High Performance Algorithms for the Control and Load Identification of Boost DC-DC Converters

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Abstract— **This paper proposes two high performance control methods and an on-line load identification algorithm for boost DC-DC converters. The first is a short-horizon model predictive control method whose solution can be computed analytically, thus rendering an algorithm that can be implemented on-line. Further, we show how to find a suitable performance index so as to achieve good response. The idea is to use the inductance current as the penalized output, instead of the output voltage. The second is a 3-level hysteresis control method which is a natural extension of the conventional hysteresis control method and has three layers in the hysteresis band. The key is to set the duty ratio in the central layer as the steady-state duty ratio. Compared with the well-known PID-PWM method, the proposed methods are able to achieve a better trade-off between rise-time and overshoot in the response of output voltage. Finally, a simple on-line identification algorithm is proposed for the load resistance. Combination of the proposed control methods and identification algorithm yields high performance robust control for boost converters.**

I. INTRODUCTION

Nowadays, power electronics is widely used in both industry and society, including vehicles. DC-DC converters are used to change the battery voltage to power car electronics and control units, while inverters are used to control the AC motors of electric vehicles. The most popular control method for switching circuits in power electronics, such as DC-DC converters, is PID-PWM[1]. However, since the parameters are fixed in PID-PWM, it is not an easy job to tune the parameters so as to achieve a good trade-off between rise time and overshoot, particularly when the load resistance varies after heating. This motivates us to invent new high performance robust control methods. The purpose is to pave a road toward better transient performance of DC-DC converters.

From an optimization point of view, this control problem is a constrained optimization problem because the duty ratio is confined in the interval $[0, 1]$. Hence, it is natural to try model predictive control (MPC)[3], [4]. However, generally in MPC the solution has to be computed based on recursive computation which can not be implemented on-line because the switching period is extremely short (microseconds). However, if we focus on the optimization over a short horizon (1, 2 or 3), and if a quadratic performance index is used, then the nice property of convexity may allow an analytic solution. It will be proved that this is indeed the case. Then the problem reduces to how to find a suitable performance index so that a better performance can be obtained. The basic idea is to get over the undershoot phenomenon stemming from the unstable zero in the transfer function from the duty ratio to the output voltage, which leads to the use of inductance current as the penalized output instead of the output voltage. This method forms the first contribution of the paper.

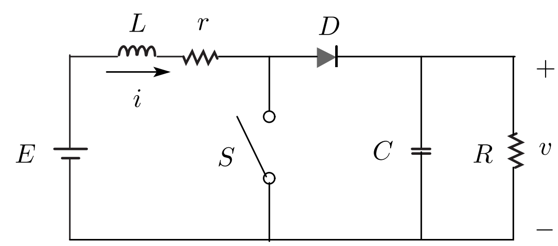

Fig. 1. Boost DC-DC converter

Secondly, a 3-level hysteresis control method is proposed which inherits the simplicity of conventional hysteresis control[1], [5], [6], [7] but overcomes its shortcomings, notably large ripples. This method is able to improve the transient response significantly and is the second contribution of this paper.

Lastly, since these two methods depend on the parameter of load which varies after heating, an on-line identification algorithm is devised. In this algorithm, the load resistance estimation involves only the average of output voltage and can be easily implemented. Combination of this identification algorithm with the proposed control methods provides high performance robust control for boost converters. This is the third contribution of the paper.

The proposed algorithms are validated through computer simulations. Comparison with PID-PWM indicates that the proposed methods do achieve a better robust performance than PID-PWM.

II. BOOST DC-DC CONVERTER

A DC-DC converter is a switching circuit which transforms the voltage of a DC source into other desired voltage in the load side. In this section, we give a brief review on the model of boost converter. The circuit of boost DC-DC converter is shown in Fig. 1.

The state vector is taken as $x(t) = [i(t)\ v(t)]'$ where i is the inductance current and v is the output voltage. Set the subscript as $j = 1$ when the switch is ON and $j = 2$ when the switch is OFF. Its state equation is described by

$$\begin{aligned} \dot{x}(t) &= A_j x(t) + b_j,\ \ j = 1, 2 \\ y(t) &= v(t) \end{aligned} \tag{1}$$

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in which

$$A_1 = \begin{bmatrix} -r/L & 0 \\ 0 & -1/CR \end{bmatrix}, \ b_1 = \begin{bmatrix} E/L \\ 0 \end{bmatrix}$$
$$A_2 = \begin{bmatrix} -r/L & -1/L \\ 1/C & -1/CR \end{bmatrix}, \ b_2 = \begin{bmatrix} E/L \\ 0 \end{bmatrix}.$$

Denote the duty ratio by d which satisfies $d \in [0, 1]$. According to the famous averaging method[1], the state-space averaged model is described by

$$\dot{x} = [dA_1 + (1-d)A_2]x + db_1 + (1-d)b_2. \quad (2)$$

This is a bilinear system with state vector x and input d. We use V_∞, I_∞ and d_∞ to denote the values of output voltage v, inductance current i and duty ratio d at the equilibrium.

Let T be the sampling period. Discretizing this averaged model, 1-step ahead output $y[k+1]$[1] and 2-step ahead output $y[k+2]$ are easily computed as

$$y[k+1] = c(I + A_2T)x[k] + Tc(A_1 - A_2)x[k]d[k] \quad (3)$$
$$y[k+2] = c(I + 2A_2T)x[k] + Tc(A_1 - A_2)x[k](d[k] + d[k+1]). \quad (4)$$

The control problem here is for the output voltage v to track a reference voltage $V_r (> E)$.

III. Short-horizon MPC algorithm

Due to the fact that the switching action has to be determined in microseconds, the control algorithm must be simple enough. So to obtain an analytic solution of MPC, we focus on short horizon predictions. Of course, the selection of the penalized output plays a key role in this case. It will be shown that for boost converters the inductance current should be penalized instead of the output voltage.

So, in the description of MPC algorithm we associate an output

$$y(t) = cx(t) \quad (5)$$

with the averaged model (2) and require the signal y to follow a constant reference y_r subject to the constraint on duty ratio d.

A. 1-step MPC algorithm

In this case, the performance index is

$$J = ||y[k+1] - y_r||^2 = \|Tc(A_1 - A_2)x[k]d[k] - (y_r - c(I + A_2T)x[k])\| \quad (6)$$

Hence, ignoring the constraint on duty ratio, the unconstrained optimal solution d^* is trivially obtained as

$$d^* = \frac{y_r - c(I + A_2T)x[k]}{Tc(A_1 - A_2)x[k]} \quad (7)$$

if $c(A_1 - A_2)x[k] \neq 0$ and arbitrary otherwise.

Since the index J is convex in d, it is easy to know that the constrained optimal solution d° is given by

1) $d^\circ = d_\infty$ if $c(A_1 - A_2)x[k] = 0$.
2) Otherwise
   a) $d^\circ = d^*$ if $0 \leq d^* \leq 1$,
   b) $d^\circ = 0$ if $d^* \leq 0$,
   c) $d^\circ = 1$ if $d^* \geq 1$.

[1]The convention $f(kT) = f[k]$ is used throughout this paper.

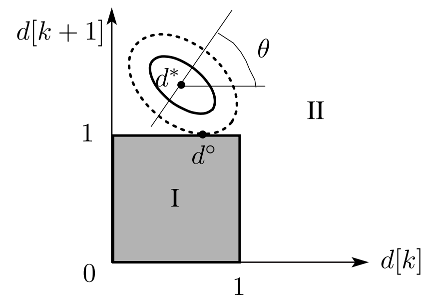


Fig. 2. Constrained optimality $\mathbf{d}^\circ$

B. 2-step MPC algorithm

In this case, the performance function is computed as

$$J = \left|\left| \begin{bmatrix} y[k+1] - y_r \\ y[k+2] - y_r \end{bmatrix} \right|\right|^2 = ||aH\mathbf{d} - w||^2 \quad (8)$$

in which

$$a = c(A_1 - A_2)Tx[k] \quad (9)$$
$$H = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, \ \mathbf{d} = \begin{bmatrix} d[k] \\ d[k+1] \end{bmatrix} \quad (10)$$
$$w = \begin{bmatrix} y_r \\ y_r \end{bmatrix} - \begin{bmatrix} c(I + A_2T) \\ c(I + 2A_2T) \end{bmatrix} x[k] \quad (11)$$

Again, its unconstrained optimal solution is computed as

$$\mathbf{d}^* = \frac{1}{a} H^{-1} w. \quad (12)$$

Further, the index can be written as

$$J = a^2 \, ||H(\mathbf{d} - \mathbf{d}^*)||^2 \,. \quad (13)$$

It is noted that $||H(\mathbf{d} - \mathbf{d}^*)||^2 \leq c^2$ is an ellipsoid centered at $\mathbf{d}^*$ and spreading outward as c increases. Since the domain of $\mathbf{d}$ is the square with unit side length as shown in Fig. 2 (region I), if $\mathbf{d}^*$ is in the square then $\mathbf{d}^\circ = \mathbf{d}^*$. Meanwhile, when $\mathbf{d}^*$ is outside the square (region II) the boundary of the ellipsoid will touch one of the four sides before entering the square. The point of tangency gives the constrained optimal solution $\mathbf{d}^\circ$ because c^2 is proportional to the value of index J. This constrained optimal solution $\mathbf{d}^\circ$ can be determined directly in accordance with the location of $\mathbf{d}^*$. The detail is omitted here.

Similar idea can be extended to solve 3-step MPC problem.

IV. Selection of performance index

As has been touched before, it is extremely important to select a suitable performance index in order to get a good overall performance since the prediction horizon is extremely short in the present approach. This selection depends on the property of converter dynamics.

For boost converter, it is discovered by simulations that when the penalized output is taken as the output voltage

v, the switch is locked in the OFF mode ($d^\circ[k] \equiv 0$) and reference tracking fails.

To reveal the reason why the proposed MPC does not track the given voltage reference, let us look at the linearized dynamics of the boost converter. Toward this goal, let the deviations from the equilibrium be denoted by $\tilde{v}$, $\tilde{i}$ and $\tilde{d}$. It is easy to see that the transfer functions $G_{\tilde{v}\tilde{d}}(s)$ from $\tilde{d}$ to $\tilde{v}$ and $G_{\tilde{i}\tilde{d}}(s)$ from $\tilde{d}$ to $\tilde{i}$ for the boost converter are as follows[1], [2]:

$$G_{\tilde{v}\tilde{d}}(s) = \frac{-\frac{I_\infty}{C}s + \frac{1}{LC}\{V_\infty(1-d_\infty) - rI_\infty\}}{s^2 + \left(\frac{1}{CR} + \frac{r}{L}\right)s + \frac{1}{LC}\{(1-d_\infty)^2 + \frac{r}{R}\}} \tag{14}$$

$$G_{\tilde{i}\tilde{d}}(s) = \frac{\frac{V_\infty}{L}s + \frac{1}{LC}\{\frac{V_\infty}{R} + I_\infty(1-d_\infty)\}}{s^2 + \left(\frac{1}{CR} + \frac{r}{L}\right)s + \frac{1}{LC}\{(1-d_\infty)^2 + \frac{r}{R}\}} \tag{15}$$

The zero z_v of $G_{\tilde{v}\tilde{d}}(s)$ and the zero z_i of $G_{\tilde{i}\tilde{d}}(s)$ are

$$z_v = \frac{R}{L}\left\{(1-d_\infty)^2 - \frac{r}{L}\right\},\; z_i = -\frac{2}{RC} \tag{16}$$

respectively. So when

$$d_\infty < 1 - \sqrt{\frac{r}{R}}, \tag{17}$$

$G_{\tilde{v}\tilde{d}}(s)$ has an unstable zero. Then, undershoot inevitably occurs in the voltage response when the input $\tilde{d}$ rises. Therefore, whenever d is raised over d_∞ undershoot occurs in the output voltage v. This implies $v < 0$ in the initial time which corresponds to a greater value of index J (because the horizon is short). So to prevent this from happening, the unconstrained duty ratio d^* gets negative ($d^* < 0$). Consequently, the constrained duty ratio becomes $d^\circ = 0$.

To overcome this problem, we note that the zero of transfer function $G_{\tilde{i}\tilde{d}}(s)$ is stable so that there is no undershoot in the response of inductance current i. So, we penalize the tracking error of inductance current i. That is, we set

$$y(t) = i(t) = [1 \;\; 0]x(t) \tag{18}$$

in the algorithm described in section III.

However, the aim of converter control is to track given output voltage V_r. Therefore, in order for this performance index to work, i and v must have a one-to-one relationship. Computation based on (2) shows that

$$V_\infty = \frac{1-d_\infty}{(1-d_\infty)^2 + r/R}E \tag{19}$$

$$I_\infty = \frac{1}{(1-d_\infty)^2 + r/R}\frac{E}{R}. \tag{20}$$

Then, the relation between V_∞ and I_∞ becomes

$$I_\infty = \frac{V_\infty}{(1-d_\infty)R}. \tag{21}$$

The characteristics of V_∞ and I_∞ with respect to d_∞ are shown in Fig. 3. From this figure, it is clear that $V_\infty(d_\infty)$ has an inflection point at

$$\bar{d} = 1 - \sqrt{\frac{r}{R}}. \tag{22}$$

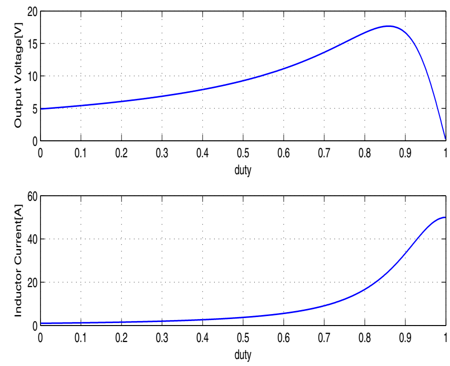


Fig. 3. (V_∞, I_∞) vs d_∞

TABLE I

SELECTION OF PERFORMANCE INDEX

Motivation	Measure
Avoiding the effect of unstable zero	Penalize i, $d[0] = 1 - \sqrt{\frac{r}{R}}$
Tracking of Vr	$0 \leq d^\circ \leq 1 - \sqrt{\frac{r}{R}}$

So when $d \leq \bar{d}$, i and v are one-to-one. Therefore, if we limit the duty ratio to

$$0 \leq d^\circ \leq 1 - \sqrt{\frac{r}{R}}, \tag{23}$$

the tracking of v can replaced by that of i. Moreover, to avoid the undershoot in the output voltage, the initial input $d[0]$ should take the maximum value. These discussions are summarized in Table. I.

A. Simulation results

In all subsequent simulations, the circuit parameters used are given in Table II.

The steady-state output voltage is set as $V_\infty = V_r$. Then the steady-state duty ratio d_∞ is obtained from (19) as

$$d_\infty = \frac{\left(2 - \frac{E}{V_r}\right) - \sqrt{\left(\frac{E}{V_r}\right)^2 - 4\frac{r}{R}}}{2} \tag{24}$$

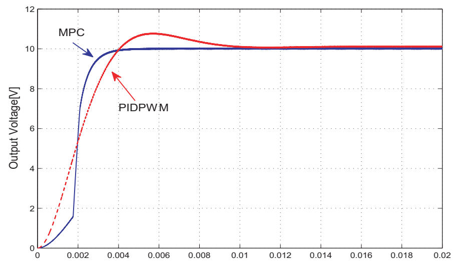
and the steady-state inductance current I_∞ can be computed via (21).

Almost no difference of response is observed in 1-step and 2-step MPC. So only the simulation result of 1-step MPC is

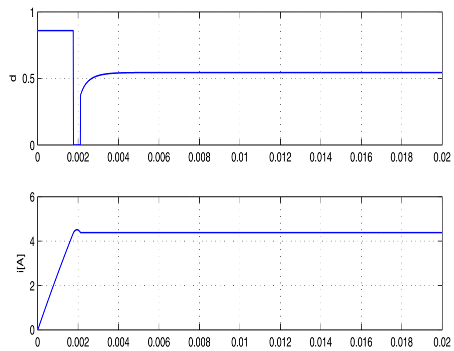
TABLE II

CIRCUIT CONSTANT OF CONVERTER

Power supply voltage	E	5[V]
Reference voltage	V_r	10[V]
Inductance	L	1.89[mH]
Capacitance	C	220[μF]
Coil resistance	R	5[Ω]
Inductance resistance	r	0.1[Ω]
Diode	D	ideal
Sampling frequency	$f = 1/T$	100[kHz]



(a) Responses of v (MPC and PID-PWM)



(b) Responses of d, i (MPC)

Fig. 4. 1-step MPC

shown in Fig. 4. As is clear from this figure, the output voltage v tracks the reference V_r satisfactorily. The rise time is short enough and there is no overshoot. Compared with well-tuned PWM-PID, the proposed MPC method achieves a better response.

V. 3-Level Hysteresis Control Algorithm

The main mechanism of conventional hysteresis control is to switch on or off in such a way that the output voltage will be contained in the hysteresis band (interval between the upper and lower bounds). Concretely, the switching action is determined in the following way:

1) if $i <$ lower bound, $d = 1$
2) if $i >$ upper bound, $d = 0$
3) else output the preceding d

The hysteresis control is robust to load uncertainty, but the switching frequency cannot be prescribed. Also, the performance is not as good as PWM-PID because it is simply a static control method and the dynamical property of converter is not used.

A. Algorithm

Our proposal is to add a central layer inside the hysteresis band, as illustrated by the shaded strip in Fig. 5. In the central layer, the duty ratio is set as its steady state value d_∞. Further, the switching decision is made at the sampling instant, i.e. the switching frequency is set as the sampling period. The concrete algorithm is:

1) if $i <$ lower bound, $d = 1$

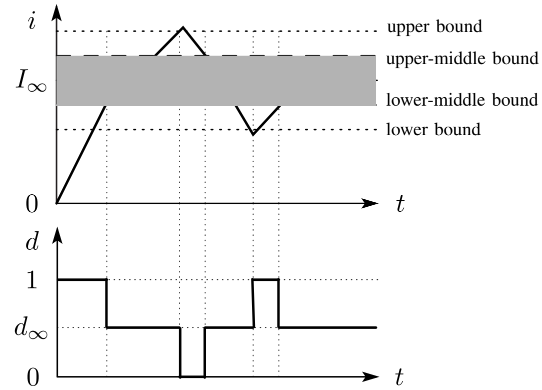


Fig. 5. 3-level hysterisis control

TABLE III
SIMULATION PARAMETERS

	3-level hysteresis control	hysteresis control
upper bound	$I_\infty + 0.45$[A]	$I_\infty + 0.45$[A]
upper-middle bound	$I_\infty + 0.01$[A]	-
lower-middle bound	$I_\infty - 0.01$[A]	-
lower bound	$I_\infty - 0.45$[A]	$I_\infty - 0.45$[A]

2) if $i >$ upper bound, $d = 0$
3) else if lower-middle bound $< i <$ upper-middle bound, $d = d_\infty$
4) else output the preceding d

In the 3-level hysteresis control the duty ratio is set as $d = d_\infty$ in the steady-state, so even when the hysteresis band is big the ripple at the steady-state will be made small. That is, a good trade-off between convergence rate and control precision can be achieved.

The guideline for the selection of parameters of each hysteresis layer is as follows.

1) Central layer (upper-middle bound, lower-middle bound): a little bit wider than the range of ripple at the steady-state.
2) Outer layers (upper bound, lower bound): via trial and error

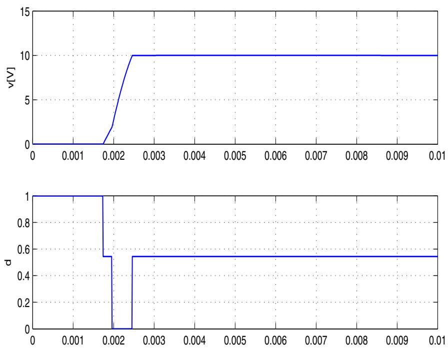
B. Simulations

The parameters used in the simulations are given at Table. III.

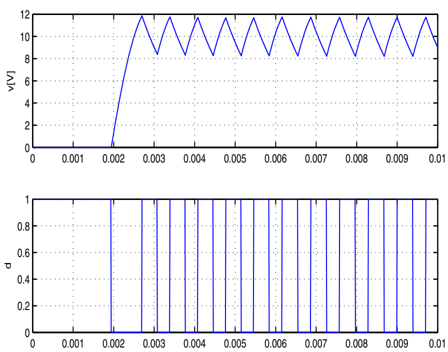
Simulation results are shown in Fig. 6. It is evident that higher control precision is obtained as compared with conventional hysteresis control even though the hysteresis bands are the same.

VI. Load Identification Algorithm

An eminent drawback of the proposed MPC and 3-level hysteresis algorithms is their sensitivity to load variation because both algorithms depend on the load resistance R. It is confirmed through simulations that when the load resistance is different from the nominal value, the voltage output converges to a constant different from the reference.



(a) 3-level hysteresis control



(b) Conventional hysteresis control

Fig. 6. Comparison with conventional hysteresis control (v, d)

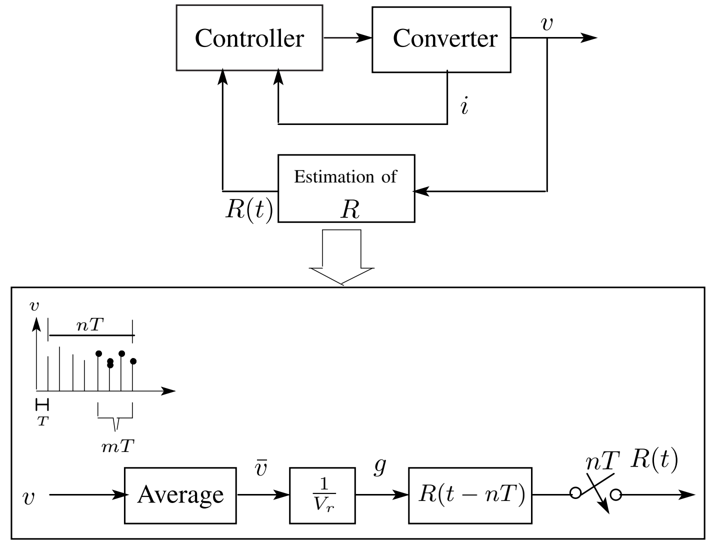


Fig. 7. Identification algorithm of R

So certain parameter identification algorithm has to be introduced to overcome this shortcoming.

A. Identifying R based on output voltage

It is observed that $v > V_r$ when the nominal R is smaller than the true R and, $v < V_r$ when the nominal is greater than the true value. This property forms the basis of the identification of R.

The algorithm is illustrated in Fig. 7. Since the output voltage has ripple even in the steady-state, we need to use averaged value. So the sampled voltage is partitioned into batches, each batch containing n samples. Also, too frequent update of resistance estimate worsens the response of states. So the estimated resistance R is renewed only once every n samples so as to prevent the bad effect of parameter identification. Suppose the ending instant of a batch is t, the averaged voltage $\bar{v}$ in this batch is computed using m sampled data $v(t-(m-1)T), \cdots, v(t)$

$$\bar{v} = \frac{1}{m}\sum_{k=0}^{m-1} v(t-kT). \tag{25}$$

Then, set a gain as

$$g = \frac{\bar{v}}{V_r}. \tag{26}$$

The estimate $R(t)$ of R at t is updated as

$$R(t) = gR(t-nT). \tag{27}$$

In this identification algorithm, no additional sensor is required and the computation time is short since the only computation burden is the averaging.

Attention should be paid to the following points:

1) Since the algorithm depends on the steady-state output voltage, it cannot be applied in the transient.
2) m should not be too big, nor too small. It is suggested to choose in the interval $m/n = 20\% \sim 40\%$.
3) The update period n should not be too small in order to lower the sensitivity of control input.

In the subsequent simulations, the first 500 samples (5[ms]) is not used and, n, m are set as $200, 50$ respectively. The resistance R is increased from $5[\Omega]$ to $10[\Omega]$ at $t = 0.05$[sec].

B. MPC

The application to MPC is shown in Fig. 8 and Fig. 9. It is clear that after the change of load the voltage tracks the reference swiftly. Also from Fig.9 the estimation of R is sufficiently precise and the convergence is fairly fast.

C. 3-level hysteresis control

The application to 3-level hysteresis is shown in Fig. 10 and Fig. 11. Again, the voltage tracks the reference quickly after the change of R and the estimation of R is successful.

D. Comparison: PWM-PID

For comparison, the gains of PID in PWM-PID are tuned as $K_P = 10^{-3}, K_I = 10^{-5}, K_D = 1.5$ so as to get a good trade-off between transient response and robustness to load variation. The response is illustrated in Fig. 12. As is seen from this figure, the robustness is achieved at the sacrifice of settling time. Compared with the responses of MPC and

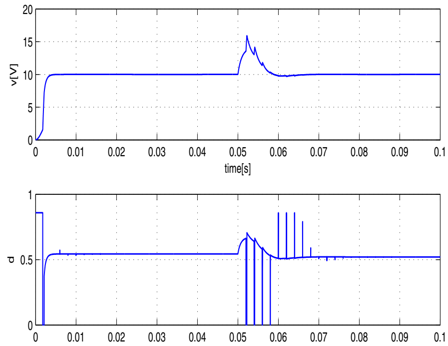


Fig. 8. Responses of v, d (MPC+R estimation)

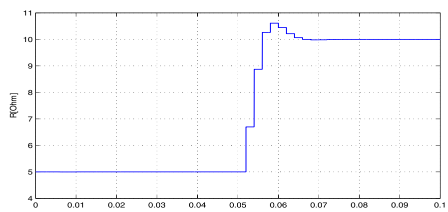


Fig. 9. Estimate of R (MPC+R estimation)

3-level hysteresis control, the performance of PWM-PID is better in the short period right after the load change. However, the overall performance of the proposed methods is better than that of PWM-PID since the convergence of PWM-PID is much longer and the tuning of PID parameters is not so easy.

Another advantage of the load identification algorithm lies in its universality: it can be combined with any control methods.

VII. Conclusion

In this paper, we have proposed two high performance control methods for boost DC-DC converters. The first one is a short-horizon MPC method, another is a 3-level hysteresis

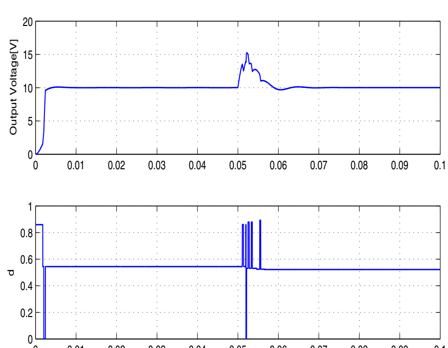


Fig. 10. Response of v, d (3-level hysteresis control+R estimation)

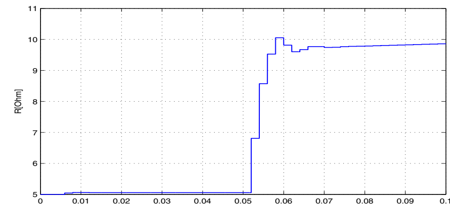


Fig. 11. Estimate of R (3-level hysteresis control+R estimation)

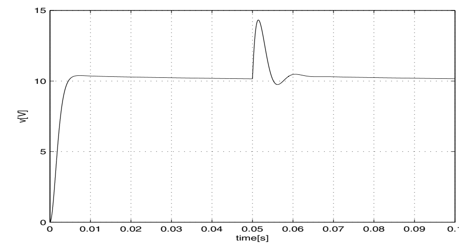


Fig. 12. Response of v (PID-PWM)

control method. These methods demonstrate response performance better than PWM-PID. Further, for load uncertainty an on-line identification algorithm has been proposed. This identification algorithm has good convergence property and can be applied with any control method.

Some future works are to investigate the robustness with respect to voltage reference as well as power source voltage.

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