Toward Analytical Solution of Optimal Control Problems for HEV energy management

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***Abstract*—In simulation, the hybrid vehicle energy management problem can be expressed as an optimal control one. The minimum principle can be used to derive efficient algorithms but the obtained solutions still require some numerical computation. In this work, a new formulation of the optimization problem allows to use analytical models and to obtain some analytical solutions of the control variables. The main benefit is the reduction of the computation requirements which may be critical if the optimization problem is solved, for example, within a real time predictive control framework.**

Keywords-Hybrid vehicle, Energy management, Optimal control.

I. Introduction

Hybrid vehicles use two energy sources for their propelling and one of them must be reversible. Usually one or more electrical machines are associated to an Internal Combustion Engine. This work focuses on energy management. Control strategies are algorithms that choose at each sample time the power split between the engine and the electrical machine(s) [1].

In simulation, the objective is to compute the minimal fuel consumption achievable by a given vehicle on a given driving cycle. Several approaches have been investigated, mainly the Dynamic Programming [2][3][4] and the Minimum Principle [5][6][7][8]. This work focuses on the latest. As the computational requirement of the resulting algorithm is relatively low (compared to the Dynamic Programming approaches), this algorithm can be embedded within a predictive control framework for the real time control of a hybrid powertrain [6]. However, even efficient, both the control variables and the initial Lagrangian parameter have to be numerically computed.

These real time control algorithms have proved their efficiency but were mostly implemented into rapid prototyping system such as the Dspace one. To be implemented on mass production ECU, it is often necessary to reduce CPU load.

In this work, a particular choice of the control variables and the use of analytical energetic models of both the electrical machine and the Internal Combustion Engine (ICE) allows to reduce CPU load significantly. The ICE is modeled using Willans approximation and a similar model is derived for the electric machine (EM) [10][11].

II. Energetic model of the considered hybrid vehicle

Let us consider the parallel single shaft arrangement, figure 1. The electric machine is coupled to the IC engine using for example a set of gears with a reduction ratio ρ.

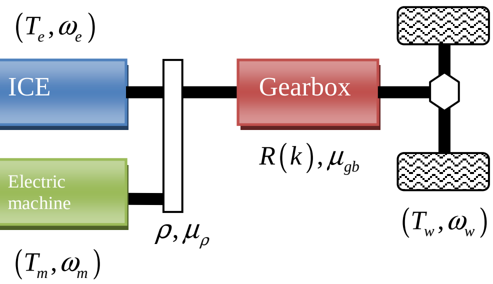

Figure 1 : Single shaft arrangement

Notation : letter ω is used for speed, T for torque, μ for efficiency, subscript m for the electric machine, e for the ICE, w for the wheel. $R(k)$ is the gearbox ratio of the k^{th} gear. ϑ is the IC engine state (1=on; 0=off). n_{gb} is the number of gearbox gears. i is the discrete time variable.

To lighten expressions, efficiencies are not taken into account $\mu_\rho=1$ and $\mu_{gb}=1$ (The reader may refer to [7] for a derivation of the algorithm with efficiencies) and the discrete time variable i is omitted when there is no ambiguity.

The driving cycle is defined by the torque and speed requested at the wheel $W(i)=\left(T_w(i)\ \omega_w(i)\right)^T$.

The ICE operating point is defined by $\left(T_e(i),\omega_e(i)\right)$ the ICE torque and speed. The EM operating point is $\left(T_m(i),\omega_m(i)\right)$.

The vehicle mechanical arrangement is defined by two equations:

$$T_w(i)=R(k(i))\cdot\left(T_e(i)\cdot\vartheta(i)+\rho\cdot T_m(i)\right) \quad (1)$$

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$$\omega_w(i) = \frac{\omega_m(i)}{R(k(i))} = \frac{\omega_e(i)}{\rho \cdot R(k(i))} \quad (2)$$

The ICE fuel flow is computed using a Willans model [10][11]:

$$T_e = e_0(\omega_e)\cdot u_f + e_1(\omega_e)\cdot u_f^2 - T_{loss}(\omega_e) \quad (3)$$

With u_f the fuel flow in g/s, T_e the IC engine torque available on the output shaft and T_{loss} the losses torque.

With $e_0(\omega_e) = e_{00} + e_{01}\cdot\omega_e + e_{02}\cdot\omega_e^2$, $e_1(\omega_e) = e_{10} + e_{11}\cdot\omega_e$, $T_{loss}(\omega_e) = p_0 + p_1\cdot\omega_e + p_2\cdot\omega_e^2$.

All the e_{ij} and p_i coefficients can be easily computed using a linear regression on the ICE fuel flow map obtained on test bench.

The IC engine speed and torque are limited:

$$0 \le T_e \le T_{e\_\max}(\omega_e) \quad (4)$$

$$\omega_{e\_\min} < \omega_e(i) < \omega_{e\_\max} \quad (5)$$

Therefore the fuel flow is also limited:

$$0 \le u_f \le u_{f\_\max}(\omega_e) \quad (6)$$

With s the sampling period, the IC engine fuel consumption over a driving cycle of length $N\cdot s$ is given by:

$$J = \sum_{i=0}^{N-1} u_f(i)\cdot\vartheta(i)\cdot s \quad (7)$$

The electric machine speed and torque are limited:

$$T_{m\_\min}(\omega_m) \le T_m \le T_{m\_\max}(\omega_m) \quad (8)$$

$$\omega_{m\_\min} \le \omega_m(i) \le \omega_{m\_\max} \quad (9)$$

A polynomial modeling of the electric machine and the whole electric chain (including battery efficiency) is considered. The battery current is then given by :

$$I_b(T_m,\omega_m) = m_0(\omega_m) + m_1(\omega_m)\cdot T_m + m_2(\omega_m)\cdot T_m^2 \quad (10)$$

The battery state of charge is given by:

$$x(i+1) = x(i) - \frac{1}{Q_{batt}}\cdot I_b(T_m,\omega_m)\cdot s \quad (11)$$

Nb: $T_{e\_\max}(\omega_e)$, $T_{m\_\min}(\omega_m)$ and $T_{m\_\max}(\omega_m)$ are usually computed using maps.

III. Optimization problem

The derivation of the optimization algorithm is classical. For more detailed information about this algorithm, the reader may refer to [7].

A. *Problem formulation*

In simulation, the driving cycle is known and so $W(i) = \begin{pmatrix} T_w(i) & \omega_w(i)\end{pmatrix}^T$ is known $\forall i \in [0, N-1]$.

Let us define $u(i) = \begin{pmatrix} u_c(i)^T & u_d(i)^T\end{pmatrix}^T$ the decision vector, composed of the continuous variable $u_c(i) = u_f(i)$ and the discrete variables $u_d(i) = \begin{pmatrix} k(i) & \vartheta(i)\end{pmatrix}^T$.

Combining the various speeds and torques constraints (4),(9) and (8),(9) with the mechanical arrangement equations (1) and(2) allows to compute the sets $\mathfrak{U}_d(W(i))$ and $\mathfrak{U}_c(W(i))$ of admissible values for the discrete and continuous decision variables. Let $\mathfrak{U}(W(i)) = \mathfrak{U}_d(W(i)) \times \mathfrak{U}_c(W(i))$ be the set of the admissible values for the whole decision vector $u(i)$.

The problem to be solved is written with only the decision variable $u(i)$ and the exogenous variable $W(i)$:

$$\min_{u(i)\in\mathfrak{U}(W(i))} \sum_{i=0}^{N-1} u_f(i)\cdot\vartheta(i)\cdot s \quad (12)$$

Under constraint:

$$x(i+1) = x(i) - I(u(i),W(i))\cdot s \quad (13)$$

$$x(N) = x(0) + \Delta x \quad (14)$$

With

$$I(u,W) = \frac{1}{Q_{batt}}\cdot I_b\left(\frac{T_{gb}}{\rho} - \frac{e_0(\omega_{gb})}{\rho}\cdot u_f - \frac{e_1(\omega_{gb})}{\rho}\cdot u_f^2 + \frac{T_{loss}(\omega_{gb})}{\rho}, \omega_m\right)$$

$\omega_m = \omega_w \cdot R(k)\cdot\rho$ and $T_{gb} = \frac{T_w}{R(k)}$.

Equation (14) allows to set the overall state of charge variation $x(N) - x(0)$.

Using (1),(3) and (10), $I(u,W)$ can be written as a fourth order polynomial in the continuous decision variable u_f :

$$\begin{aligned} I(u,W) = {} & \alpha_0(u_d,W) + \alpha_1(u_d,W)\cdot u_f + \alpha_2(u_d,W)\cdot u_f^2 \\ & + \alpha_3(u_d,W)\cdot u_f^3 + \alpha_4(u_d,W)\cdot u_f^4 \end{aligned} \quad (15)$$

B. *Minimum principle*

Let us define $\lambda(i)$ the Lagrangian parameters associated to the system dynamics (13). The Hamiltonian of problem (12)-(14) is given by:

$$\begin{aligned} & H(u(i),W(i),\lambda(i+1),x(i)) \\ & \quad = u_f(i)\cdot\vartheta(i)\cdot s - \lambda(i+1)\cdot\big(x(i) - I(u(i),W(i))\big)\cdot s \end{aligned} \quad (16)$$

The discrete variables $u_d(i)$ can be fixed using empirical rules, for example to enhance the vehicle drivability. In this case, $card(\mathfrak{U}_d(W(i))) = 1$ and necessary conditions can be derived using the minimum principle. If the discrete variables

$u_d(i)$ are optimized then the proposed algorithm is suboptimal.

For a given discrete decision vector u_d, the Hamiltonian is a fourth order polynomial of the control variable u_f :

$$H(u,W,\lambda,x)= \begin{pmatrix} -\lambda\cdot x+\lambda\cdot\alpha_0(u_d,W)+(\vartheta+\lambda\cdot\alpha_1(u_d,W))\cdot u_f \\ +\lambda\cdot\alpha_2(u_d,W)\cdot u_f^{\ 2}+\lambda\cdot\alpha_3(u_d,W)\cdot u_f^{\ 3} \\ +\lambda\cdot\alpha_4(u_d,W)\cdot u_f^{\ 4} \end{pmatrix}\cdot s \quad (17)$$

Along the optimal trajectory x^*, the optimal control $u^*(i)$ verify:

$$u^*(i)=\arg\min_{v\in U(W(i))} H(v,W(i),\lambda(i+1),x^*(i)) \quad (18)$$

$$\frac{\partial H(u^*(i),W(i),\lambda(i+1),x^*(i))}{\partial x^*(i)}=-\lambda(i)\Leftrightarrow\lambda(i)=\lambda(i+1) \quad (19)$$

From (18) and (19), it is sufficient to choose the initial Lagrangian parameter $\lambda(0)$ to compute the control sequence and thus the whole state trajectory. The value of the $\lambda(0)$ parameter that ensures constraint (14) is usually computed using a dichotic search [7]. This algorithm is denoted as “Proposed algorithm”.

Hamiltonian evaluation is usually one of the most computationally intensive tasks. It is therefore necessary to reduce the number of its evaluations for its minimization.

For the pure electric mode $\vartheta=0$, only the gear number can be optimized, the continuous variable is fixed by (1). For the hybrid mode, $\vartheta=1$, as $H(u,W,\lambda,x)$ is a fourth order polynomial in u_f, for each possible $u_d(i)$ the minimum of the Hamiltonian belongs to a set of real values that can be computed analytically, for example using, Cardan’s method.

Let us note $U_{sol}(W(i),u_d(i))$ the set of admissible solutions for the continuous variable:

$$U_{sol}(W(i),u_d(i))=\{u_1(u_d(i)),u_2(u_d(i)),u_3(u_d(i)), 0,u_{f\_\max}(\omega_w(i)\cdot\rho\cdot R(k(i)))\}\cap\mathbb{R} \quad (20)$$

with $\{u_1(u_d(i)),u_2(u_d(i)),u_3(u_d(i))\}\subset\mathbb{C}$ the three solutions of :

$$\frac{\partial H\left(\left(u_f\ u_d(i)^T\right)^T,W(i),\lambda(0),x^*(i)\right)}{\partial u_f}=0 \quad (21)$$

Thus (18) is simplified:

$$u^*(i)=\arg\min_{\substack{u_d(i)\in\mathcal{U}_d(W(i))\\ v\in U_{sol}(W(i),u_d(i))}} H\left(\left(v\quad u_d(i)^T\right)^T,W(i),\lambda(0),x^*(i)\right) \quad (22)$$

With the proposed decision variable, it is possible to reduce the computation of (18) to the evaluation of only $card(U_{sol}(W,u_d)\times u_d)\le 6\cdot n_{gb}$ possible values (with n_{gb} the number of available gears, 5 values for each gear when $\vartheta=1$ and n_{gb} more values for the pure electric mode $\vartheta=0$).

Moreover, in practice $card(\{u_1(u_d(i)),u_2(u_d(i)),u_3(u_d(i))\}\cap\mathbb{R})=1$ so (22) can be computed by only $card(U_{sol}(W,u_d)\times u_d)\le 4\cdot n_{gb}$ evaluations of the Hamiltonian.

IV. SIMULATION RESULTS

The proposed algorithm is applied to a hybrid vehicle built at the LAMIH, figure 2 in collaboration with PSA Peugeot Citroën and the financial support of the ADEME, the FEDER and the Nord Pas de Calais Region.

Figure 2 : The prototype built at the LAMIH

The considered powertrain is made up of:

- an IC engine with peak power of 55kw. It is a 1.4l gasoline engine.
- a DC electric motor (EM) able to supply up to 43kw of electric power.
- an energy storage system: it consists of twenty modules of 26Ah/12v lead acid batteries which ensures about 35km of autonomy in pure electric mode.
- a gear box: It has only two gears $(n_{gb}=2)$ corresponding approximately to the second and the fifth gear of a conventional car.

The vehicle weight is 1600kg, i.e., 300 kg more than the weight of conventional cars.

The control law is computed using the model described by equations (1), (2),(4) and (8) but is applied to a more detailed simulation model similar to ADVISOR [9]. The main differences between the model used for control synthesis and the simulation model are a more detailed battery model and additional dynamics of the IC engine.

The IC engine energetic behavior is simulated using maps over speed and torque. A detailed battery model and electrical machine (with its converter) is used. Therefore, the analytical models (3) and (10) are only used to compute the control $u^*(i)$.

A. *Validity of analytical models versus original maps*

As the proposed approach rely on energetic modeling of the IC engine and the electrical machine, it is mandatory to evaluate the validity of the considered models. Model parameters can be easily obtained using a linear regression on the original maps.

Figure 3 and figure 4 present the comparison of the fuel flow rate and the specific fuel consumption obtained using Willans model (equation (3)) and the original map. The maximum error $\left|T_e - e_0(\omega_e)\cdot u - e_1(\omega_e)\cdot u^2 + T_{loss}(\omega_e)\right|$ is 6 Nm and the RMS error is 1.94 Nm.

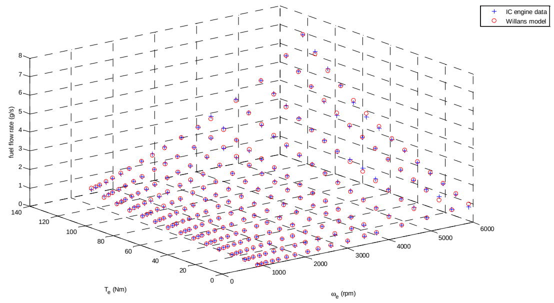

Figure 3 : Comparison of fuel consumption obtained using original data and the Willans model

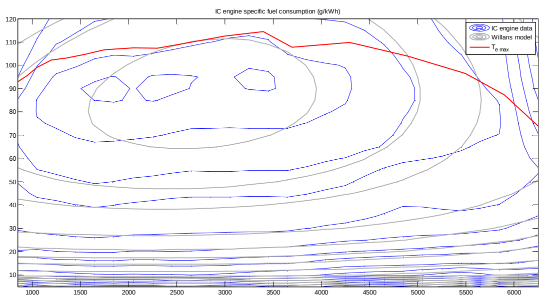

Figure 4 :Validation of the IC Specific fuel consumption

For the electrical machine, the maximum error $\left|I_b(T_m,\omega_m) - m_0(\omega_m) - m_1(\omega_m)\cdot T_m - m_2(\omega_m)\cdot T_m^{\,2}\right|$ is 6.16 A and the RMS error is 1.45 A.

B. *Simulation results for the NEDC driving cycle.*

The considered speed cycle is the Normalized European Driving Cycle. Its duration is $N\cdot s = 1180\ s$. As an example, the chosen overall state of charge variation is $\Delta x = 0$. In this case vehicle energy propelling is only drawn from fuel. The battery is only used as an energy buffer and the fuel consumption is comparable with the one measured on conventional vehicles. Using a dichotic search, the value $\lambda(0) = 1604$ that ensures $x(N) - x(0) = \text{-}3.38\cdot 10^{-2}\%$ is obtained. The corresponding fuel consumption is $5.31\, l/100km$. Simulation results are given figure 5.

C. *Computation efficiency*

To evaluate the computation efficiency of the “Proposed Algorithm”, a “Reference Algorithm” is considered. The reader may refer to [7] for a detailed description. The optimization problem is similar to the presented one, but the considered decision variable is the “classical” vector: $u(i) = \begin{pmatrix} T_e(i) & k(i) & \vartheta(i)\end{pmatrix}^T$. In this case the Hamiltonian is:

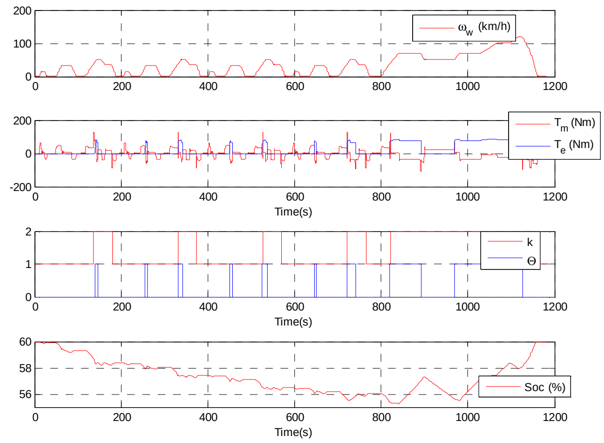

Figure 5 : Results on the NEDC driving cycle

$$H\big(u(i),W(i),\lambda(i+1),x(i)\big) = \dot{m}_{map}\big(u(i),W(i)\big)\cdot\vartheta(i)\cdot s - \lambda(i+1)\cdot\big(x(i) - I_{map}(u,W)\big)\cdot s \quad (23)$$

$\dot{m}_{map}(u,W)$ and $I_{map}(u,W)$ are functions evaluated with the original maps (over speed and torque) of the IC engine fuel consumption and the battery current. The continuous variable T_e belongs to a set of admissible values obtained by combining the various constraints (4),(5),(8) and (9) with the mechanical arrangement equations (1) and (2):

$$T_e(i) \in \mathcal{U}_{T_e}\big(W(i),u_d(i)\big) \quad (24)$$

With $\mathcal{U}_{T_e}\big(W(i),u_d(i)\big) = \left[T_{e\_min}{}'\big(w_e(i)\big), T_{e\_max}{}'\big(w_e(i)\big)\right]$

Along the optimal trajectory x^*, the optimal control value $u^*(i)$ satisfies:

$$u^*(i) = \underset{\substack{u_d(i)\in\mathcal{U}_d(W(i))\\ v\in\mathcal{U}_{Te}(W(i),u_d(i))}}{\arg\min}\ H\Big(\big(v \quad u_d(i)^T\big)^T, W(i), \lambda(0), x^*(i)\Big) \quad (25)$$

If the fuel consumption $\dot{m}_{map}$ and battery current I_{map} are computed using linear interpolation based on the original maps, then $\dot{m}_{map}(u,W)$ and $I_{map}(u,W)$ are piecewise linear with respect to the ICE torque T_e that belongs to a more restricted set:

$$T_e \in \mathcal{U}_{sol}{}'(W,u_d) \subset \mathcal{U}_{T_e}(W,u_d) \quad (26)$$

With $\mathcal{U}_{sol}{}'(W,u_d) = \left\{\mathcal{U}_{vertices}(W,u_d), T'_{e\_min}(w_e), T'_{e\_max}(w_e)\right\}$

And $\mathcal{U}_{vertices}\big(W(i),u_d(i)\big)$ the set obtained by joining all the torque vertices of the fuel consumption map and the battery current map with the mechanical arrangement equations (1) and (2). So equation (25) can be simplified:

$$u^*(i) = \underset{\substack{u_d(i)\in\mathcal{U}_d(W(i)) \\ v\in\mathcal{U}_{sol}'(W,u_d)}}{\arg\min}\ H\left(\left(v \quad u_d(i)^T\right)^T, W(i), \lambda(0), x(i)\right) \tag{27}$$

With n_{Tice} and n_{Tm} the vertices number for the IC engine fuel consumption map and battery current map, in hybrid mode $(\vartheta=1)$, there are $n_{Tice}+n_{Tm}$ possible solutions on the map vertices and 2 bounds on the continuous variables. In pure electric mode $(\vartheta=0)$, there are n_{gb} additional values to be considered:

$$card\left(\mathcal{U}_{sol}'(W,u_d)\times u_d\right) \le n_{gb}\cdot\left(n_{Tice}+n_{Tm}+3\right) \tag{28}$$

For the considered vehicle and maps, $n_{Tice}=18$ and $n_{Tm}=26$ and therefore, at each sampling time, the maximum number of values of the continuous variable considered for the minimization of the Hamiltonian is :

- $n_{gb}\cdot\left(n_{Tice}+n_{Tm}+3\right)=98$ for the Reference Algorithm
- $4\cdot n_{gb}=8$ for the Proposed Algorithm.

Remark: in practice, according to the actual driving conditions $W(i)$ at sampling time $i\cdot s$, this maximum number of Hamiltonian evaluations is not likely to be reached since $card\left(\mathcal{U}_d\left(W(i)\right)\right)$ is usually lower than $2\cdot n_{gb}$: the available gears k and IC engine state ϑ are restricted by constraints (1) ,(2),(4),(5),(8) and (9).

The computation time of the simulation model and the control law is below 6 seconds on a standard laptop (Windows XP, Intel Core 2 Duo 2.66 GHz processor, 4 Go) for both algorithms. It is highly variable depending on the operating system activity (disk cache, tasks management, etc.). Still, most of the computation time is used by the simulation solver rather than the control law computation.

Therefore, n_{add}, the number of additions/subtractions, and n_{mul}, the number of multiplications/divisions, required to compute all the control values along the NEDC driving cycle are chosen as a criterion to compare the computation efficiency of both algorithms.

Let us recall that the same control algorithm is used except, of course, for the Hamiltonian evaluation. The simulation model is the same in both cases. The obtained results are given in table 1.

	Proposed Algorithm	Reference Algorithm	Improvement (%)
n_{add}	98 400	836 704	88.2 %
n_{mul}	201 794	461 681	56.3 %
$n_{add}+n_{mul}$	309 194	1 298 385	76.2 %
$\frac{n_{add}}{N}$	84	710	88.2 %
n_{mul}/N	179	392	56.3 %
$\left(n_{add}+n_{mul}\right)/N$	262	1101	76.2 %

Table 1 : computation efficiency of both algorithms

The proposed algorithm allows a 74.3% reduction of the number of total operations (additions, subtractions, multiplications, divisions). A careful analysis of the source codes shows that interpolation routines require most of the CPU load. It should be noticed that even for the Proposed Algorithm, some interpolation routines are needed, for example to compute the torque limits, equations (4) and (8).

The computational time can be critical for example, if this optimization algorithm is embedded within a predictive framework for the real time control of a real hybrid powertrain [19]. The average number of operations on this driving cycle for the Proposed Algorithm is about 300 per second for the proposed algorithm which is far below the limits of current ECU available to automobile manufacturers.

D. *Fuel consumption comparison*

At last, to validate the proposed algorithm, it should be checked that the difference between the analytical models and the original maps does not have a significant impact on the obtained fuel consumption.

As the hybrid vehicle has two energy sources, it important to express its fuel consumption as a function of the overall state of charge variation $\Delta x = x(N)-x(0)$. The comparison of results obtained using the Proposed Algorithm and the Reference Algorithm is given figure 6. The fuel consumption difference do not exceed $0.15\, l/100km$ and its RMS value is $1.6\cdot10^{-1}\, l/100km$. It can be therefore concluded that the proposed models do not have a significant impact on the fuel consumption.

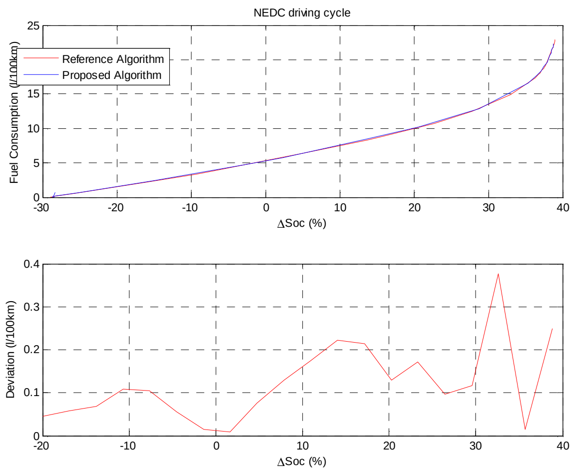

Figure 6 : Results on the NEDC driving cycle

V. CONCLUSION

The energetic management problem of hybrid vehicle has been written as an optimal control problem. Using the minimum principle, efficient algorithms have been derived. $u(i)=\left(T_e(i) \quad k(i) \quad \vartheta(i)\right)^T$ is a classical choice of the control variable. In this case, the Hamiltonian minimization (which is the core of the algorithm) requires its evaluation for a huge set of values, constructed by considering the torque vertices of the original fuel consumption map and the battery current map.

If a Willans model of the IC engine is used together with second order polynomial model of the battery current, a convenient decision vector includes the fuel consumption : $u=\left(u_f \quad k \quad \vartheta\right)^T$. In this case the Hamiltonian is a fourth order polynomial of the fuel consumption and its possible local minima can be analytically computed. This allows a significant reduction of the computational requirement.

On the NEDC driving cycle, the number of additions, subtractions, multiplications and divisions required by the control law computation is reduced by 74.3%. Simulations have shown that the obtained fuel consumption increase is not greater than $0.17\ l/100km$.

Future work will be devoted to the use of the analytical minima of the Hamiltonian to estimate the initial Lagrangian parameter $\lambda(0)$ without using a dichotic search. This work will also be applied to the BELHYSYMA project.

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