Backstepping Control Design of a Supercapacitor Storage Subsystem for Traction Applications

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Abstract—**A backstepping approach is used for control design of a supercapacitor storage and braking system for a series hybrid electric vehicle. The step by step design procedure allows defining a control structure to guarantee stability for the overall system using a recursive Lyapunov design. Inherent nonlinearities in the model, and in particular involving the control input for the supercapacitor are dealt with by properly defining the errors and internal reference signals. Simulation results demonstrate the effectiveness of the control structure.**

I. Introduction

A supercapacitor (SC) storage system for a series hybrid electric vehicle was developed and control strategies were proposed to maximize energy usage in the SC in [1] and references therein. Due to their high power density, SC systems are now being proposed for various applications in traction, including electric and hybrid electric vehicles, and railway traction systems. The control strategy proposed in [1] was developed using the Energetic Macroscopic Representation (EMR) and its associated inversion-based control design procedure [2]. Whilst this method constitutes an efficient way to describe and develop control structures for complex systems, the overall stability of the system remains an issue.

The backstepping technique proposed in [3] is an important result for control law development and for recursive Lyapunov design [4]. The methodology leads to variations of cascade control structures and provides guidelines for controller tuning in order to systematically build functions to guarantee stability of the system.

In this paper, a backstepping approach is used to develop a globally stabilizing controller for the SC storage subsystem. The model of the system is briefly described in Section II and the backstepping control design is presented in Section III. The resulting control structure has strong similarities with the inversion-based control developed in [1] with the additional advantage of guaranteed stability. Simulation results demonstrate the performance of the control strategy in Section IV and Section V concludes the paper.

II. Model of the Storage and Braking Subsystems

The studied system was developed for a series hybrid electric vehicle [1]. The generation and traction parts of the vehicle are represented by equivalent current sources and the study concentrates on the SC and the associated braking subsystem (Fig. 1). The model is described by [1]:

dc-bus

$$C\frac{d}{dt}u_C = i_{gen} - i_{coup1} \tag{1}$$

Subsystems couplings

$$i_{coup1} = i_{coup2} + i_{tract} \tag{2}$$

$$i_{coup2} = i_{chopB} + i_{chopS} \tag{3}$$

Supercapacitor subsystem

$$L\frac{d}{dt}i_L = u_{chopS} - u_{Scaps} - r_L i_L \tag{4}$$

$$u_{chopS} = \alpha_{chopS}\, u_C \tag{5}$$

$$i_{chopS} = \alpha_{chopS}\, i_L \tag{6}$$

Braking subsystem

$$i_{RB} = \frac{u_{chopB}}{R_B} \tag{7}$$

$$u_{chopB} = \alpha_{chopB}\, u_C \tag{8}$$

$$i_{chopB} = i_{RB} \tag{9}$$

where the main variables are shown on Fig. 1, r_L is the internal resistor of the inductor, and α_{chopS} and α_{chopB} are respectively the modulation index of the SC chopper and braking resistor chopper, which are described by their average model.

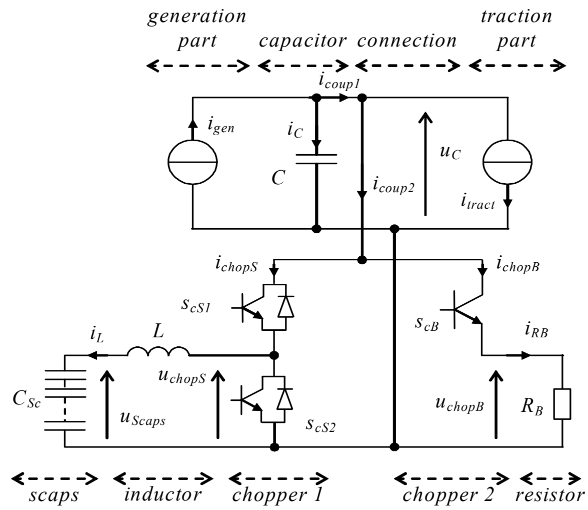

Figure 1: Supercapacitor storage subsystem

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III. Backstepping Control Design

The idea behind backstepping is to recursively design locally stabilizing controllers while building an energy function that will allow, at the last step, to design a globally stabilizing controller for the system. Henceforth, at each step:

1. consider a subsystem starting from the output;
2. define the error;
3. define an energy function as a function of the error (Lyapunov candidate function - *lcf*) — at each step, the previously defined *lcf* is augmented by additional terms;
4. determine a local controller to stabilize the subsystem assuming that the next steps will stabilize the inner loops errors — stabilization is obtained by forcing negativity of the time derivative of the energy function;
5. repeat the procedure until the last loop is reached. Then, define a control input that renders the time derivative of the overall *lcf* negative or semi-negative.

The main control objective is to maintain the dc-bus voltage constant at its reference value. The control inputs are the SC and braking choppers modulation indices. Load sharing between the two choppers is defined by a strategy presented in [1] to maximize the energy storage of the SC. The corresponding weighting signal is not defined analytically and is hence considered as an external input to the control law. Moreover, generator and traction subsystems' models are not used for control design and the corresponding currents are considered as perturbations to the storage subsystem. The generator and traction currents are assumed to be measurable during controller development.

A dc-voltage stabilization

Given the dc-voltage reference signal u_C^*, define the output error and its time derivative using (1):

$$e_1 = u_C - u_C^*, \quad (10)$$

$$\dot{e}_1 = \frac{1}{C}\left(i_{gen} - i_{coup1}\right) - \dot{u}_C^*. \quad (11)$$

Define a *lcf* corresponding to the energy in the error system, and an integral term with $K_{I1} \geq 0$ to force inclusion of an integrator into the controller, and evaluate its time derivative using (11):

$$V_1 = \frac{C}{2} e_1^2 + \frac{K_{I1}}{2}\left(\int_0^t e_1 \, d\tau\right)^2 \quad (12)$$

$$\begin{aligned} \dot{V}_1 &= C\, e_1 \dot{e}_1 + K_{I1}\, e_1 \int_0^t e_1 \, d\tau \\ &= e_1\left(i_{gen} - i_{coup1} - C\, \dot{u}_C^* + K_{I1} \int_0^t e_1 \, d\tau\right) \end{aligned} \quad (13)$$

Define the SC and braking current error as

$$e_{coup2} = i_{coup2} - i_{coup2}^* \quad (14)$$

Substituting (2), (3) and (14) into (13) results into

$$\dot{V}_1 = -e_1 e_{coup2} - e_1\left(i_{coup2}^* - i_{gen} + i_{tract} + C\, \dot{u}_C^* - K_{I1} \int_0^t e_1 \, d\tau\right) \quad (15)$$

Defining the total SC and braking current reference as

$$i_{coup2}^* = i_{gen} - i_{tract} - C\, \dot{u}_C^* + K_{P1} e_1 + K_{I1} \int_0^t e_1 \, d\tau \quad (16)$$

results into

$$\dot{V}_1 = -e_1 e_{coup2} - K_{P1} e_1^2 \quad (17)$$

so that for V_1, and henceforth e_1, decreases to zero if $K_{P1}>0$ and e_{coup2} converges to zero. The next steps aim at meeting this last condition.

B Current repartition

Considering (3) and (14), we define the specific errors for the SC and braking subsystems:

$$e_{coup2} = e_{21} + e_{22} \quad (18)$$

$$e_{21} = i_{chopB} - i_{chopB}^* \quad (19)$$

$$e_{22} = i_{chopS} - i_{chopS}^* \quad (20)$$

Following the strategy from [1], the reference signals are defined as follows, with function k_{ED} being defined in [1]:

$$i_{chopB}^* = k_{ED}\, i_{coup2}^* \quad (21)$$

$$i_{chopS}^* = \left(1 - k_{ED}\right) i_{coup2}^* \quad (22)$$

Such that (17) becomes

$$\dot{V}_1 = -e_1 e_{21} - e_1 e_{22} - K_{P1} e_1^2 \quad (23)$$

C Braking subsystem

Using (7), (8), and (9), (19) is written as

$$e_{21} = \frac{\alpha_{chopB}\, u_C}{R_B} - i_{chopB}^* \quad (24)$$

To avoid division by the dc-bus capacitor voltage in the control law, use (10) to rewrite (24) as

$$e_{21} = \frac{\alpha_{chopB}\, e_1}{R_B} + \frac{\alpha_{chopB}\, u_C^*}{R_B} - i_{chopB}^* \quad (25)$$

Considering that the braking subsystem contains no energy storage element, and assuming accurate knowledge of R_B, the current error can be removed from (23) by choosing

$$\alpha_{chopB} = \frac{R_B}{u_C^*} i_{chopB}^* + \frac{R_B}{u_C^*} K_B\, e_1 \quad (26)$$

such that, using (25) and (26), (23) simplifies to

$$\dot{V}_1 = -e_1 e_{22} - \left(\frac{\alpha_{chopB}}{R_B} + K_B + K_{P1}\right) e_1^2 \quad (27)$$

which is of the same form as (17) and has the same sign properties considering that $0 \leq \alpha_{chopB} \leq 1$.

D Supercapacitor subsystem

Define the current error in the SC subsystem as

$$e_{22} = i_{chopS} - i_{chopS}^* = \alpha_{chopS}\left(i_L - \frac{i_{chopS}^*}{\alpha_{chopS}}\right) = \alpha_{chopS}\, e_{221} \quad (28)$$

$$e_{221} = i_L - \frac{i_{chopS}^*}{\alpha_{chopS}} \quad (29)$$

Defining the *clf* as

$$V_2 = V_1 + \frac{L}{2} e_{221}^2 + \frac{K_{I2}}{2}\left(\int_0^t e_{221} d\tau\right)^2 \tag{30}$$

with $K_{I2}>0$ and trying to impose a function of the form

$$\dot{V}_2 = -\left(\frac{\alpha_{chopB}}{R_B} + K_B + K_{P1}\right) e_1^2 - K_{P2} e_{221}^2 \tag{31}$$

results into a nonlinear differential equation for α_{chopS}, which raises issues about initial conditions, stability, and implementation of the solution for α_{chopS}. However, the steady state of α_{chopS} can be approximated by

$$\alpha_{chopS} \cong \frac{u_{Scaps}}{u_C^*} \cong \frac{u_{Scaps}^*}{u_C^*} \tag{32}$$

so that we define

$$\hat{\alpha}_{chopS} = \frac{u_{Scaps}^*}{u_C^*} \tag{33}$$

$$e_{22} = \alpha_{chopS}\left(i_L - \frac{i_{chopS}^*}{\hat{\alpha}_{chopS}}\right) \tag{34}$$

$$e_{221} = i_L - \frac{i_{chopS}^*}{\hat{\alpha}_{chopS}} \tag{35}$$

$$\dot{e}_{221} = \frac{1}{L}\left(u_{chopS} - u_{Scaps} - r_L i_L\right) - \frac{d}{dt}\left(\frac{u_C^* \, i_{chopS}^*}{u_{Scaps}^*}\right) \tag{36}$$

Using (5), (27), (34), (35), and (36), time derivative of (30) results into

$$\begin{aligned}\dot{V}_2 = &-\left(\frac{\alpha_{chopB}}{R_B} + K_B + K_{P1}\right) e_1^2 + e_{221}\left(\alpha_{chopS} u_C^* - u_{Scaps}\right) \\ &+ e_{221}\left(-r_L i_L - L\frac{d}{dt}\left(\frac{u_C^* \, i_{chopS}^*}{u_{Scaps}^*}\right) + K_{I2}\int_0^t e_{221} d\tau\right)\end{aligned} \tag{37}$$

which equals (31) by choosing

$$\begin{aligned}\alpha_{chopS} = &\frac{1}{u_C^*}\left(-K_{P2} e_{221} - K_{I2}\int_0^t e_{221} d\tau + u_{Scaps} + r_L i_L\right) \\ &+ \frac{L}{u_C^*}\frac{d}{dt}\left(\frac{u_C^* \, i_{chopS}^*}{u_{Scaps}^*}\right)\end{aligned} \tag{38}$$

Function (30) is positive definite and (31) is negative definite if $K_{P2}>0$. Then the error system is globally stable under hypotheses that the inputs are continuous and bounded.

IV. Control Performance Evaluation

The system described in [1] was simulated to evaluate the performance of the control law to disturbance rejection and parameter uncertainties (Fig. 2): i_{gen} and i_{tract} are not used in (16) and inductor L is under evaluated by 20% in the controller. All the errors converge to zero for constant disturbances and the responses are very close to those obtained by simulation and experimentally in [1]. However, as opposed to the approach in [1], stability could be formally proven for the complete system by using the backstepping design approach.

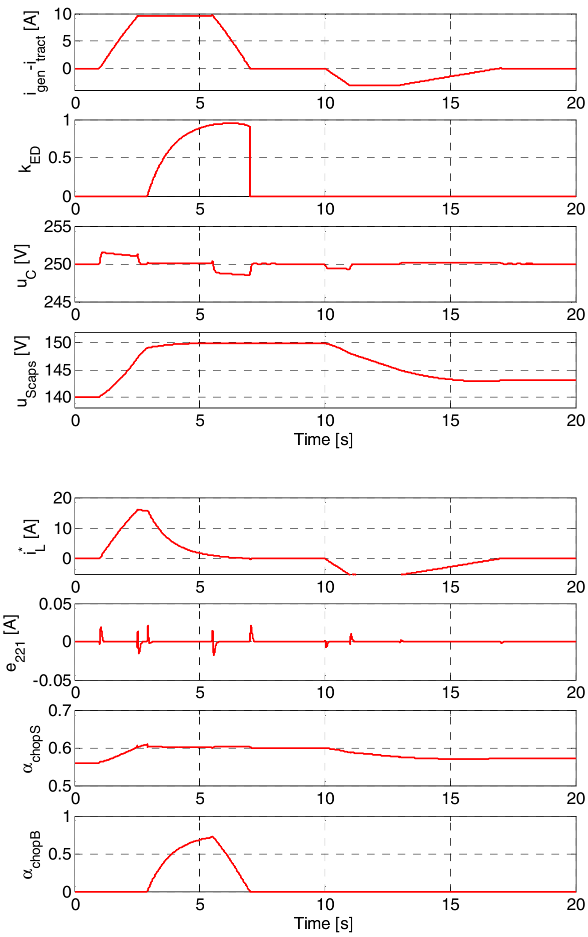

Figure 2: Simulation results of the SC system ($u_C^* = 250V$, K_B=0).

V. Conclusion

Backstepping is a recursive approach that allows structuring the control scheme in complex systems. The design offers flexibility in the choice of the stabilizing controller. Nonlinearities in the control input of the model are dealt with by approximating the nonlinearity model to define an inner reference signal. Whilst the final control scheme has strong similarities with the inversion-based control structure obtained from the Energetic Macroscopic Representation in [1], stability of the complete system is guaranteed with the backstepping approach.

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