Estimation of Individual In-cynlinder air mass flow via Periodic Observer in Takagi-Sugeno form

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Abstract— **In this paper, we propose to estimate the individual in-cylinder air mass flow for a gasoline IC engine. In order to achieve this goal a periodic observer for a class of non-linear models in the discrete Takagi-Sugeno form is designed. The adopted framework to prove the stability of the observer is based on the Lyapunov theory and uses linear matrix inequalities (LMI) formalism. Some simulations and experimental results are provided to show the efficiency of the proposed method.**

Keywords-In-cylinder air mass flow estimation; gasoline IC engine; periodic observer; Takagi-Sugeno design

I. INTRODUCTION

The high demand for consumption, emission and drivability on modern internal combustion engines call for control concepts that cannot be carried out without the current status of the engine being detected. Since many of the variables required for control can only be measured. If at all, using an expensive sensors (meaning sensors that are not suited for series production), there is completely need for novel and effective ways of engine controlling, estimation and modeling.

The cylinder by cylinder air mass flow estimation (or individual in-cylinder estimation) problem is an interesting and challenging task for the engine control community. Indeed, an accurate estimation of each in-cylinder air mass flow could allow reaching an accurate control of the fuel injection associated.

Various methodologies exit in the literature and some examples are given below. A first approach [1] presents a way to estimate the air charge mal-distribution in the individual cylinders of a four cylinder spark ignition (SI) engine based on a nonlinear discrete observer using only the measurement of the manifold pressure. In [2] and [3], the authors propose to use the manifold pressure and the air mass flow sensors to obtain the individual cylinder air charge form a multi-rate sampling method. In [4], the authors propose several observers design (high gain observer, etc…) to estimate the air mass flow in each cylinder from the estimation of the variation of the volumetric efficiency. In [5], the authors propose the use of an unknown input observer and in [6] a method based of each cylinder air mass flow prediction is given. Other tools are available mainly based on speed density equation with different algorithms ([7, 8, 9, 10 and 11].

In this paper, we propose an original approach using a Takagi-Sugeno (TS) [12] periodic observer to estimate the air mass flow rate in cyclic manner with respect to the cylinder firing sequence. The proposed observer is designed in the crank angle domain and permits to take into account the nonlinearity of the air model using the TS representation. The stability of the estimation error is proved from the Lyapunov theory which results in linear matrix inequalities (LMI) [13] stability condition.

The paper is organized as follows. First, the model used for to represent the four cylinder engine air dynamics is developed. Then, section III presents the design of periodic TS observer which allows estimating the in-cylinder air mass flow, where a discrete periodic TS model in the crank angle domain is developed and stability conditions are provided. In section IV, some simulation and experimental results are proposed to show the efficiency of the method. Finally, some conclusions and further works are given in the last section.

II. MODEL OF THE AIR MASS FLOW

This section present the model of the air flow which will be used to estimate the air flow entering each cylinder, the first part present the dynamic of the manifold pressure, the throttle and the runner vale flow, the last part explain the domain of the present framework.

A. Dynamic of the manifold pressure

The dynamic of the manifold pressure P_{man} is given by the following expression [14]:

$$\frac{dP_{man}}{dt} = \frac{RT_{man}}{V_{man}}\left(D_{thr}(t) - \sum_{i=1}^{n_{cyl}} D_{cyl_i}(t)\right) \tag{1}$$

Where T_{man} is the manifold temperature, V_{man} the manifold volume, R the perfect gas constant, D_{thr} the mass air flow rate

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across the throttle, $D_{cyl_i}(t)$ the air cylinder flow in the i^{th} cylinder, and n_{cyl} is the number of the cylinders.

The mass flow rate across the throttle is modeled based on the following adiabatic orifice flow [13]:

$$D_{thr}(t) = C_d . A_{th} \cdot \left(\frac{P_{atm}}{\sqrt{RT}} \right) . d(P_{atm}, P_{man}) \quad (2)$$

The function A_{th} with the discharge coefficient C_d expresses the geometric flows characteristics for the throttle. The differential pressure function $d(P_{atm}, P_{man})$ is defined by:

$$d(P_{atm}, P_{man}) = \begin{cases} (P_r)^{1/\gamma} . \sqrt{\frac{2\times\gamma}{\gamma-1} . \left(1-(P_r)^{\gamma-1/\gamma}\right)} & if \ P_r > \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} \\ \sqrt{\gamma} . (2/\gamma+1)^{\frac{\gamma+1}{2(\gamma-1)}} & if \ P_r \le \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} \end{cases} \quad (3)$$

with the constant $\lambda = 1.4$ and the pressure ratio $P_r = \frac{P_{man}}{P_{atm}}$ between the atmospheric pressure P_{atm} and the manifold pressure.

The air mass flow into one cylinder, is obtained the same way as (2):

$$D_{cyl_i}(t) = A_v(Lift_i(\theta)) . \left(\frac{P_{man}}{\sqrt{RT}} \right) . d(P_{man}, P_{cyl_i}) \quad (4)$$

The geometric flow characteristics across the intake valve for each cylinder $A_v(Lift_i(\theta))$ are modeled as linear functions of the valve lift $A_v = \alpha_i . Lift_i(\theta)$ (the scalar α_i is the maximum surface opening), with [15]:

$$Lift_i(\theta) = IVL \sin^2\left(\frac{180}{IVD} \left(\theta - 90(i-1) - IVO \right) \right) \quad (5)$$

The function is characterized by the intake valve open timing (IVO), the maximum lift (IVL) and the intake valve open duration (IVD) and in the same manner as (3), the differential pressure function with $P_r = \frac{P_{cyl_i}}{P_{man}}$ is given by:

$$d(P_{man}, P_{cyl_i}) = \begin{cases} (P_{r_i})^{1/\gamma} . \sqrt{\frac{2\times\gamma}{\gamma-1} . \left(1-(P_{r_i})^{\gamma-1/\gamma}\right)} & if \ P_{r_i} > \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} \\ \sqrt{\gamma} . (2/\gamma+1)^{\frac{\gamma+1}{2(\gamma-1)}} & if \ P_{r_i} \le \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} \end{cases} \quad (6)$$

In this paper, we only consider the case where equation (6) is a constant.

B. Dynamic of the sensors

The observer is based on the measurement of the air mass flow rate provided by a hot wire anemometer, and on the measurement of the manifold pressure provided by a piezoelectric pressure sensor situated after the throttle.

The dynamic of the air mass flow sensor is modeled as a first order dynamic:

$$\tau_{thr} \frac{dD_{thr_m}(t)}{dt} + D_{thr_m}(t) = D_{thr}(t) \quad (7)$$

Where $D_{thr_m}(\theta)$ is the measured mass air flow, $D_{thr}(\theta)$ is the actual mass air flow through the throttle and τ_{thr} the time constant of the sensor ($20\,ms$).

The dynamic of the pressure is also defined as a first order:

$$\tau_{man} \frac{dP_{man_m}(t)}{dt} + P_{man_m}(t) = P_{man}(t) \quad (8)$$

Where $P_{man_m}(\theta)$ is the measured manifold pressure, and τ_{man} the time constant of the pressure sensor ($10\,ms$).

C. Non-linear continuous model:

In order to compute the nonlinear continuous model used to derive an observer, we rewrite equation (4) as follows:

$$D_{cyl_i}(t) = \alpha_i . Lift_i(\theta) . \left(\frac{1}{\sqrt{RT}} \right) . \beta_i(t) \quad (9)$$

with $\beta_i(t) = P_{man}(t) . d(P_{man}(t), P_{cyl_i}(t))$ (10)

If we consider only the constant part of $d(P_{man}, P_{cyl_i})$ and using equation (1), the dynamic of the variables β_i can be written as:

$$\begin{aligned} \dot{\beta}_i(t) &= \sqrt{\gamma . (2/\gamma+1)^{\frac{\gamma+1}{\gamma-1}}} \times \dot{P}_{man}(t) \\ &= \sqrt{\gamma . (2/\gamma+1)^{\frac{\gamma+1}{\gamma-1}}} \times \frac{RT_{man}}{V_{man}} \left(D_{thr}(t) - \sum_{i=1}^{n_{cyl}} D_{cyl_i}(t) \right) \end{aligned} \quad (11)$$

Or :

$$\dot{\beta}_i(t) = \sqrt{\gamma . (2/\gamma+1)^{\frac{\gamma+1}{\gamma-1}}} \times \frac{RT_{man}}{V_{man}} \left(D_{thr}(t) - \sum_{i=1}^{n_{cyl}} \alpha_i . Lift_i(\theta) . \left(\frac{1}{\sqrt{RT}} \right) . \beta_i(t) \right) \quad (12)$$

So, the problem of the estimation of the in-cylinder air mass flow can be transformed into the estimation of the state variables $\beta_i(t)$. From the previous differential equations of the air flow model, a non-linear model can be deduced considering the state vector:

$X^T(t) = \left[P_{man}(t) \ \ P_{man_m}(t) \ \ D_{thr_m}(t) \ \ \beta_1(t) \ \ \beta_2(t) \ \ \beta_3(t) \ \ \beta_4(t) \right]^T$,

a possible continuous sate space representation of the model is given here:

$$\dot{X}(t)=\begin{bmatrix} 0 & 0 & 0 & \lambda_1 & \lambda_2 & \lambda_3 & \lambda_4 \\ -1/\tau_{man} & 1/\tau_{man} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1/\tau_{thr} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \mu_4 \end{bmatrix}.\dot{X}(t)+\begin{bmatrix} \frac{RT_{man}}{V_{man}} \\ 0 \\ 1/\tau_{thr} \\ \eta \\ \eta \\ \eta \\ \eta \end{bmatrix}.D_{thr}(t) \quad (13)$$

with: $\lambda_i=\alpha_i.Lift_i(\theta).\left(\frac{\sqrt{RT_{man}}}{V_{man}}\right)$, $\eta=\sqrt{\gamma.(2/\gamma+1)^{\frac{\gamma+1}{\gamma-1}}}\times\frac{RT_{man}}{V_{man}}$,

and $\mu_i=\sqrt{\gamma.(2/\gamma+1)^{\frac{\gamma+1}{\gamma-1}}}.\frac{\sqrt{RT_{man}}}{V_{man}}.\alpha_i.Lift_i(\theta)$ (14)

And the output equation:

$$\begin{bmatrix} P_{man_m}(t) \\ D_{thr_m}(t) \end{bmatrix}=\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}X(t) \quad (15)$$

Note that, if we consider directly the observation of the model (13) with the output equation (15), it is not observable. To solve this problem a periodic representation of (13) in the crank angle domain will be considered.

D. Crank angle Domain transformation

The opening and closing timing of intake valve of each cylinder are produced periodically on each cylinder. Those timing are varying in the time domain with the engine speed. However, if we consider the crank angle domain, the opening and closing of the intake valve are produced at the same angle.

For that reason, it is convenient to formulate equation (1), (2), (4), (7) and (8) in crankshaft angle using the following transformation:

$$\frac{dX(t)}{dt}=\frac{dX(\theta)}{d\theta}\left(\frac{d\theta}{dt}\right)=\frac{dX(\theta)}{d\theta}\dot{\theta} \quad (16)$$

It has been argued that all dynamics, except for fuel dynamics, varies less in the crank angle domain [16,17], also the sampling period is fixed in the crank angle domain but will vary in the time domain if the engine speed changes.

III. Design of the Periodic TS observer

In this part, the design of the periodic TS observer is presented in three steps. The first step is the development of a discrete periodic TS model derived from the continuous nonlinear model of the section II. Then, the design of the associated observer which ensures the stability of the state estimation error is exposed. The last of this section is dedicated to the estimation of the individual In-cylinder air mass flow from the results of the previous observer.

A. Development of a discrete periodic TS model

For implementation in the embedded engine control, the estimation must be cast in discrete form. A discrete TS model in the crank angle domain is deduced from the previous section. A simple Euler method is used for the discretisation of the continuous model. In order to take into account the nonlinearity of the model (13) and to preserve the periodicity of the engine, a periodic Takagi-Sugeno (TS) form is adopted which is written in this form [18, 19]:

$$\begin{cases} x(\theta+1)=\sum_{i=1}^{r}h_i(z(\theta))\left(A_i^{(c)}x(\theta)+B_i^{(c)}u(\theta)\right)=A_{z(\theta)}^{(c)}x(\theta)+B_{z(\theta)}^{(c)}u(\theta) \\ y(\theta)=\sum_{i=1}^{r}h_i(z(\theta))C_i^{(c)}x(\theta)=C_{z(\theta)}^{(c)}x(\theta) \end{cases} \quad (17)$$

$c=\theta \bmod p$.

with A , B and C are p-periodic matrix, p is the period of the model and "mod" stands for the modulo function. The vectors $x(\theta)$, $z(\theta)$ and $u(\theta)$ are respectively the state, premise and control vector. $z(\theta)$ is assumed to be measurable. r is the number of linear sub-models (or rules) and $h_i(z(\theta))$ are a non linear functions satisfying the convex sum property $\sum_{i=1}^{r}h_i(z(\theta))=1$ and all of them are positive $h_i(z(\theta))\geq 0$.

A way to derive such TS models is to use the so-called sector nonlinearity approach [19]. It consists in representing a bounded nonlinearity, i.e. $\underline{f_i}\leq f_i(\cdot)\leq\overline{f_i}$ using two functions verifying the convex sum property:

$$\forall i\in\{1,2\}, w_i^1(t)=\frac{\overline{f_i}-f_i(.)}{\overline{f_i}-\underline{f_i}}=1-w_i^2(t) \quad (18)$$

Considering the hypotheses that the manifold temperature stays constant, one non linear function is choose for the TS model:

$$NL=\sum_{i=1}^{4}\delta_j(i)\times lift_i(\theta) \quad (19)$$

Where $\delta_j(i)=1$ when $i=j$, otherwise $\delta_j(i)=0$.

From (17), (18) and (19), a two rules periodic TS representation of the non-linear model (13) is given by:

$$\begin{cases} X(\theta+1)=\sum_{i=1}^{2}h_i(z(\theta)).A_i^{(c)}X(\theta)+B^{(c)}.D_{pap}(\theta) \\ \begin{bmatrix} P_{man_m}(\theta) \\ D_{thr_m}(\theta) \end{bmatrix}=CX(\theta) \end{cases} \quad (20)$$

where the matrices $A_i^{(c)}$ and $B^{(c)}$ are given in appendix.

B. Design of the discrete TS periodic observer

A periodic fuzzy observer of the model can be considered as:

$$\begin{cases} \hat{x}(\theta+1) = A_{z(t)}^{(c)}\hat{x}(\theta) + B_{z(t)}^{(c)}u(\theta) + S_{z(t)}^{(c)^{-1}} K_{z(t)}^{(c)}\left(y(\theta) - \hat{y}(\theta)\right) \\ \hat{y}(\theta) = C_{z(t)}^{(c)}\hat{x}(\theta) \end{cases} \quad (21)$$

The dynamic of the prediction error becomes:

$$\tilde{x}(\theta+1) = \left(A_{z(\theta)}^{(c)} - S_{z(\theta)}^{(c)^{-1}} K_{z(\theta)}^{(c)} C_{z(\theta)}^{(c)}\right)\tilde{x}(t) = \bar{A}_{z(\theta)}^{(c)}\tilde{x}(\theta) \quad (22)$$

The next Lemma gives a first result obtained using the following quadratic periodic Lyapunov function:

$$V_l(\tilde{x}) = \tilde{x}^T(\theta)P^{(l)}\tilde{x}(\theta) \text{ with } l \in \{0,\ldots,p-1\}. \quad (23)$$

The prediction error is *p*-stable if the following inequality holds along the trajectory [20]:

$$\begin{aligned} &V_0(\tilde{x}(\theta)) > V_1(\tilde{x}(\theta+1)) > \ldots > V_{p-1}(\tilde{x}(\theta+p-1)) \\ &> V_0(\tilde{x}(\theta+p)) \end{aligned} \quad (24)$$

Consider the following quantity:

$$\Upsilon_{ij}^{(l)} = \begin{bmatrix} -P^{(l \bmod p)} & (*) \\ \bar{A}_{ij}^{(l)} & \Theta^{(l+1)} \end{bmatrix} < 0 \quad (25)$$

With $\bar{A}_{ij}^{(l)} = S_j^{(l \bmod p)} A_i^{(l \bmod p)} - K_j^{(l \bmod p)} C_i^{(l \bmod p)}$

$\Theta^{(l+1)} = -S_j^{(l \bmod p)} - \left(S_j^{(l \bmod p)}\right)^T + P^{((l+1) \bmod p)}$ and $P^{(p)} = P^{(0)}$

Theorem1 [18]: The prediction error (22) is globally asymptotically *p*-stable if there exits matrices $P^{(l)} > 0$, $S_i^{(l)}$ and $K_i^{(l)}$, $l \in \{0,\ldots,p-1\}$ and $i \in \{1,\ldots,r\}$ such that the following LMI conditions (26) and (27) hold for all $l \in \{0,\ldots,p-1\}$ and $\Upsilon_{ij}^{(l)}$ defined in (25):

$$\Upsilon_{ii} < 0,\ i \in \{1,\ldots,r\} \quad (26)$$

$$\frac{2}{r-1}\Upsilon_{ii} + \Upsilon_{ij} + \Upsilon_{ji} < 0\ i, j \in \{1,\ldots,r\},\ i \neq j \quad (27)$$

In the particular case of the air flow model (20), the periodicity of the parameters $\left(p = 720^{\circ}\right)$ is very large compared to the sample angle of the model $\left(\theta_s = 6^{\circ}\right)$, the LMI problem (25) associated with the observation of the model can become quickly infeasible. In order to reduce the conservatism of stability conditions, the next lemma is used to design our observers which minimize the number of LMI based on the application of the same observations gains for all the sample angles between 0° and 180°, the same thing for the second, the third and the fourth phase.

$$\Omega_{ij}^{m} = \begin{bmatrix} -P^{(m)} & (*) \\ S_j^{(m)}A_i^{(m)} - K_j^{(m)}C_i^{(m)} & -S_j^{(m)} - \left(S_j^{(m)}\right)^T + P^{(m)} \end{bmatrix} < 0 \quad (28)$$

$$\Upsilon_{ij}^{m} = \begin{bmatrix} -P^{(m)} & (*) \\ S_j^{(m)}A_i^{(m)} - K_j^{(m)}C_i^{(m)} & -S_j^{(m)} - \left(S_j^{(m)}\right)^T + P^{(m+1)} \end{bmatrix} < 0 \quad (29)$$

$\forall\, i, j \in \{1,\cdots,r\}^2$, $P^{(p)} = P^{(0)}$.

Let us define m_i is the number of sample of phase i with $i \in \{1,\ldots,np\}$, np is the number of phase ($m_{np} = p$).

Lemma1 [18]: The prediction error (22) is globally asymptotically *p*-stable if there exits matrices $P^{(m)} > 0$, $S_i^{(m)}$ and $K_i^{(m)}$, $i \in \{1,\ldots,r\}$ such that and holds for all $m \in \{m_1,\ldots,m_{np}\}$ with Υ_{ij}^{m} and Ω_{ij}^{m} defined in (28) and (29)

Proof: The proof is obvious because the LMI conditions provided by lemma 1 are included in Theorem 1.

C. *Estimation of the Individual In-cylinder air mass flow*

As was mentioned earlier, the estimation of individual In-cylinder air mass flow is deduced from the estimation of the state variables $\beta_i(\theta)$ using equation(9).

IV. EXPERIMENTAL RESULTS

In this section, experimental results are given. The engine used in the experiments is an inline four-cylinder four stroke gasoline engine. Considering the observer (21) and the model (20), the application of Lemma 1 gives the following gains:

$$K\{1,1\} = \begin{bmatrix} 2.0430 & 0 \\ 635.0841 & 0 \\ 0 & 517.1135 \\ -2.1404 & 0 \\ -1.9929 & 0 \\ -2.2282 & 0 \\ 5.9784 & 0 \end{bmatrix}, K\{1,2\} = \begin{bmatrix} 2.0374 & 0 \\ 635.0844 & 0 \\ 0 & 517.1135 \\ -2.1488 & 0 \\ -1.9929 & 0 \\ -2.2282 & 0 \\ 5.9784 & 0 \end{bmatrix},$$

$$K\{2,1\} = \begin{bmatrix} 2.0115 & 0 \\ 635.0387 & 0 \\ 0 & 517.1135 \\ 5.9874 & 0 \\ -1.9495 & 0 \\ -1.3772 & 0 \\ -1.9335 & 0 \end{bmatrix}, K\{2,2\} = \begin{bmatrix} -5.7189 & 0 \\ 651.1277 & 0 \\ 0 & 517.1135 \\ 6.0646 & 0 \\ -2.0352 & 0 \\ -1.3934 & 0 \\ -1.9096 & 0 \end{bmatrix}$$

$$K\{3,1\} = \begin{bmatrix} 1.9483 & 0 \\ 635.0631 & 0 \\ 0 & 517.1135 \\ -2.4646 & 0 \\ 5.9954 & 0 \\ -2.3159 & 0 \\ -2.1991 & 0 \end{bmatrix}, K\{3,2\} = \begin{bmatrix} 1.9390 & 0 \\ 635.0635 & 0 \\ 0 & 517.1135 \\ -2.4646 & 0 \\ 5.9954 & 0 \\ -2.3255 & 0 \\ -2.1991 & 0 \end{bmatrix},$$

$$K\{4,1\} = \begin{bmatrix} 1.9634 & 0 \\ 635.0829 & 0 \\ 0 & 517.1135 \\ -1.3740 & 0 \\ -2.0433 & 0 \\ 5.9308 & 0 \\ -1.8363 & 0 \end{bmatrix}, K\{4,2\} = \begin{bmatrix} 1.9533 & 0 \\ 635.0833 & 0 \\ 0 & 517.1135 \\ -1.3740 & 0 \\ -2.0433 & 0 \\ 5.9308 & 0 \\ -1.8458 & 0 \end{bmatrix}$$

$$S\{1\}=\begin{bmatrix} 407.5055 & -308.8860 & 0 & 7.6880 & 7.6328 & 7.6596 & 7.7333 \\ -6.0909 & 643.5909 & 0 & 0.1916 & 0.1907 & 0.1913 & 0.1932 \\ 0 & 0 & 530.3728 & 0 & 0 & 0 & 0 \\ 7.9730 & -2.9759 & 0 & 491.0987 & -11.5317 & -11.5083 & -11.5358 \\ 7.8958 & -1.0047 & 0 & -11.5317 & 340.4809 & -11.5320 & -11.4950 \\ 7.8798 & 0.7448 & 0 & -11.5083 & -11.5320 & 360.8030 & -11.5355 \\ 7.8986 & 3.0230 & 0 & -11.5358 & -11.4950 & -11.5355 & 323.6297 \end{bmatrix},$$

$$S\{2\}=\begin{bmatrix} 407.4692 & -308.8394 & 0 & 7.7307 & 7.6735 & 7.6409 & 7.6595 \\ -6.0885 & 643.5449 & 0 & 0.1931 & 0.1913 & 0.1908 & 0.1913 \\ 0 & 0 & 530.37 & 0 & 0 & 0 & 0 \\ 7.8958 & 3.0887 & 0 & 491.0987 & -11.5317 & -11.5083 & -11.5358 \\ 7.9718 & -3.0399 & 0 & -11.5317 & 340.4809 & -11.5320 & -11.4950 \\ 7.8960 & -0.6487 & 0 & -11.5083 & -11.5320 & 360.8030 & -11.5355 \\ 7.8707 & 1.1174 & 0 & -11.5358 & -11.4950 & -11.5355 & 323.6297 \end{bmatrix}$$

$$S\{3\}=\begin{bmatrix} 407.4590 & -308.8765 & 0 & 7.6561 & 7.7302 & 7.6760 & 7.6326 \\ -6.0899 & 643.5709 & 0 & 0.1912 & 0.1931 & 0.1913 & 0.1906 \\ 0 & 0 & 530.37 & 0 & 0 & 0 & 0 \\ 7.8798 & 0.6400 & 0 & 491.0987 & -11.5317 & -11.5083 & -11.5358 \\ 7.8960 & 3.0313 & 0 & -11.5317 & 340.4809 & -11.5320 & -11.4950 \\ 7.9722 & -3.0504 & 0 & -11.5083 & -11.5320 & 360.8030 & -11.5355 \\ 7.8984 & -1.1094 & 0 & -11.5358 & -11.4950 & -11.5355 & 323.6297 \end{bmatrix},$$

$$S\{4\}=\begin{bmatrix} 407.4017 & -308.8624 & 0 & 7.6403 & 7.6568 & 7.7306 & 7.6747 \\ -6.0910 & 643.5906 & 0 & 0.1908 & 0.1912 & 0.1930 & 0.1914 \\ 0 & 0 & 530.37 & 0 & 0 & 0 & 0 \\ 7.8986 & -0.7528 & 0 & 491.0987 & -11.5317 & -11.5083 & -11.5358 \\ 7.8707 & 1.0133 & 0 & -11.5317 & 340.4809 & -11.5320 & -11.4950 \\ 7.8984 & 2.9543 & 0 & -11.5083 & -11.5320 & 360.8030 & -11.5355 \\ 7.9742 & -3.0310 & 0 & -11.5358 & -11.4950 & -11.5355 & 323.6297 \end{bmatrix}$$

The figures 1 to 3 present a trial for the throttle valve input given figure 1. The figure 2 and 3 gives respectively the observation of the variables $P_{man}(\theta)$, $P_{man_m}(\theta)$, $D_{thr_m}(\theta)$ and the estimation the cylinder 1 and 2 air mass flow. For the other cylinders the results are approximately the same. The maximal relative errors on the estimation figure 3 is about 10% which is acceptable in order to design a controller based on this observation method.

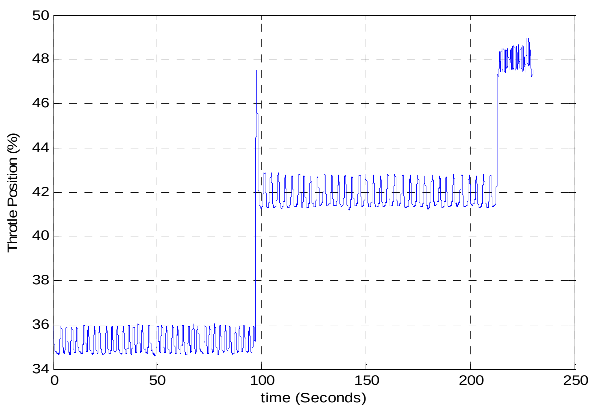

Fig 1. Throttle valve position

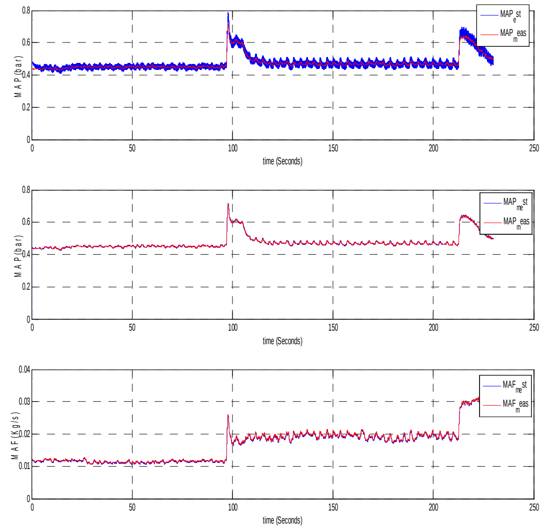

Fig 2. Observation of $P_{man}(\theta)$, $P_{man_m}(\theta)$ and $D_{thr_m}(\theta)$

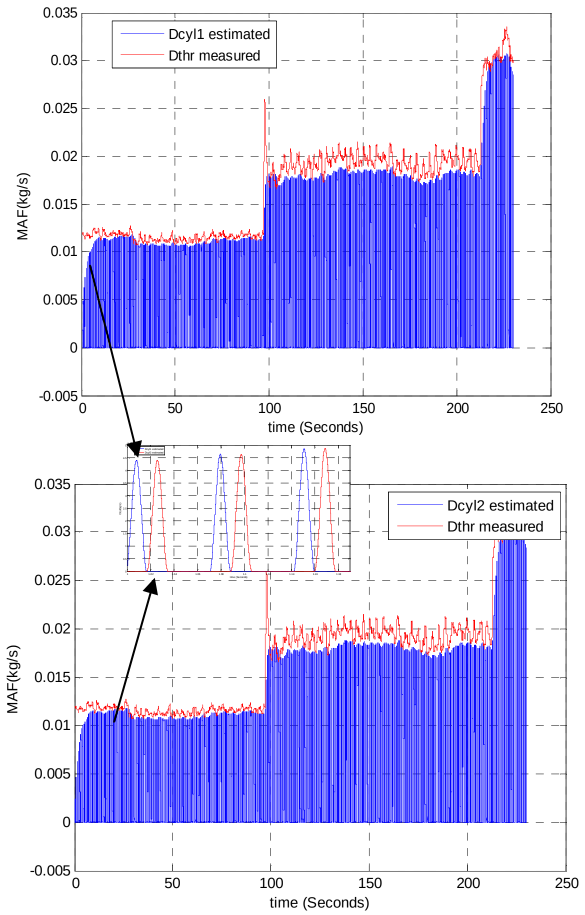

Fig 3. Estimation of cylinder 1 and 2 air mass flow

V. CONCLUSION

This work presents a new method to estimate the in-cylinder air mass flow using a periodic non linear discrete observer. Future work will expand the model and incorporate the cylinder pressure, the phenomenon of induction ram, and taking into account the overlap period between the different cylinders. This work will lead to the development of a controller design for the air fuel ratio of each cylinder.

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APPENDIX

Considering the cylinder firing sequence, with no loss of generality, a firing order 1-2-3-4 is assumed, as an example, the matrices for $0 < \theta < 180$ are given by:

$$A_1^{(\theta)} = \begin{bmatrix} 1 & 0 & 0 & \lambda_1\left[\bar{f}_1\right] & 0 & 0 & 0 \\ \left(\frac{\theta_s}{N_e \times \tau_{au1}}\right) & \left(1-\frac{\theta_s}{N_e \times \tau_{au1}}\right) & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \left(1-\frac{\theta_s}{N_e \times \tau_{au2}}\right) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1-\left(\frac{\theta_s}{N_e}\right).\mu_1\left[\bar{f}_1\right] & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2^{(\theta)} = \begin{bmatrix} 1 & 0 & 0 & \lambda_1\left[\underline{f}_1\right] & 0 & 0 & 0 \\ \left(\frac{\theta_s}{N_e \times \tau_{au1}}\right) & \left(1-\frac{\theta_s}{N_e \times \tau_{au1}}\right) & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \left(1-\frac{\theta_s}{N_e \times \tau_{au2}}\right) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1-\left(\frac{\theta_s}{N_e}\right).\mu_1\left[\underline{f}_1\right] & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

with $\theta_s = 6^\circ$ is the sample angle of the model, $\mu_i\left[\bar{f}_i\right]$ means that in equation (14) the non-linearity $Lift_i(\theta) = \bar{f}_i$, respectively for $\lambda_i\left[\bar{f}_i\right]$.

and the constant matrices: $C = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$.

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