# Electrical Loss Minimization Strategy for Interior Permanent Magnet Synchronous Motor Drives

#### Meifen CAO

Department of Electrical Engineering
Tokyo Metropolitan College of Industrial Technology
<a href="mailto:cmf@s.metro-cit.ac.jp">cmf@s.metro-cit.ac.jp</a>

Abstract-An electrical loss minimization strategy for interior permanent magnet synchronous motor drives is proposed in this paper. Until now a Look-up Table based maximum torque per armature current control strategy was proposed by authors. That is a control to minimize copper losses. Meanwhile, the efficiency deteriorates as a result that the iron losses increase sharply in high speed area. In this paper, a model considering iron losses is discussed and the relationship between the electrical losses and d-q axes current, motor torque, motor speed is clarified at first. Next, measurement experiments are made to prove that there is an optimal armature current phase which minimize the electrical losses for given motor speed and load torque. The current vector trajectories at which the copper losses will be minimal or the electrical losses will be minimal are given. Then, a Look-up Table based electrical loss minimization strategy is proposed. The driving efficiency with  $i_{x} = 0$  control,

maximum torque per armature current control and electrical loss minimization control are compared with experiment results. The effectiveness of the proposed control is verified by experiments.

#### I. INTRODUCTION

The concerns over global environmental problems, such as increased CO2 in the atmosphere, have prompted researches for vehicles free from fossil fuel resources. Electric vehicles (EV) play an important role in the solution to energy and environmental problems. Besides being environment-friendly and requiring small parking space, electric scooters contribute a lot to the reduction of traffic congestion because of their small space on the road. From an environmental point of view, electric scooters are excellent vehicles. Recently, the electric scooters industry is growing very rapidly in China. The possession number of electric scooters in China is over 50 million in 2008 [1]. It will be possible that electric scooters will be spread not only in China but also in worldwide in near future.

On the other hand, the driving distances per charge of EV are not long enough to meet the needs of users now. Because the number of on-board batteries of an electric scooter is limited, it is very important to realize high efficiency. Permanent magnet synchronous motors (PMSM) tend to become the mainstream for EV because of their high power density and small-sized drive systems [2]. Furthermore, interior permanent magnet synchronous motors (IPMSM) have been increasingly developed and widely used to meet the high efficiency and performance requirement. Because of

#### Nobukazu HOSHI

Department of Electrical Engineering Tokyo University of Science <a href="mailto:nhoshi@rs.noda.tus.ac.jp">nhoshi@rs.noda.tus.ac.jp</a>

high energy density permanent magnets are implanted structurally, IPMSM have good mechanical intensity as well as the ability to produce reluctance torque in addition to the magnet torque.

It is known from the relation between motor torque and armature current phase that there is an optimum current phase at which the maximum motor torque is reached for a given armature current. This means it is a condition of generating torque most effectively for a given armature current. Until now a Look-up Table based maximum torque per armature current control strategy was proposed by authors [3]. However, it is a control to minimize copper losses because the armature current is minimized for a given load. Meanwhile, the efficient deteriorates as a result that the iron losses increase sharply in high speed area.

The optimal direct-axis current can be calculated by differentiating the relation between the electrical losses and dq axes current, motor torque, and motor speed according to the motor torque and speed [4]. However, in the case of  $L_d \neq L_q$ , for example, in the case of an IPMSM with magnetic saliency, it is very difficult to solve the equations and it seems impossible to obtain the closed solutions. Even an approximate solution is derived [5], it seems to be difficult to finish the calculation in a control period. Otherwise, an online loss minimization control [6] was proposed. In this method, the optimal armature current phase is calculated online with motor parameters. The variation of the motor parameters in operation will make a difference in efficiency.

In this paper, a model considering iron losses is discussed and the relationship between the driving efficiency and d-q axes current, motor torque, motor speed is clarified at first. Next, measurement experiments are made to prove that (1) there is an optimal armature current phase which minimize the copper losses, (2) there is an optimal armature current phase which minimize the electrical losses for given motor speed and load torque. The current vector trajectories which are used as the Look-up Table (LUT) are given. Using the optimum current phase in LUT as the references of the driving control, the copper losses will be minimal or the electrical losses will be minimal. The driving efficiency for  $i_d = 0$  control, maximum torque per armature current control and electrical losses minimization control are compared with experiment results. The effectiveness of the proposed control is verified by experiments.

# II. MATHEMATICAL MODELLING OF IPMSM CONSIDERING IRON LOSSES

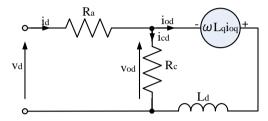
The equivalence circuit in which the iron losses are approximately described by an equivalent resistance  $R_c$  is shown in Fig. 1.

With reference to Fig.1, the voltage equations of IPMSM are shown as,

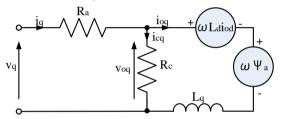
$$\begin{bmatrix} v_d \\ v_q \end{bmatrix} = R_a \begin{bmatrix} i_{od} \\ i_{oq} \end{bmatrix} + (1 + \frac{R_a}{R_c}) \begin{bmatrix} v_{od} \\ v_{oq} \end{bmatrix} + p \begin{bmatrix} L_d & 0 \\ 0 & L_q \end{bmatrix} \begin{bmatrix} i_{od} \\ i_{oq} \end{bmatrix}$$
(1)

where

$$\begin{bmatrix} v_{od} \\ v_{oq} \end{bmatrix} = \begin{bmatrix} 0 & -\omega L_q \\ \omega L_d & 0 \end{bmatrix} \begin{bmatrix} i_{od} \\ i_{oq} \end{bmatrix} + \begin{bmatrix} 0 \\ \omega \psi_a \end{bmatrix}$$
 (2)



(a) Equivalent circuit of d-axis.



(b) Equivalent circuit of *q*-axis.

Fig. 1. Equivalent circuits with iron-core losses [7].

$$i_{od} = i_d - i_{cd}, \quad i_{oq} = i_q - i_{cq}, \quad \vec{i}_{oa} = \vec{i}_{od} + \vec{i}_{oq}$$
 (3)

$$i_{cd} = -\frac{\omega L_q i_{oq}}{R_c}, \quad i_{cq} = \frac{\omega (\psi_a + L_d i_{od})}{R_c}$$
(4)

The motor torque is given by

$$T = \vec{i}_{oa} \times P_n \vec{\Psi}_o = P_n \Psi_a i_{oq} + P_n (L_d - L_q) i_{od} i_{oq}$$
(5)

The copper losses  $W_c$  and the iron losses  $W_i$  are shown as,

$$W_{c} = R_{a} I_{a}^{2}$$

$$= R_{a} \left\{ (i_{od} - \frac{\omega L_{q} i_{oq}}{R_{c}})^{2} + (i_{oq} + \frac{\omega (\psi_{a} + L_{d} i_{od})}{R_{c}})^{2} \right\}$$
(6)

$$W_{i} = R_{c} (i_{cd}^{2} + i_{cq}^{2}) = \frac{\omega^{2} \{ (\psi_{a} + L_{d} i_{od})^{2} + (L_{q} i_{oq})^{2} \}}{R_{c}}$$
(7)

The electrical losses  $W_{loss}$  can be given as,

$$W_{loss} = W_c + W_i \tag{8}$$

where,

$$I_a$$
 Armature current  $(=|i_a|=\sqrt{i_d^2+i_q^2})$ 

 $P_n$  Number of pole pairs

Ψ<sub>a</sub> Total interlinkage magnetic flux

Ψ<sub>a</sub> Permanent magnet flux

 $R_a$  Armature winding resistance

 $R_{\circ}$  Iron losses resistance

 $L_d$ ,  $L_a$  d – and q –axis inductances

 $i_d$ ,  $i_a = d$  – and q –axis components of armature current

 $i_{cd}$ ,  $i_{ca}$  d – and q-axis components of iron losses current

If iron losses can be neglected, (5) can be written to (9),

$$T = P_n \left\{ \Psi_a I_a \cos \beta + \frac{1}{2} (L_q - L_d) I_a^2 \sin 2\beta \right\}$$
 (9)

Here,  $\beta$  is the current phase shown in Fig.2.

The optimum current phase  $\beta_m$ , at which the maximum motor torque [7] is reached for a given armsture current, can be derived from the derivative of T with respect to  $\beta$  in (9).

$$\beta_m = \sin^{-1} \left( \frac{-\Psi_a + \sqrt{\Psi_a^2 + 8(L_d - L_q)^2 I_a^2}}{4(L_q - L_d)I_a} \right)$$
(10)

If the current phase is controlled to be  $\beta_m$ , the armature current will be minimized for a given load torque and the copper losses will be minimal.

When iron losses are considered, from (5),  $i_{oq}$  can be expressed as a function of torque T and current  $i_{od}$ . So the electrical losses can be expressed as a function of torque T, current  $i_{od}$  and speed  $\omega$  from (6) (7) and (8). It can be shown as bellow,

$$W_{loss}(T, i_{od}, \omega) = W_c(T, i_{od}, \omega) + W_i(T, i_{od}, \omega)$$
(11)

From (11), it is known that at given torque T and speed  $\omega$ , the electrical losses depend on the direct-axis current  $i_{od}$  only. So the electrical loss minimization condition at steady state for given torque T and speed  $\omega$  condition can be obtained by,

$$\frac{\partial W_{loss}}{\partial i_{od}}\bigg|_{T,\omega} = 0 \tag{12}$$

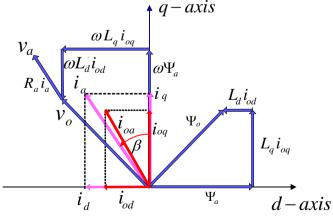


Fig. 2. Vector diagram of IPMSM.

To solve (12), the optimal direct-axis current  $i_{od}$  can be obtained. Thus the electrical losses can be minimized by control the direct-axis current. However, in the case of  $L_d \neq L_q$  , for example, in the case of IPM with magnetic saliency, it is very difficult to solve (12) and it seems impossible to obtain the closed solution of  $i_{od}$ . Otherwise from (13), it is known that when the electrical losses are minimized, the driving efficiency  $\eta$  can reach maximum.

$$\eta = \frac{P_{out}}{P_{in}} \times 100\% = \frac{P_{out}}{P_{out} + W_{loss} + W_m} \times 100\%$$
 where,  $W_m$  is the mechanical losses. (13)

#### III. PROPOSED CONTROL STRATEGY

In order to make the problem simple, the  $\beta$  shown in fig.2 is used to instead of the  $i_{od}$  in the proposed strategy. The proposed strategy is shown with steps A and B as below.

# Measurement Experiments of Relationships between $\beta$ to $I_a$ , $\beta$ to $W_{loss}$

For a given load torque T the amplitude of armature current  $I_a$  to different current phase references  $\beta^*$  can be measured. Furthermore, for a given load torque T and a given speed  $\omega$  the electrical losses  $W_{loss}$  to different current phase references  $\beta^*$  can be measured. The block diagram of the measurement is shown in Fig. 3. The experiments are made with a motor-generator set, the load is a DC generator connected with a variable resistance load. In order to keep the load torque and motor speed constant as the given load torque and speed during every measurement, a speed controller is included. Specifically, (9) can be rewritten to (14) and (11) can be rewritten to (15),

$$I_{a_{k}l} = f_{k}(T_{k}, \beta_{kl}^{*})$$
 (14)

$$W_{loss\_kil} = g_{ki}(T_k, \omega_{i,} \beta_{kil}^*)$$
(15)

where the subscript "k" means the k -th load torque, "i" means i -th speed and "l" means the l -th  $\beta^*$ . When references  $\beta^*$  is set to  $\beta_{k1}^*$  ,  $\beta_{k2}^*$  ...  $\beta_{km}^*$  , the  $I_{a\_k1}, \quad I_{a\_k2}, \quad \dots, \quad I_{a\_km}$  and the  $W_{loss\_ki1}, \quad W_{loss\_ki2},$  . ,  $W_{loss\_kim}$  can be measured. Using polynomial approximation for  $I_{a \ kl}$  and  $W_{loss \ kil}$  (l=1,2,...,m),  $\widetilde{f}_{\boldsymbol{k}}$  and  $\widetilde{g}_{\boldsymbol{k}\boldsymbol{i}}$  which are the approximate expression of  $f_{\boldsymbol{k}}$ and  $g_{ki}$  can be obtained. The optimum current phase  $\beta_{Im}$  at which  $I_a$  is minimal can be derived from the derivative of  $\widetilde{f}_k$  with respect to  ${\beta_k}^*$ . The optimum current phase  ${\beta_{ei}}_m$ at which the electrical losses  $W_{loss}$  is minimal can be derived from the derivative of  $\widetilde{g}_{ki}$  with respect to  ${\beta_{ki}}^*$ .

With the measurement experiments, the following items can be verified,

- There is an optimal  $\beta_{l_m}$  where the armature current is minimal for a given motor torque
- There is an optimal  $\beta_{eim}$  where the electrical losses are minimal for a given motor torque and a given speed.

#### Electrical Loss Minimization Control

From the measurement experiments it is known that  $\widetilde{f}_k$  and  $\widetilde{g}_{ki}$  are unimodal functions between  $\beta_{k1}$  to  $\beta_{km}$ and  $\beta_{kil}$  to  $\beta_{kin}$ . So a minimum value exists between  $\beta_{kl}$  to

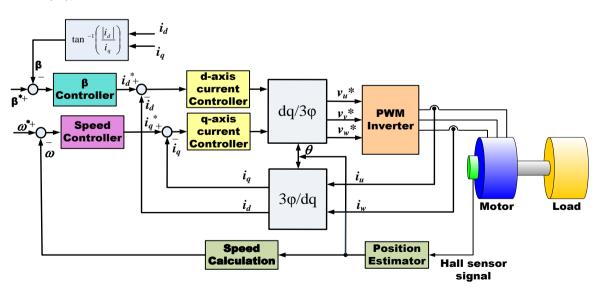


Fig.3. Block diagram of measurement experiments

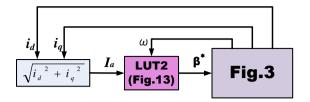


Fig.4. Block diagram of the electrical loss minimization control

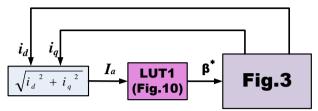


Fig.5. Block diagram of the armature current minimization control

 $\beta_{km}$  or  $\beta_{ki1}$  to  $\beta_{kin}$  for k-th load torque and i-th motor speed. The block diagram of the proposed electrical loss minimization control strategy is shown in Fig. 4. In this strategy, the current phase reference  $\beta^*$  is created from the LUT2 developed in section A according to the load torque and the motor speed. Otherwise, the block diagram of the armature current control is shown in Fig. 5. Here, the current phase reference  $\beta^*$  is created from the LUT1 developed in section A according to the load torque.

#### EXPERIMENTAL SETUP

In the experiments hardware setup, an IPMSM for electric motorcycles is used as a control object and a DC generator connected with a variable resistance load is used as the IPMSM's load. Their specifications are shown in Table I. The d- and q-axis inductances are measured at stand still condition using a LCR meter. The IPMSM-DCG set is shown in Fig.6. The DC generator used as a variable load is shown in Fig.7.

The control system set up shown in Fig.8 consists of inverter and MOSFET module (FM600TU-3A, by Mitsubishi). The inverter is composed of power board, control board, interface board and gate board. The SH7047 is used as a CPU on the control board.

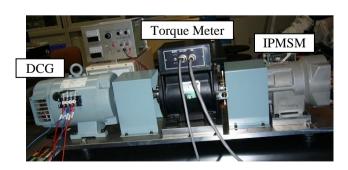


Fig.6. IPMSM-DCG set

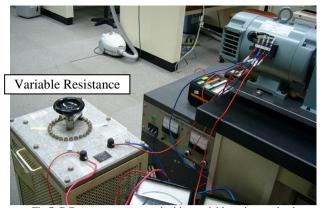


Fig.7. DC generator connected with a variable resistance load

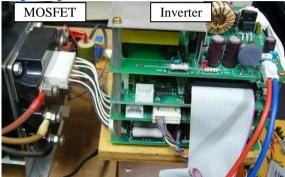


Fig.8. Control system

TABLE I SPECIFICATIONS OF IPMSM-DCG SET

Test motor					
Type: IPMSM	$L_d$	0.193[mH]			
Rated power: 580[W]	$L_q$	0.361[mH]			
Number of pole pairs: 2	$R_a$	$0.0374[\Omega]$			
$\Psi_a$ : 0.042[Wb]	$R_c$	50[Ω]			
Load generator					

Туре	Separately excited DCG
Rated power	1[kW]
Rated speed	5000[rpm]

### IV. EXPERIMENTAL RESULTS

## A Results of the Measurement Experiments

The measurement experiments for  $I_a$  -  $\beta$  (Fig.9) are performed as follows,

- 1. The test IPMSM is operated in a load condition.
- 2.  $I_a$  is measured for every  $\beta^*$  which is set from 0° to 25° with every 5°.
- 3. In the neighborhood of the minimum of  $I_a$ ,  $\beta^*$  is set with every 1° for additional measurements.
- 4. Plot  $I_a$  with respect to  $\beta^*$  and using polynomial approximation and minimal derivation, the optimum current phase  $\beta_m$  is obtained.
- 5. Repeat 1 to 4 for other load conditions.

The measurement experiments for  $W_{loss}$ - $\beta$  (Fig.11 and Fig.12) are performed as follows,

- 1. The test IPMSM is operated in the given load torque and motor speed.
- 2.  $W_{loss}$  is measured for every  $\beta^*$  which is set from 0° to 25° with every 5°.
- 3. In the neighborhood of the minimum of  $W_{loss}$ ,  $\beta^*$  is set with every 1° for additional measurements.
- 4. Plot  $W_{loss}$  with respect to  $\boldsymbol{\beta}^*$  and using polynomial approximation and minimal derivation, the optimum current phase  $\beta_m$  are obtained.
- 5. Repeat 1 to 4 for other load torques and motor speeds. Because a DC generator connected with a variable resistance load is used as the test IPMSM's load, the speed feedback control is necessary to keep the load torque constant during the measurement. The control block

diagram is shown in Fig.3.

The measured  $I_a$  -  $\beta$  and  $W_{loss}$  -  $\beta$  characteristics and the fitted curve are shown in Fig. 9, Fig. 11 and Fig. 12. The measured current vector trajectory at which the armature current will be minimal for a given load torque are shown in Fig.10 (LUT1). The measured current vector trajectory at which the electrical losses  $W_{loss}$  will be minimal for a given load torque are shown in Fig.13 (LUT2).

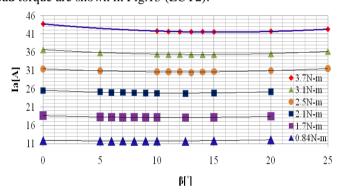


Fig.9. Measured  $I_a$  -  $\beta$  characteristics in the case of different load torque

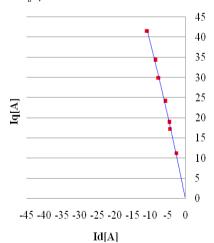


Fig. 10. Measured current vector trajectory at which the armature current will be minimal for a given load torque.

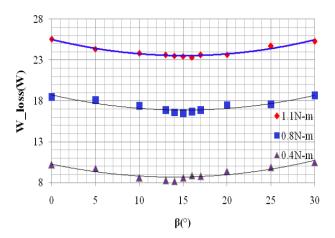


Fig.11. Measured  $W_{loss}$  -  $\beta$  characteristics (N=5000min<sup>-1</sup>)

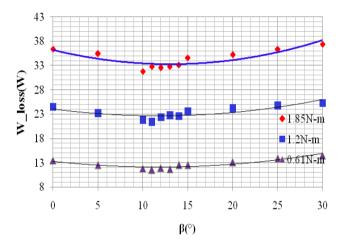


Fig.12. Measured  $W_{loss}$ - $\beta$  characteristics (N=3000min<sup>-1</sup>)

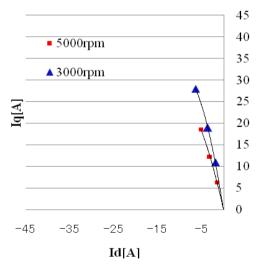


Fig.13. Measured current vector trajectory at which the electrical losses  $W_{loss}$  will be minimal for a given load torque and a given speed.

#### B Results of the Proposed Electrical Loss Minimization Control

The verification experiments of the proposed electrical loss minimization control are performed. The load resistance is adjusted to set up the load torque. The motor speed is controlled by the speed controller. The test IPMSM is driven under three control strategies. (A) The armature current phase reference  $\beta^*$  is set to 0 to achieve  $i_d = 0$  control. (B) The armature current phase reference  $\beta^*$  is created from the maximal torque per armature current control using LUT1 developed in experiment A. (C) The armature current phase reference  $\beta^*$  is calculated from the electrical loss minimization control using LUT2 obtained in experiment A.

A driving efficiency comparison of (A), (B) and (C), when the IPMSM is driven in the case of load torque T=1.1N-m when motor speed N=5000rpm, T=1.8N-m when N=3000rpm and T=3.7N-m when N=1500rpm, are shown in Table II.

TABLE II.

DRIVING EFFICIENCY COMPARISON OF (A), (B) AND (C)

Load condition	T=1.1N-m,	T=1.8N-m,	T=3.7N-m,
	5000rpm	3000rpm	1500rpm
$\eta$ [%] ( $\beta^* = 0^\circ$ )	91.5	91.2	91
$ \eta $ [%] $ (\beta^* = \beta_m \text{ from LUT 1}) $	93.1	93.5	93.8
$ \eta  [\%] \\ (\beta^* = \beta_m  \text{from LUT 2}) $	94.2	93.9	93.8

The results show that the driving efficiency is maximal when the test IPMSM is driven under the electrical loss minimization control. Because the  $L_d$  and  $L_q$  of the test IPMSM are small, about one-tenth of the normal one, the differences of the efficiency between the electrical loss minimization control (with LUT2) and the maximal torque per armature current control (with LUT1) are no so obvious. It is because the dependence of iron losses on current phase is not so obvious especially during low to middle speed range.

# V. CONCLUSIONS

In this paper, a model considering iron losses is discussed and the relationship between the electrical losses and d-q axes current, motor torque, motor speed is

clarified at first. Next, measurement experiments are made to prove that (1) there is an optimal armature current phase which minimize the copper losses, (2) there is an optimal armature current phase which minimize the electrical losses for given motor speed and load torque. The current vector trajectories which are used as the Look-up Table (LUT) are given. Using the optimum current phase in LUT as the references of the driving control, the copper losses will be minimal or the electrical losses will be minimal. The driving efficiency for  $i_d=0$  control, maximum torque per armature current control and electrical losses minimization control are compared with experiment results. The effectiveness of the proposed control is verified by experiments.

However, it can be concluded that for IPMSM with small  $L_d$  and  $L_q$  the influence of copper losses is larger than iron losses. In this case, it is sufficiently to use the LUT1 (maximal torque per armature current control) to drive IPMSM efficiently.

#### ACKNOWLEDGMENT

This research is supported financially by Grant-in-Aid for Scientific Research (B) of Japan Society for the Promotion of Science (21360137).

#### REFERENCES

- [1] "2008-2009 Annual Report on the Development of China's Automobile Industry" (in Chinese 汽车蓝皮书——中国汽车产业发展报告 2009), <a href="http://www.lgc.cn/xinwen/14.htm">http://www.lgc.cn/xinwen/14.htm</a>
- [2] S. Okuma, "Vehicular Technology," (in Japanese) T.IEE Japan, Vol. 122-D, No.7, 2002
- [3] M. Cao, J. Egashira and K. Kaneko, "High Efficiency Control of IPMSM for Electric Motorcycles," *IEEE 6<sup>th</sup> International Power Electronics and Motion Control*, Wuhan, China, 2009
- [4] S. Morimoto, Y. Yong and etc., "Loss Minimization Control of Permanent Magnet Synchronous Motor Drives," *IEEE Trans. 1E*, Vol.41, No.5, pp.511-517, 1994
- [5] C. Mademlis and N. Margaris, "Loss Minimization in Vector-Controlled Interior Permanent-Magnet Synchronous Motor Drives," *IEEE Trans. IE*, Vol.49, No.6, pp.1344-1347, 2002
- [6] M. Cao, "Online Loss Minimization Control of IPMSM for Electric Scooters", International Power Electronics Conference –ECCE ASIA-IPEC-Sapporo 2010, Sapporo, Japan, 2010
- [7] Y. Takeda, N. Matsui, S. Morimoto and Y. Honda, Design and Control of Interior Permanent Magnet Motors, (in Japanese) Ohmsha, 2001