Sensitivity Analysis of LuGre Friction Model for Pneumatic Actuator Control

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***Abstract*—This paper discusses the analytical model of a pneumatic actuator. Nonlinear hysteresis phenomenon resulting from the friction is also presented in this article. Hysteresis phenomenon governed by the friction in the actuator is modeled with dynamic friction (LuGre) model. Moreover, effects of friction model parameters on the over all model are also discussed. This new approach allows for the sensitivity analysis for the pneumatic actuator.**

I. Introduction

Pneumatic actuators are devices that convert power of compressed air into mechanical energy. Their low price and high power to weight ratio give them an edge on electrical actuators. Air pressure is controlled in one or both chambers of pneumatic actuator via a pressure converter. Detailed analysis of a single acting pneumatic actuator (pressure variation in one chamber, only), has been discussed in [GCGD09], [LM90], [Mor99] and [vVB97]. Whereas, Working and control of double acting pneumatic actuator (pressure variation in both chambers) is detailed in [LSFB05], [MGG05] and [TYK06].

Hysteresis is an important phenomenon experienced in the actuators. Hysteresis occurs due to friction between two bodies in contact. A brief review about different friction models has been presented in the article by Olsson [OA98]. Astrom [AdW08] in his work summarizes the contributions of Amonton, Coulomb and Stribeck [Str02]. Static and dynamic friction models have been developed in the literature. Static friction models have the limitations of simulation and control at zero velocities (See for example [OA98] and [AdW08]).
Dahl [Dah77], Bliman and Sorien (See [CO95]) etc, presented different dynamic models for friction. Dahl model is the simplest dynamic model (See for example [Dah77]). Some other models like LuGre model, Bliman and Sorien, presented by Hlouvry [HD94], Canudas [CO95], Wenjing [Wen07] and Astrom [AdW08], cover other phenomenons like stick-slip friction and stribeck effect. LuGre model is more comprehensive, but its parameter identification is relatively difficult as compared to other models.
There always exists some constrains in model development due to computational limitations, assumptions and knowledge gaps. A perfect model that accounts for every aspect of reality is impossible to determine. Sensitivity analysis (SA) is a useful tool to see the impact of system parameters on the model. Especially, when parameters are difficult to identified or there is uncertainty due to lack of knowledge about the system. Different techniques for sensitivity and uncertainty analysis are detailed in [GFP$^+$09]. In [Che08] sensitivity analysis of electro-hydraulic actuator has been studied using the method of parameter variation. The local sensitivity analysis techniques for dynamic systems are proposed in [PWB06] and [Par69].
In this article, an analytical model of the pneumatic actuator is presented. Hysteresis in the actuator is studied with the help of Dynamic friction model. Parameters of the LuGre friction model are identified by performing different identification tests and nonlinear least square method. Sensitivity of model parameters is also studied in this work, as nonlinear friction model does not have a unique solution. Model is validated with identified parameters and it is shown that model predict the experimental results with accuracy.
This paper is organized as follows: In Section II, the analytical form of the pneumatic actuator is developed. In Section III, A dynamics friction model is presented and its identification is discussed. In Section IV, sensitivity analysis of the friction model is conducted using the analytical form. Model validation is discussed in section V.

II. Pneumatic Actuator Model

The pneumatic actuator consists of a linear linkage shaft, connected to a diaphragm. The diaphragm divides the internal actuator space into two chambers via a membrane. Pressure difference in the chambers moves the diaphragm and the linkage linearly. This motion is converted to rotary motion via a rotating unison ring in the VNT; that actuates all VNT vanes together. These guided vanes regulate exhaust gas flow to the turbocharger. In Fig.1(a) an industrial pneumatic actuator and its schematic diagram is shown. While, all the forces acting on its diaphragm are shown in Fig.1(b).
The mechanical subsystem for the diaphragm can be modeled as a mass, spring and damper system. Applied force to the diaphragm is pressure difference between actuator chambers. Using Newton's second law of motion.

$$m_d\ddot{x}_d = F_{act} - b_d\dot{x}_d - F_{sm} - F_{aero} - F_f \qquad (1)$$

Where x_d and F_{act} are diaphragm position and force due to pressure difference across diaphragm of pneumatic actuator. F_s is the some of pre-loaded force F_0, and spring stiffness

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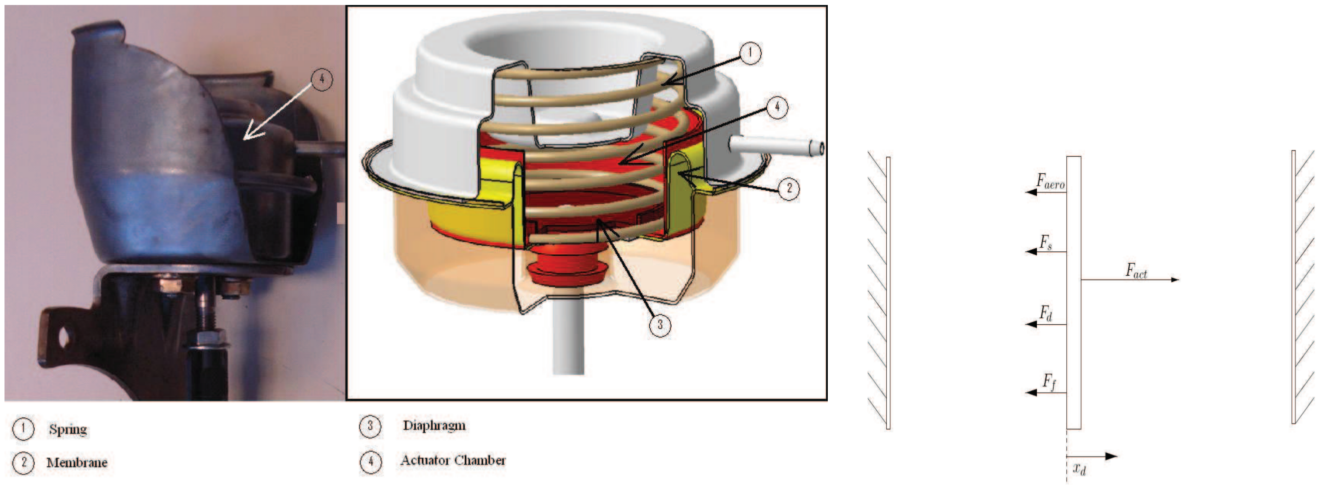


(a) Industrial pneumatic actuator

(b) Free body diagram of Diaphragm with all the forces acting on it

Fig. 1. A pneumatic actuator with its schematic and free-body diagram

force. Damping force is the product of diaphragm velocity and damping coefficient b_d (See Fig.1).

$$\begin{aligned} F_{act} &= (p_{atm} - p_{act})\, A_d \\ F_{sm} &= F_0 + k_{sm} x_d \end{aligned}$$

Where A_d is the area of the diaphragm. One of the actuator chambers is at atmospheric pressure p_{atm}. The pressure in the second chamber, p_{act} can be adjusted to attain the desired position. k_{sm} is combined stiffness of the spring and membrane. F_f and F_{aero} are the force of friction and external perturbation force which is neglected here. Friction force will be discussed in detail in section III. Actuator pressure dynamics depend on the air mass flow is discussed below

A. Mass flow model

Pressure inside the actuator chamber varies due to the air mass entering or leaving the chamber. Air mass flow can be modeled as the air passing through the duct using orifice flow equation. Pressure regulator builds a link between the actuator chamber and the source reservoir or atmosphere. Hence, A path is formed between the actuator chamber and external sources (Vacuum reservoir and atmosphere). Air mass flow from actuator to vacuum reservoir, $\dot{m}_{act-out}$ is given by following equation,

$$\dot{m}_{act-out} = A_{eff(act-res)} p_{act} \sqrt{\frac{\gamma}{RT_{act}}} . \Phi\left(\frac{p_{res}}{p_{act}}\right) \tag{2}$$

In the configuration when actuator chamber is connected to atmosphere, air mass flows from atmosphere to actuator chamber $\dot{m}_{act-in}$.

$$\dot{m}_{act-in} = A_{eff(atm-act)} p_{atm} \sqrt{\frac{\gamma}{RT_{atm}}} . \Phi\left(\frac{p_{act}}{p_{atm}}\right) \tag{3}$$

Here $A_{eff(act-res)}$ and $A_{eff(atm-act)}$ are the effective flow areas between actuator-reservoir and atmosphere-actuator respectively. $\Phi(p_r)$ is a function which depends on the pressure ratio p_r of the upstream and down stream pressure. Air mass flow is called choked flow when this ratio is ≤ 0.528 (See for example [Hey88] and [OC97]), air mass flow becomes constant at this point. Expression for $\Phi(p_r)$ is given in the following equation

$$\Phi(p_r) = \begin{cases} \sqrt{\left\{ (p_r)^{2/\gamma} - (p_r)^{(\gamma+1)/\gamma} \right\} \dfrac{2}{\gamma - 1}} , & p_r \geq \left(\frac{2}{\gamma+1}\right)^{\gamma/(\gamma-1)} \\ \sqrt{\left(\frac{2}{\gamma+1}\right)^{\gamma+1/\gamma-1}} , & p_r \leq \left(\frac{2}{\gamma+1}\right)^{\gamma/(\gamma-1)} \end{cases} \tag{4}$$

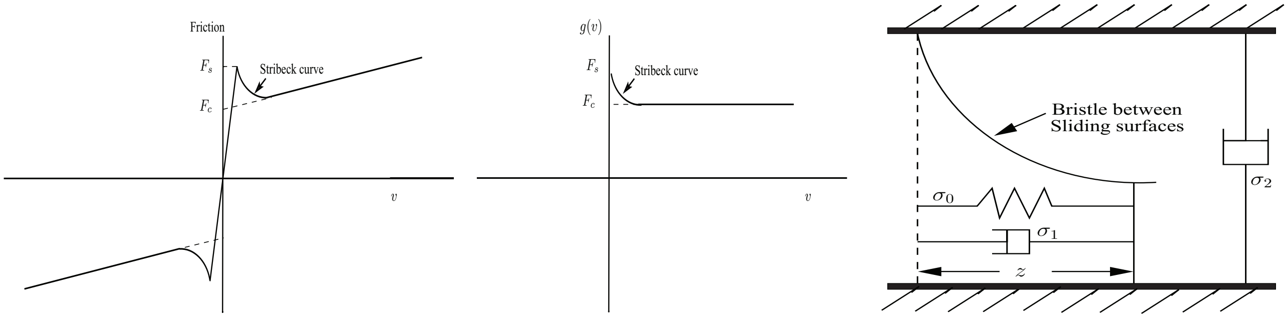
The total mass flow to the actuator is the difference of air mass flowing out from the actuator to reservoir and air mass flowing to the actuator from atmosphere.

$$\dot{m}_{act} = \dot{m}_{act_{in}} - \dot{m}_{act_{out}} \tag{5}$$

III. Dynamic Friction Modeling

Dynamic models are more useful in simulations since static models tend to be discontinuous. They have simulation advantages on the static models. Many dynamic models for friction have been proposed by different researches. Some examples of dynamic friction models are Dahl model [Dah77], Bouc-Wen model [RGC+09] and LuGre Model [CO95] etc. These models cover a variety of phenomena like hysteresis, stick slip, Stribeck effect caused by friction. These models are discussed below.

1) LuGre Friction Model: LuGre model, (See for example [Alt04] and [OA98]), is an extension of the Dahl model that captures the Stribeck effect and thus can describe stick-slip motion. Compared with the Dahl model, the LuGre model has a velocity-dependent function $g(v)$ instead of a constant, an additional damping σ_1 associated with micro-displacement [AdW08], LuGre model can be presented by state space



(a) (i) Friction Vs velocity, (ii) Function g(v) that captures coulomb and stribeck friction

(b) Parameters of the LuGre friction model

Fig. 2. LuGre friction model for diaphragm velocity (See equation (6))

equation

$$F_f = \sigma_0 z + \sigma_1 \dot{z} + f(v) \tag{6}$$

$$\dot{z} = v - \sigma_0 \frac{|v|}{g(v)} z \tag{7}$$

Where F_f is the friction force and σ_0 and σ_1 are coefficients. z is the internal state of model that represents micro displacement between bristles [CO95]. $f(v)$ is the velocity dependant damping force, that prevent the model to behave as a linear spring at small displacements. Friction model takes into consideration the Stribeck effect through the function g(v)[SAS07]. Function g(v) is given below

$$g(v) = \left(F_c + (F_s - F_c)\exp^{-(|v|/v_s)}\right) \tag{8}$$

v_s is the sliding speed coefficient determines the stribeck curve and $g(v)$ is such that $F_c \leq g(v) \leq F_s$

2) LuGre Model Identification: Further study of the experimental results will permit us to identify parameters for LuGre model. Considering the force balance equation(1) of the actuator at the steady state and high velocity (when $v \gg v_s$ and $\ddot{x}_d = 0$). Furthermore, derivative of the bristle displacement $\dot{z} = 0$ in above mentioned conditions. See [Thi05] for more details.

$$F_{act} - F_{sm} = F_f = \sigma_0 z + \sigma_2 v \tag{9}$$

Where F_{act} and F_s are the applied force and spring equivalent force respectively. σ_0 and σ_2 are LuGre model parameter for friction. Equation (6) can be simplified for $\dot{z} = 0$

$$\frac{dz}{dt} = v - \frac{\sigma_0}{g(v)} z|v| = 0 \tag{10}$$

Using above equation and equation (8) and eliminating $g(v)$ term.

$$\sigma_0 z = g(v) = \Big(F_c + (F_s - F_c)\exp^{-(|v|/v_s)}\Big)\frac{v}{|v|} \tag{11}$$

Using above equation in equation (9) force balance equation can be written as

$$F_{act} - F_{sm} = \Big(F_c + (F_s - F_c)\exp^{-(|v|/v_s)}\Big)\frac{v}{|v|} + \sigma_2 v \tag{12}$$

Preforming the test at $v \gg v_s$ above equation can be simplified to find F_c and σ_2

$$F_{act} - F_{sm} = F_c \frac{v}{|v|} + \sigma_2 v \tag{13}$$

Using equation (13), we identified F_c and σ_2 with condition that $v \gg v_s$. Once both parameters are identified equation (12) is used to determine F_s and v_s. Remaining two parameters (σ_0 and σ_1) are determined using non linear least squares approach. Since these parameters exert influence in the pre-sliding region, only. All parameters are given in the Table.I. LuGre and Dahl model studied here shows similar results for simulation. However LuGre model is difficult to identify due to more unknown parameters. On the other hand, Dahl model does not address all friction phenomenon.

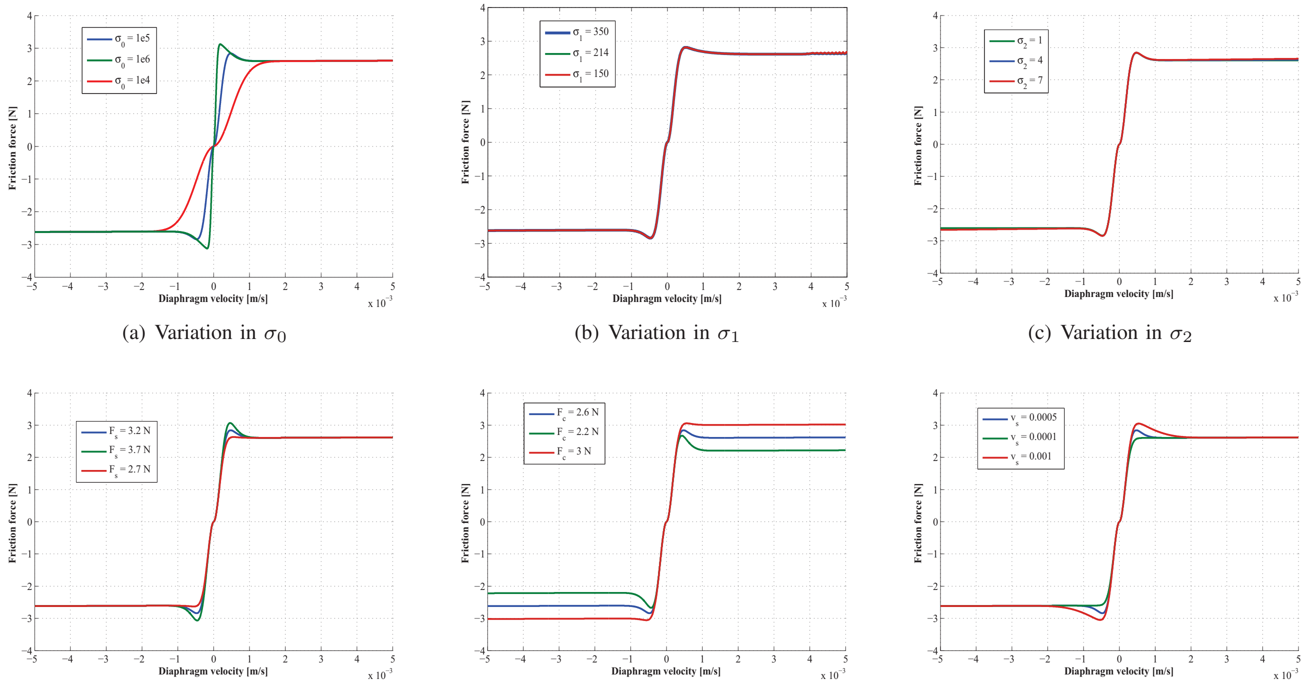
A. Sensitivity of Parameters

Design sensitivity plays a critical role in identification studies, as well as numerical optimization, and reliability analysis. Sensitivity analysis should be used in the process of model development. It is important to understand that how closely a model approximate a real system, which can not be depicted by comparing model results with experimental data. There are certain analysis which should be carried out, to improve model performance. Two main approaches for model analysis are

- Sensitivity analysis, which involves studying how changes in a models input values or assumptions affect its output or response.
- Uncertainty analysis, which investigates how a model might be affected by the lack of knowledge about a certain population or the real value of model parameters.

There exist some sensitivity analysis techniques for example montecarlo method, sobul method, method of partial derivative and variance based sensitivity analysis. Variance based methods, normally calculate sensibility index (SI) using the covariance of input and output. Standardized Regression Coefficient (SRC) is a variance based method that calculate sensibility index of all the inputs with respect to output.

$$SRC = \left[\frac{Cov(x_i, y)}{\sqrt{Var(x_i)Var(y)}}\right]^2 \tag{14}$$



(a) Variation in σ_0

(b) Variation in σ_1

(c) Variation in σ_2

(d) Variation in static friction F_s

(e) Variation in coulomb friction F_c

(f) Variation in stribeck velocity v_s

Fig. 3. LuGre friction model for diaphragm velocity

Another approaches is partial derivative method in which partial derivative of output is analyzed with respect to all inputs. Since, the friction model discussed here is nonlinear and complex we preferred parameters varying method, in which influence of the variation in the input parameters is observed on the output. Friction model is identified using nonlinear least square method, which gives approximate parameters for the system. The formalization of the variation in model parameters is stated as In the Fig.3 all the parameters are varied to see their impact on the over all friction model. It is seen that parameters σ_0, σ_1 and v_s has influence on the dynamics of the friction model. Whereas, other parameters effects the steady state error of the friction model.

$$\begin{aligned}\sigma_{0x} &= \sigma_0 + \delta_{\sigma_0}\ ,\ \sigma_{1x} = \sigma_1 + \delta_{\sigma_1}\\ \sigma_{2x} &= \sigma_2 + \delta_{\sigma_2}\ ,\ v_{sx} = v_s + \delta_{v_s}\\ F_{sx} &= F_s + \delta_{F_s}\ ,\ F_{cx} = F_c + \delta_{F_c}\end{aligned} \tag{15}$$

Where δ_{σ_0}, δ_{σ_1}, δ_{σ_2}, δ_{v_s}, δ_{F_s} and δ_{F_c} are variation in the all six friction model parameters. Models response after changing different input parameters is shown in Fig.3. Where, identified parameters are varied one by one and there impact on the model is studied. It is seen that model parameters F_s and F_c are more sensitive as compared to other parameters.

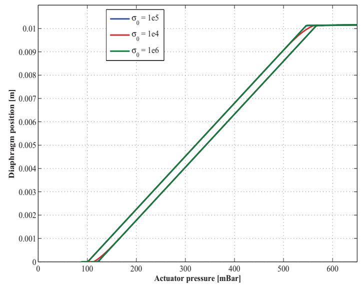
IV. Model validation

Experimental results were obtained using the test bench shown in the Fig.4. System consists of two Druck pressure sensors, A pneumatic actuator and a pressure regulator. Pressure inside the single acting actuator is changed with the help of pressure regulator (EPC). Actuator pressure is measured with Druck pressure sensor while actuator diaphragm position is measured with potentiometer.

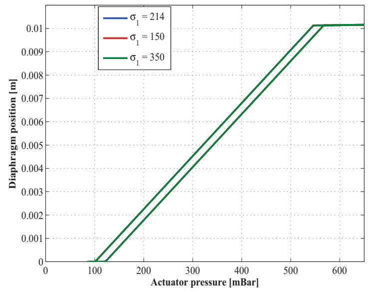
Simulation results between applied pressure and diaphragm position by changing LuGre model parameters is shown in the Fig.5. In Fig.5(a) effect on hysteresis curve, by changing stiffness coefficient σ_0, is shown. It can be seen that variation in the value of σ_0 is important if we change its value from more than 50%. In the Fig.5(b), effect of variation of σ_1 is shown. σ_1 is less sensitive as compared to σ_0 as the variation up to 68% has little significance on the results. Fig.5(c) represents variation in damping constant σ_2. Effect of σ_2 is negligible as by changing its value up to 90% produce little variation in the results.

In Fig.5(d), graph shows the effect of static friction on applied pressure and diaphragm position curve. Coulomb friction is varied up to 15.36% and its impact is shown in Fig.5(e). Static friction is most important parameter as the variation in the value of 16% has an impact on the hysteresis curve. Effect of stribeck velocity on the hysteresis curve is shown in Fig.5(f). Increase or decrease in the stribeck velocity effects the results as we change it from nominal position.

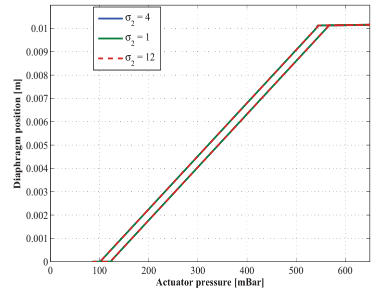
In the Fig.6, hysteresis curve is shown that is obtained from experiments. Friction model is validated using approximative LuGre model parameters. The coherence between experimental result and simulations have demonstrated the effectiveness of the proposed method. Identified parameters are given in the the Table.I.



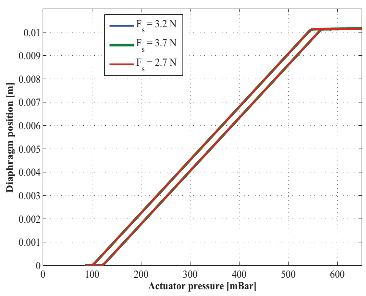
(a) Variation in σ_0



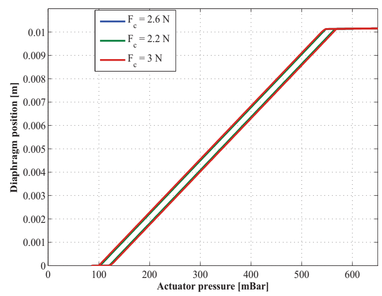
(b) Variation in σ_1



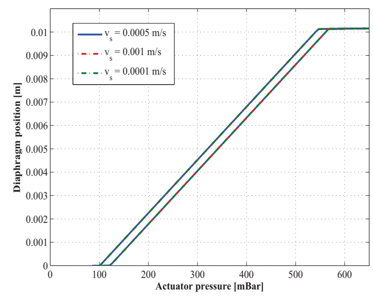
(c) Variation in σ_2



(d) Variation in static friction F_s



(e) Variation in coulomb friction F_c



(f) Variation in stribeck velocity v_s

Fig. 5. LuGre friction model for diaphragm velocity

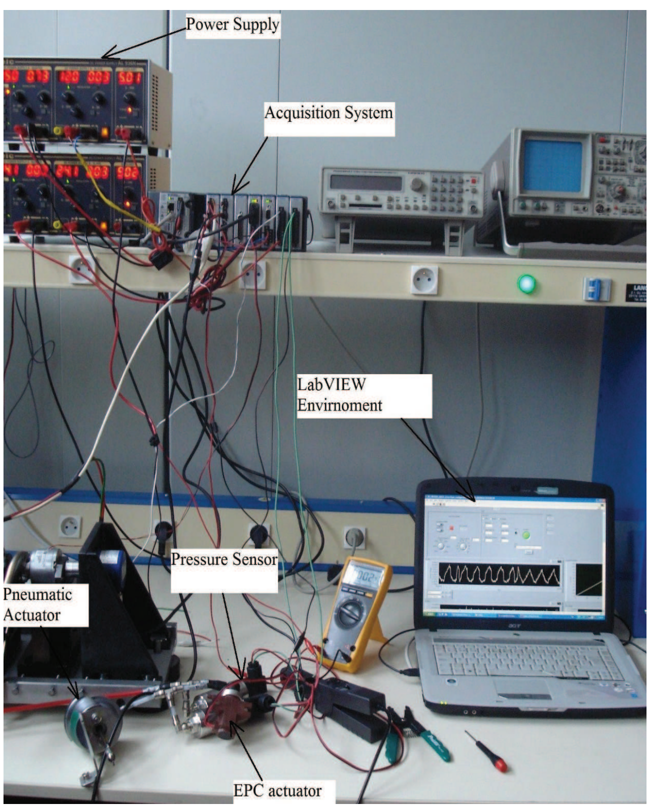


Fig. 4. Experimental setup for pneumatic actuator with pressure regulator

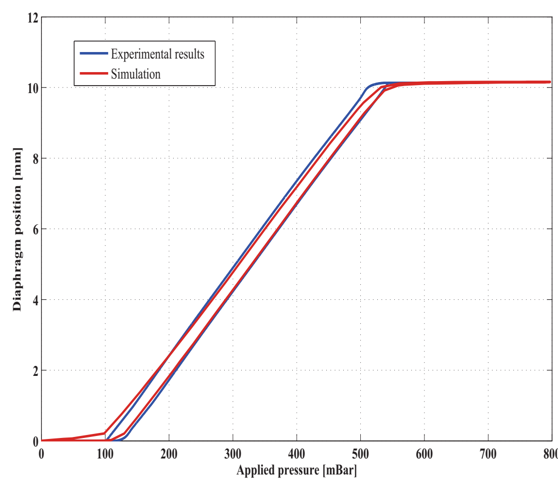


Fig. 6. comparison between experimental results and simulation for hysteresis observed at no load on turbine

V. Conclusion

The results presented in this paper validate a modeling scheme for the pneumatic. This work allows for the synthesis of pneumatic actuators controlled by electro-pneumatic pressure converters. Pressure inside the actuator chamber is modeled from air mass flow equation. Hysteresis due to friction is modeled using LuGre friction models. Model parameters are identified from nonlinear least square method. A comprehensive analysis of LuGre parameters sensitivity is discussed. Impact of all parameters on the friction model is presented. Identified parameters are simulated and compared with the experimental results.

ACKNOWLEDGMENT

This work is partially sponsored by project SIMBA. Author is grateful to the project partners for their support.

APPENDIX

Parameters used for the simulation of pneumatic actuator are shown in Table.I.

TABLE I
PARAMETERS VALUES FOR THE DIAPHRAGM

Parameter	abbreviation	Value
Diaphragm mass	m_d	0.63 kg
Spring preloaded force	F_0	32.68 N
Spring(+membrane) constant	k_{sm}	12850 N/m
Damping constant	b_d, σ_2	4 N-s/m
Static friction force	F_s	3.2 N
Coulomb friction force	F_c	2.6 N
Stiffness constant	σ_0	1e5
Bristle constant	σ_1	214
Stribeck velocity	v_s	0.0005 m/s
Diaphragm volume	V_0	$60e-6m^3$
Diaphragm cross sectional area	A_d	$2.92246e-3m^2$
Atmospheric pressure	p_{atm}	$1e5Pa$

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