Stochastic Analysis on the Energy Constraint of V2G Frequency Regulation

Sekyung Han[1], Soo Hee Han[2], Kaoru Sezaki[3]

[1] Institute of Industrial Science, The University of Tokyo, Tokyo, Japan (arnold@mcl.iis.u-tokyo.ac.jp)
[2] Department of Electrical Engineering, Konkuk University, Seoul, Korea (shhan@konkuk.ac.kr)
[3] Center of Spatial Information Science, The University of Tokyo, Tokyo, Japan (sezaki@iis.u-tokyo.ac.jp)

Abstract—**Energy constraint for V2G frequency regulation is illustrated in terms of state-of-charge (SOC) of the pertaining vehicle battery. Actual regulation signal is investigated, and energy deviation caused by a single regulation signal is obtained. With the derived energy deviation model, a probability distribution of successful regulation is estimated. Random walk theory is employed for stochastic analysis of the distribution. For the derived probability distribution, an approximation to the normal distribution is made to perform practical calculation on a digital computer. Estimated probability distribution is averaged over the unit contract time, usually an hour, to yield a weight function that represents the energy constraint. Finally, simulations are provided to with various parameters.**

Keywords-component; V2G; Regulation; Battery; Optimal Control; Smart Grid

I. INTRODUCTION

Success of the hybrid electric vehicles (HEVs) and global warming crisis are accelerating the propagation of the electric vehicles (EVs). Especially, plug-in type electric vehicles such as plug-in hybrid electric vehicle (PHEV) charge the battery directly from power grid at slow rate, and thus the vehicles keep plugged-in for most of the parking time. To utilize the huge energy storages for grid side benefit, several applications have been investigated [1-3]. In particular, frequency regulation is considered as one of the most promising and practical V2G service [4].

In the previous work of the authors, an aggregator was developed to perform an optimal control of each vehicle for the V2G frequency regulation [5]. During the development, they deployed a weight function to reflect an energy constraint caused by lack of the energy capacity of the battery. Although energy deviation caused by the regulation is zero-mean in the long term, the regulation could still be restricted depending on the current state-of-charge (SOC) in short-term. For instance, a vehicle with fully charged battery cannot accommodate regulation down at all since it requires charging of the battery. If SOC is 90%, the battery can accommodate both of the regulation up and down, but regulation may soon be incapable due to the small margin to full-charge. If the battery has more margin than a certain value, however, regulation would not be restricted at all.

In [5], the energy constraint was intuitively devised as a linearly decreasing weight function after a marginal point. The marginal point was employed to distinguish the boundary region of SOC where the energy constraint exists from the energy constraint free region in the middle range. Fig. 1 depicts the weight functions for regulation up and down, respectively. The marginal point, however, was left to be obtained experimentally. Moreover, the validity of the weight function was not thoroughly investigated.

In this paper, we derive the weight functions mathematical y to reflect the energy constraint more precisely. Specifically, a physical impact of the energy capacity is investigated quantitatively and is represented with respect to SOC through a stochastic analysis. In section 2, real-world regulation signal is analyzed to build an energy deviation model. Using the result, probability of successful regulation is estimated in the following section. During the estimation, random walk theory is employed, and the probability is approximated to the normal distribution for the calculation. Finally, simulations are provided with various parameter configurations in section 4.

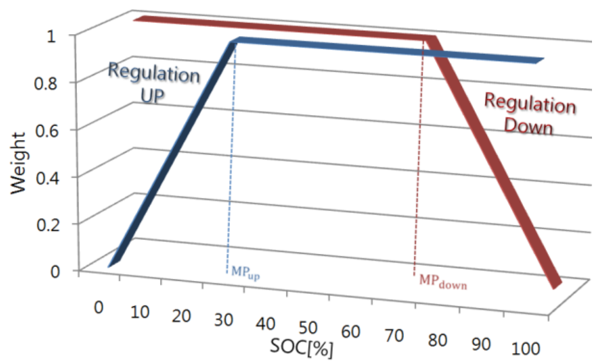

Figure 1. Conventional weight functions representing the energy constraint.

II. MODELING OF REGULATION SIGNAL DISTRIBUTION

Regulation signal is calculated and dispatched by a grid operator's computer system (usually referred to as the Energy Management System, or EMS). As shown in Fig. 2, the nature of regulation is that the fluctuations of power changes between positive and negative, which correspond to discharging and charging respectively, are fairly frequent to avoid a large cumulative energy deviation [2]. However, there are cases when the demands from the EMS may require extended periods at the upper or lower limit of the contracted regulation

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power. Fig. 2 depicts a statistical distribution of regulation signals from PJM. The x-axis represents ratio of actually requested power capacity to the contracted size, while the y-axis is normalized call rate. At a glance, the pattern of Fig. 2 seems as if they follow typical curve of the normal distribution. Due to the intensive calls at both ends, however, it is difficult to represent the signal with conventional probability distributions. Nevertheless, the calls are evenly distributed between positive and negative, and thus it is possible to approximate the original distribution to the Bernoulli distribution with the success probability of 0.5 in terms of energy deviation. Consequently, Fig. 2 can be redrawn as Fig. 3. From the perspective of energy capacity, the converted signal is identical with the original one. In the following section, this converted distribution is used to build a stochastic model of the probability distribution of successfully carrying out the regulation.

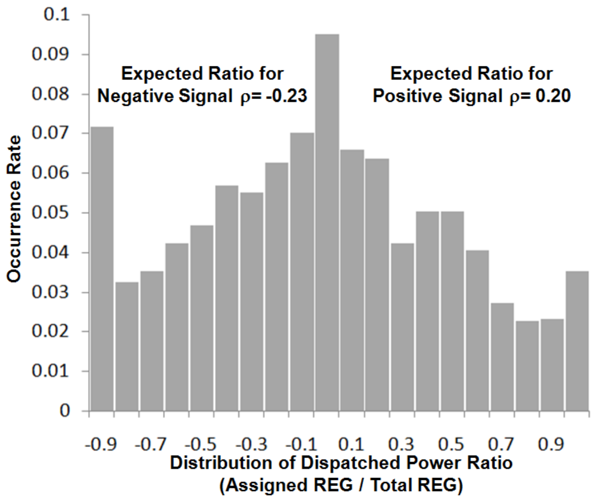

Figure 2. Distribution for the ratio of actually dispatched power to the contracted power capacity.
(Sampled data during June 01-06, 2007 for every 5 minutes from PJM)

III. ENERGY CONSTRAINT FORMULATION

The energy capacity of a battery can be described in terms of the battery state-of-charge (SOC). For each of regulation up and down, SOC represents totally opposite aspects. Regulation up signal results in discharge of the battery. Thus, the higher the SOC is, the more the battery can accommodate regulation up signal, and vice versa for regulation down.

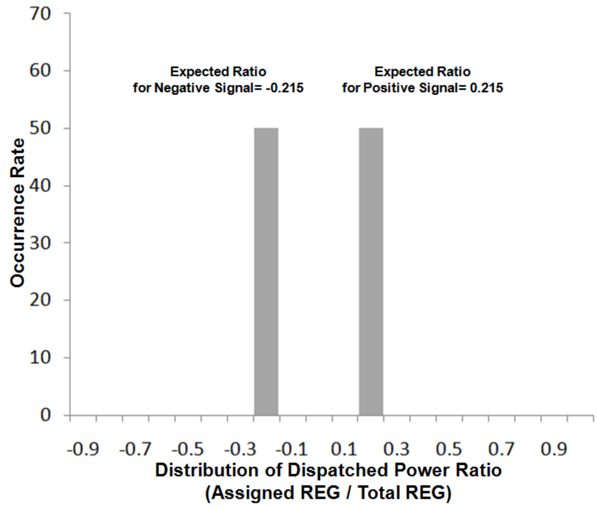

Figure 3. Approximated Bernoulli distribution of Fig. 2. Note that the scale of y-axis is different from Fig. 2 and the expected ratio is set to eqaul for both positive and negative signal.

If we assume that aggregator allocates the given regulation to the vehicles in proportion to the power capacity of each vehicle, Fig. 3 still holds for individual vehicle because it merely indicates the dispatched ratio to the contracted power capacity. Considering that the regulation signal is periodically sent, energy deviation of a vehicle caused by each regulation signal can be described as follows:

$$\Delta E\,[kWh] = \rho \times INTV\,[h] \times PCAP\,[kW] \tag{1}$$

where ρ is utilization rate of the contracted power capacity which is depicted in Fig. 3. *INTV* is signal period from grid operator and *PCAP* is power capacity that the vehicle is in charge for each regulation call. *PCAP* is generally maximum power capacity of the vehicle battery. Note that we assumed symmetric power capacity between charge and discharge. Otherwise, (1) should be separated for regulation up and down with corresponding *PCAP*.

TABLE I. EXAMPLE CONDITIONS

Items	Value
ρ	0.215
INTV [h]	0.001 hour (= 3.6 sec)
PCAP [kW]	20 kW
Battery Size [kWh]	4.3 kWh

As an example, assume that a plug-in hybrid electric vehicle is performing V2G regulation under the conditions described in TABLE 1. The vehicle is expected to charge or discharge 0.0043kWh ($= 0.215 \times 0.001 \times 20$) for every regulation signal. Since the battery size (maximum energy capacity) of this vehicle is 4.3kWh, the maximum signal count that the vehicle can afford for the continuous limit signals is 1000 times. In other words, the minimum affordable time of the vehicle for the extreme case would be up to 1 hour (1000 times of signal =

3600 seconds = 1 hour). Although such long-term biased signal is unrealistic, the chance of failure for providing the regulation is certainly not zero. In most cases, SOC of the battery is in the somewhere around middle range, and thus it is more likely to have less energy capacity than the battery size. Typically, the affordable time of the regulation is directly related to the current SOC. Consequently, empty battery would have the maximum success probability for regulation down, and the least for regulation up. Likewise, a battery with 50% SOC can accommodate both of the regulation up and down with same success probability, but it is more likely to fail than the empty battery for regulation down.

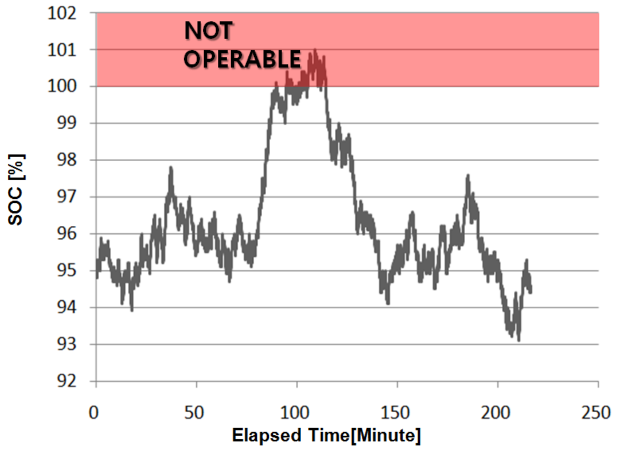

Figure 4. Trajectory of the SOC performing V2G regulation. Initial SOC is 95%, ΔE and battery size is 0.0043kWh and 43kWh respectively.

From the discussion so far, SOC deviation could be represented in terms of regulation signal. With evenly distributed regulation signals between up and down, we can employ the theory of random walk to illustrate the probability distribution of SOC trajectory. Fig. 4 shows the simulated trajectory of SOC with the initial value of 95% with the conditions described in TABLE 1. Even with the fairly distributed, short term (=3.6sec) regulation signals, it is highly likely to hit the upper limit due to the small margin for the regulation down. To address it in a quantitative manner, distribution of the success probability for regulation up and down should be estimated in terms of SOC. At any moment, regulation is affordable only when the battery has room for the expected amount of deviation in SOC. From the result of (1), the SOC deviation by a regulation signal can be calculated as:

$$\Delta SOC = \frac{100 \times \Delta E}{Battery\ Size} \tag{2}$$

Regulation is affordable only when $0 \le SOC + \Delta SOC \le 100$. From the random walk analysis, the probability that regulation down is manageable at n-th signal starting at a given SOC can be described as follows:

$$\mathrm{P_{RD}}(n, SOC) = \sum_{i=k}^{n} \frac{{}_nC_i}{2^n} \text{ for k>0, otherwise 1} \tag{3}$$

where $k = \left\lceil \dfrac{n - (100 - SOC)/\Delta SOC}{2} \right\rceil$.

Considering hourly regulation market, success probability of the regulation down for 1 hour with a given initial SOC can be represented as follows:

$$\mathrm{W_{RD}}(SOC) = INTV \times \sum_{i=1}^{\frac{1}{INTV}} P_{RD}(i, SOC) \tag{4}$$

where $INTV$ is interval between the regulation signals as defined in (1).

Meanwhile, (3) includes n-*th* power of two, as well as factorial of the n. Since signal period of the regulation is in the order of seconds, the reciprocal of $INTV$ can reach up to more than a thousand, and so is n. Consequently, it is practically impossible to calculate the probability from (3) directly. Coincidently, however, (3) is in the form binomial distribution with success probability of 1/2. With the large enough n, it is well known that the binomial distribution can be given by the normal distribution as:

$$B(n, p) \cong N(np, np(1-p)) \tag{5}$$

where p is success probability of each Bernoulli trial. As a result, (3) can be represented in the form of the normal distribution as follows:

$$\mathrm{P_{RD}}(n, SOC) \cong 1 - \int_{-\infty}^{k} f(x)dx \tag{6}$$

From the same reasoning, the success probability of regulation up can be obtained as follows:

$$\mathrm{P_{RU}}(n, SOC) = \sum_{i=0}^{k} \frac{{}_nC_i}{2^n} \cong \int_{-\infty}^{k} f(x)dx$$

$$\text{for k < n, otherwise 1} \tag{7}$$

where $k = \left\lfloor \dfrac{n + SOC/\Delta SOC}{2} \right\rfloor$.

and

$$\mathrm{W_{RU}}(SOC) = INTV \times \sum_{i=1}^{\frac{1}{INTV}} P_{RU}(i, SOC) \tag{6}$$

The functions $\mathrm{W_{RD}}$ and $\mathrm{W_{RU}}$ represent expected probability that the aggregator can successfully manage regulation request during the unit contract time. Therefore, these functions directly can be used as weight functions to reflect the energy constraint. However, the exact form of the weight function may vary depending on the required usage. For example, one may need $\mathrm{W_{RD}}(100) = 0$ to indicate that the fully charged battery is no worth in terms of regulation down. In our case, $\mathrm{W_{RD}}(100)$ is not zero because $P_{RD}(n,100)$ never yields zero for any n. From (3) and (4), it can be inferred that the minimum of $P_{RD}(n,100)$ is 0.5 when SOC is 100, and hence $W_{RD}(100) = 0.5$. In this case, the value between $W_{RD}(100)$ and $W_{RD}(0)$ can be normalized to conform to 0 and 1, and vice versa for regulation up.

IV. SIMULATION

As seen in (1), energy deviation is proportional to the utilization rate and power capacity of the vehicle battery. That is, if the grid operator calls for the regulation with more intensive power capacity, the energy deviation would increase as well. Likewise, a vehicle with bigger power capacity is allocated with bigger portion for each regulation call, and hence the greater energy deviation. As a result, with fixed battery size (in terms of energy capacity, Ah), it can be inferred that the energy constraint would be severer as the energy deviation is increased, and it should be reflected to the derived weight function as well.

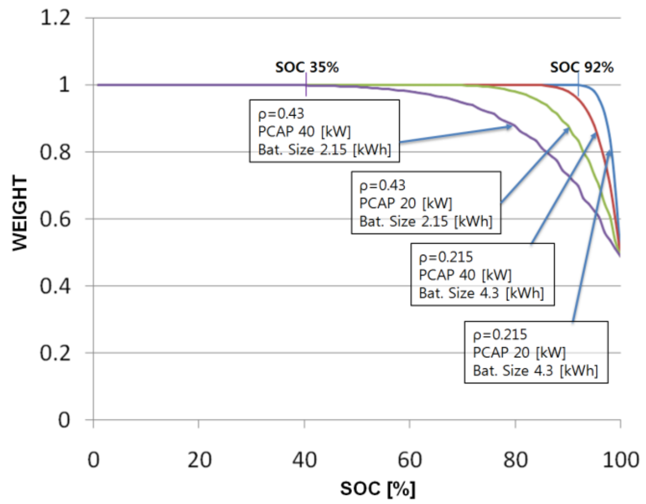

Figure 5. Weight functions for regulation down. Four different parameter configurations are used.

For verification, we actually derived the weight function with various parameters for regulation up and down, respectively. Fig. 5 illustrates weight functions for regulation down. Four different parameter configurations were used. As expected, as ρ and PCAP get bigger, the curve remains below one (1.0) in wider range, representing stronger energy constraint. Specifically, the one with smallest energy deviation condition (ρ =0.215, *PCAP* = 20 kW) remains constant as 1.0 before 92% of SOC. Contrarily, the one with biggest energy deviation remains below 1.0 between 35% and 100%. That is, a V2G vehicle under this condition may suffer the energy constraint within this wide range. Moreover, the minimum of all curves are 0.5 at 100% of SOC as discussed in the previous section. The probability that regulation down signal will come is 50% for any moment. Thus, a fully charged battery can still have 50% of chance of managing the regulation signal.

Fig. 6 illustrates weight functions for regulation up. In this case, all the discussion we made for regulation down works exactly the same. Only the direction is opposite.

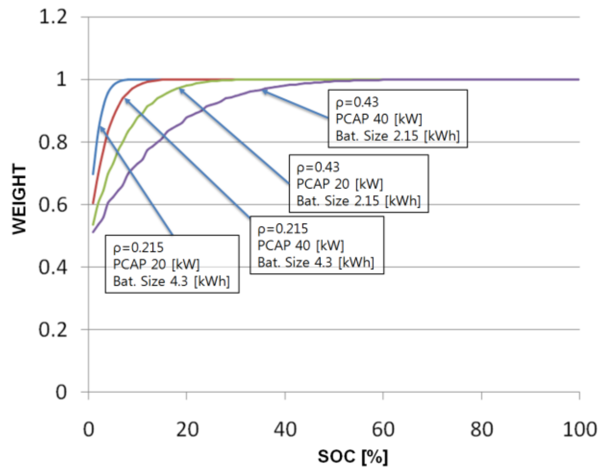

Figure 6. Weight functions for regulation up. The same four parameter configurations are used as in Fig. 5.

V. CONCLUSION

We estimated the energy constraint in terms of remained battery state-of-charge (SOC) in a quantitative manner. Using the facts that the regulation signals are periodic and symmetric between positive and negative, the probability of successfully carrying out a regulation signal was derived. It was averaged to form a weight function so that it can be reflected to a cost function to rate the current value of regulation. The derived equations are analyzed, and the result was verified through actual simulations. One may analyze and model the energy constraint from different perspective resulting in different weight functions. It could be meaningful to compare those results by actually applying the weight functions to the cost function for the V2G regulation control in the other work of the authors [5].

[1] J. Tomic and W. Kempton, "Using fleets of electric-drive vehicles for grid support," *Journal of Power Sources*, 2007

[2] W. Kempton and S. E. Letendre, "Electric vehicles as a new power source for electric utilities," *Transpn Res.-D*, vol. 2, no. 3, pp. 157–175, 1997.

[3] W. Kempton and J. Tomic, "Vehicle-to-grid power implementation: From stabilizing the grid to supporting large-scale renewable energy," *Journal of Power Sources*, 2005.

[4] A. N. Brooks, "Vehicle-to-grid demonstration project: Grid regulation ancillary service with a battery electric vehicle," AC Propulsion, Inc.,Tech. Rep., 2002.

[5] S. K. Han, S. H. Han, and K. Sezaki, "Design of An Optimal Aggregator for Vehicle-to-Grid Regulation Service" in IEEE PES Conference on Innovative Smart Grid Technologies(ISGT), Gaithersburg, MD, USA, 19-21 January 2010.

[6] W. Kempton, V. Udo, K. Huber, K. Komara, S. Letendre, S. Baker, D. Brunner, and N. Pearre, "A test of vehicle-to-grid(v2g) for energy storage and frequency regulation in the pjm system," Univ. of Delaware and Pepco Holdings,Inc and PJM Interconnect and Green Mountain