Exercises Module 2
Statics

Several of these exercises are borrowed from Chapter 2 of Duffy's book "Statics and Kinematics with Applications to Robotics". That is why forces and distances are given in English Engineering units, i.e., inches (in), and pound-forces (lbf), respectively. Note that 1 in = 0.0254 m, and 1 lbf = 4.4482 N.

Problems

Problem 1

1. On separate figures draw the lines that join the pairs of points $(-1, 3)$, $(3, 2)$; and $(1, 1)$, $(-2, -2)$. Also, using Grassmann's rule, determine the Plücker coordinates $\{L, M; R\}$ of each line. In each figure draw the triangle $O12$ and determine the signed area of this triangle using the determinant of the three points O, 1, and 2. Compare these values with the corresponding values of R you have obtained. Express each set of Plücker coordinates in the unitized form $\{c, s; p\}$, and write the corresponding equation for each line. For each line compute its angle θ with the X axis, and its signed distance p from the origin O.

2. Using Grassmann's rule for computing line-line intersections, compute the coordinates of the intersections of the pairs of lines $\{3.03, 1.75; 3.5\}$, $\{-1.03, 2.82; -6\}$; and $\{1, 1; 0\}$, $\{-1, -1; -\sqrt{2}\}$. Verify your results by drawing each pair of lines.

Problem 2

1. Write the Plücker coordinates for the X' and Y' axes in the OXY coordinate system, and the Plücker coordinates for the X and Y axes in the $O'X'Y'$ coordinate system (Fig. 1a). From such coordinates, construct the matrices $[A]$ and $[A']$ for the induced force transformations

$$\begin{bmatrix} L \\ M \\ R \end{bmatrix} = [A] \begin{bmatrix} L' \\ M' \\ R' \end{bmatrix} \qquad \begin{bmatrix} L' \\ M' \\ R' \end{bmatrix} = [A'] \begin{bmatrix} L \\ M \\ R \end{bmatrix}$$

for the pure translation indicated in the figure. Show that $[A'] = [A]^{-1}$

2. Write the Plücker coordinates for the X'' and Y'' axes in the $OX'Y'$ coordinate system, and the Plücker coordinates for the X', Y' axes in the $OX''Y''$ coordinate system (Fig. 1b). Determine the matrices $[B']$ and $[B'']$ for the induced force transformations

$$\begin{bmatrix} L' \\ M' \\ R' \end{bmatrix} = [B'] \begin{bmatrix} L'' \\ M'' \\ R'' \end{bmatrix} \qquad \begin{bmatrix} L'' \\ M'' \\ R'' \end{bmatrix} = [B''] \begin{bmatrix} L' \\ M' \\ R' \end{bmatrix}$$

for the pure rotation indicated in the figure. Show that $[B''] = [B']^{-1}$.

3. Determine $[e] = [A]\,[B']$.

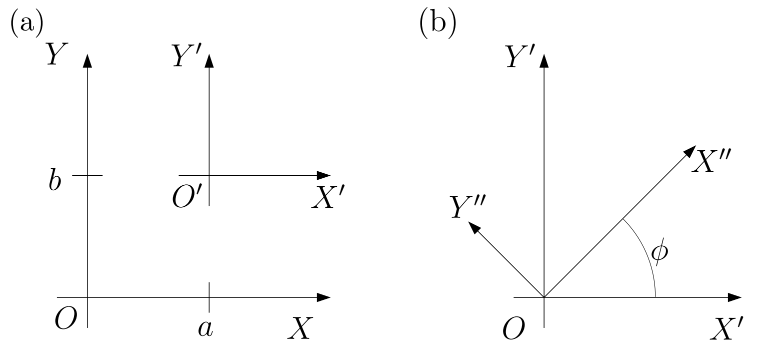


Figure 1: Translation (a) and rotation (b) of coordinate systems.

Problem 3

1. For the 3-RPR manipulator in Fig. 2a:
   (a) Determine the unitized coordinates of the lines that join the connectors B_1C_1, B_2C_2, and B_3C_3 using the indicated coordinate system XY. Thus, determine the matrix j of the mapping $\hat{w} = j\lambda$ for this manipulator.
   (b) Suppose that only the vertical force of 10 lbf applied through point C_1 in Fig. 2a is acting on the upper platform. Compute the equilibrating connector forces f_1, f_2, and f_3, and state whether each connector is in compression or in tension.
   (c) Do the same but assuming that only the force of 10 lbf applied through point C_2, C_3 in Fig. 2a is being applied to the platform.
2. For the 3-RPR manipulator in Fig. 2b:
   (a) Determine the unitized coordinates of the lines that join the connectors B_1C_1, B_2C_2, and B_3C_3 using the indicated coordinate system XY. Thus, determine the matrix j of the mapping $\hat{w} = j\lambda$ for this manipulator.
   (b) A vertical force of 5 lbf is applied one inch to the right of point C_1. Compute the equilibrating connector forces and state whether each connector is in compression or in tension.
   (c) Compute the equilibrating connector forces when the vertical force of 5 lbf is applied at C_1, and then at C_2.

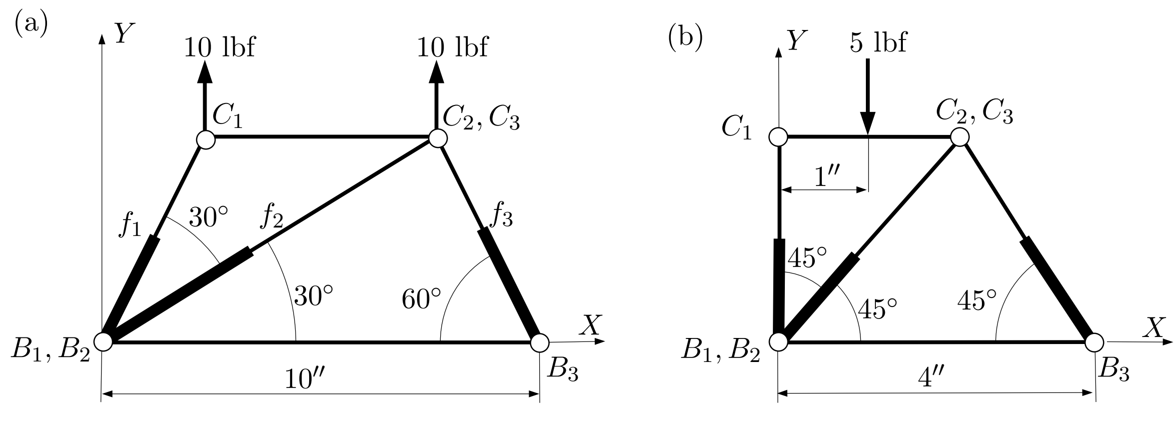


Figure 2: Two 3RPR manipulators.

Problem 4

This exercise requires using a programming language like Matlab or GNU Octave. Fig. 3 illustrates a 3-RPR manipulator with special geometry, with the moving platform in an initial position for which the coordinates of points 1, 2, and 3 are, respectively $(0, 3)$, $(0.4\sqrt{2}, 2.4526)$, and $(0.95\sqrt{2}, 1.7)$.

1. Write the unitized coordinates $[c, s, p]^T$ of the force $\mathbf{f}$ that passes through point 2 and is parallel to the X axis, as shown in the figure.

2. Give the platform self-parallel displacements in increments of 0.1 inches away from the inital position, in the positive X direction, up to 2.9 inches. For each increment, compute the coordinates of points 1, 2, and 3.

3. Compute the corresponding connector lengths.

4. Use Grassmann's rule to compute the corresponding sets of unitized line connector coordinates, in order to determine the matrix j for each position, and, correspondingly, compute j^{-1}.

5. Compute the resulting leg forces for each position (i.e., the signed magnitudes of the leg forces) assuming that the magnitude of the force $\mathbf{f}$ is of 1 unit of force.

6. Plot the equilibrating force in each of the three connectors against the horizontal position x. Label each curve and state whether a connector is in tension or compression.

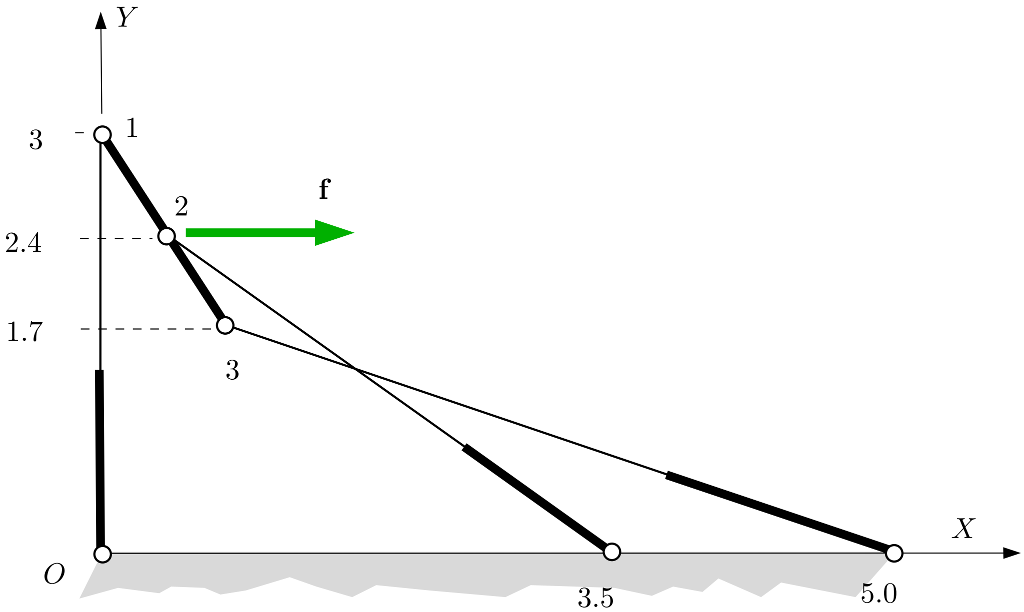


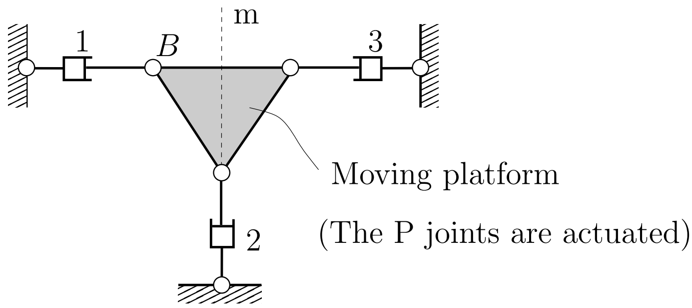
Figure 3: A 3RPR manipulator whose moving platform begins to translate to the right.

Problem 5

Let $\hat{w}_1$ and $\hat{w}_2$ be two wrenches ("Torsors" in Catalan) and denote by $\$_1$ and $\$_2$ their respective lines of application, which intersect at some point P. Prove that a linear combination $\hat{w} = \lambda_1 \hat{w}_1 + \lambda_2 \hat{w}_2$, where λ_1 and λ_2 are scalars, is a wrench whose line of application intersects P. Show that the result holds independently of whether $\$_1$ and $\$_2$ are skew or parallel.

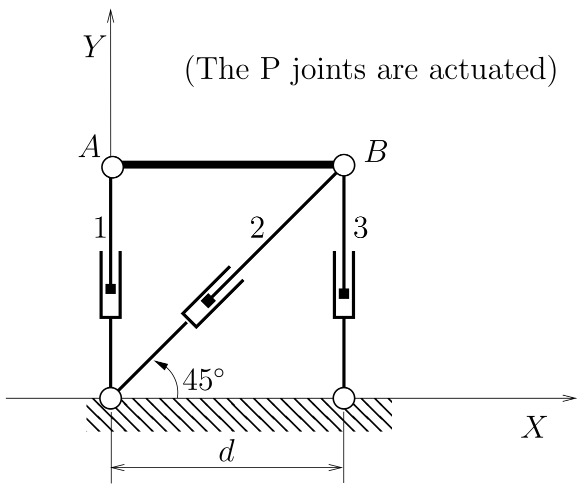
Short questions

Short question 1
Mark the incorrect statement relative to the 3-RPR robot in the following figure:



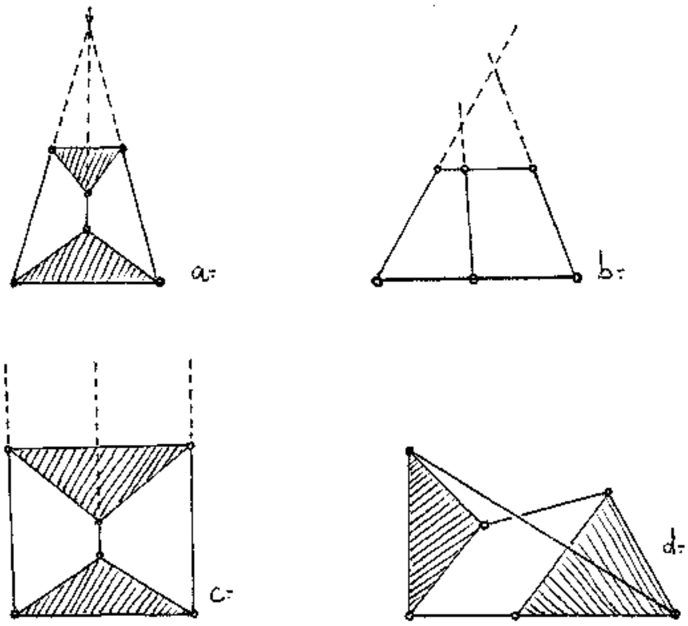
1. The robot is in a singular configuration.
2. The robot can equilibrate any vertical force on the line of action m, applied to the platform.
3. It is sufficient to actuate leg 2 in order to equilibrate any vertical force with line bound m.
4. Any vertical force applied on B can be equilibrated by actuating leg 2 exclusively.
5. The robot can equilibrate any horizontal force applied on B.

Short question 2
Mark the correct statement relative to the 3RPR robot in the following figure:



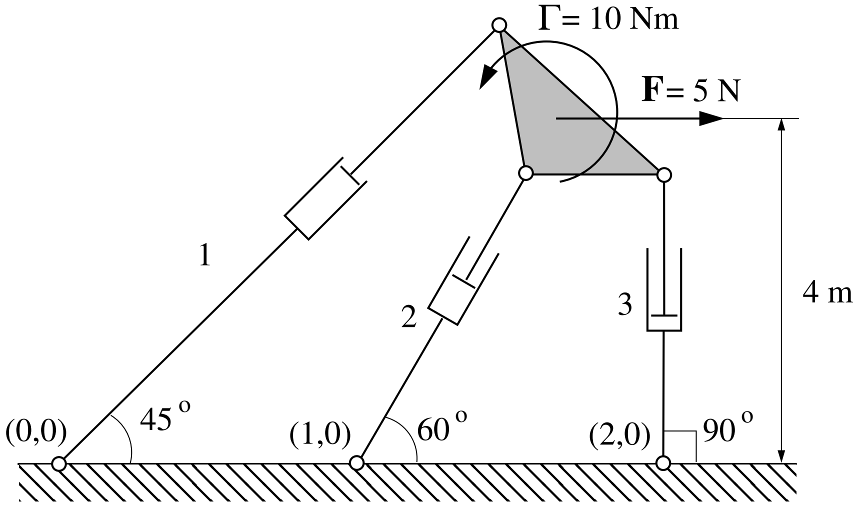
1. The robot is in a singular configuration.
2. In the coordinate system XY the Jacobian matrix associated with the shown configuration is
$$\begin{bmatrix} 0 & \sqrt{2}/2 & 1 \\ 1 & \sqrt{2}/2 & 0 \\ 0 & 0 & d \end{bmatrix}.$$
3. It sufficies to actuate leg 1, in order to equilibrate a vertical force applied on A.
4. If legs 1 and 3 respectively apply forces $\vec{F}_1$ and $\vec{F}_3$ on the platform (and leg 2 does not apply any force), then these forces can only be equilibrated by an external force $-(\vec{F}_1 + \vec{F}_3)$ applied on B.
5. This robot can never reach a singular configuration.

Short question 3
Which of the following configurations are singular?



1. **a** and **b**.
2. **a**, **b** and **d**.
3. **a** and **c**.
4. **a**, **b** and **c**.
5. **b** and **c**.

Short question 4
At the shown configuration, 3-RPR robot of the figure is in equilibrium, counteracting the external force F and torque Γ indicated, applied to the platform.



Mark the correct statement:

1. Leg 1 is in tension with $5\sqrt{2}$ N, and legs 2 and 3 work in compression, both with 5 N.
2. Legs 1 and 2 are in tension with $5\sqrt{2}$ N, and leg 3 is in compression with 5 N.
3. Leg 1 is in tension with $5\sqrt{2}$ N, leg 2 is unloaded, and leg 3 is in compression with 5 N.
4. All legs are working in tension.
5. A singular configuration can be reached by varying the orientation and magnitude of the extenal applied force.