Optimal Energy Management for a Hybrid Energy Storage System for Electric Vehicles Based on Stochastic Dynamic Programming

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***Abstract*—For electric and hybrid electric cars, commonly nickel-metal hydride and lithium-ion batteries are used as energy storage. The size of the battery depends not only on the driving range, but also on the power demands for accelerating and braking and life-time considerations. This becomes even more apparent with short driving ranges, e.g. in commuter traffic. By hybridization of the storage, adding double layer capacitors, the battery can be relieved from the stress of peak power and even downsized to the energy demands instead of power demands.**

The dimensioning of the storage is performed by a parametric study via Deterministic Dynamic Programming. To determine an energy management to control the power flows to the storage online during operation which considers the stochastic influences of traffic and the driver, Stochastic Dynamic Programming is investigated and compared to the optimal strategy found during the dimensioning.

***Index Terms*—Energy management, dynamic programming, hybrid energy storage system, electric vehicle**

I. Introduction

The typical electrical energy storage of today's electric and hybrid-electric vehicles usually consists solely of nickel-metal hydride (NiMH) or lithium-ion (Li-ion) batteries. In order to substitute conventional cars, the capacity is often dimensioned for driving ranges up to 250 km, as with the Tesla Roadster or BMW Mini-E. This leads to an expensive, heavy and bulky energy storage.

A further typical area of operation are secondary cars used for short distance trips or commuting. The average driving range is only approx. 30 km, with 80% of the trips being shorter than 60 km [1]. For these cars, the energy capacity of the storage can be designed much smaller.

However, as even traction batteries offer mainly a high energy density but only limited power density, such a battery storage sized for smaller energy amounts has limited or insufficient power capabilities to drive urban and suburban drive cycles, especially when recuperating braking power. Hybridization of the storage by additional double layer capacitors (DLC) can increase the power capabilities significantly as well as release the battery from the dynamic stress of peak power.

When designing the hybrid energy storage system (HES), two questions arise: Firstly the sizing of the battery and the DLC, and secondly the determination of an online, causal energy management for the distribution of the traction power to both storages which takes the stochastic influences of traffic and the driver into account. For the former, the optimal optimization method of Deterministic Dynamic Programming is performed on recorded driving profiles in a parametric study to both optimize the energy management strategy and the size of the HES at the same time. For the latter, Stochastic Dynamic Programming is a suitable way to find a causal operating strategy considering the stochastics.

This paper is structured as follows: After a short description of the vehicle, the drive cycles and the possible energy storage technologies in Section II, the optimization problem and the dimensioning of the HES are described in Section III. The calculation of a causal operating strategy is then explained in detail in Section IV and simulation results are presented in Section V.

II. System Description

A. Vehicle

In this contribution, an energy storage dimensioned for a reduced driving range of 60 km for a vehicle based on the BMW Mini-E [2] is investigated. The BMW Mini-E is an all electric powered car field-tested in the United States, United Kingdom and Germany since 2009. It is propelled by a 150 kW induction motor. Its original energy storage consists solely of lithium ion batteries featuring a driving range of about 175 km. Further data are specified in Table I.

TABLE I
DATA OF THE STUDIED VEHICLE

Tare weight w/o storage	1224 kg
Payload	475 kg
Maximum Drive Power	150 kW @ 7000-8000 min^{-1}
Maximum Torque	220 Nm @ 0-5000 min^{-1}
Drag Coefficient	0.35
Reference Cross Section Area	2 m^2

B. Data Acquisition/Driving Profiles

For the dimensioning of the energy storage and the design of an operating strategy, several driving profiles have been recorded by GPS in urban and sub-urban traffic in Paderborn, Germany. This contribution is based on a data base of 7 measured profiles on the same route, but two different

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driving directions. The length of the trips is approx. 9.4 km, comprising a difference in altitude of 57 m uphill in profiles 1-3 and downhill in profiles 4-7. The power profiles of the electric car were then simulated by a model of the BMW Mini-E, taking into account the variable weight of the energy storage to be designed. The peak power at full payload was 91 kW for acceleration and up to 60 kW for recuperation.

C. Energy Storage

Common energy storages for vehicular applications are batteries or double layer capacitors (DLC) [3], [4]. Traction batteries, both lithium-ion (Li-ion) and nickel-metal hydride (NiMH), offer a high energy density but only a poor power density and small number of full load cycles. DLC on the other hand offer a high power density and high number of cycles without degradation at the costs of only a low energy density. Typical characteristics of the storage examined are listed in Table II.

Applying the recorded power profiles for the power rating and an energy consumption of 54 MJ/100 km based on statistical data from [5], the required masses of the respective storage technologies for driving ranges of 60 km and 100 km are stated in Table III.

It is apparent that both batteries and DLC solely do not satisfy the requirements of a high power and energy density, especially when recuperating braking power. However, combining both energy storage technologies to a hybrid energy storage system (HES) complements the high energy of the batteries and the high power of the DLC, permitting a downsizing of the battery storage and a higher peak power at the same time.

For the BMW Mini-E based vehicle, a HES featuring NiMH batteries and DLC is investigated in the following. In order to control the power flows of each storage individually and to adjust the state of charge dependent voltage variations, both storages are each connected to the drive train via bidirectional power converters. The resulting structure of the HES is depicted in Fig. 1.

TABLE II
CHARACTERISTICS OF CONSIDERED TRACTION ENERGY STORAGES

Storage type	Spec. energy (kJ/kg)	Spec. power (W/kg) (disch./charge)	Full load cycles
NiMH Batt.	165	230/50	500-2000
Li-ion Batt.	380	700/100-400	500-3000
DLC	15	2000/2000	>500000

TABLE III
NECESSARY STORAGE DEVICE MASSES FOR THE INVESTIGATED URBAN DRIVE CYCLES

Storage	Driving range in km	Mass of storage in kg sized for	
		energy	peak power (discharge/charge)
NiMH Batt.	100	330	395/1200
	60	200	395/1200
Li-ion Batt. (energy cell)	100	145	130/600-150
	60	85	130/600-150
DLC	60	2160	45/30

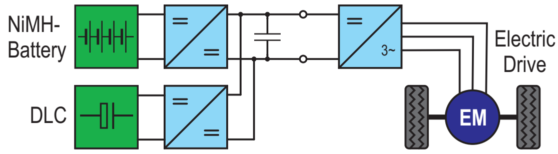

Fig. 1. Structure of the Hybrid Energy Storage System

III. DIMENSIONING OF THE HYBRID ENERGY STORAGE

The hybrid structure of the HES offers a degree of freedom for the distribution of the driving power to the distinct energy storages as well as for the dimensioning of the storage. As the size of the storage has direct influence on the operating strategy, both have to be optimized at the same time.

Several objectives may be important for both the dimensioning and the operating strategy, e.g. a high efficiency, low deterioration or increased availability. In this paper, we consider the objective of minimizing the energy losses e_{v} in the storage system. The determination of the operating strategy can then be described as an optimization problem:
Find an optimal strategy $\pi(t)$ for the power distribution to both storages which minimizes

$$e_{\mathrm{v}} = \int_{T_{\mathrm{start}}}^{T_{\mathrm{end}}} p_{\mathrm{losses}} \, \mathrm{d}t \qquad (1)$$

with p_{losses} being the power dissipation of the HES. Constraints are the maximum and minimum power of both energy storages and the state of charge of the DLC.

To limit the influence of the operating strategy on the dimensioning, an optimal optimization method like *Deterministic Dynamic Programming (DDP)* is well suited. It is easy to implement, has a limited computational effort and it can serve as a benchmark for the causal but sub-optimal strategies to be developed in Section IV.

The basic idea of DDP is to decompose a complex optimization problem into several smaller, easier to solve subproblems. The sequential solution of the subproblems then leads to the optimum solution of the overall problem [6], [7].

For the application of DDP, the driving cycles and related power profiles have been discretized in time. The vehicle was described by its state x_k, comprising the velocity v_k, the power demand $P_{\mathrm{el},\,k}$ and the state of charge $SOC_{\mathrm{DLC},k}$ of the DLC. At each time step k, the algorithm of DDP has to find the optimal strategy for the power distribution to both storages that minimizes the overall energy losses.

For the dimensioning, a parametric study of the energy losses was performed by DDP on a detailed model of the HES, taking into account the effect of the variable weight of the storage on the power profiles. Two different storage configurations have been examined and compared to a pure battery storage of 400 kg capable of providing both the propulsion power and energy of the driving cycles:

Configuration 1: A HES with the same weight of 400 kg as the pure battery, but with part of the battery replaced by DLC.

Configuration 2: A downsized battery of 200 kg which meets only the energy demands, and additional DLC.

In both configurations, a maximum DLC mass of 200 kg is feasible because of the required energy of the HES and the maximum payload of the vehicle.

The results for the different driving profiles and the average values for uphill, downhill and all cycles are displayed in Fig. 2 and Fig. 3. The losses are normalized to the losses of the reference battery of 400 kg. Recuperation power which cannot be stored is dissipated via a shunt, increasing the losses. Compositions of the HES with DLC smaller than 32 kg in config. 2 are not realizable as the peak power of the HES is too small for at least some of the power profiles.

It is apparent for both configurations that the energy losses can be reduced significantly by replacing part of the reference battery sized for power demand by DLC (configuration 1) or by adding extra DLC to the small battery downsized only for energy demands (configuration 2). The losses first decrease significantly with additional DLC, but rise again after attaining a minimum due to a less favorable ratio of DLC and battery power or the higher weight of the storage and thus the vehicle.

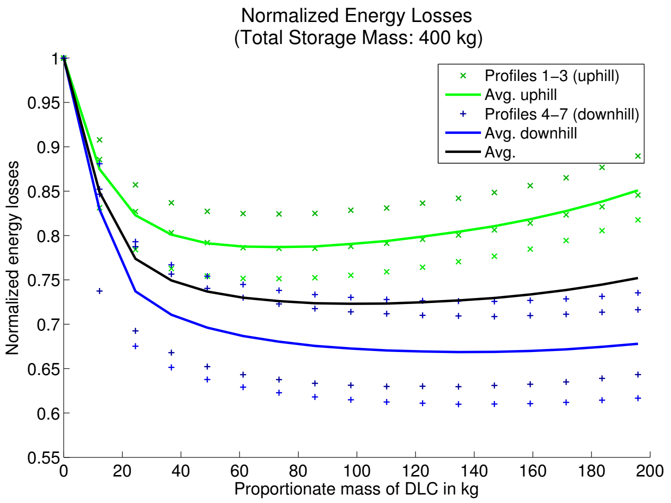

Fig. 2. Energy Losses for Configuration 1 (Constant Storage Mass of 400 kg, with Part of the Batteries Replaced by DLC)

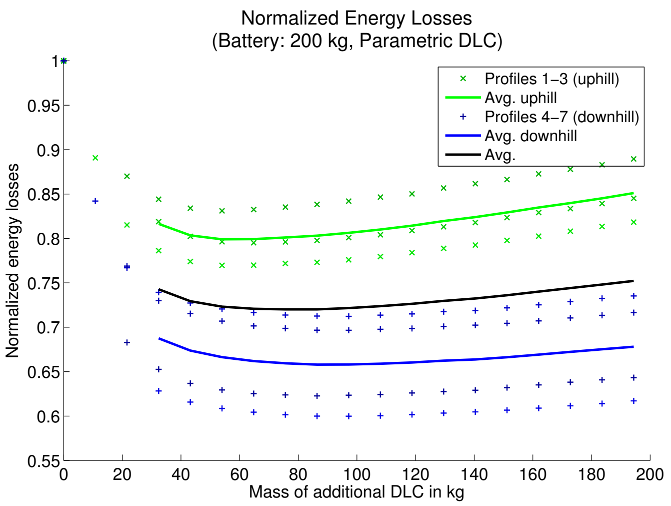

Fig. 3. Energy Losses for Configuration 2 (Constant Battery Mass of 200 kg and Variable Mass of Additional DLC)

As the power and efficiency of the battery are lower when recuperating, the reduction of losses is distinctly higher by up to 18 percentage points on downhill cycles which comprise 31 % more recuperation work and 20 % less traction work.

For configuration 1, a reduction of the losses by 17.5-39 % compared to a pure battery storage is possible, on average by 28 %. As the costs for DLC are distinctly higher than for batteries, a battery weight of 340 kg and DLC weight of 60 kg was selected. This implies that 15 % of the original battery have been replaced by DLC, resulting in a possible reduction of the losses by 17.5-25 % (average 21 %) on uphill cycles and 25.5-37 % (average 31.5 %) on downhill cycles with higher recuperation.

For configuration 2, the losses can be reduced by a similar amount as with configuration 1. A DLC weight of 54 kg in addition to the constant battery weight of 200 kg is a good compromise to allow reductions of the losses by 17-23 % (avg. 20 %) on uphill cycles and by 28-39 % (avg. 33.5 %) on downhill cycles. The weight of the storage decreases by 146 kg or 36.5 %, additionally resulting in a lower consumption of the car of about 6 %.

For both configurations and the selected dimensioning, the maximum power of the HES in contrast to the reference battery meets the power requirements of 150 kW of the original car. Careful cost assessment shows that the HES of config. 2 can be cost-neutral for NiMH batteries, but details depend on the future development of storage costs. The selected HES configurations are summarized in Table IV.

TABLE IV
SELECTED HYBRID ENERGY STORAGE SYSTEMS

Parameter	Config. 1	Config. 2
Battery weight in kg	340	200
DLC weight in kg	60	54
Storage weight in kg	400	254
Total storable energy in MJ	56.9	33.7
Energy storable in battery in MJ	56.1	33
Energy storable in DLC in MJ	0.8	0.74
Max. power in kW	198	154
Avg. loss reduction uphill in %	21	20
Avg. loss reduction downhill in %	31.5	33.5

IV. CAUSAL ENERGY MANAGEMENT

For the sizing of the HES, an optimal, non-causal operating strategy was applicable which required the exact knowledge of future driving power. However, for the operation of the vehicle in urban traffic, a causal energy management is necessary which can manage the stochastic influences on the driving cycle and hence the power demand.

Operating strategies for a causal energy management can be obtained by numerous ways. A conventional strategy is the limitation of the battery power to reduce the peak power stress, often combined with a velocity dependent adaptation of the state of charge of the DLC [8], [9]. Another approach is to partition the demanded power to a higher and a lower frequency share and then distribute it to the respective storage accordingly [10]. More sophisticated strategies apply heuristics based on boolean or fuzzy rules [11], [12] or use knowledge of short term future power demand [13].

These approaches are often easy to implement, but they make no use of the stochastics of the driving cycle. Hence, to determine a causal energy management which takes stochastic influences into account, we use the method of *Stochastic Dynamic Programming (SDP)* [14], [15], [7].

A. Stochastic Dynamic Programming

Instead of the exact power profile as with the DDP, only the stochastic process of the driving cycle has to be known. As with the DDP, the cycle is divided into short sections k and the system is again described by its state $x_k = \{v_k, P_{\mathrm{el},k}, SOC_{\mathrm{DLC},k}\}$ comprising the velocity and power of the car and the state of charge of the DLC. The power distribution to both storages is controlled by the control signal u, which in our case is the value discretized battery current. Based on the recorded driving cycles and simulations of the vehicle power and the HES, for each value of the control variable u the transition probability of the state variables at the end of each section are precalculated and modeled as a homogeneous Markov chain. This implies that future states only depend on the actual state x_k, not on previous states. As the Markov chain is homogeneous, the probability for a transition from one state to another is independent on time and thus the position of the vehicle.

The cost function $J_\pi(x_0)$ to be minimized by SDP can only be defined as an expectancy value of the final costs. This is composed of the expected costs $c_k(x_k, \pi(x_k), \omega_k)$ of the sections $k = 0, 1, \ldots, N-1$ with knowledge of the probability distribution ω_k and the costs $c_N(x_N)$ of the last state x_N, e.g. for a SOC of the DLC deviating from the initial SOC [11]:

$$J_\pi(x_0) = E_{\omega_k}\left\{c_N(x_N) + \sum_{k=0}^{N-1} c_k\left(x_k, \pi(x_k), \omega_k\right)\right\} \tag{2}$$

$\pi(x_k)$ denotes the possible strategies, i.e. the power distribution u to both storages, at section k. The costs c_k consist of the energy losses $e_\mathrm{v}(x_k, u)$ at this section and a term which preserves a medium SOC of the DLC:

$$c_k(x_k, u) = e_\mathrm{v}(x_k, u) + \alpha(SOC_{DLC,k+1} - SOC_{DLC,med})^2 \tag{3}$$

This additional term is necessary as the prediction of the SDP only covers a short horizon. Thus, without this term, higher losses caused by an empty or full DLC beyond this prediction horizon could not be prevented. Furthermore, an approximately balanced SOC of the DLC can be achieved at the end of the driving cycle.

B. Algorithm

The optimal strategies are calculated by a modified policy-iteration algorithm described in [15], [16]. The objective is to find a strategy $\pi(x)$ for each state $x = \{v, P_\mathrm{el}, SOC_\mathrm{DLC}\}$ of the state space X which selects the appropriate power distribution u to both storages to minimize the expectancy value of the costs stated in (2). The algorithm iterates the following four steps for each state x:

1) Initial guess: Set the iteration index $i = 1$. For each state x set the strategy $\pi_1 = 0$ (which in our case means the complete vehicle power is provided by the DLC). Set the cost function $J_{\pi_0} = 0$.
2) Evaluate the strategy: Calculate the expected truncated costs $J_{\pi_i}(x)$ of the strategy π_i for N time steps k ahead by iterating
$$J_{\pi_i}(x_k) = c(x_k, u) + \lambda \sum_{x_{k+1}\in X} \mathbb{P}(x_{k+1}|x_k, u) J_{\pi_i}(x_{k+1}) \tag{4}$$
backwards, starting with $J_{\pi_N} = J_{\pi_{i-1}}$ (the costs of the former iteration) and finishing with the expected costs $J_{\pi_i}(x_0)$. The strategy $\pi_i = u$ is kept invariant for all iteration steps. λ is a discount factor to limit the influence of future costs and to ensure convergence of the costs. All costs $c(x_k, u)$ and transition probabilities $\mathbb{P}$ have been calculated in advance for speedup of the algorithm. The iteration of the second step terminates when the costs $J_{\pi_i}(x_k)$ converge or when N steps are completed.
3) Improvement of the strategy: Find a strategy π_{i+1} which minimizes the expected costs for the next two time steps, using the costs estimated in step 2:
$$\pi_{i+1}(x_0) = \arg\min_u \left\{c(x_0, u) + \lambda \sum_{x_1\in X} \mathbb{P}(x_1|x_0, u) J_{\pi_i}(x_1)\right\} \tag{5}$$
$$J_{\pi_{i+1}}(x_0) = \min_u \left\{c(x_0, u) + \lambda \sum_{x_1\in X} \mathbb{P}(x_1|x_0, u) J_{\pi_i}(x_1)\right\} \tag{6}$$
4) Break iteration if costs converge:
If $|J_{\pi_{i+1}}(x_0) - J_{\pi_i}(x_k)| < \varepsilon$, the optimal strategy $\pi(x) = \pi_{k+1}(x)$ is found. If costs did not converge, increase the iteration index i and repeat from step 2.

C. Application

The algorithm results in a look-up table stating for each actual state x of the vehicle the optimal power distribution to both storages minimizing the expected costs. This can easily be implemented e.g. on a microcontroller. When driving the electric vehicle, the actual state is identified, i.e. the velocity, the propulsion power and the state of charge of the DLC. The corresponding power distribution to both storages is then selected from the look-up table and applied to the HES.

TABLE V
SIMULATION RESULTS (PROFILES 1-3: LOW RECUPERATION POWER, 4-7: HIGH RECUPERATION POWER)

Profile no.	Energy losses of energy storage system including converters					Efficiency of storage including converters		
	Pure battery vehicle in kJ	Config. 1		Config. 2		Pure battery in %	Config. 1 in %	Config. 2 in %
		in kJ	in % vs. pure bat.	in kJ	in % vs. pure bat.			
1	784.4	692.9	88.3	696.9	88.8	85.2	86.7	85.8
2	795.2	749.4	94.2	752.2	94.6	85.6	86.3	85.4
3	651.3	632.3	97.1	631.3	96.9	86.8	87.2	86.4
Average	**743.6**	**691.5**	**93.2**	**693.4**	**93.5**	**85.9**	**86.7**	**85.9**
4	794.9	578.0	72.7	545.9	68.7	78.3	83.2	83.2
5	659.1	569.3	86.4	539.8	81.9	81.2	83.3	83.3
6	528.1	446.0	84.5	430.2	81.5	81.4	83.8	83.7
7	793.6	601.5	75.8	574.2	72.4	80.8	84.7	84.7
Average	**693.9**	**548.7**	**79.8**	**522.5**	**76.1**	**80.4**	**83.8**	**83.7**

V. SIMULATION RESULTS

The *Stochastic Dynamic Programming* was performed on the data base of the 7 drive cycles. The time period of the sections k was chosen as 5 s, as the FFT-spectra of the cycles mainly contain frequencies above 0.2 Hz. Thus, with this interval the expected cost are predicted long enough to cover long-term changes in speed and power, while it is short enough to include detailed state transitions.

The calculation of SDP consumes significant amount of memory, so the resolution of the state variables has to be limited. As the SOC of the DLC depends on the supplied power and the SOC at the former time step, this quantity should have a higher resolution than the velocity and the power demand. With a memory of 12 GB, resolutions of 33 states for the SOC, 13 states for velocity, 19 states for power and 19 control values for the power distribution have turned up applicable.

The generated look-up table was applied to the individual profiles. The results are listed in Table V. Profiles 1-3 comprise a difference in altitude of 57 m uphill, while profiles 4-7 are downhill. The average energy consumption uphill is 4.5 MJ for the pure battery car and config. 1 and 4.2 MJ for the car with reduced battery weight (config. 2). The downhill consumptions are 2.84 MJ and 2.7 MJ respectively.

For both storage configurations, a significant reduction of the losses on average by 6.5 % uphill for both configurations and downhill by 20 % for config. 1 and 24 % for config. 2 is possible. As presented with the DDP, the HES increases the efficiency of the storage especially on profiles with higher recuperation power as the power of the pure battery is limited when recuperating.

This also becomes apparent with the efficiency of the storage, which is increased by 3.3 percentage points for the HES compared to a pure battery when using the profiles with higher recuperation power. For the profiles with lower recuperation power, efficiency is increased by almost 1 percentage point for configuration 1. For the HES with reduced weight (config. 2), the efficiency is similar compared to the pure battery, but the overall energy demand of the car can be reduced by 6 % due to the reduced weight. Compared to the results of Deterministic Dynamic Programming, the results of the causal SDP strategy decrease by approx. 14 percentage points uphill and 10 percentage points downhill due to the limited knowledge of the driving cycles.

The resulting trajectories of DDP (green) and SDP (red) are exemplarily displayed for profile 4 and storage configuration 2 in Fig. 4. For both strategies, the trajectories of the battery power are very similar. The battery only delivers a lower share of the traction power of up to 20 kW with DDP and 29 kW with SDP. The DLC is primarily recharged by regenerative braking: recuperated energy is solely stored in the DLC when applying DDP, but with the causal SDP strategy, a small part of the recuperated energy has to be stored in the battery. A charging of one storage by the other scarcely takes place as it is costly due to the poor efficiency of two power converters involved. A wider range of the SOC of the DLC is used with DDP, as the future power is known exactly and thus can be considered, while the SDP algorithm has to preserve a medium SOC due to the limited knowledge of future power.

VI. CONCLUSION

Using a hybrid energy storage system combining batteries and double layer capacitors for electric vehicles instead of a pure traction battery results in a significant reduction of losses in the energy storage.

Via Deterministic Dynamic Programming, the size of the hybrid storage can be optimized. A downsizing of the battery to the energy demands of the vehicle is possible with relatively low additional DLC weight of 54 kg, reducing the investment costs for the battery, weight of the car and thus energy consumption.

Stochastic Dynamic Programming offers a way to determine a causal strategy to distribute the power to both storages, taking the stochastic influences of traffic into account. A reduction of losses up to 24 % is possible.

Further work will address the consideration of multiple competing objectives and the applicability of the determined strategies to drive cycles which are not part of the data base.

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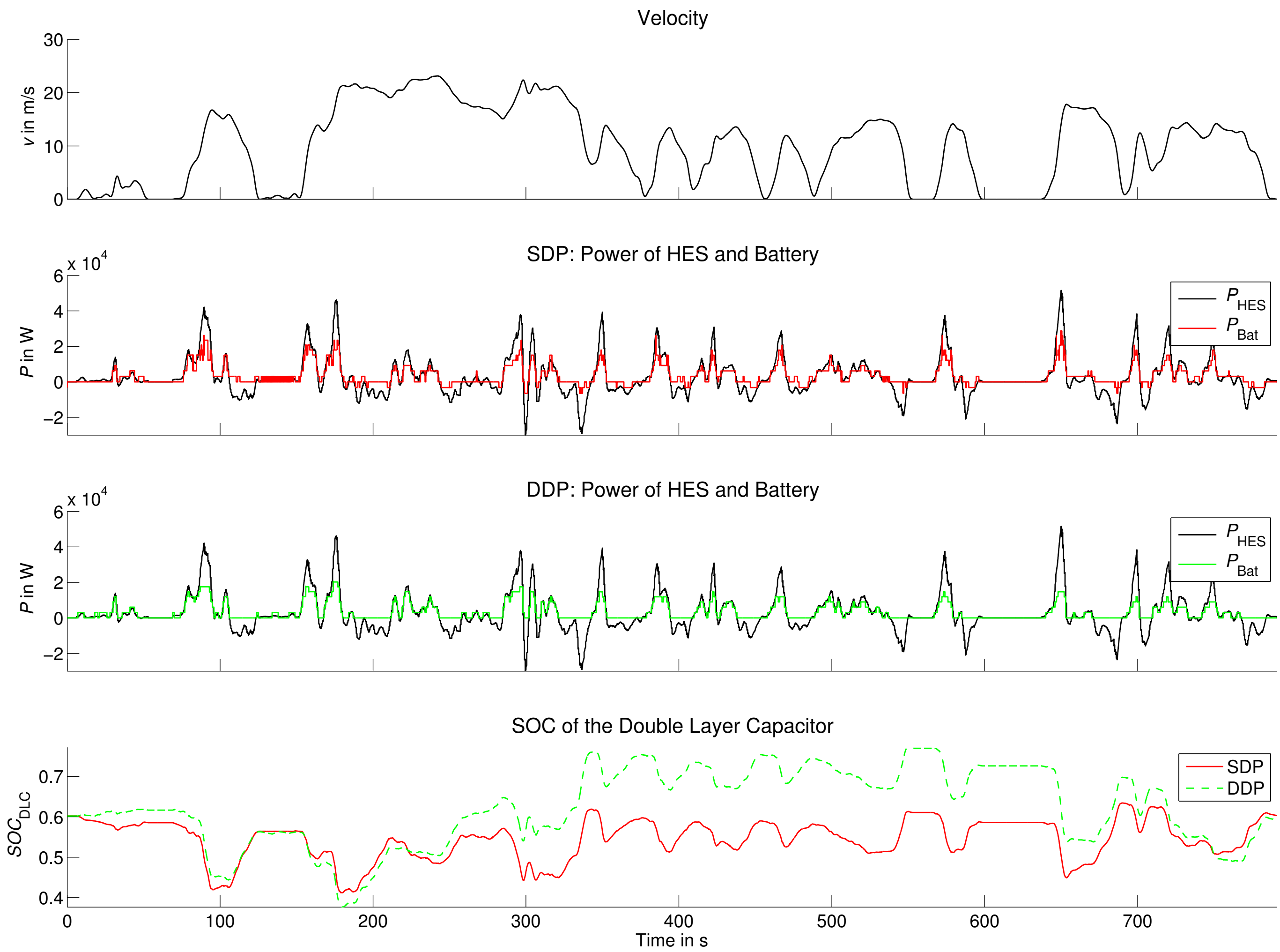

Fig. 4. Comparison of Power and SOC Trajectories Resulting from DDP and SDP Strategy

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