# The Bond Graph – an excellent modelling tool to study abstraction level and structure comparison

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Abstract—In this paper the Bond Graph (BG) technique is used for modelling dynamical systems. Thus a typical modern power oriented representation ensures energy independency, a view inside the studied system and preservation of the physical structure based on special bidirectional connections. A comparison of scalar and vectorial connections between the components and a more detailed classification for vectorial models are firstly given. Thereafter it will be shown that the BG may be used as an excellent tool both for defining different abstraction levels of the same modelling artefact still without using the principle of subsystems and for the detection of the same power flow structure hidden behind different hardware solutions for one given problem. That way the paper proves that scalar, vectorial or compact modelling provides specific advantages respectively and that the definition of two conjugated power variables is a basic principle of structure comparisons. The electrical machine and the task of power split are used as examples close to the subject of vehicle power and propulsion.

Keywords- modelling; Bond Graph; power flow; bidirectional connection; scalar; vectorial; level; structure study

## I. INTRODUCTION

Modelling vehicles and propulsion systems takes advantage of applying graphical description methods based on power flow. This principle of action and reaction realize methods as for instance Energetic Macroscopic Representation (EMR) [1], [2], Power Flow Oriented Graph (POG) [3], [4] and Bond Graph (BG) [5], [6]. Although each method features specific advantages there are a lot of similarities and close relations including same parameter definitions partly [7], [8]. In particular all these mentioned power flow oriented methods clearly model the inner structure of the dynamical system and thus enable a view inside, represent the physical structure and support to study the power exchange between the different basic elements and subsystems. The deduction of control schemes is an EMR feature which has to be pointed out for instance, whereas the system analyses using state space equations highlights the POG for one. The BG method in particular equates to a tool which is very well qualified for easily enabling different abstraction level models and structure comparisons. This is due to the strictly energy domain independency in conjunction with bidirectional operating connections (bonds) between the basic elements. Comparisons of EMR, POG and BG were specified in detail already [4], [6] - [10]. The BG modelling technique is of particular interest for automotive applications [11] - [14] which typically deal with several energy domains, make use of subsystems of different detailedness and need to compare various hardware solutions as well as to evaluate energy efficiency. Especially eyecatching, graphical marking of causality and points of constancy of one of the conjugated power variables are unique propositions of BG's. Furthermore easy handling of subsystems and abstraction levels as well as the compact bidirectional identification of the power flow are of great practical interest for modelling. Thus the modelling of complex systems such as a HEV may be advantageously implemented [11] via BG's making use of various detailedness and subsystems [12]. Therefore the paper focuses on BG and in particular upon abstraction level and structure comparison. It is organized as following. Section II describes and compares basic connection types and features; sec. III introduces the doubly fed induction machine as a good example for defining different modelling levels whereas sec. IV applies the power split task as useful example for hidden power flow structures of similar type and sec. V explains how to establish simulation based on BG. The paper will show that the BG may claim to be a valuable tool which enables some supplementary perceptions studying vehicle power and propulsion.

# II. BOND GRAPH BASICS AND SYSTEMATIZATION OF MODELLING CONNECTION TYPES

#### A. Modelling using BG's

Connections between BG elements, so-called bonds, own conjugated power variables of types effort (e) and flow (f), show a half arrow to mark the preferred energy transfer direction as well as a causality stroke at one side to mark the starting side for the flow transfer regarding a selected bond by definition. The product of effort, as for instance voltage or torque, and flow, as for instance current or angular velocity, has to be a power. Please see appendix I for definition of BG basic elements, causality, icon and resolved equation representation. The latter explains the belonging mathematical operation of the element depending on causality and parameter. Prefixes "M" of particular elements (MGY, MTF, MSE, MR) would symbolise non-constant parameters and effect an additional unidirectional powerless entrance to transfer the changing parameter. Diverse rules and hints are available to

apply and reshape BGs. Appendix I makes clear that modelling via BG neither does involves programming of equations lying under the icons nor copy and paste of transfer functions and mixing points. Instead of the user copies, connects via nodes and parameterizes BG elements (icons). This draws an essential distinction to EMR and POG. Even though BG's take advantage of bidirectional connections an add-on library enables BG's to be simulated by means of standard software [15], [16] using graphical user interfaces for customizing and thus drastically reducing the number of library elements.

## B. The basic connection types

Obviously there may be found the two types scalar and vectorial if studying possible connections between power flow modelling elements. Table I highlights the features of both types. In doing so following features are conspicuous. Firstly, once being familiar with BG modelling easy scalar cases. especially in electrical and mechanical domain, may be directly modelled and simulated without any mathematical equation. This important feature may cause various positive effects by concentration on the inner mode of operation, i.e. the power flow, at the outset and on the contrary to other methods assured equation generation after that only. Secondly, even if type I scalar is applied there exist cases requiring the description of mutual influences of scalar parts of the system. Thus already scalar modelling may need the definition of non-basic field elements IF, CF, RF using multi port definitions – cf. appendix II. The lift magnet serves as a proper example for modelling non-linear parameters in this regard. Thirdly to connect scalar and vectorial parts of a modelling requires TF or GY elements applying parameters of type vector. Because of the bidirectional bonds there have to exist two appendant equations one of them using the transposed parameter. Missing one of these adequate conjugated equations yields a strong hint for an incomplete or wrong system description.

## C. The vectorial system modelling

A more detailed study of the vectorial modelling results in table II. It may be understood to distinguish between vectorial modelling of type II and III. According table II it becomes clear that graphical system descriptions of type II are based on coordinate transforms and representations by means of components. An continuative study of type II systems leads to a subdivision in systems which may be folded and therefore easily allow differing modelling levels and systems with holonomic constraints which take an order reduction and therefore may be modelled by means of several completely equivalent variants of same level. The graphical representation of type III systems takes advantage of scalar described basic segments in a first step. An arbitrary number of such segments may be incorporated via very compact vectorial modelling in a second step. The Kelvin-Voigt element [18] as basic segment for belt conveyor modelling representative demonstrates the idea of type III systems. However, this paper focuses on systems of type II.

# III. DIFFERENT DETAILED MODELLING LEVELS OF THE SAME OBJECT

This paragraph uses the doubly fed induction machine (DFIM) as an example. On the one hand the DFIM stands for a special proper example related to the aim of this paragraph, on the other hand DFIM represents an essential part of a possible series HEV topology [13]. Equation system (1) - (5) specifies the usual mathematical description. The rotor  $u_{Rd}$ ,  $u_{Rq}$  and stator  $u_{Sd}$ ,  $u_{Sq}$  voltage components lead to corresponding current components  $i_{Rd}$ ,  $i_{Rq}$ ,  $i_{Sd}$ ,  $i_{Sq}$  by means of resistances  $R_R$ ,  $R_S$  and inductances  $L_R$ ,  $L_S$  of the rotor and stator windings. Whereat  $\omega_S$  and  $\omega$  represent the synchronous and mechanical angular velocity,  $L_K$  symbolises the coupling inductance between stator and rotor,  $z_P$  stands for the number of pole pairs and  $\Psi$  symbolises the flux linkage.

TABLE I. COMPARISON OF MODELLING CONNECTION TYPES

Type I: Scalar	Type: Vectorial	
a) All connections between the elements are of type scalar	a) Connections between elements typically or partly of type vectorial	
b) Direct modelling of easy systems without math. equations is possible	b) Summing up of components of same background to vectorial connections	
c) Different parts of the system may be connected by field elements with parameters of type matrix	c) Parameters typically or partly of type vector respectively matrix d) Parameters of type vector connect scalar and vectorial model parts	
Examples: - Ia/b: DC machine; Elastic shaft [6]	Examples: - Electrical three phase machines	
<ul> <li>Ia/c: Lift magnet [17], [18]</li> </ul>	<ul> <li>Multidimensional motion</li> </ul>	

TABLE II. COMPARISON OF VECTORIAL MODELLINGS WITH REGARD TO THE SYSTEM CONSTRUCTION

Type II: By means of different components of a system	Type III: By means of a number of same basic segments	
<ul> <li>a) System description based on a coordinate system</li> <li>b) Coordinate transform optionally included</li> <li>c) Folding of similar parts of the system for a most compact modelling</li> <li>d) Definition of equivalent systems based on holonomic constraints</li> </ul>	<ul><li>a) Subdivision of greater, extensive systems of typically homogenous state in segments of equal conditions</li><li>b) The parameters of each segment are typically equal, but it is no precondition</li><li>c) The system may feature a feedback, but this is no precondition</li></ul>	
Examples: - IIa/d: Double pendulum [17] - IIb/c: Electrical machines [20] - IId: Power split [22], [24]	Examples: — IIIa/b: Fluid tank system [5] — IIIa/b/c: Belt conveyor [18] — IIIa/b: Transformer heating [19]	

$$\Delta \underline{u} = \underline{u} - \underline{M}_R \cdot \underline{i} + \omega_S \cdot \underline{M}_{S1} \cdot \underline{\Psi} - z_P \cdot (\underline{M}_{ZP} \cdot \underline{\Psi}) \cdot \omega \qquad (1) \qquad \Delta T = T_M - T_L - K_d \cdot \omega; \quad \omega = J_M^{-1} \int \Delta T \, dt \qquad (3)$$

$$\underline{i} = \underline{M}_{L} \underbrace{\int \Delta \underline{u} \, dt}_{\underline{\Psi}}; \quad T_{M} = z_{P} \cdot (\underline{M}_{ZP} \cdot \underline{\Psi})^{T} \cdot \underline{i}$$

$$(2) \quad \underline{x} = \begin{bmatrix} x_{Sd} & x_{Sq} & x_{Rd} & x_{Rq} \end{bmatrix}^{T} \text{ where } \underline{x} = \in \left\{ \Delta \underline{u} \ \underline{i} \ \underline{u} \ \underline{\Psi} \right\}$$

$$(4)$$

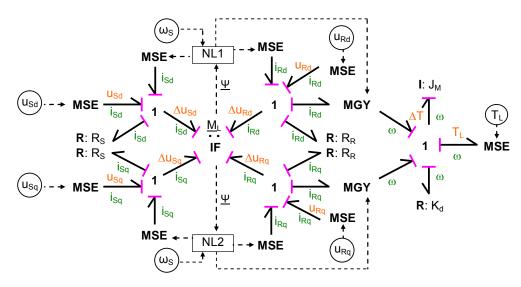


Fig. 1a scalar model via representation of all voltage and current components in particular

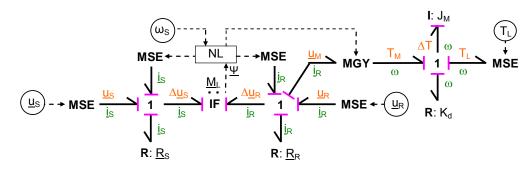


Fig. 1b vectorial model via vertical folding of stator and rotor components respectively

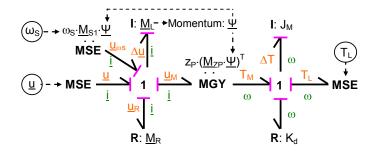


Fig. 1c compact model via horizontal folding of stator and rotor side [20]

Figure 1. Different abstraction levels for same problem using example DFIM (NL: nonlinearity)

The motor torque  $T_M$  is obtained from (2), whereas (3) describes the motion equation, where  $J_M$ ,  $K_d$  and  $T_L$  are inertia, friction coefficient and load torque. Please note, the Bond Graph modelling will take advantage of the idea of a modified power conserving Park transform from electric three-phase system to the d,q two-axis coordinate system as an essential postulate [20]. There is no need to model three phases [14] because typical control methods are based on a transform as for instance the field-oriented control. Since the system description is based on both d-q components and stator / rotor construction two folding (subsuming) steps offer three abstraction levels – fig. 1.

Using a scalar model representation permits an immediate overview to all particular physical processes, as for instance storages and losses. Links of more than two elements are apparently visible - see fig 1a: IF. This causes the most comprehensive drawing. The vertical folding according fig. 1b groups power flow components of hardware elements. Thus the point of view focuses on the functionality of the hardware elements. Drawings get easier, but more background knowledge is necessary to understand. Figure 1c applying the horizontal folding of similar constructed hardware elements effects the most compact modelling. The focus now changes over to the basic functionality of the studied artefact. That step results in easy modelling structures assuming consolidated background knowledge. The method of subsystems is not still used. All three variants could constitute the content of a subsystem.

# IV. HIDDEN POWER FLOW STRUCTURES OF DIFFERENT HARDWARE SOLUTIONS

The Bond Graph method may support a classification system for varying architectures of different energy storage systems of different types, such as electrical, hydraulic or kinetic [21] as well as support to detect same power flow structures of different hardware solutions. This paragraph deals with the latter. In doing so BG's display their special

advantages such as highlighted causality for fast and easy understanding, direct view and graphical differentiation of two types of balance equations or separated representation of storage and loss operations. Assuming in general one combustion engine (torque  $T_{CE}$ , angular velocity  $\Omega_{CE}$ , inertia  $J_{CE}$ , friction  $f_{CE}$ ) and two electrical machines  $(T_{MI}, T_{M2}, \Omega_{M1},$  $\Omega_{\rm M2},~J_{MI},~J_{M2},~f_{MI},~f_{M2})$  a typical task constructing Hybrid Electrical Vehicles is a power split regarding these three units. Possible solutions for this issue are for instance planetary gear [22] - [24], two concentric electrical machines [24], [25], series hybrid [22] or parallel hybrid architectures [22]. Table III summarises the belonging holonomic constraints regarding the angular velocities, states the gear ratios  $k_R$  and  $k_S$  as well as specifies the vectorial equation of motion reduced to second order just based on these holonomic constraints. Whereas  $J_{eq}$ symbolizes the vehicle mass m equivalent inertia converted by means of the wheel radius  $r_{wh}$  and the fixed ratio  $k_{trans}$  of the reduction gear. This very simple model takes into account a general resisting torque  $T_{Res}$  only but neglects any slip or cornering. Nevertheless it is sufficiently for this purpose which focuses on power split hardware.

Another effect of the constraints is the free choice of the two state variables [23]. Hence fig. 2 features two columns exemplarily both with  $\Omega_{\rm M1}$  as the first state variable. The left column introduces  $\Omega_{CE}$  whereas the right column uses  $\Omega_{M2}$  as the second state variable. Resulting components of the motion equation may be understood from table IV. Although the same BG structure has to arise from the functionality – see fig. 2 – the belonging parameters lead to heterogeneous application properties. The right representation ensures a common description model of all four hardware solutions while a division by  $k_S$  hinders that for the left representation if one takes up a mathematical position - table V. The physical position would state that  $\Omega_{M1}$  and  $\Omega_{CE}$  cannot lead to a calculation of a nonexistent angular velocity  $\Omega_{M2}$ . The right BG in fig. 2 also handles the further order reduction by omission of one of the electrical machines in the case of parallel hybrid architecture [23].

TABLE III. HOLONOMIC CONSTRAINTS FOR THE FOUR SYSTEMS IN [22] - [24] AND COMMON DEFINITIONS

a, planetary gear:	$\Omega_{\rm CE} = k_{\rm R}\Omega_{\rm M1} + k_{\rm S}\Omega_{\rm M2}$	$T = \frac{d\Omega}{d\Omega} + f = \Omega$
b, two concentric machines:	$\Omega_{\mathrm{CE}} = \Omega_{\mathrm{M1}} - \Omega_{\mathrm{M2}}$	$\underline{\mathbf{T}} = \underline{\mathbf{J}} \cdot \frac{\mathrm{d}\underline{\Omega}}{\mathrm{d}t} + \underline{\mathbf{f}} \cdot \underline{\Omega}$
c, series hybrid	$\Omega_{\rm CE}  =  k_{\rm S} \Omega_{\rm M2};  k_{\rm R}  =  0; \ k_{\rm S}  \neq  0 \label{eq:omega_ce}$	$\left \begin{array}{c} \underline{\mathrm{T}} = \left  \begin{array}{c} \mathrm{T_1} \\ \mathrm{T} \end{array} \right ;  \mathrm{J_{eq}} = \mathrm{m} \frac{\mathrm{r_{wh}^2}}{\mathrm{t^2}} $
d, parallel hybrid	$\Omega_{\mathrm{CE}}  =  k_{\mathrm{R}}^{} \Omega_{\mathrm{M1}}^{};  k_{\mathrm{S}}^{}  =  0; \ k_{\mathrm{R}}^{}  \neq  0$	$- [T_2]'$ $eq$ $k_{trans}^2$

TABLE IV. MODELLING APPROACH REGARDING  $\Omega$  including parameters of the equivalent systems

$\underline{\Omega} = \begin{bmatrix} \Omega_{\mathrm{M1}} \\ \Omega_{\mathrm{CE}} \end{bmatrix};  \begin{array}{l} T_1 = T_{\mathrm{M1}} - K_{12} \cdot T_{\mathrm{M2}} - T_{\mathrm{Res}} \\ T_2 = T_{\mathrm{CE}} + K_{23} \cdot T_{\mathrm{M2}} \end{array}$	$\underline{\Omega} = \begin{bmatrix} \Omega_{\mathrm{M1}} \\ \Omega_{\mathrm{M2}} \end{bmatrix};  \begin{array}{l} T_1 = T_{\mathrm{M1}} - K_{10} \cdot T_{\mathrm{CE}} - T_{\mathrm{Res}} \\ T_2 = T_{\mathrm{M2}} + K_{20} \cdot T_{\mathrm{CE}} \end{array}$
$\underline{J} \; = \begin{bmatrix} J_{M1} \! + \! J_{eq} \! + \! K_{12}^2 J_{M2} & \! - \! K_{sd} J_{M2} \\ \! - \! K_{sd} J_{M2} & \! J_{CE} \! + \! K_{22} J_{M2} \end{bmatrix}$	$\underline{J} = \begin{bmatrix} J_{M1} + J_{eq} + K_{10}^2 J_{CE} + K_2 J_{M2} & -K_{10} K_{20} J_{CE} \\ -K_{10} K_{20} J_{CE} & (1-K_2) J_{M2} + K_{20}^2 J_{CE} \end{bmatrix}$
$ar{ ext{f}} \; = egin{bmatrix}  ext{f}_{ ext{M1}} +  ext{K}_{12}^2  ext{f}_{ ext{M2}} & - ext{K}_{ ext{sd}}  ext{f}_{ ext{M2}} \ - ext{K}_{ ext{sd}}  ext{f}_{ ext{M2}} &  ext{f}_{ ext{CE}} +  ext{K}_{22}  ext{f}_{ ext{M2}} \end{bmatrix}$	$\underline{f} = \begin{bmatrix} f_{M1} + K_{10}^2 f_{CE} + K_2 f_{M2} & -K_{10} K_{20} f_{CE} \\ -K_{10} K_{20} f_{CE} & (1-K_2) f_{M2} + K_{20}^2 f_{CE} \end{bmatrix}$

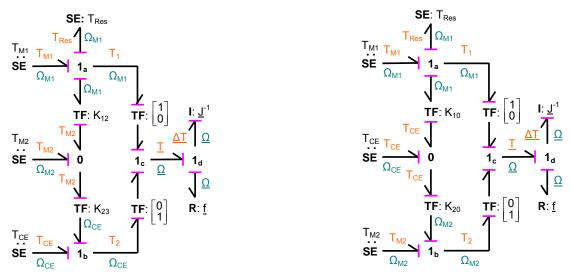


Figure 2. BG modelling - Same Bond Graph structures but different parameters depending on modelling approaches

TABLE V. DIFFERING DEFINITIONS OF COMMON BG SWITCHING PARAMETERS

Var. \ Par.	$\mathbf{K}_{12}$	$\mathbf{K}_{23}$	$K_{sd}$	$\mathbf{K}_{22}$	K <sub>10</sub>	$\mathbf{K}_{20}$	$\mathbf{K}_2$
a,	$k_R/k_S$	$1\!/k_{_{\rm S}}$	$\mathbf{K}_{12}\mathbf{K}_{23}$	$\mathrm{K}^2_{23}$	-k <sub>R</sub>	$k_{\rm S}$	0
b,	-1	-1	0	0	-1	-1	1
c,	0	$1/k_{_{\rm S}}$	0	$\mathrm{K}^2_{23}$	0	$k_{\rm S}$	0
d,	-	-	-	-	-k <sub>R</sub>	0	0

A second equation then effects nothing but ensures consistent modelling by parameter re-setting only. By comparison with EMR representations [22], [24] follows that the EMR coupling device icon for energy distribution may both summarize and split functionality of the BG elements TF at the right sides of fig. 2 and the sum up and distribution functionality of node  $\rm l_c$  whereas the BG representation strictly distinguishes balance equations (nodes) and transformations (TF). The same is true regarding losses and energy storage of parts of a dynamic system - cp. node  $\rm l_d$ . On the other hand the EMR port definition splits the task of node  $\rm l_a$  into two icons.

The compact representation of fig. 1c may be treated as another example for a same BG structure of different hardware solutions. If the order of vectors and matrices as well as the parameters are modified then it may be used to describe other usual types of electrical machines such as induction machines and electrically or permanently excited synchronous as well as DC machines [20]. This summarises the basic functionality of the electric machine as an electromagnetic converter clearly. Hence such BG representations may result in very compact models of high information density.

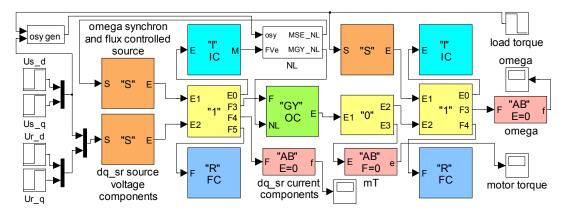


Figure 3. Bond Graph of fig. 1.c modelled via Simulink add-on block library BG V.2.1 [15], [16]

#### V. SIMULINK BOND GRAPH LIBRARY

There is no need for special software solutions or automatic equation reconfiguration to simulate BG's. For instance add-on library BG V.2.1 [15], [16] enables Simulink to handle BG models. Nine menu-driven blocks only ensure essential BG functionalities including fields, vectorial operations, subsystems or non-linear parameters. If applicable optionally block outputs provide generalized displacement respectively momentum or power.

The latter drastically simplifies computations of the energy efficiency. Source and activated bond blocks allow connections to standard unidirectional operating Simulink blocks and the entire LTI functionality remains available as for instance bode diagrams or transfer functions. The software organizes the bidirectional operation in background without any user action. Apparent misconnections of effort and flow variables cause error messages. All power variables are available using measurement nodes. As an example fig. 3 introduces the Simulink representation of fig. 1c. The structure of the BG can be recognized easily. Blocks of type "AB" serve as measurement devices. Thus the 0-type node block may be cancelled if there is no measurement of the motor torque desired.

#### VI. CONCLUSION

Bond Graph based power flow modelling provides valuable structure data of the inner state of a dynamical system to work out comparisons of power flow structures of different models for the same subject, such as scalar, vectorial or compact models, respectively highlights hidden equivalent power flow structures of different solutions for the same problem. It is shown that exactly the BG method is an appropriate one regarding such type of topics. Example DFIG clearly points up the accentuation of losses and makes a point of one constant conjugated power variable as well as energy conversion and storage everything independently of the abstraction level using BG's. The latter enables the variation of directly visible components and complexity. Moreover scalar, vectorial or compact representations may build up subsystems again. On the other hand example power split hints at the advantages of BG's regarding accentuation of same power flow structures on different hardware solutions for a given aim. There emerge not only well-arranged drawings via bidirectional connections but also eve-catching graphical representations of constraints and causal chains by means of node causality symbols.

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BG element	Causality	Icon	Resolved Representation
I-type energy storage 1-port	integral	e 1: K <sub>1</sub>	$\underbrace{f(t) = \frac{1}{K_I} \int e(t)dt + f(0)}_{f(t)}$
C-type energy storage 1-port	integral	e C: Kc	$\underbrace{f} = \underbrace{e(t) = \frac{1}{K_C} \int f(t)dt + e(0)}$
GY- gyrator energy transformer 2-port	outside	$\begin{array}{c c} & K_{GY} \\ \hline f_1 & GY & f_2 \end{array}$	$ \begin{array}{c c} e_1 & e_2 = K_{GY} \cdot f_1 & \xrightarrow{e_2} \\ \hline f_1 & e_1 = K_{GY} \cdot f_2 & \xrightarrow{f_2} \end{array} $
TF-transformer energy transformer 2-port	left	$\begin{array}{c} e_1 \\ f_1 \end{array} \begin{array}{c} K_{TF} \\ \vdots \\ f_2 \end{array}$	$ \begin{array}{c c} \stackrel{e_1}{\longleftarrow} e_1 = K_{TF} \cdot e_2 \\ \stackrel{f_1}{\longrightarrow} f_2 = K_{TF} \cdot f_1 \\ \end{array} $
SE-type energy source 1-port	effort source flow destination	SE: K <sub>E</sub> e	$ \begin{array}{c c} & e=K_E \\ \hline K_E & f \\ \hline \end{array} \begin{array}{c} \text{identical to:} \\ \text{destination} \\ \text{flow (DF)} \end{array} $
R-type energy loss 1-port	flow	F R: K <sub>R</sub>	$\underbrace{\stackrel{e}{f}}_{e=K_R \cdot f}$
1-node energy distribution multi port	at node side exactly once no causality stroke	e <sub>1</sub> f <sub>0</sub> e <sub>n</sub> f <sub>1</sub> : f <sub>n</sub>	$e_0 = (e_1 + + e_m)$ $-(e_{m+1} + + e_n)$ $f_0 = f_1 = = f_n$
Activated bond effort measurement 1-port	"input" flow = = 0; power balance not influenced	e f=0 AB: e	$ \begin{array}{c c} e \\ \hline f=0 \\ \hline f=0 \end{array} $

Appendix II: The I-type field "IF" as an Example of a nonbasic BG element

BG element	Causality	Icon	Resolved Representation
IF: I-type energy storage field, example 2-port IF	integral	$ \begin{array}{c c} \underline{e_1} & \underline{\underline{M}} \\ \hline f_1 & F \\ \end{array} $	$\frac{\begin{bmatrix} e_1 & e_2 \end{bmatrix}^T}{\overset{[f_1 & f_2]^T}{}} \underbrace{ \begin{bmatrix} \underline{f} & \underline{M} \int \underline{e}  \mathrm{d}t + \underline{f}(0) \end{bmatrix}}$