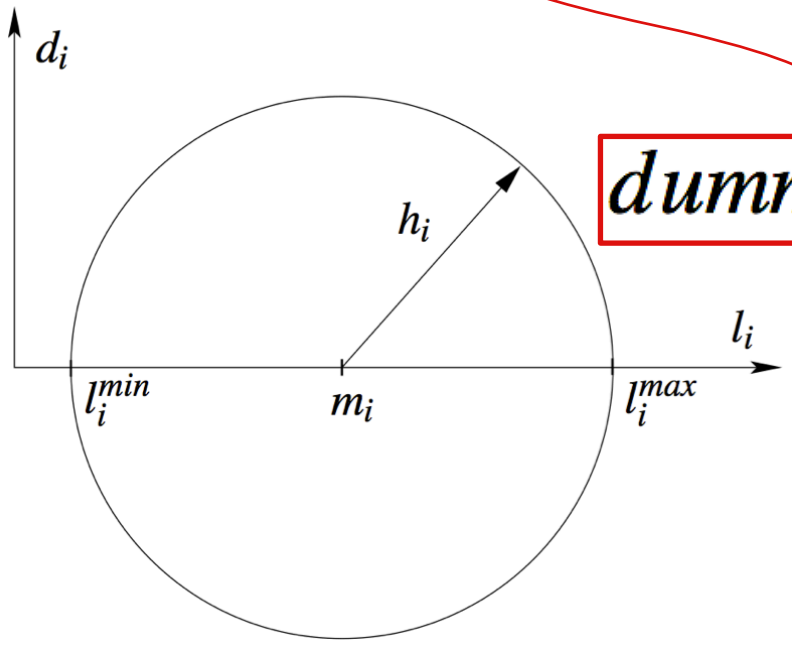


WORKSPACE EQUATIONS

ACTIVE JOINTS

$$L_i \in \left[L_i^{min}, L_i^{max}\right]$$

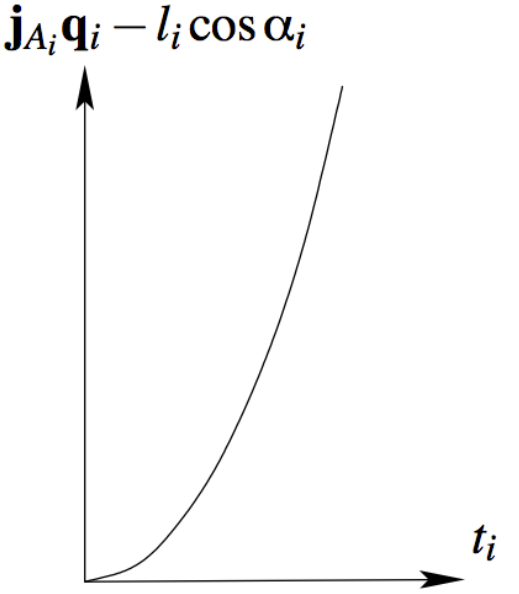
$$(l_i - m_i)^2 + d_i^2 = h_i^2$$



PASSIVE JOINTS

$$\boldsymbol{j}_{A_i}\boldsymbol{q}_i \geq l_i \cos\alpha_i$$

$$\boldsymbol{j}_{A_i}\boldsymbol{q}_i - l_i \cos\alpha_i = t_i^2$$



dummy variables

BOUNDARY EQUATIONS

$$\prod_{i=1}^{6}(dummy\ \ variables) = 0$$

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BOUNDARY ISOLATION

NUMERICAL METHOD BASED ON LINEAR RELAXATIONS

QUADRATIC FORM

INITIAL BOUNDING BOX

BOUNDARY ISOLATION

QUADRATIC FORM

$$q_i \qquad q_i q_j \qquad q_i^2$$

$$c_\tau = \cos\tau$$
$$s_\tau = \sin\tau$$
$$c_\tau^2 + s_\tau^2 = 1$$

$$p_k = y_i^2$$
$$w_k = y_i y_j$$

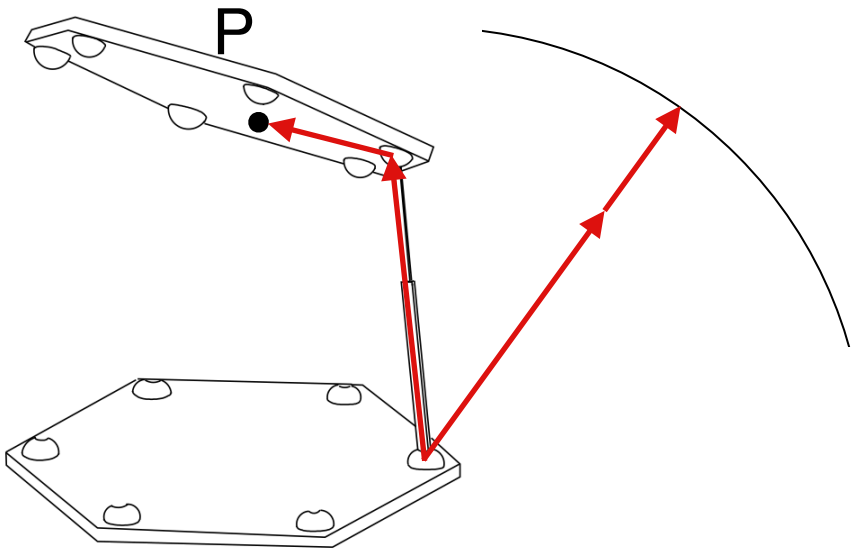
BOUNDARY ISOLATION

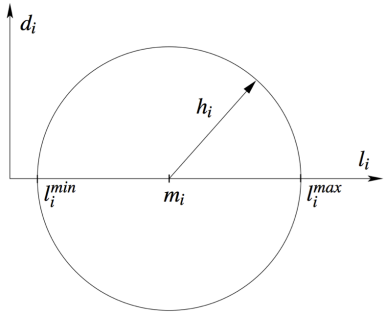
INITIAL BOUNDING BOX

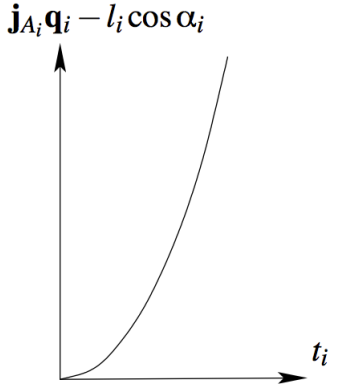
$$\boldsymbol{R}, c_\tau, s_\tau \longrightarrow [-1, 1]$$

$$x, y, z \longrightarrow [x^{max}, x^{min}]$$

$$dummy\ variables \longrightarrow [d^{max}, d^{min}]$$

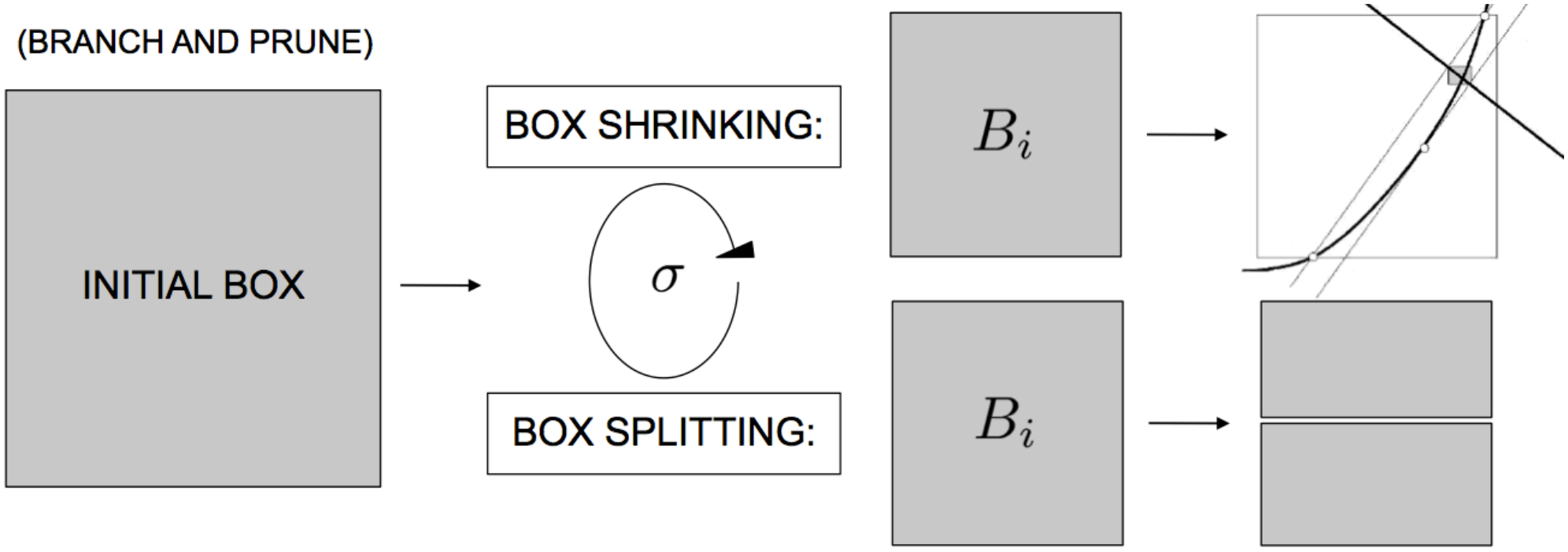






BOUNDARY ISOLATION

NUMERICAL SOLUTION



(Porta et al., 2009)

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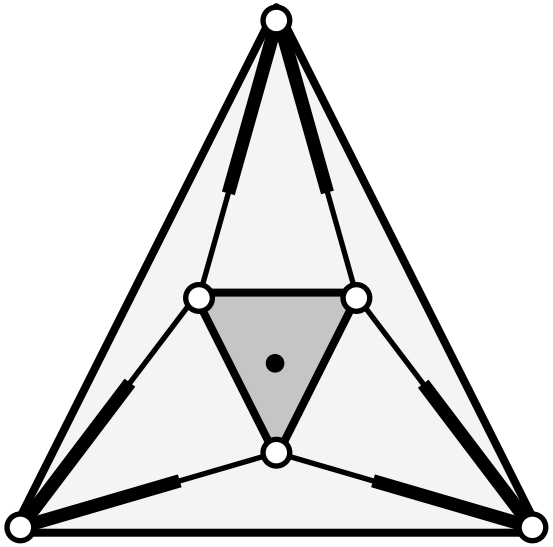
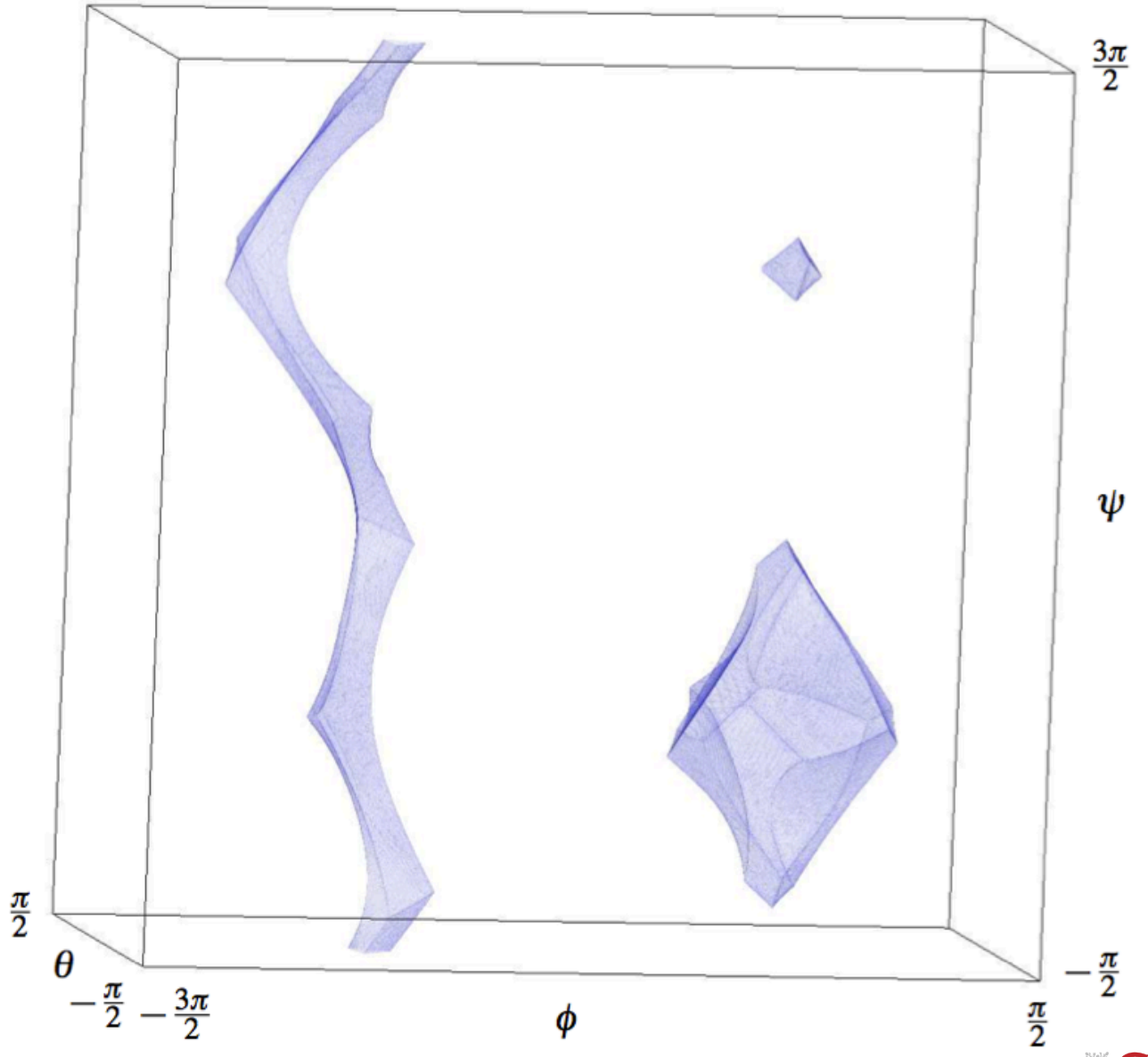
EXAMPLES

STANDARD PLATFORM



CONSTANT POSITION SLICE (ϕ, θ, ψ)



EXAMPLES

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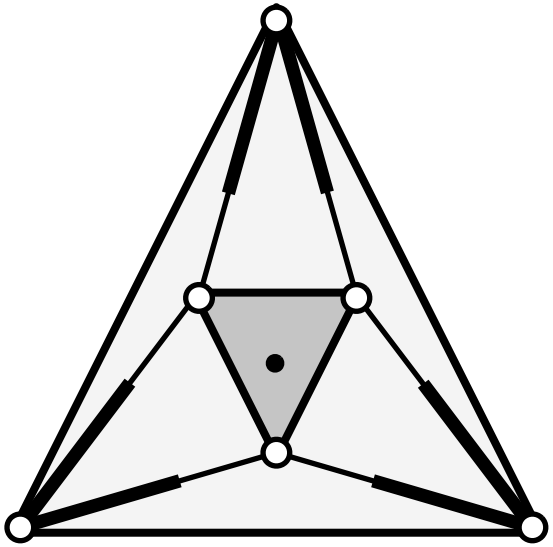

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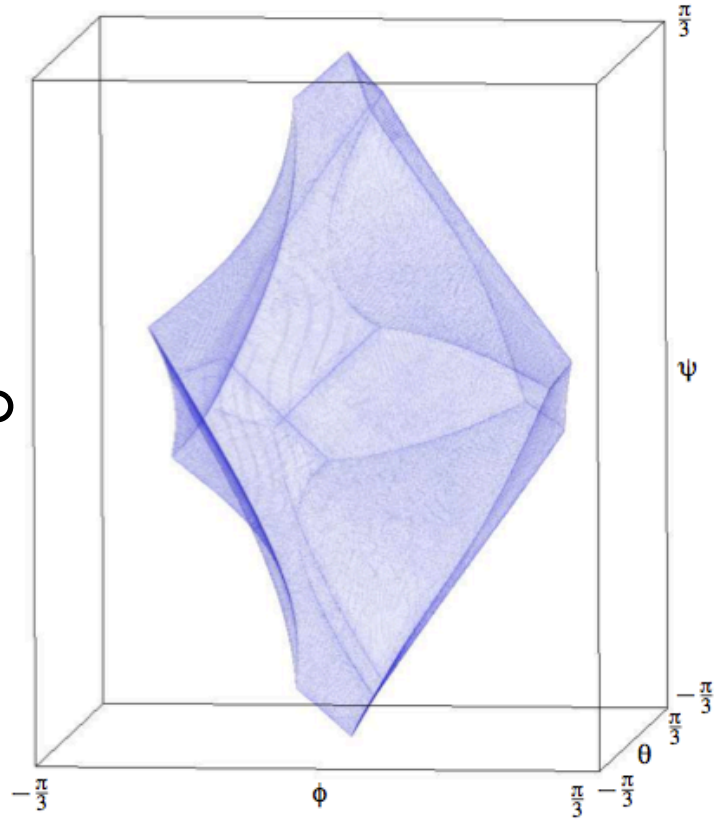
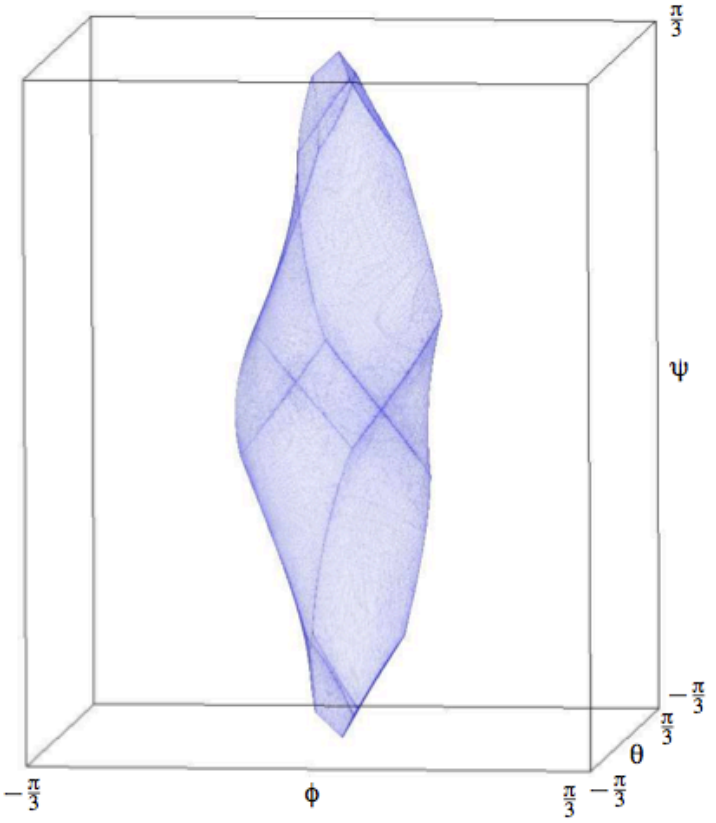
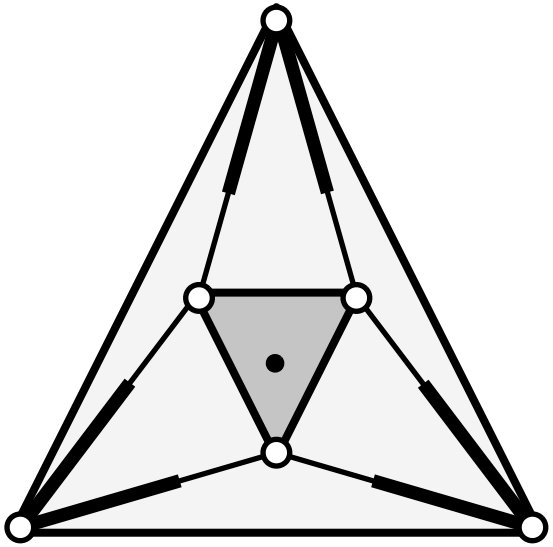


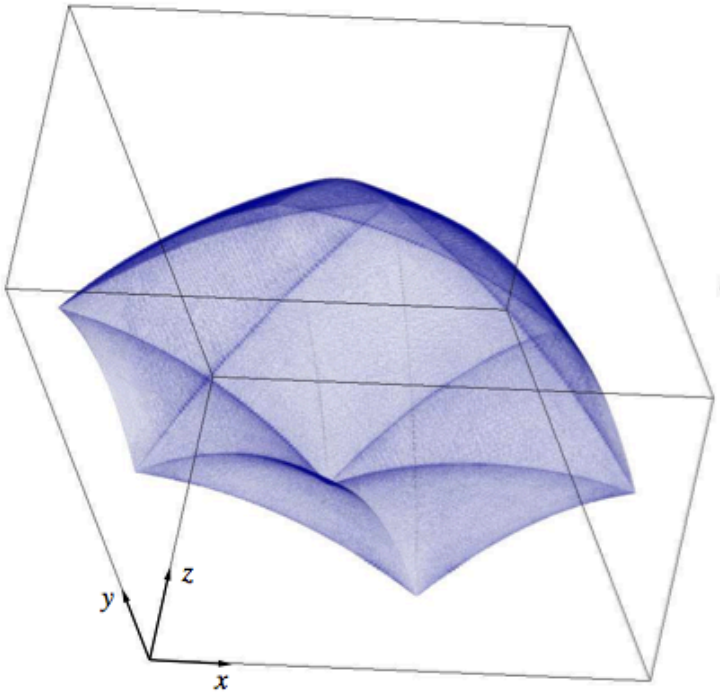


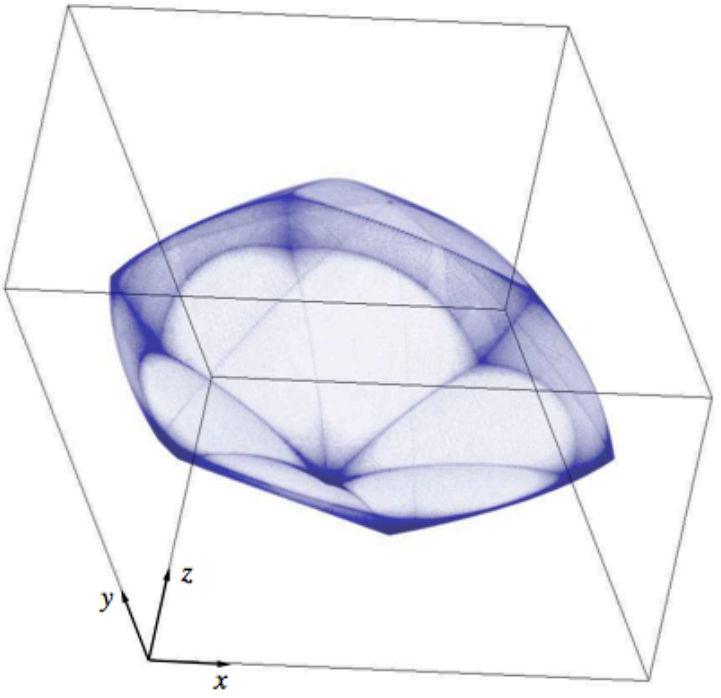
EXAMPLES

STANDARD PLATFORM



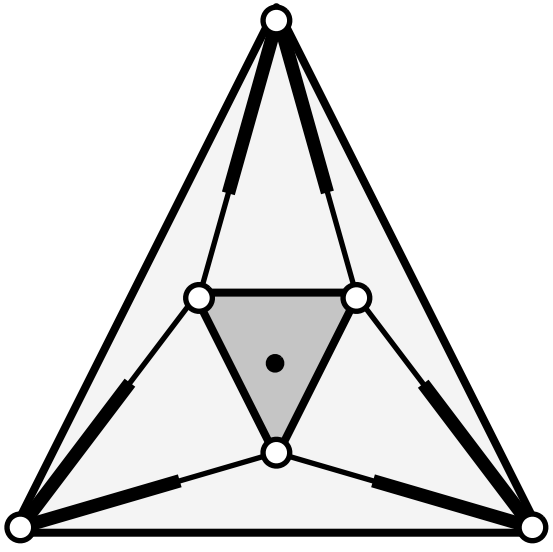
CONSTANT ORIENTATION SLICE (x, y, z)



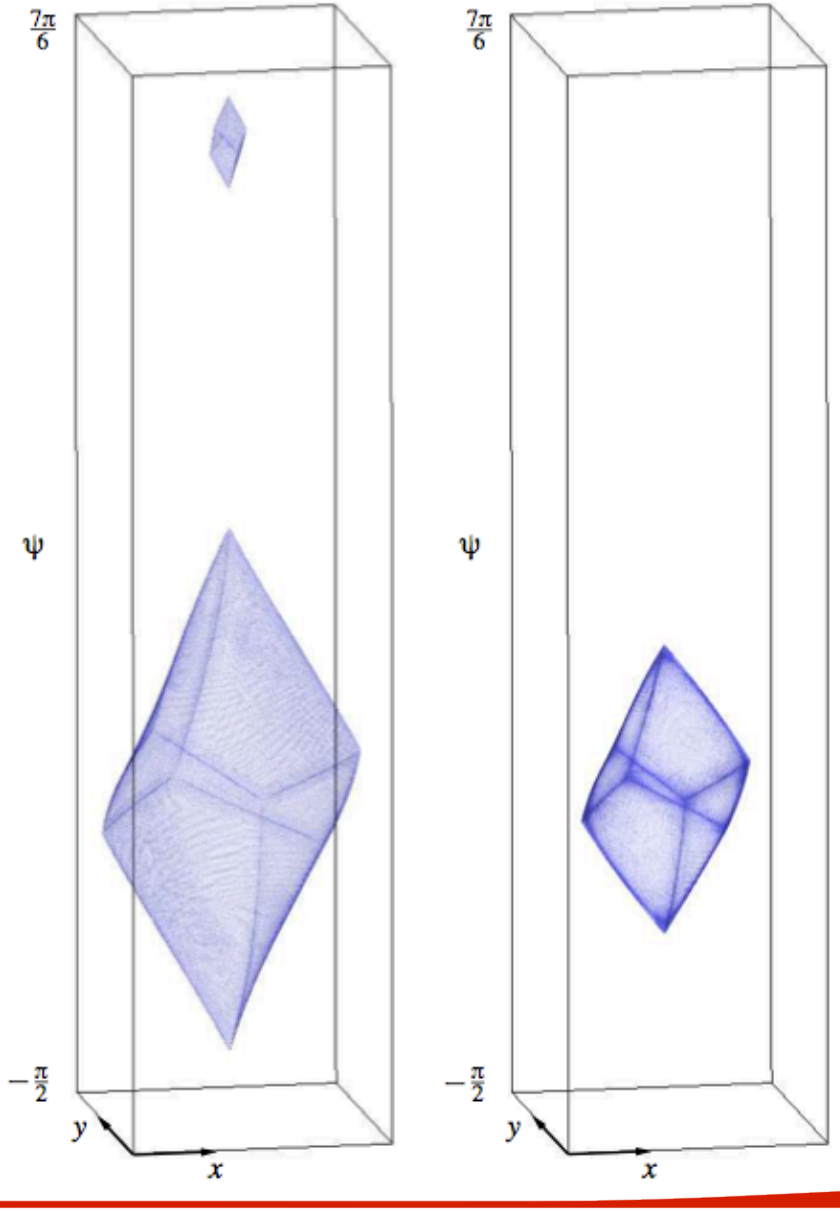


EXAMPLES

STANDARD PLATFORM

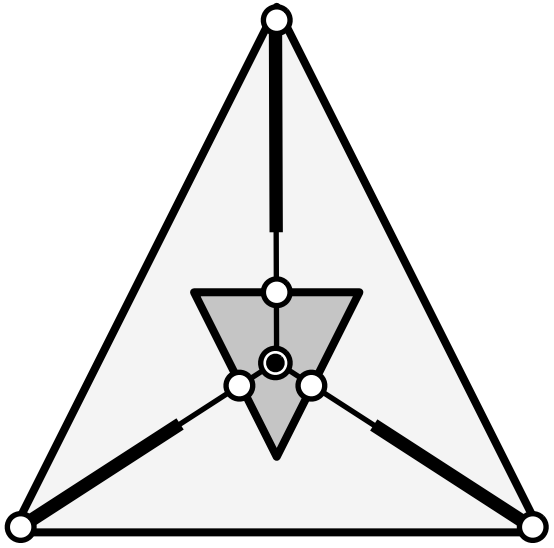


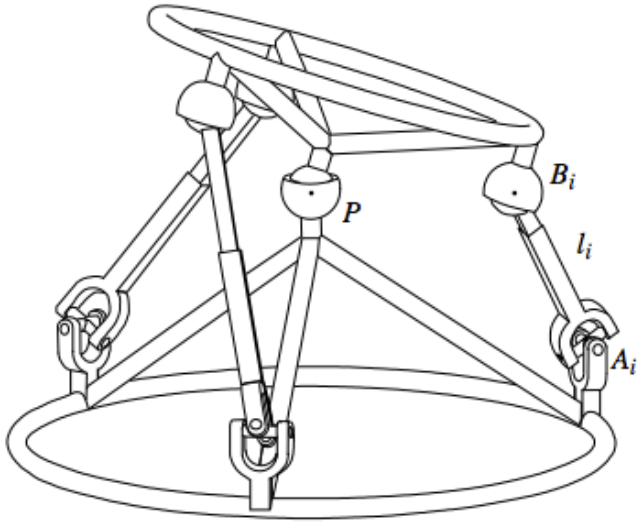
PLANAR MODE SLICE (x, y, ψ)



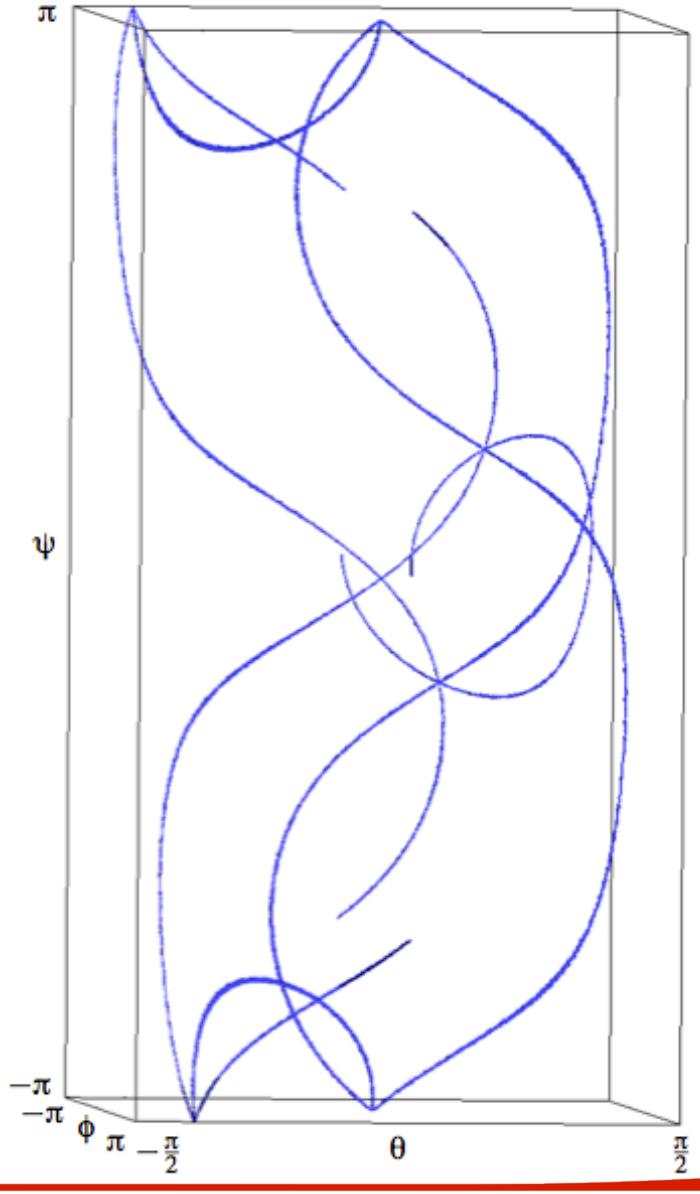
EXAMPLES

SPECIAL PLATFORM





CONSTANT POSITION SLICE (ϕ, θ, ψ)



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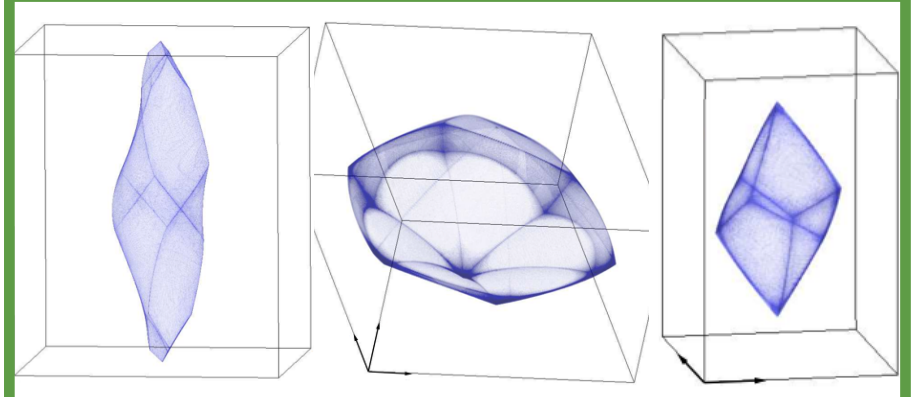
CONCLUSION

NEW APPROACH FOR COMPUTING WORKSPACES OF STEWART PLATFORMS

UNIFIED



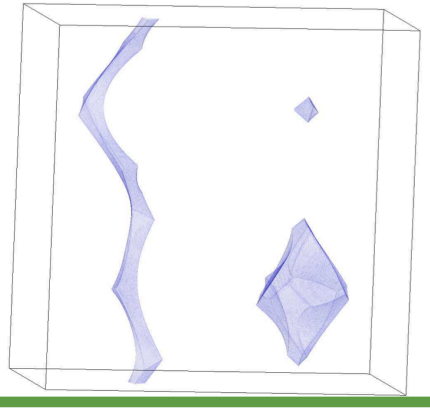
GENERAL

Arbitrary geometry

Active and passive joint limits

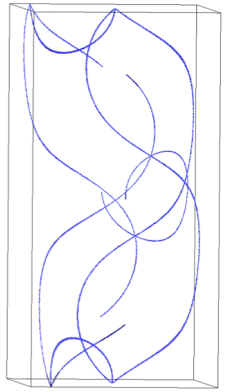
COMPLETE

Several connected components



ACCURATE

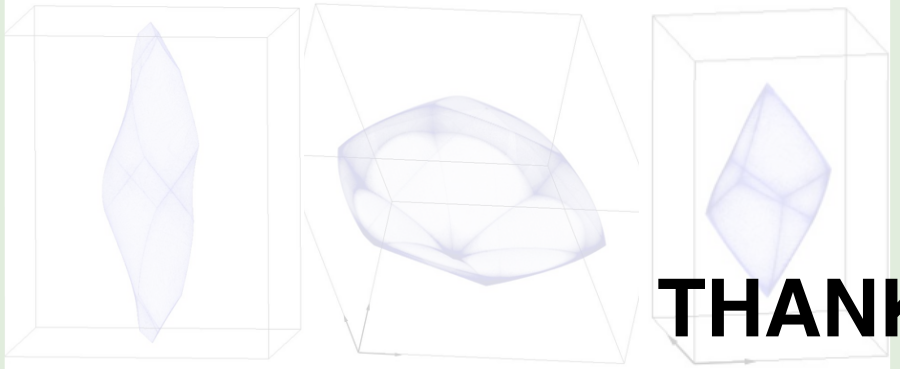
Interior barriers



CONCLUSION

NEW APPROACH FOR COMPUTING WORKSPACES OF STEWART PLATFORMS

UNIFIED



GENERAL

Arbitrary geometry

Active and passive joint limits

THANK YOU!

COMPLETE

Several connected components

ACCURATE

Interior barriers