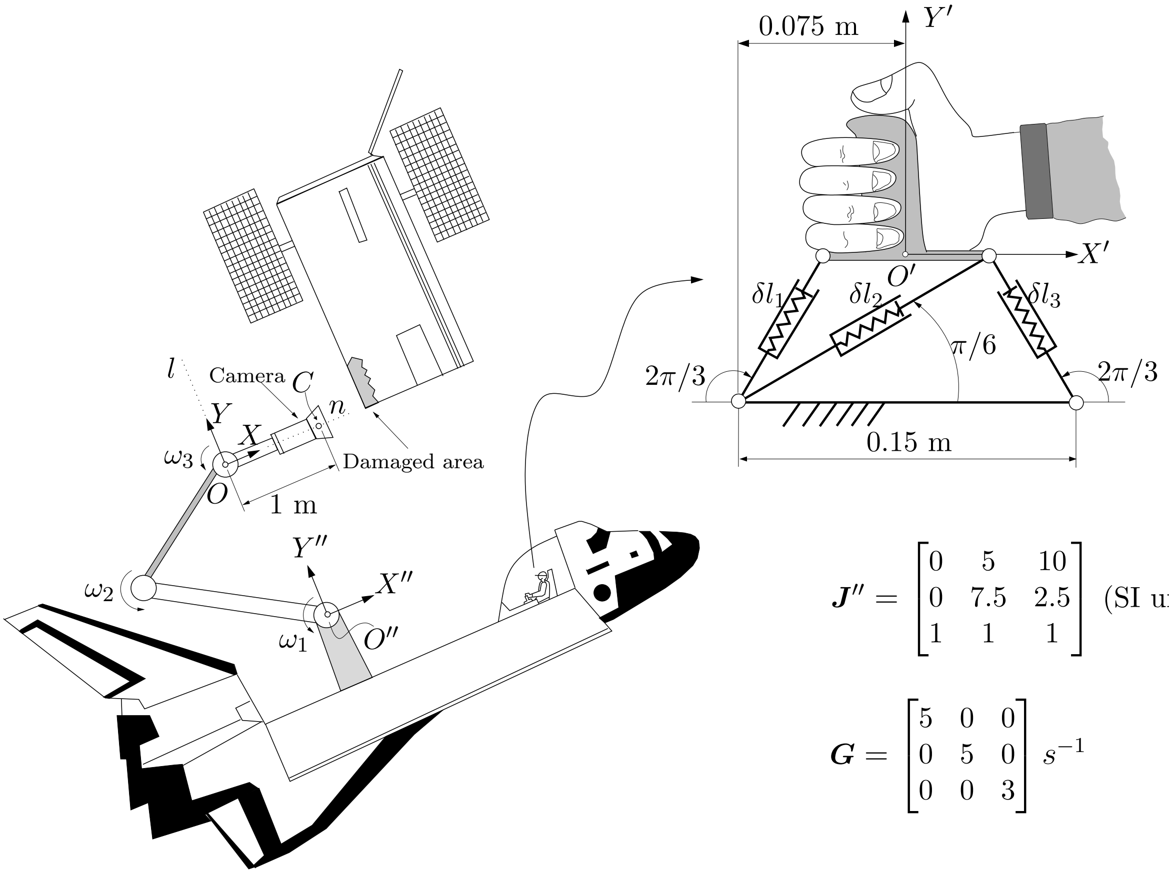
Mini-project 2
Analysis of the Space Shuttle Atlantis

Part A: Motion control of the Canadarm

The Hubble orbital telescope has received the impact of a small meteorite and the Atlantis shuttle has been sent to evaluate the damage and to repair it (Fig. 1). The shuttle is equipped with the *Canadarm*, a 3R arm which is used to do inspection and maintenance operations. The actuators of this arm can rotate at maximum angular velocities of 0.5 rad/s, and can exert torques of 5 Nm, in any sense. Its kinematic Jacobian $\boldsymbol{J}''$ is shown in Fig. 1, expressed in the $O''X''Y''$ reference frame.



$$\boldsymbol{J}'' = \begin{bmatrix} 0 & 5 & 10 \\ 0 & 7.5 & 2.5 \\ 1 & 1 & 1 \end{bmatrix} \text{ (SI units)}$$

$$\boldsymbol{G} = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 3 \end{bmatrix} s^{-1}$$

Figure 1: The Atlantis shuttle inspecting the Hubble telescope.

The spaceship evolves until it achieves the position of Fig. 1. The end-effector of the Canadarm carries a camera with which the surface of the telescope has to be filmed on the damaged area. From the Atlantis cargo bay, an astronaut commands the motion of the camera with a passive *joystick* 3RPR, whose joints P have springs with constant $k = 100$ N/m. The control system of the arm guarantees that at any moment the twist $\hat{T}$ of the camera with respect to the Atlantis is proportional to the infinitesimal displacement $\delta\hat{D}$ of the joystick with respect to its rest position, according to the equation

$$\hat{T} = \boldsymbol{G} \cdot \delta\hat{D}, \tag{1}$$

where $\hat{T}$ and $\delta\hat{D}$ are given in the given reference frames OXY and $O'X'Y'$, respectively, and $\boldsymbol{G}$ is the gain matrix indicated in Fig. 1. The joystick legs have sensors that measure the small longitudinal displacements $\delta l_1, \delta l_2, \delta l_3$ that correspond to the displacement $\delta\hat{D}$.

In the configuration of Fig. 1, determine:

A The static Jacobian of the joystick expressed in the reference $O'X'Y'$.

B The kinematic Jacobian of the Canadarm expressed in the reference OXY.

Starting at the configuration of Fig. 1, the astronaut applies a certain displacement $\delta\hat{D}$ to the Joystick, such that its sensors are reading the values $\delta l_1 = 0.017321$ m, $\delta l_2 = 0.01$ m, and $\delta l_3 = 0.017321$ m. Determine:

C The twist $\hat{T}$ of the camera, and the velocity at which point C of the camera moves.

D The angular velocities ω_1, ω_2, ω_3, that the control system will have to demand to the actuators of the Canadarm to produce the twist $\hat{T}$.

E The wrench that the astronaut has to apply to the joystick to maintain it in the new position, assuming that all springs work near their unloaded configuration.

During the planning of this mission, the engineers of the Jet Propulsion Lab had to decide a good configuration from which to start filming the damaged area of the telescope. It was necessary to record a video of such area by displacing the distal joint of the Canadarm along a trajectory l parallel to the telescope, with constant velocity of 0.5 m/s, and maintaining the axis n of the camera perpendicular to the trajectory (Fig. 1). The configuration in Fig. 1 was finally chosen, but configurations C_1 i C_2 of Fig. 2 had also been considered. Answer the following questions:

F Why configuration C_1 was quickly discarded? What happens in this configuration with the velocities?

G Configuration C_2 seemed to be initially adequate, but after doing some computations it was discarded. Reproduce these computations and justify why this configuration was discarded.

H Give a base of the space of twists of freedom of the camera in configurations C_1 and C_2, in the reference frame $O''X''Y''$.

I If in configuration C_1 the astronaut produces the displacement $\delta\hat{D} = [0.01, 0, 0]^\mathsf{T}$ on the Joystick, can the arm move with the corresponding twist? And if he produces the displacement $\delta\hat{D} = [0.01, 0.01, 0]^\mathsf{T}$?

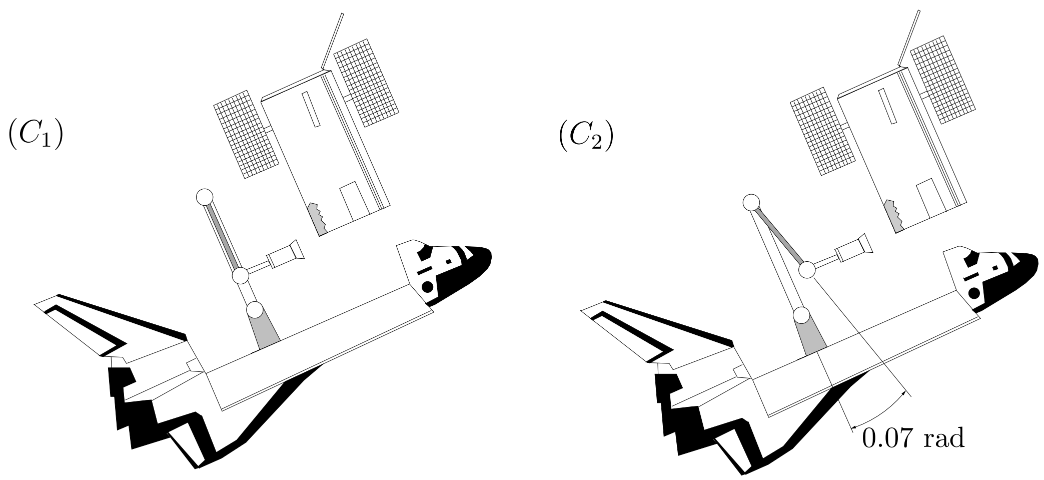
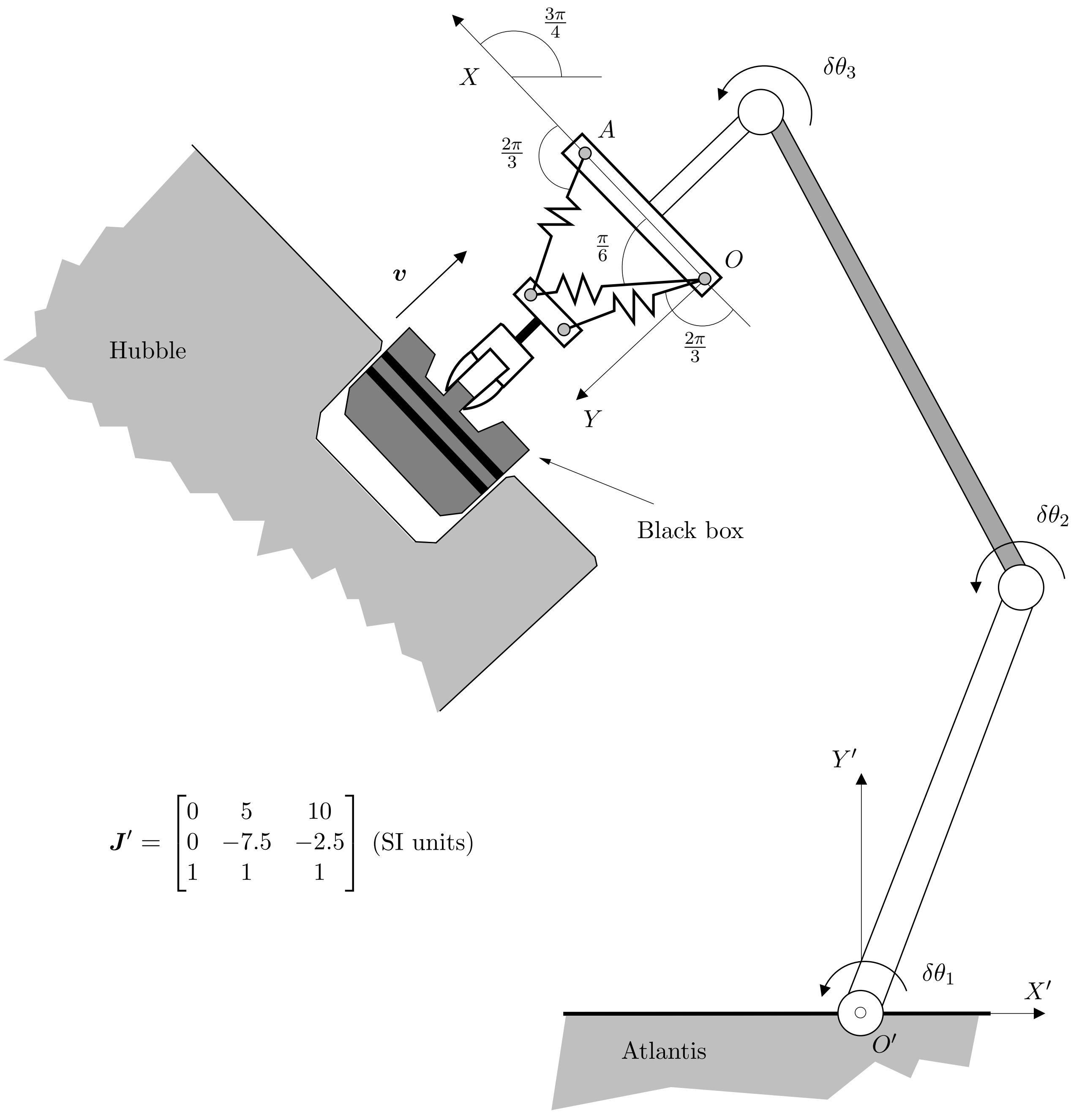


Figure 2: Two alternative initial configurations.

Part B: Hybrid control of the Canadarm-Hubble contact

The meteorite damaged the telecommunications system of the Hubble and the last observations - which could confirm the existence of extraterrestrial life - could not be transmitted to the Earth. To recover the data, the black box of the telescope has to be extracted. The end-effector of the Canadarm is equipped with a gripper and the arm is tele-operated until the handle of the box is hold (Fig. 3). The Hubble frees automatically the box and the Canadarm initiates its extraction, letting it slide along the shown direction $\boldsymbol{v}$. The box-Hubble contact is planar without friction.



$$\boldsymbol{J}' = \begin{bmatrix} 0 & 5 & 10 \\ 0 & -7.5 & -2.5 \\ 1 & 1 & 1 \end{bmatrix} \text{ (SI units)}$$

Figure 3: Extraction of the black box of the Hubble with the Canadarm (the drawing is not at scale).

In order not to affect the orbit, the box-telescope interaction forces have to be minimized, using a hybrid-control strategy. The Canadarm carries a flexible 3RPR wrist that gives the wrench that the Hubble makes on the box at every instant. The Atlantis maintains its position and orientation fixed relative to the Hubble. Gravity effects are negligible and the springs of the wrist 3RPR have a length which is very close to the rest length. The spring constants are all equal, of 200 N/m.

In the configuration of the figure, determine:

A The rigidity matrix of the 3RPR wrist in the reference frame OXY, knowing that the distance OA is of 0.3 m. Note: the wrist is not drawn at scale.

B The Jacobian $\boldsymbol{J}$ of the Canadarm in the reference frame OXY, knowing that the Jacobian $\boldsymbol{J}'$ in the system $O'X'Y'$ is the one indicated in the Fig. 3, and the coordinates of point O in the reference frame $O'X'Y'$ are $[0.15, 7.35]$ m.

C The space of controllable displacements $\delta\hat{D}$ (of the box with respect to the Hubble), in the reference frame OXY.

D The space of controllable force variations $\delta\hat{w}$ (of the Hubble on the box), in frame OXY.

At the time instant of the figure, the Hubble telescope is applying a clockwise pure torque of 0.1 Nm on the box. The box has to perform a translation of 0.01 m in the direction $\boldsymbol{v}$. Determine:

E The wrench $\delta\hat{w}$ that expresses the force variation (of the Hubble telescope on the box) which has to be produced to eliminate the previous torque, and the small displacement $\delta\hat{D}$ associated with the desired translation, both in the reference frame OXY.

F The small increments of the angle $\delta\theta_1$, $\delta\theta_2$, and $\delta\theta_3$ that the joints of the Canadarm have to perform to cause simultaneously the variation of force $\delta\hat{w}$ and displacement $\delta\hat{D}$.

Figure 4: Image of the real context of parts A and B.