Exercises Module 3
Kinematics

Problems

English Engineering units (inches for distance, inches/second for speed) are assumed throughout.

Problem 1

1. A lamina is instantaneously rotating counterclockwise with angular velocity ω about a point C with coordinates (x_C, y_C). The magnitude v_P of the velocity $\boldsymbol{v}_P$ of point $P = (x_P, y_P)$ is known. Obtain an expression for ω in terms of v_P and the coordinates of C and P. Also, derive an expression for the velocity vector $\boldsymbol{v}_O$ of a point in the lamina coincident with the origin O and draw $\boldsymbol{v}_O$ in Fig. 1.

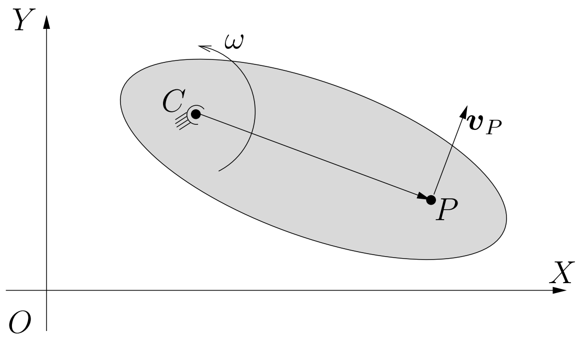


Figure 1: A rotating lamina.

2. Consider the robot in Fig. 2, whose end-effector is instantaneously rotating about a point C with coordinates $(x_C, y_C) = (0, 2)$ at an angular velocity ω. The velocity $\boldsymbol{v}_P$ of point P has magnitude $v_P = 1$ in/sec. The coordinates of P are $(x_P, y_P) = (1, 2)$ in. Use the expressions from part 1 to determine ω and the velocity $\boldsymbol{v}_O$ of a point in the end-effector coincident with the origin point O. Compute the joint velocities ω_1, ω_2, and ω_3.

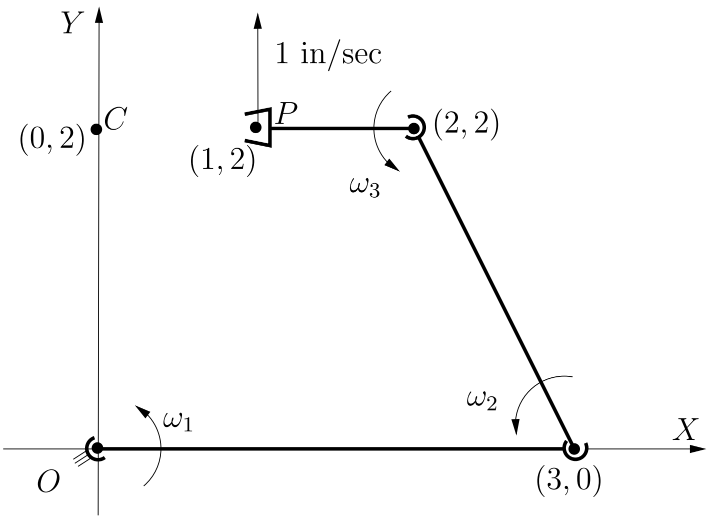


Figure 2: A 3R serial robot.

3. For the manipulator in Fig. 3 compute the joint velocities ω_1, v_2, and ω_3 corresponding to the twists

$$\hat{T} = \omega \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad \hat{T} = v_x \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad \hat{T} = v_y \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix},$$

and write a short comment on each result explaining how it makes sense physically.

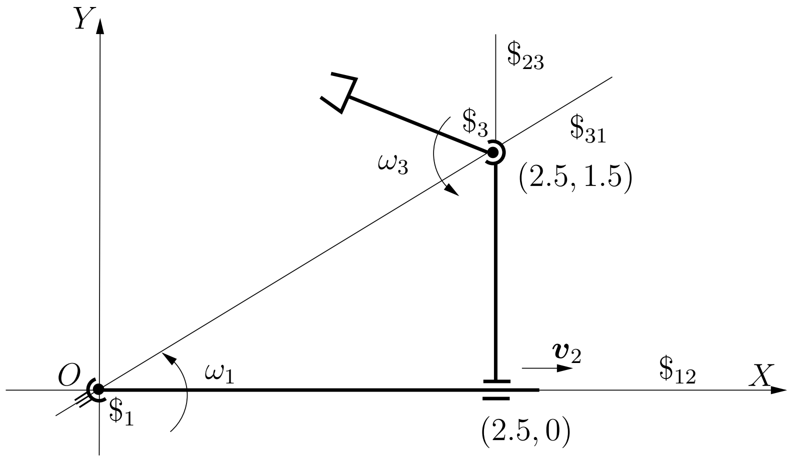


Figure 3: A RPR serial manipulator.

4. For the $3R$ robot shown in Figure 4 do the following:

(a) Use Grassmann's rule to obtain the coordinates of $\hat{s}_{23}$, $\hat{s}_{31}$ and $\hat{s}_{12}$ of the lines $\$_{23}$, $\$_{31}$ and $\$_{12}$.

(b) Write the coordinates $\hat{S}_1$, $\hat{S}_2$ and $\hat{S}_3$ of the lines $\$_1$, $\$_2$ and $\$_3$ through the points $(0, 0)$, $(3, 0)$, and $(2, 2)$ which are perpendicular to the XY plane.

(c) Show that $\hat{s}_{23}^T \hat{S}_2 = \hat{s}_{23}^T \hat{S}_3 = 0$, $\hat{s}_{31}^T \hat{S}_3 = \hat{s}_{31}^T \hat{S}_1 = 0$, and $\hat{s}_{12}^T \hat{S}_1 = \hat{s}_{12}^T \hat{S}_2 = 0$

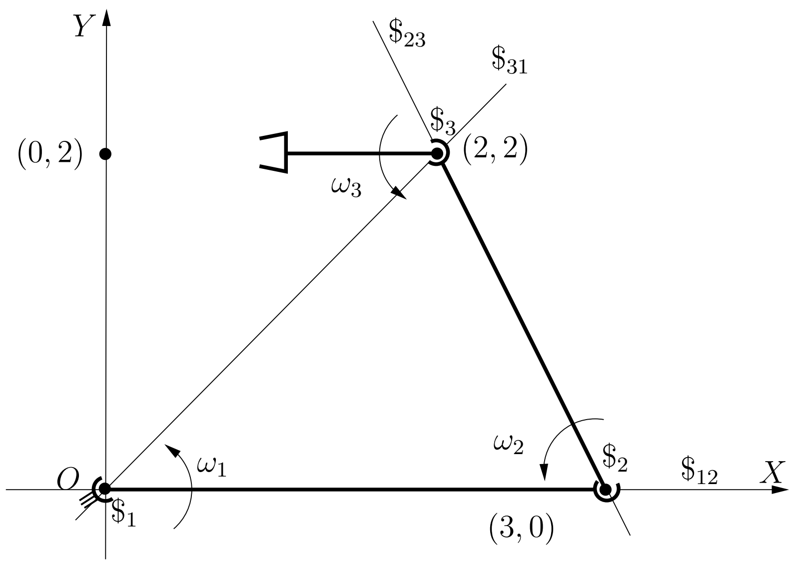


Figure 4: A 3R serial manipulator.

Problem 2

1. For the $2R$ planar manipulator in Fig. 5:

   (a) Use parallel projection to obtain expressions for the coordinates (x_2, y_2), (x_3, y_3) of the points 2 and 3 in terms of θ_1 and θ_2.

   (b) Perform a complete reverse displacement analysis, i.e., obtain expressions for θ_1 and θ_2 in terms of (x_3, y_3).

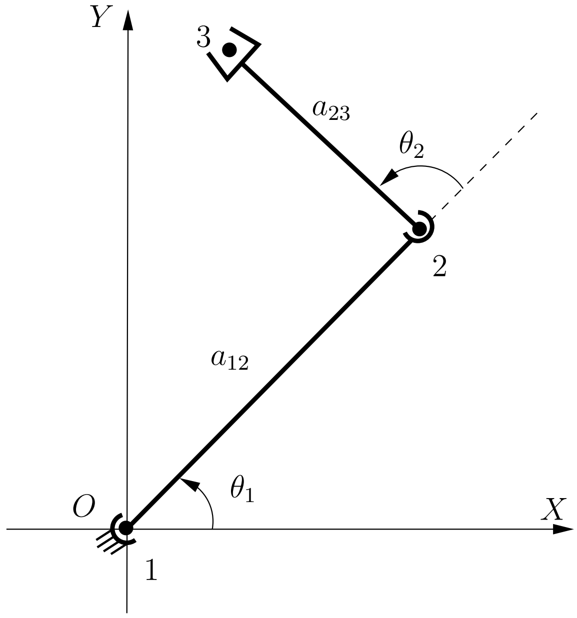


Figure 5: A 2R serial manipulator.

2. The end effector of the $3R$ manipulator in Fig. 6 is moving with a pure translational constant velocity $\boldsymbol{v}$ parallel to the x axis, as illustrated, of magnitude 1 in/sec.

   (a) Give the end-effector self-parallel displacements in increments δ of 0.2 feet away from the initial position $(x_3, y_3) = (0, 2.5)$ until it reaches the final position on the right.

   (b) For each set of coordinates (x_3, y_3) compute corresponding sets of θ_2 and θ_1 using the reverse analysis in Part 1. Then compute the corresponding values for the coordinates (x_2, y_2).

   (c) Compute the matrix

$$J = \begin{bmatrix} 0 & y_2 & y_3 \\ 0 & -x_2 & -x_3 \\ 1 & 1 & 1 \end{bmatrix}$$

   for each increment and correspondingly compute J^{-1}.

   (d) Compute the corresponding angular velocities of the joints using $[\omega_1, \omega_2, \omega_3]^{\mathsf{T}} = J^{-1}\,\hat{T}$.

   (e) Plot ω_1, ω_2 and ω_3 against x_3 and comment the results.

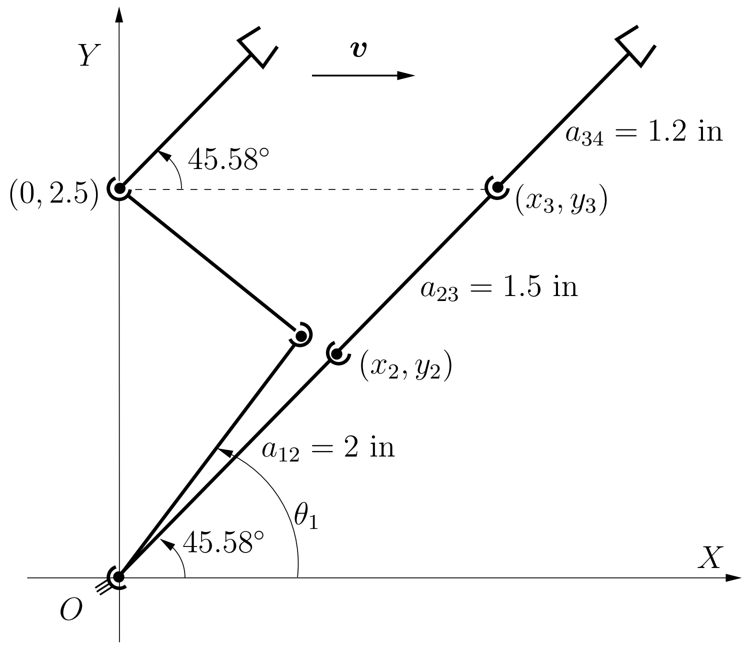


Figure 6: 3R serial manipulator.

3. (a) Write the matrix J for the PRR manipulator (see Fig. 7). Show that $\det J = 0$ when $x_2 = x_3$. Draw this configuration and comment on your result.

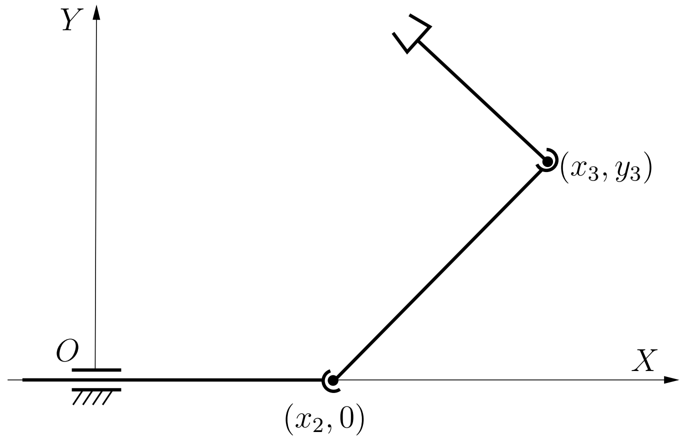


Figure 7: PRR serial manipulator.

(b) Write the matrix J for the RPR manipulator shown in Figure 8. Verify that

$$\begin{aligned} x_3 &= a_{12} \cos\theta_1 + S_2 \cos\gamma, \\ y_3 &= a_{12} \sin\theta_1 + S_2 \sin\gamma, \\ \gamma &= \theta_1 + \pi/2 \end{aligned}$$

Use these expressions to show that $\det J = 0$ when $S_2 = 0$. Draw this configuration and comment on your results.

(c) Write the matrix J for the RRP manipulator shown in Figure 9. Verify the expressions $x_2 = a_{12} \cos\theta_1$, $y_2 = a_{12} \sin\theta_1$, $\gamma = \theta_1 + \theta_2 + \pi/2$, and show that $\det J = 0$ when $\sin\theta_2 = 0$. Draw these two configurations and comment on your results.

(d) Write the J matrices for each one of the possible $2P - R$ manipulators and obtain their singularity conditions.

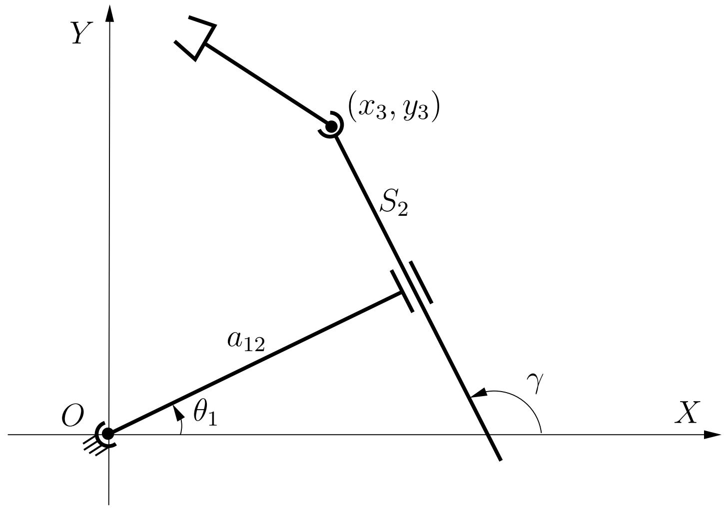


Figure 8: RPR serial manipulator.

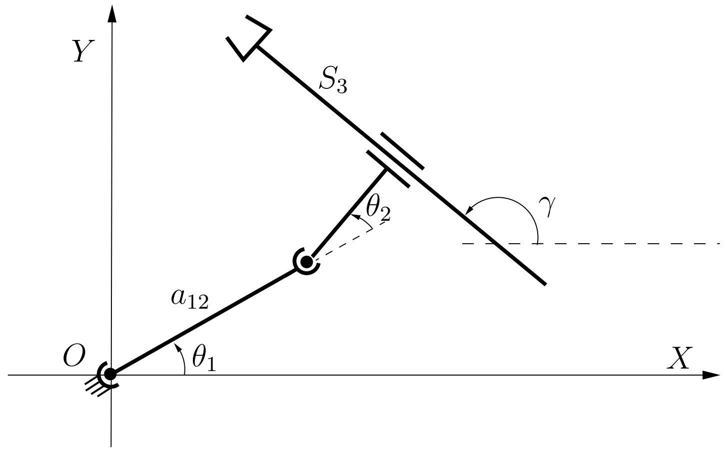


Figure 9: RPR serial manipulator.

Problem 3

1. The input crank of the $4R$ mechanism in Fig. 10 is undergoing a counterclockwise rotation of 10 rad/sec relative to the ground. Compute the angular velocities ω_1, ω_2 and ω_3.

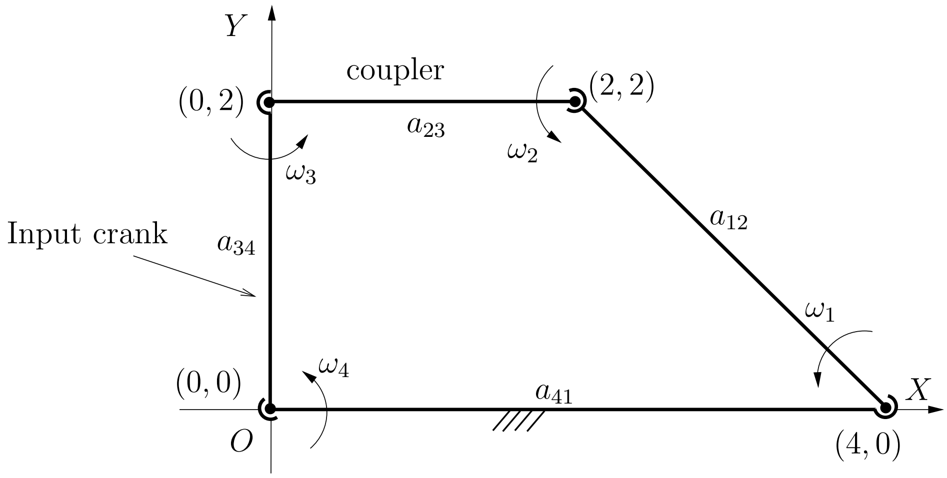


Figure 10: A 4R mechanism.

2. Compute the twist of the coupler link relative to the XY coordinate system by disconnecting the mechanism at the third joint and by considering the coupler to be the end effector of the $2R$ manipulator (see Fig. 11 (a)).

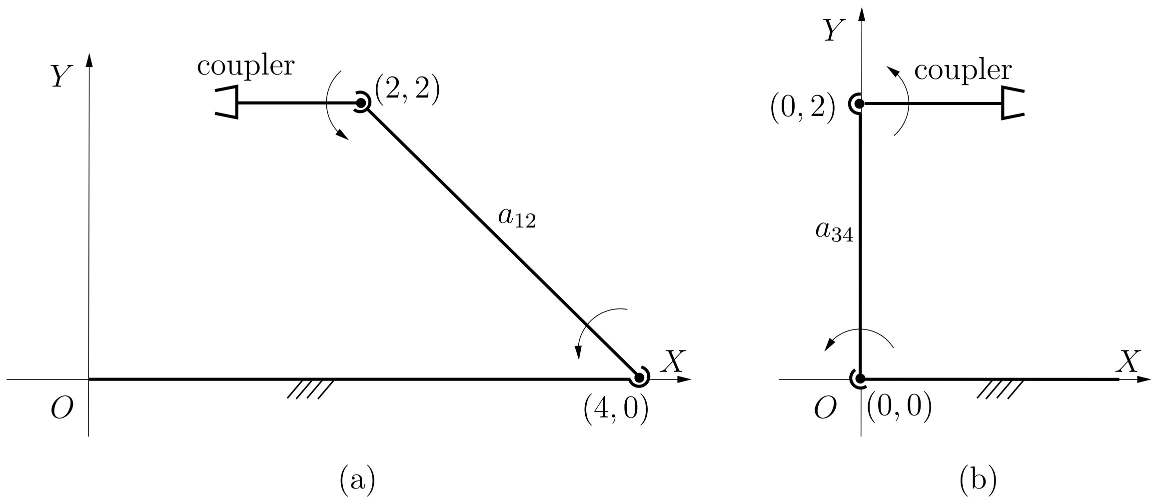


Figure 11: Two 2R manipulators.

3. Repeat part 2 by disconnecting the mechanism at the second joint (see Fig. 11 (b)).

4. Compare the results obtained in parts 2 and 3 and draw the instant center on your figures.

Short questions

Short question 1
Consider the following Jacobian matrix of a serial manipulator:

$$J = \begin{pmatrix} 1 & 0 & \cos\gamma & m \\ 0 & -b & \sin\gamma & -n \\ 0 & 1 & 0 & 1 \end{pmatrix}$$

Mark the incorrect statement:

1. It is a $PRPR$ manipulator.
2. It is a redundant mechanism.
3. The second joint is at point $(0, b)$.
4. The fourth joint is at point (n, m).
5. The instantaneous center of rotation relative to the first joint is located at infinity.

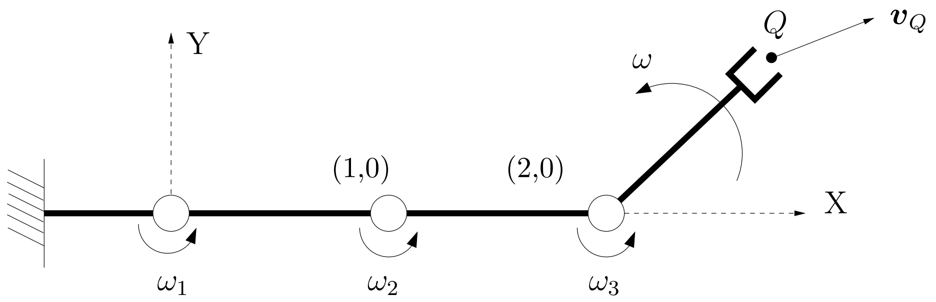
Short question 2
Consider a lamina moving in the plane, and a given coordinate system. The velocity of the origin, considered as a point on the lamina, is $\boldsymbol{v}_o = (2, 0)$ m/s. Mark the incorrect statement:

1. If the angular velocity of the lamina is 0.2 rad/s (counterclockwise), the instant center of rotation is at point $(0, 10)$.
2. If the angular velocity of the lamina is 2 rad/s (clockwise), the instant center of rotation is at point $(0, -1)$.
3. If the velocity of any point on the lamina is $\boldsymbol{v} = (2, 0)$ m/s, the angular velocity is null and the instantaneous center of rotation is at infinity, in the direction of the OX axis.
4. If the twist of the lamina is $\hat{T} = [2\text{m/s}, 0\text{m/s}, \text{-}0.5 \text{ rad/s}]$, the velocity at point $P = (0, -4)$ is $\boldsymbol{v}_P = (0, 0)$ m/s.
5. If the twist of the lamina is $\hat{T} = [2\text{m/s}, 0\text{m/s}, \text{-}0.5 \text{ rad/s}]$, the velocity at point $Q = (0, 4)$ is $\boldsymbol{v}_Q = (4, 0)$ m/s.

Short question 3
Joe and Ken find themselves by accident on an ice floe, floating adrift on the sea. Bob observes them from a balloon, and is able to measure their instantaneous velocities, $\boldsymbol{v}_{joe}$ and $\boldsymbol{v}_{ken}$, relative to the sea. Joe and Ken are on different locations on the floating ice. Mark the correct statement:

1. $v_{joe} = 2$ pointing to the West, and $v_{ken} = 1$ pointing to the East. They are positioned in such a way that their common line is perpendicular to their velocities. The instant center of the ice floe is on the line joining their positions, closer to Joe than to Ken.
2. $v_{joe} = 2$ in the North direction, and $v_{ken} = 1$ pointing to the East. The instant center is on the line joining their positions, closer to Ken than to Joe, and just below Bob's position.
3. $v_{joe} = 2$ and $v_{ken} = 1$, both pointing to the South, and the instant center is at infinity.
4. $v_{joe} = 2$ and $v_{ken} = 2$, both pointing to the East. Bob realizes that all points on the ice floe have the same velocity as Joe and Ken.
5. Bob decides to land on the ice to rescue his colleagues. Just at the moment when the balloon touches the ground, at a point equidistant to Joe and Ken, Joe is on the left side of Bob, with $v_{joe} = 2$ pointing forwards, and Ken is on the right side of Bob, with $v_{ken} = 1$ pointing backwards. Bob gets quickly sick, as he has just landed on the instant center of the floe.

Short question 4
Consider the following configuration of a 3R manipulator, where ω_i is the angular velocity of joint i, and $\boldsymbol{v}_Q$ and ω are the velocity of point Q and the angular velocity of the end effector, respectively. Both $\boldsymbol{v}_Q$ and ω are assumed relative to the absolute coordinate system XY displayed in the figure:



Which one of the following statements is false?

1. The manipulator is in a singular configuration.
2. Modifying ω_1, ω_2, and ω_3 arbitrary velocities $\boldsymbol{v}_Q$ can be produced in the XY plane.
3. Given desired values for $\boldsymbol{v}_Q$ and ω, a set of angular velocities ω_1, ω_2, and ω_3 can always be found that produce them.
4. With the locations of the joints shown in the Figure, the twist of the end effector, given in coordinate system XY, has the form
$$\begin{bmatrix} 0 \\ -\omega_2 - 2\omega_3 \\ \omega_1 + \omega_2 + \omega_3 \end{bmatrix}$$
5. Necessarily, the instantaneous center of rotation of the end effector will be located on the X axis.