# Causal-based generation of velocity reference for automatic subways

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*Abstract-* In this paper, a trajectory generator based on causal modeling is presented. This strategy is applied to the velocity constraint trajectory generation of an automatic subway. The constraints take into account of passengers comfort by the use of imposed maximum jerk and acceleration values. The proposed velocity generator is enhanced by the use of torque model of the DC machines. The proposed velocity planning method is applied to a model of the VAL subway and comparisons with the present control structure are provided.

Keywords- Causal Ordering Graph, Trajectory generation, Automatic Subway, Control

### I. INTRODUCTION

Nowadays, transportation systems are a key issue of the challenges to face in terms of energy saving and CO2 emission [1]. Thus more and more tramways, subways and trains are developed to ensure an increasing mobility without too environmental impacts [2][3].

Automatic subways have been developed thirty years ago to ensure more secure operations and more efficient traffics in urban centers. The first objective was to obtain the best dynamical performances in order to achieve the requested traffic performances. Nowadays, more considerations are taken into account on energy consumption and mechanical stresses [4]-[6]. It should be notice that the generation of the velocity reference is of prime importance. Speed regulation involves more than matching actual to command speed. It also includes control of acceleration and jerk limiting (controlling the rate of change of acceleration). Specific algorithms has been defined in order to limit the maximum acceleration and jerk values for comfort purpose.

In this paper, a new velocity reference generation is proposed for an automatic subway. This new generation system is based on a causal modeling approach: Causal Ordering Graph [7]. An application to the VAL subway is presented. Its actual control includes a complex algorithm for acceleration and jerk limitation. Moreover, this control is sensitive to noisy disturbances; The new causal-based generation aims to propose a simpler and efficient algorithm. In section II, the actual approach is described. The new causal-based velocity generator is detailed in section III. To illustrate the efficiency of this method, trajectories resulting from this strategy are tested on a model of the VAL subway in section IV.

# II. INITIAL GENERATION SYSTEM OF THE VELOCITY REFERENCE

# A. Studied subway traction system

The studied system is the automatic subway VAL, which has been developed in the 80's (Fig. 1). VAL is a type of automatic rubber-tired people mover technology and was the first automatic subway in the world. A supply rail delivers a constant DC voltage of 750 V to the subway traction system and a specific signal track automatically delivers the velocity reference to control the subway all along the line.



Fig. 1. VAL subways.

# B. Velocity reference and limitations

The velocity reference, noted  $v_{ref}$ , is integrated in the guide track, and delivers velocity steps according to the subway position and the operational modes defined by the Centralized Control Centre. These references induced acceleration discontinuities that subways cannot physically follow without excessive shocks. It is important to control not only acceleration but also jerk, so named because of the uncomfortable (and potentially hazardous) effect produced by abrupt changes in acceleration or speed. Control of jerk, more than control of acceleration itself, contributes to a smooth ride and, for the standing passenger. Thus, the maximum acceleration value and the maximum jerk value (time derivative of acceleration) have to be limited. Maximum values have been defined for comfort performances by  $\gamma_{max} = 1,3 \text{ m/s}^2$  and  $J_{max} = 0,65 \text{ m/s}^3$ .

Initially, these constraints are taken into account directly in the velocity loop. Fig. 2 shows a simplified functional representation of the present velocity control structure. A specific algorithm has been developed on the base of the velocity measurement, noted  $v_{meas}$ . The velocity control yields an acceleration reference,  $\gamma_{ref}$ . Then, an acceleration control loop generates a jerk reference  $J_{ref}$ . This jerk reference is first limited  $J_{lim}$ , and secondly integrated to obtain a new acceleration reference  $\gamma_{ref2}$ . This new acceleration reference is limited  $\gamma_{lim}$ , and a new acceleration control loop leads to the torque reference  $T_{ref}$ . One can note that the velocity measurement is derived to obtain an acceleration values and to define the required torque  $T_{ref}$ . Moreover, this initial control structure contains some empirical functions.

Such solution suffers from three major drawbacks:

- 1) The velocity control system has to deal with the regulation aspects and the limitations. Thus, loop gains result from a compromise between the two functionalities.
- 2) The acceleration limitation requires the feedback of the time derivative of a noisy velocity measure, noted  $v_{meas}$ , which induced more noise in the torque control and can conducted to unwanted oscillatory behaviors.
- 3) Fig 2 shows a very simplified functional representation of the initial velocity control. In practice, the synthesis of the control requires the tuning of more than fifteen loops parameters.



Fig. 2. Present control structure of the Val subway (simplified).

### III. CAUSAL-BASED GENERATION SYSTEM

### A. Causal Ordering Graph

The Causal Ordering Graph is a graphical description which is exclusively based on integral causality (i.e. physical causality) [7]. Such a graphical formalism has been used to develop inversion-based controls of various electromechanical systems.

With COG formalism, two kinds of relationships are considered: rigid relationships (non-time dependant, doublearrow pictogram) and causal relationship (time dependant, i.e. integral, single-arrow pictograms). In order to achieve the control scheme of the system (blue pictograms), specific inversion rules are used: direct inversion for rigid relationship, and indirect inversion (control loop) for causal relationships.



Fig. 3. Causal Ordering Graph pictograms.

### B. Causal-based generation of the velocity reference

First, a causal description of the relationships between the velocity, the acceleration and the Jerk is defined (Fig. 4). In order to obtain the control of the velocity, a cascaded control loop could be derived from the inversion-based methodology (Fig. 5): a velocity control loop (using a velocity controller) and an acceleration control loop (using an acceleration controller). The measurements of the velocity and of the acceleration are required. We can notice that a velocity v is obtained from a velocity reference  $v_{ref}$ .



Fig. 4. Causal description between Jerk and velocity.



Fig. 5. Cascaded control structure resulting from causal inversion principle.

This principle is used in order to obtain the smooth velocity reference  $v_{ref2}$  from the track reference  $v_{ref}$ . In this case the real system (orange) is replaced by an estimation model (purple) and limitation operations (double ovals) are inserted to limit the jerk and the acceleration. The resulting causalbased control can be split into the two separated functionality, i.e. the velocity generation and the velocity control, as depicted in Fig. 6. Each loops of the velocity generator acting on a pure integrator, the two controllers, denoted by relations  $R\gamma$  and  $R\nu$ , are simple proportional gains, noted respectively  $k_{\gamma}$  and  $k_{V}$ .



Assuming zero initial conditions for acceleration and velocity, the velocity profile resulting from this strategy can be decomposed into three stages detailed as follow.

*1. Linear acceleration stage (positive slope)* During this first stage, the condition

$$\left(v_{ref} - v_{ref\,2}\right) > \frac{\gamma_{max}}{k_{v}},\tag{1}$$

is verified. Thus, the acceleration constraint is reached and  $\gamma_{lim} = \gamma_{max}$ . The reference acceleration  $\gamma_{ref2}$  is given by the inner acceleration loop. While

$$\left(\gamma_{max} - \gamma_{ref\,2}\right) > \frac{J_{max}}{k_{\gamma}},$$
 (2)

then

$$\gamma_{ref\,2}\left(s\right) = \frac{J_{max}}{s^2}\,,\tag{3}$$

with s denoting the continuous time domain Laplace variable. During this time, the acceleration is linear. As soon as the condition (2) is not satisfied, the reference acceleration is given by

$$\gamma_{ref2}(s) = \frac{\gamma_{max}}{s} \frac{1}{1 + \frac{s}{k_{\gamma}}} \to \gamma_{max} .$$
(4)

One notes that this first acceleration stage can be assumed to be perfectly linear if  $k_{\gamma}$  is high. This gain has to be chosen accordingly to system accuracy (or quantization error of the measurement system). In this case, the relation (4) is verified during an insignificant time. Therefore, the first stage time  $T_1$ can be expressed as

$$T_1 = \frac{\gamma_{max}}{J_{max}} \,. \tag{5}$$

### 2. Constant acceleration stage

During this stage, the condition (1) is still satisfied, and the reference acceleration reached at the end of the preceding stage is given by

$$\gamma_{ref2}(s) = \frac{\gamma_{max}}{s} \tag{6}$$

# 3. Linear acceleration stage (negative slope)

As soon as the condition (1) is not verified,  $\gamma_{lim}$  becomes equal to

$$\gamma_{lim} = k_v \left( v_{ref} - v_{ref\,2} \right),\tag{7}$$

which is lower than  $\gamma_{ref2}$ . Then, we can have the same behavior (linear acceleration) than during the first stage, but with a negative slope.

The variation of the velocity during the first stage can be calculated as

$$\Delta v = \frac{T_1 \gamma_{max}}{2} = \frac{\gamma_{max}^2}{2J_{max}} \,. \tag{8}$$

Thus, for symmetry reason, the same variation has to be imposed during the last stage. Equaling (7) with  $\Delta v$  given by the condition (1), conducts to the relation

$$\Delta v = \frac{\gamma_{max}}{kv} = \frac{\gamma_{max}^2}{2J_{max}}.$$
(9)

As a result, the tuning condition on  $k_v$  is

$$k_{\nu} = \frac{2J_{max}}{\gamma_{max}} \,. \tag{10}$$

Fig. 7 presents an example of velocity trajectory and its time derivatives resulting from this strategy. The acceleration and jerk constraints are satisfied. The result can be extend for smoother profile (higher order constraints). Such methodology leads to a trajectory composed of piecewise polynomial with order equal to n, with n the higher order of the trajectory time derivative to be limited.

Thus, a jerk and acceleration limited velocity profile (n = 2) is here efficiently generated with the causal-based velocity generator, without any analytical description of the theoretical jerk profile. Moreover, this methodology only requires a fine tuning of one proportional gain, given by (10), according to jerk and acceleration constraints and can be extend to trajectory generation for positioning systems.

# C. Integration of torque limitations

One can note that the previous methodology for trajectory generation can be obtained by the used of two cascaded Finite Impulse Response averaging filter. But in the causal inversion-based method, the generator structure can be easily extended to take into account other aspects of the system dynamics. For example, the evolution of the available DCmachine torque according to the velocity of the subway can be experimentally derived and introduced in the velocity generator to limit the reference of the acceleration. In the same manner, the mean traveling resistance can be reasonably identified and used to limit the acceleration too.



Fig. 7. Example of velocity planning and its time derivatives resulting from the causal-based velocity generation ( $v_{ref} = 10$ m/s,  $\gamma_{max} = 1,3$  m/s<sup>2</sup>,  $J_{max} = 0,65$  m/s<sup>3</sup>).

Fig. 8 presents the causal description of the velocity generator with the use of a conditional switch for the limitation of acceleration according to the available torque. If  $\gamma_{lim}$  is higher than the predicted acceleration  $\gamma_{ref\beta}$  coming from a look-up table, then  $\gamma_{lim1}$  is equal to  $\gamma_{ref\beta}$ . In the case of the studied subway, such a situation can happened when the reference of the velocity is higher than 12 *m/s*. The resulting velocity, acceleration and jerk profiles are described in Fig. 9 for  $v_{ref}$  equal to 18 m/s, i.e. the maximum authorized value for the subway velocity.

The proposed velocity generator gives a physically realizable trajectory to the velocity control loop. Consequently, the velocity control has mainly to deal with the disturbance rejection [6]. Moreover, the resulting control structure did not use the noisy estimated acceleration and the tuning methodology is very simplified as compared to the initial control structure.



Fig. 8. Causal description of the velocity generator including the torque limitations (relation  $R_T$ ).



Fig. 9. Velocity planning and its time derivatives resulting from the causalbased velocity generation including the torque limitations ( $v_{ref} = 18$ m/s,  $\gamma_{max} = 1.3$  m/s<sup>2</sup>,  $J_{max} = 0.65$  m/s<sup>3</sup>).

### IV. SIMULATION RESULTS

In this section, the effectiveness of the method will be demonstrated by testing it on a model of the VAL subway. The electrical part of subway VAL is composed of 6 choppers and 4 DC machines with field winding. The mechanical part of the subway VAL is composed of 4 bogies with differentials and 8 wheels. The developed model, detailed in [6], includes the dominating phenomena acting on the subway dynamics, such as the torque dynamics of the DC machines, the low stiffness of the mechanical differential, the viscous friction on the track and the tire stiffness, backslashes and the contact law between the rail and the wheels. Fig. 10 shows a constant lumped parameters model of a bogie. The whole traction system was simulated using Matlab-Simulink<sup>TM</sup>. A validation of this model is provided in comparison with experimental results obtained from the actual process in [6]. The main data used are as follows: the total mass of the subway is 16 *tons* (no passenger), and the requested velocity is 18 *m/s*.



Fig. 10. Scheme of a boogie of the VAL subway.



Fig. 11. Comparative simulation of required torque Tref ( $v_{ref} = 18$ m/s,  $\gamma_{max} = 1,3$  m/s<sup>2</sup>,  $J_{max} = 0,65$  m/s<sup>3</sup>).

Fig. 11 compares the reference torque calculated by the initial control structure and by the new trajectory profile generator coupled with a simple velocity loop. Fig. 12 shows the velocity, acceleration and jerk behavior of the subway for the two control structures. At low velocity, the behavior of the system is nearly the same for the two controllers. When

approaching the cruise velocity, the initial reference torque being unrealizable by the system, the subway acceleration is not smooth and conducts to high jerk value far beyond the limitation. This torque is limited in the current control loop [6], which is not presented in this paper. The new algorithm provides a reference torque, which can be perfectly follow by the subway and the constraints are satisfied all along the trajectory.



Fig. 12. Comparative analysis of the simulated subway velocity and its time derivatives ( $v_{ref} = 18$ m/s,  $\gamma_{max} = 1,3$  m/s<sup>2</sup>,  $J_{max} = 0,65$  m/s<sup>3</sup>).

### V. CONCLUSIONS

In this paper, an original and systematic method for trajectory generation based on the causal inversion principle is presented. The causal-based generator methodology is a generic tool for trajectory planning. It can easily deal with constraints on the time derivatives of the reference to be planned. This study is specifically applied to the synthesis of the velocity reference for an automatic subway. The main interest is to show the deductive aspect and the simplicity of this approach, as compared to the complexity of the initial control structure. Simulations conducted on a model of the VAL subway demonstrate the effectiveness of this methodology as compared to the initial control scheme. Firstly, the causal based control leads to a smooth subway behavior, which verified the constraints imposed by passengers comfort. Secondly, the resulting control structure dissociates the velocity generation from the velocity control, leading to an easier synthesis of the controller.

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