

# Minimum-Copper-Loss Control Over Full Speed Range of an IPMSM Drive for Hybrid Electric Vehicle Application

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**Abstract** — Since the efficiency is very important in automotive applications, in this paper a new minimum-copper-loss control scheme, over full speed range for an IPMSM drive used for hybrid electric vehicle application under both current and voltage limits is presented. This paper presents an algorithm to get the smallest current vector command in both field-weakening and deep field-weakening regions for a certain amount of torque. Furthermore, this control scheme is robust to dc-link voltage and load torque variations. Moreover, no machine parameters are required in field-weakening control mode. Finally, the simulation results demonstrate the feasibility and performance of the proposed technique.

**Keywords** — *Automotive application, Drive, Electrical drive, Electric vehicle, Hybrid electric vehicle (HEV), Packaging, Permanent magnet motor, Traction application.*

## I. INTRODUCTION

Increasing awareness of air quality and interest in innovative vehicles stimulate research activity to improve the propulsion system by reducing the vehicle emissions. In recent years, considerable developments have been obtained in the area of electric and hybrid electric vehicles, whose use solves the environmental problems and fuel consumption caused by the use of internal combustion engine vehicles. Moreover, the torque generated by the electric motor can be appropriately controlled so that the vehicle stability and safety are greatly improved.

The interior permanent-magnet synchronous machines have become increasingly popular in recent years for automotive applications due to their capabilities such as high efficiency, high torque density and extended speed range. Since the machine efficiency is a key performance characteristic due to the limited power source, e.g., a battery/a fuel cell, the maximum efficiency control can be one of the most attractive criteria [1].

Not like a grid-source drive system, an HEV/FCV has a variable voltage source, e.g., a battery or a fuel cell. So the control strategy should be properly modified to detect the magnitude of the DC bus voltage as quickly as possible to keep the system stable and to operate it in the high-efficiency condition again.

For a pure or hybrid electric vehicle drive, the variation of the parameters are unavoidable, and they may cause some unwanted situations such as instability and performance-degradation problems, particularly in the field weakening region [2].

Some investigations for increasing efficiency can be classified into two categories; power-measuring method [3] and loss modeling methods [4]. The former searches minimum input power resulting in maximum efficiency by adapting step/ramp-wise d-axis current reference and is robust to parameter variations. However, any adaptive algorithm should guarantee the system stability, and it is difficult to set the optimal time interval for difference calculation and steady-state condition. The latter typically represents copper loss and iron loss as each corresponding resistance of an equivalent circuit in the rotor reference frame. Although, this approach is vulnerable to parameter variation, it can be alleviated by parameter compensation methods using a lookup table or an online estimation technique [1].

In some approaches the online algorithms are used to search the optimal current reference that minimizes the copper loss. These methods are based on using the numerical methods to obtain a current reference from a given torque command. Although, they can be effective even to the magnetic saturation [5], but in these investigations, the variation of the DC bus voltage is not considered and it is assumed to be constant. Recently, a field-weakening control for IPMSM drive was presented in [2] that is based on the voltage-constraint-tracking. This scheme can automatically change to a proper control mode and can maintain the minimum copper-loss operation. Also in this scheme there is no need to know the machines parameters variations in the field-weakening control mode and is robust to the DC bus voltage variation, but this scheme is designed for the IPMSM with  $\lambda_{PM}/L_d > I_{sm}$  which is not used for the HEV application.

In this paper, a new minimum-copper-loss control scheme, over full speed range for an IPMSM drive used for the hybrid electric vehicle application under both the current and voltage limits is presented. Particularly, this control scheme is robust to the DC-link voltage

variations. Moreover, no machine parameters are required in the field-weakening control mode.

## II. MATHEMATICAL MODEL AND OPERATING CONSTRAINTS OF AN IPMSM

The steady-state voltage equations of an IPMSM in the rotor reference frame can be expressed as follows [2]:

$$V_{ds} = R_s i_{ds} - \omega_{re} L_q i_{qs} \quad (1)$$

$$V_{qs} = R_s i_{qs} + \omega_{re} (L_d i_{ds} + \lambda_{PM}) \quad (2)$$

Where  $V_{ds}$  and  $V_{qs}$  are the stator d- and q-axis voltages, respectively;  $i_{ds}$  and  $i_{qs}$  are the stator d- and q-axis currents, respectively;  $R_s$ ,  $\omega_{re}$ ,  $L_q$ ,  $L_d$ , and  $\lambda_{PM}$  are the stator resistance, the electrical angular frequency, the q-axis inductance, the d-axis inductance, and the PM flux linkage, respectively. The motor torque equation can be expressed as

$$T_e = \frac{3P}{2} [\lambda_{PM} i_{qs} + (L_d - L_q) i_{ds} i_{qs}] \quad (3)$$

Where  $T_e$  and  $p$  are the generated torque and the number of poles, respectively.

For a practical motor drive system, the operating states should stay within the system limits, considering both motor and inverter ratings. They can be expressed in terms of the current and the voltage constraints, respectively, as

$$i_{ds}^2 + i_{qs}^2 \leq I_{sm}^2 \quad (4)$$

$$v_{ds}^2 + v_{qs}^2 \leq V_{sm}^2 \quad (5)$$

Where  $I_{sm}$  and  $V_{sm}$  are the maximum line-current and the maximum phase-voltage amplitudes, respectively. The current constraint (4) on  $i_{ds} - i_{qs}$  plane can be represented by a circular region with the following circular boundary:

$$i_{ds}^2 + i_{qs}^2 = I_{sm}^2 \quad (6)$$

Substituting (1) and (2) into (5), one can also obtain the voltage constraint which can be represented by an elliptical region with the following elliptical boundary:

$$\begin{aligned} & (R_s i_{ds} - \omega_{re} L_q i_{qs})^2 \\ & + [R_s i_{ds} - \omega_{re} (L_d i_{ds} + \lambda_{PM})]^2 \\ & = V_{sm}^2 \end{aligned} \quad (7)$$

Thus from (7), it is seen that when  $i_{ds} = i_{qs} = 0$ , then one can obtain the corresponding speed, called the critical angular frequency  $\omega_{rc}$  [6]

$$\omega_{rc} = \frac{V_{sm}}{\lambda_{PM}} \quad (8)$$

Similarly, from (7), when  $i_{ds} = -I_{sm}$  and  $i_{qs}$  then one can obtain the corresponding speed, called the extreme angular frequency  $\omega_{re}$  [6]

$$\omega_{re} = \frac{\sqrt{V_{sm}^2 - (R_s I_{sm})^2}}{\lambda_{PM} - L_d I_{sm}} \quad (9)$$

The corresponding operating points of  $\omega_{rc}$  and  $\omega_{re}$  on the  $i_{ds} - i_{qs}$  plane are marked in fig. (1).

To keep the drive operating at a safe and stable state, the available operating point according to the desired speed and desired torque should be inside the overlapped region

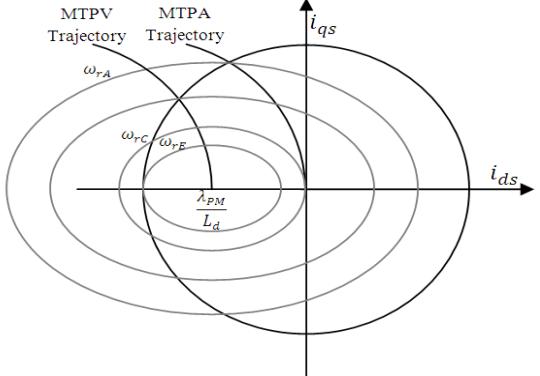


Fig. 1. Operating region of an IPM synchronous machine

defined by (4) and (5). As the motor speed runs higher, the voltage-limit ellipse shrinks, and the area of the available operating region gets smaller. IPMSM used as traction motors for hybrid electric vehicles, due to the packaging constraint in the vehicular powertrain architecture, are designed into pancake shape, thus the  $\lambda_{PM}/L_d$  is smaller than  $I_{sm}$  [7]. So in beyond the extreme angular frequency, the overlapping area between the peak current constraint circle and peak voltage constraint ellipse decrease to inside the peak current circle. Correspondingly, the drive system operates on a smaller peak current constraint circle that is defined by the relationship given in (10) [7]

$$I_{sm} = \frac{V_{sm} + \omega_{re} \lambda_{PM}}{\omega_{re} L_d} \quad (10)$$

When  $\omega_r \in (0, \omega_{rA})$ , where  $\omega_{rA}$  is the maximum speed of the positive constant torque limit, the MTPA control can be applied to achieve high performance. This operating region is in fact the constant torque region. When  $\omega_r \in (\omega_{rA}, \omega_{rc})$ , the corresponding operating region is called the partial field-weakening region. In this region, depending on the load condition, either the MTPA or the field-weakening control mode can be applied. When the  $\omega_r$  is beyond the critical speed, the corresponding region is known as the full field-weakening region [2]. Which in this region field-weakening or deep field-weakening is applicable.

## III. MINIMUM-COPPER-LOSS CONTROL

### A. Control in constant torque region

In the constant torque region, the maximum-torque-per-ampere control strategy was introduced to minimize copper loss control [5]. The amplitude of stator current  $i_s$

is directly linked to the torque production [8]. Thus the d-q axis component of the current vector for the maximum-torque-per-ampere are derived as follows [8]:

$$i_{ds}^* = \frac{\lambda_{PM} - \sqrt{\lambda_{PM}^2 + 8(L_q - L_d)^2 i_s}}{4(L_q - L_d)} \quad (11)$$

$$i_{qs}^* = sign(i_s) \sqrt{i_s^2 - i_{ds}^2} \quad (12)$$

control in this region is under current constraint in (4).

### B. Control in field-weakening region

Control in field-weakening region is under both peak current and voltage constraint and in this control mode, it had been proven that the intersection point of the torque-demand and voltage-limit curves is the theoretical minimum copper-loss operating point [6]. Thus considering a stable drive operation, the available current vector according to desired speed and torque should be inside the overlapped region defined by (4) and (5). Hence, as the current vector is located inside the available operating region but not at the intersection point of the torque-demand and voltage-limit curves, the demanded current vector should be moved right to approach the voltage-limit curve to achieve the minimum copper-loss operation. It is implemented by assigning the d-axis current command as

$$i_{ds}^*(t) = i_{ds}^*(t_0) + \int_{t_0}^t K_a |\omega_r^* - \omega_r| dt \quad (13)$$

to obtain no steady-state error, where  $K_a$  is a positive integration constant [2]. On the other hand, as the commanded voltage vector is located outside the available operating range, the current controllers tend to lose control because of the limited dc-link voltage. Hence, an effective solution is the realization of an outer voltage loop, which deliver a field-weakening component  $\Delta i_{df}$  that is determined by deviation of actual voltage from maximum phase voltage amplitude in outer voltage regulating loop and it is added to d-axis component of current vector in constant torque region [8].

$$i_{ds}^* = i_{ds(mtpa)} + \Delta i_{df} \quad (14)$$

Thus this method of field-weakening is robust to sudden variation of DC bus voltage in transient operation and also this scheme utilizes not the model of the IPMSM and thus it is robust and insensitive to load condition and the machines parameters.

By use of this method in field-weakening region, the operating point can get very close to intersection point of the torque-demand and voltage-limit curves.

### C. Control in deep field-weakening region

As the speed increases further, the rated voltage is insufficient to produce constant maximum current, thus the field weakening current has to fold back in order to

satisfy both voltage and current constraints at the corresponding speed. Hence peak constant power operation ceases and the optimal current trajectory approach to characteristics current, as speed further increases. Minimum copper loss operation of IPMSM in deep field-weakening region is determined from any operating point where the torque curve is tangent to the voltage ellipse [1] that is calculated form the following condition:

$$\left. \frac{\partial T_e}{\partial i_{ds}} \right|_{\omega_{re}} = 0 \quad (15)$$

$$\left. \frac{\partial (V_{sm}/\omega_{re})_e}{\partial i_{ds}} \right|_{\omega_{re}} = 0 \quad (16)$$

and using (3) and (7), the equation that determines the  $i_{ds}$  current for minimum copper loss operation is given by(17)

$$a = (L_d - L_q)(L_d^2 + L_q^2) \quad (17)$$

$$b = L_d \lambda_{PM} \left[ \frac{3}{4} P L_d + (L_d - L_q) \right]$$

$$c = \frac{3}{4} P \lambda_{PM}^2 L_d - L_q^2 (L_d - L_q) i_s^2$$

$$i_{ds}^* = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

In this region, the field-weakening is considerable and the q- axis component of current vector is low. Therefore, the influence of magnetic saturation is insignificant and hence saturation can be neglected [9]. Also the field-weakening in this region is only under the voltage constraint.

## IV. PROPOSED MINIMUM-COPPER-LOSS CONTROL SCHEME

Based on the analysis above, the operational principle of the proposed minimum-copper-loss control scheme can be explained by using the block diagram and the flowcharts, as shown in Figs 2 and 3, respectively.

As shown in Fig. 2, the output signal from speed controller is current vector of stator and the input signals of minimum-copper-loss control block are  $\Delta V_s$ ,  $i_{df}$ ,  $\omega_r^*$ ,  $\omega_r$  and  $i_s^*$  where  $\Delta V_s$  and  $i_{df}$  are determined by deviation of actual voltage from maximum phase voltage amplitude in outer voltage regulating loop. Also the maximum pure sinusoidal voltage vector that can be produced for a given dc-link is  $.577V_{dc}$ , but  $K_v$  must be less than 0.577 [10] and, thus in this paper, a value of  $K_v = 0.56$  has been used. Now, consider Fig. 1. First  $i_{dx}$  and  $i_{qx}$  are calculated by using (11) and (12). Then the flowchart checks whether  $\Delta V_s$  is smaller than zero, if the inverter output voltage is saturated, the value of an index named “status” is assigned to be equal to zero to indicate that the field-weakening control should be activated. It should be mentioned that the value of “status” is assigned to be one as the program is in the initialized stage. By checking the value of “status,” the

decision process divides into the MTPA control and the field-weakening control routes. As to the succeeding field-weakening control process, it is further divided into two flowcharts named A and B by checking again whether the  $\Delta V_s$  is smaller than zero. If the phase voltage is not saturated yet, the program will go to flowchart A. On the other hand, the flowchart B will be selected.

In flowchart A, the inverter output voltage still does not reach  $V_{sm}$ . This condition reveals that the motor drive may either be in the constant torque-limit region or just in the left vicinity of the voltage-limit curve corresponding to the present rotor speed. Therefore,  $i_{dx}^*$  is used to compare with  $i_{ds}^*$  to decide whether the MTPA or the field-weakening control mode should be chosen. Which  $i_{ds}^*$  in flowchart A is calculated by using the previous value if  $i_{ds}^*$  plus the absolute value of the rotor speed error ( $\omega_r^* - \omega_r$ ) times a positive constant  $\alpha$ . An empirical

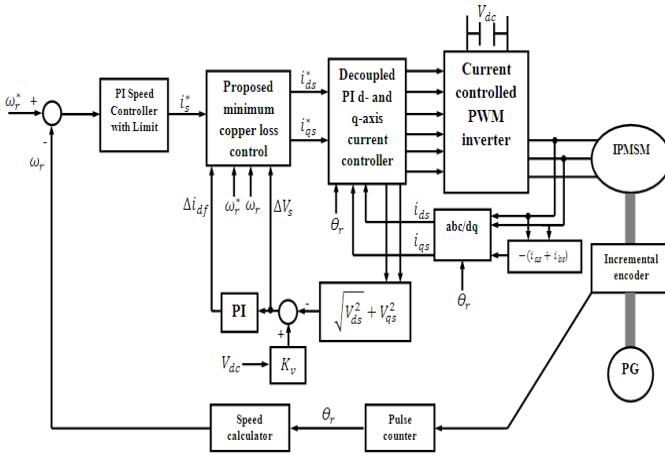


Fig. 2. Block diagram of proposed minimum-copper-loss control scheme

method is used to properly select the value of  $\alpha$  to guarantee a stable operation. From the geometrical interpretation about the operation principle of the drive in the previous section, it is seen that, by this scheme, the corresponding current vector will move right to approach the voltage-limit curve. Also  $i_{qMax}$  in this flowchart is the maximum allowable value of q- axis component of current vector that is calculated from current limit circle. In the flowchart B, the inverter output voltage already reaches  $V_{sm}$ . This condition reveals that the operating point of drive is either on or the right of the voltage-limit curve corresponding to the present rotor speed. In order to prevent the unstable operation,  $i_{ds}^*$  is calculated from  $i_{dx}$  in constant torque region added to  $\Delta i_{df}$ , that is determined by deviation of actual voltage from maximum phase voltage amplitude which is passed from PI controller in outer voltage regulating loop. Then the value of  $i_{df,deep}$  is calculated from (17) and that is compared to  $i_{ds}^*$ , if the values of  $\Delta i_{df}$  and  $i_{ds}^*$  satisfy the condition  $i_{ds}^* \geq i_{df,deep}$ , the motor operates in field-weakening control mode. On the contrary, if the condition is not satisfied, the motor operates in deep field-weakening control mode, and the  $i_{qMax}$  is calculated from voltage limit ellipse.

In flowchart C, as the absolute value of q- axis component of current vector is greater than  $i_{qMax}$ , for prevention of exceed from constraints in heavy load, the value of  $i_{qX}$  is limited by  $i_{qMax}$ .

## V. SIMULATION RESULTS

The proposed minimum-copper-loss control scheme has been investigated through computer simulation by Matlab/Simulink for IPMSM used in hybrid electric vehicle with parameters that is shown in Table. I.

From Table. I, one can observe that  $\lambda_{PM}/L_d < I_{sm}$ , indicating that in the tested motor constant power operation is limited to a finite speed range but field-weakening operation is available up to theoretically infinite speed. Also, according to (8), (9) and part. IV,  $\omega_{rc} = 4179.93$ ,  $\omega_{rE} = 2713.18$  and  $V_{sm} = 168$  V.

TABLE I  
PARAMETERS OF TESTED IPM SYNCHRONOUS MACHINE

Number of Phases	3
Number of Poles	12
$R_s$	0.03Ω
$L_d$	0.65 mH
$L_q$	0.97 mH
$\lambda_{PM}$	0.064 Wb
Peak Current Constraint	250 A
Rated DC Bus Voltage	300 V
Peak Transient Torque	130 Nm
Peak Transient Power	25 kW

Fig. 4 show the test result of a step speed command of  $\omega_r^* = 2600$  starting from rest at  $t = 0.1$ s and with a steady-state load torque of 60 N.m at 2600 rpm. At  $t=0$ s,  $V_{dc}$  is initially kept at 300 V and a sudden change of  $V_{dc}$  from 300 V to 200 V occurs at  $t = 0.5$ s.

As shown in Fig. 4, the motor starts accelerating by MTPA in constant torque region. As the speed reaches beyond base speed, the stator voltage magnitude is increases to  $V_{sm}$ , which indicates field-weakening control. So the field-weakening is activated and the magnitude of  $i_{ds}$  is increases. As the motor speed runs higher, in order to prevent the operating point exceeding from voltage-limit ellipse, the control mode is enter into deep field-weakening region and the magnitude of d-axis current is decreased. Because the control mode is located in partial field-weakening region, the operating point is return to MTPA in steady-state. As shown in Fig. 4(a), for  $t > 0.5$ s,  $V_{dc}$  is reduced to 200 V( $V_{sm} = 112$  V) and the control automatically switches back to the field-weakening mode and the magnitude of  $i_{ds}$  also increases significantly. Also as shown in Fig. 4(a), the voltage amplitude is close to  $V_{sm}$  in beyond base speed, which indicates minimum copper loss in field-weakening and deep field-weakening regions and with voltage variation.

Fig. 5 show the test result of a step speed command of  $\omega_r^* = 3500$  starting from rest at  $t = 0.1$ s and with a steady-state load torque of 40 N.m at 3500 rpm. At  $t=0$ s,  $V_{dc}$  is initially kept at 300 V and a sudden change of  $V_{dc}$  from 300 V to 400 V occurs at  $t = 0.5$ s.

As shown in Fig. 5, for  $t > 0.5$  s,  $V_{dc}$  is elevated to 400 V but  $V_s$  is smaller than  $V_{sm}$  in this position and so the control automatically switches back to the MTPA mode

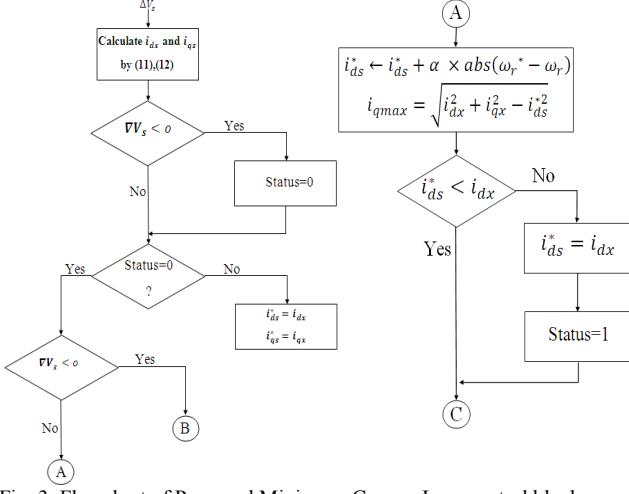


Fig. 3. Flowchart of Proposed Minimum-Copper-Loss control block

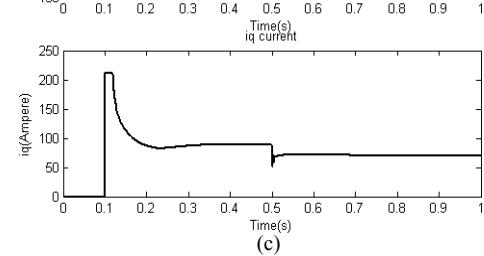
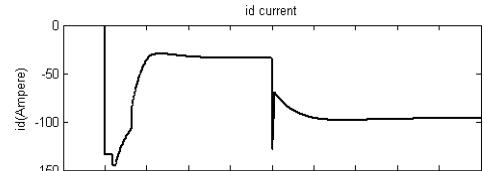
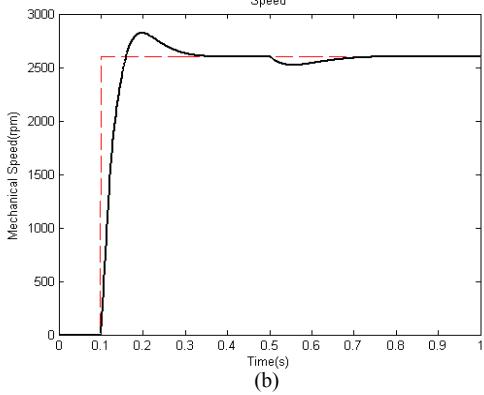
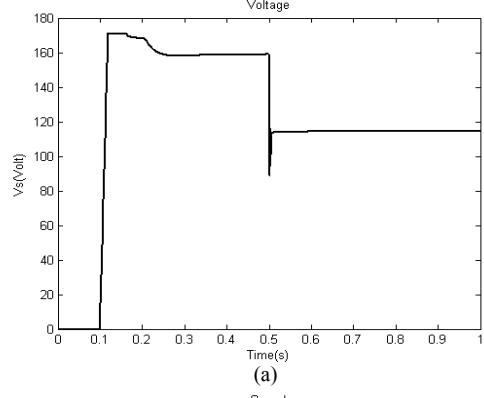


Fig. 4. Simulation result of  $V_s$  (a),  $\omega_r$  (b) and  $i_{ds}$ ,  $i_{qs}$  (c) under reduction of  $V_{dc}$ .

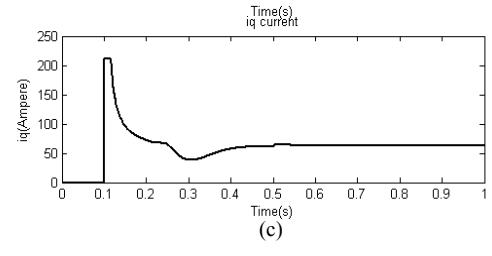
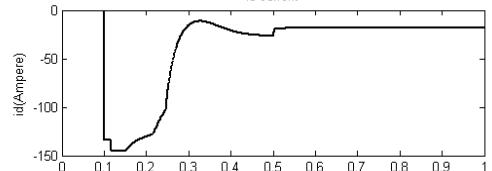
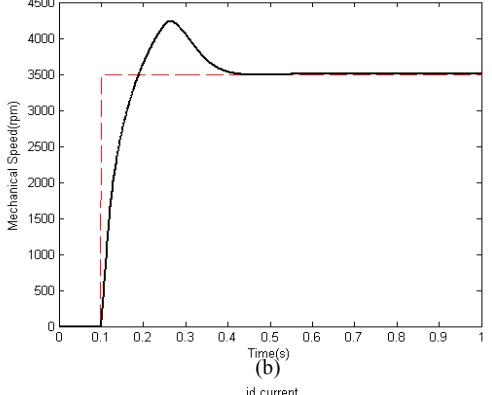
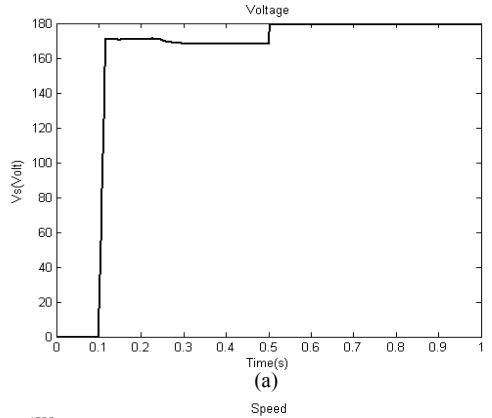
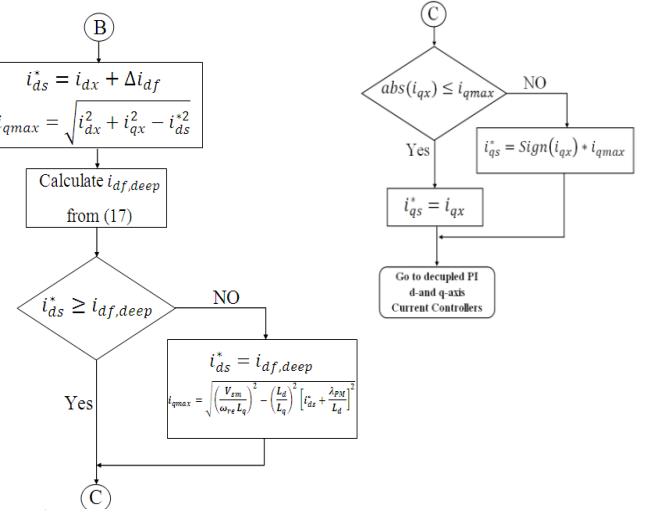


Fig. 5. Simulation result of  $V_s$  (a),  $\omega_r$  (b) and  $i_{ds}$ ,  $i_{qs}$  (c) under elevation of  $V_{dc}$ .

and the magnitude of  $i_{ds}$  also decreases significantly.

To show the performance under variation of the load torque in the full field-weakening region, a step speed command of  $\omega_r^* = 4500$  is applied to the drive starting from rest at  $t = 0.1$ s, Fig. 6 is shown the drive operation with sudden load torque 20N.m at  $t = 0.5$ s that reduce to 10N.m at  $t=1$ s.

As shown in Fig. 6, at  $t=0.5$ s, speed response is suddenly decreased slightly and then quickly recovers to steady-state. Similarly, at  $t=1$ s, speed response is increased slightly when the load is suddenly decreased. From Fig. 6(c), one can see that, with load variations the stator voltage is not exceeded from  $V_{sm}$  and in full field-weakening region the stator voltage is equal to  $V_{sm}$  in any speed and load. So the control not switches back to MTPA but in order to prevent the operating point exceeding from voltage-limit ellipse the magnitude of  $i_{ds}$  is increased and decreased with load variations.

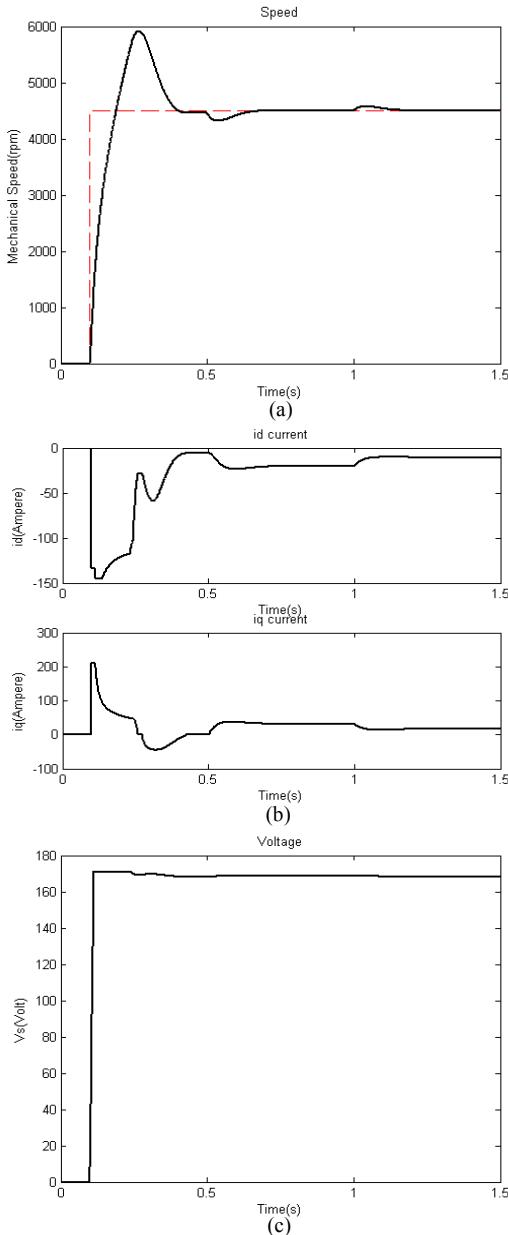


Fig. 6. Simulation result of  $\omega_r$  (a),  $i_{ds}$ ,  $i_{qs}$  (b) and  $V_s$  (c) under load variations in full field-weakening region.

## VI. CONCLUSION

A new minimum-copper-loss control scheme, over full speed range for an IPMSM drive used for hybrid electric vehicle application under both current and voltage limits is presented in this paper. The proposed scheme can achieve minimum copper loss in field-weakening and deep field-weakening regions. In this scheme the operating point can track automatically the intersection point of the torque-demand curve and the voltage-limit ellipse in field-weakening region and it can track automatically the tangential point of the torque-demand curve and the voltage-limit ellipse in deep field-weakening region. Furthermore, as the drive is operated in partial and full field-weakening regions despite the variations of inverter dc-link voltage and load torque, the transitions between MTPA and field-weakening control mode and vice versa are quite smooth and without exceeding from maximum voltage limit.

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