RECIPROCAL PRODUCT METHODS

We next illustrate a number of techniques grouped under the name "reciprocal product methods", which turn out to be very useful in obtaining closed-form solutions for some inverse problems, and for performing the kinetostatic analysis of a large family of parallel manipulators. The section has four subsections

(A) Closed-form solution to the inv. static prob. of the 3-RP̲R robot
(B) " " " " " " kinematic " " " 3R robot
(C) Alternative method to obtain $\vec{v} = J^T \hat{t}$ in a 3-RP̲R robot
(D) Kinetostatic analysis of fully-parallel manipulators

A Closed-form solution to the inverse static problem of the 3-RP̲R robot

We recall the following relationship between the prismatic joint forces f_i, and the platform wrench (module 2):

$$\hat{w} = f_1 \hat{s}_1 + f_2 \hat{s}_2 + f_3 \hat{s}_3 \qquad (\hat{w} = J \vec{f})$$

in ray coords. → normalized coords. of the leg lines $\$_1, \$_2, \$_3$

This expression gives the solution to the forward statics problem. We next show a "trick" to compute a closed-form expression for the values f_i as a function of $\hat{w}$, i.e., a solution to the inverse static problem. This solution avoids the inversion of matrix J.

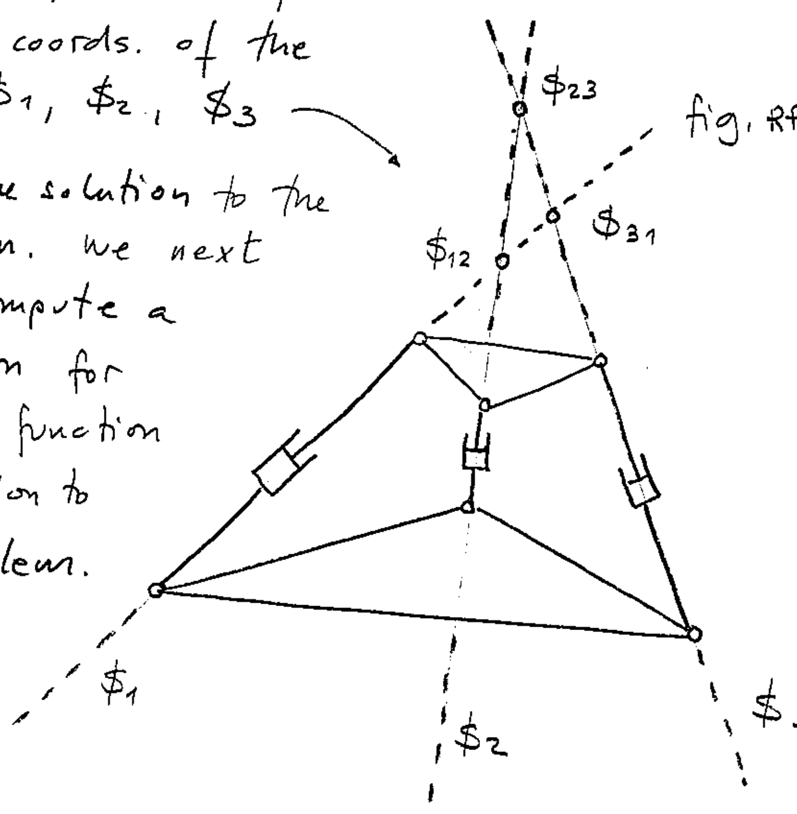


fig. RP1

These products are called "reciprocal products"

Consider the vertical line $\$_{23}$ through the point of intersection of $\$_2$ and $\$_3$ (fig. RP1). The Plücker vector of this line is denoted by $\hat{S}_{23}$ (in axis coords., but not necesarily normalized). Multiplying the previous equation by $\hat{S}_{23}^T$ we obtain

$$\hat{S}_{23}^T \cdot \hat{w} = f_1 \hat{S}_{23}^T \hat{s}_1 + f_2 \underbrace{\hat{S}_{23}^T \hat{s}_2}_{=0} + f_3 \underbrace{\hat{S}_{23}^T \hat{s}_3}_{=0}$$

Because the respective lines intersect and thus these mutual moments vanish

and hence

$$\boxed{f_1 = \frac{\hat{S}_{23}^T}{\hat{S}_{23}^T \hat{s}_1} \hat{w}}$$

Analogously, multiplying the expression by $\hat{S}_{31}^T$ and $\hat{S}_{12}^T$ we obtain

$$\boxed{\begin{aligned} f_2 &= \frac{\hat{S}_{31}^T}{\hat{S}_{31}^T \cdot \hat{s}_2} \cdot \hat{w} \\ f_3 &= \frac{\hat{S}_{12}^T}{\hat{S}_{12}^T \cdot \hat{s}_3} \cdot \hat{w} \end{aligned}}$$

WARNING: Note that these expressions for f_1, f_2, f_3 are valid as long as the denominators do not vanish. E.g. the term $\hat{S}_{23}^T \hat{s}_1$ vanishes when the line $\$_1$ intersects the line $\$_{23}$, i.e, when the three lines $\$_1$, $\$_2$, $\$_3$ are concurrent (at a proper or an improper point). Hence the expressions are valid only when $\det J \neq 0$

OBS: Note that $\hat{S}_{12}$, $\hat{S}_{23}$, and $\hat{S}_{31}$ are easily obtained as follows:

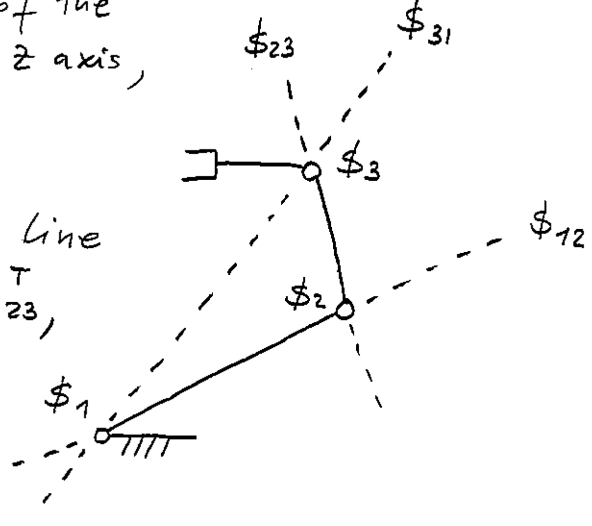
$$\hat{S}_{12} = \hat{s}_1 \times \hat{s}_2$$
$$\hat{S}_{23} = \hat{s}_2 \times \hat{s}_3$$
$$\hat{S}_{31} = \hat{s}_3 \times \hat{s}_1$$

B. Closed-form solution to the inverse kinematic problem of the 3R robot

In a similar way consider the velocity equation for the 3R manipulator

$$\hat{T} = \omega_1 \hat{S}_1 + \omega_2 \hat{S}_2 + \omega_3 \hat{S}_3$$

normalized coords. of the joint lines (‖ to z axis, in axis coords.)



Multiplying this equation by the line coordinates of the line $\$_{23}$, $\hat{s}_{23}^T$, we obtain

$$\hat{s}_{23}^T \hat{T} = \omega_1 \hat{s}_{23}^T \hat{S}_1$$

because $\hat{s}_{23}^T \hat{S}_2 = 0$ and $\hat{s}_{23}^T \hat{S}_3 = 0$. Hence

$$\boxed{\omega_1 = \frac{\hat{s}_{23}^T}{\hat{s}_{23}^T \hat{S}_1} \cdot \hat{T}}$$

As before these expressions are only valid if $\det J \neq 0$

and, analogously, using products with $\hat{s}_{31}^T$ and $\hat{s}_{12}^T$:

$$\boxed{\begin{aligned} \omega_2 &= \frac{\hat{s}_{31}^T}{\hat{s}_{31}^T \cdot \hat{S}_2} \cdot \hat{T} \\ \omega_3 &= \frac{\hat{s}_{12}^T}{\hat{s}_{12}^T \cdot \hat{S}_3} \cdot \hat{T} \end{aligned}}$$

OBS: the coordinates of lines $\$_{12}$, $\$_{23}$, $\$_{31}$ can be easily obtained as

$$\hat{s}_{12} = \hat{S}_1 \times \hat{S}_2$$
$$\hat{s}_{23} = \hat{S}_2 \times \hat{S}_3$$
$$\hat{s}_{31} = \hat{S}_3 \times \hat{S}_1$$

which provide the solution to the instantaneous inverse kinematic problem.

C. Alternative method to obtain $\vec{v} = J^T \cdot \hat{T}$ in a 3-R$\underline{P}$R robot

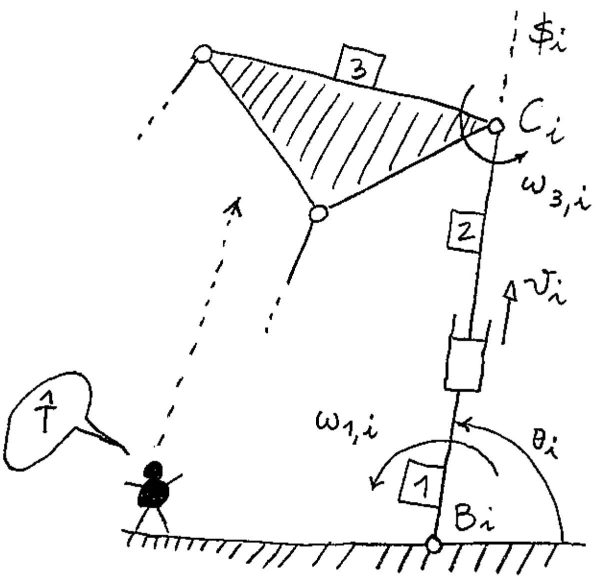
We obtained the velocity equation of a 3-R$\underline{P}$R robot using the principle of virtual power. This equation can also be obtained by forming reciprocal products on twist equations. It is worth studying such a method because:

(1) It is often used in the literature, in the case of spatial robots

(2) It gives insight into the velocity components of the leg anchor points, and this insight is necessary for later developments (especially for module 5 "hybrid control")

(3) This method will be generalised in section D below, to deal with a large family of manipulators.

So let us consider one leg of a 3-R$\underline{P}$R manipulator, and let us label the leg links as $\boxed{1}$, $\boxed{2}$, $\boxed{3}$, like in the figure.



The i-th leg of a 3-R$\underline{P}$R manipulator

If we view the leg as a serial chain, we can write

$$\hat{T} = \hat{T}_{1i} + \hat{T}_{2i} + \hat{T}_{3i} \qquad (1)$$

Twist of $\boxed{3}$ relative to ground

Twist of $\boxed{1}$ relative to ground

Twist of $\boxed{2}$ relative to $\boxed{1}$

Twist of $\boxed{3}$ relative to $\boxed{2}$

We can expand the previous Eq. (1) as follows

$$\hat{T} = \omega_{1,i}\, \hat{S}_{1,i} + v_i\, \hat{S}_{2,i} + \omega_{3,i}\, \hat{S}_{3,i} \qquad (2)$$

where

- $\omega_{1,i}$ = angular velocity of $\boxed{1}$ relative to ground
- $\omega_{3,i}$ = " " " $\boxed{3}$ " " body $\boxed{2}$
- v_i = magnitude of the velocity of the P joint
- $\hat{S}_{1,i}$ = normalized coords. of the vertical line through B_i
- $\hat{S}_{3,i}$ = " " " " " " " C_i
- $\hat{S}_{2,i}$ = " " " " " " corresponding to the twist of the P joint

To isolate v_i in terms of $\hat{T}$, we multiply Eq. (2) by $\hat{s}_i^T$, the normalized coords. of line $\$_i$, obtaining

$$\hat{s}_i^T \cdot \hat{T}_i = \hat{s}_i^T \cdot \hat{S}_{2,i} \cdot v_i, \qquad (3)$$

because $\hat{s}_i^T \cdot \hat{S}_{1,i} = 0$ and $\hat{s}_i^T \cdot \hat{S}_{3,i} = 0$. Since $\hat{s}_i^T = [c_i, s_i, p_i]$, where $c_i = \cos(\theta_i)$ and $s_i = \sin(\theta_i)$, we obtain

Thus: $$\hat{s}_i^T \cdot \hat{T} = [c_i, s_i, p_i] \cdot \begin{bmatrix} c_i \\ s_i \\ 0 \end{bmatrix} \cdot v_i = v_i$$

$$\boxed{\hat{s}_i^T \cdot \hat{T} = v_i} \qquad (4)$$

Therefore, for all legs:

$$\boxed{\begin{aligned} v_1 &= \hat{s}_1^T \cdot \hat{T} \\ v_2 &= \hat{s}_2^T \cdot \hat{T} \\ v_3 &= \hat{s}_3^T \cdot \hat{T} \end{aligned}} \qquad (5)$$

which can be expressed in matrix form as

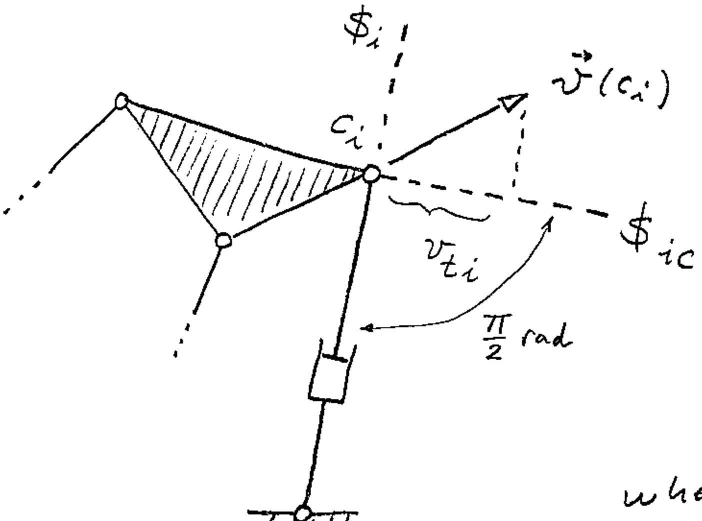
$$\boxed{\vec{v} = J^T \cdot \hat{T}} \qquad (6)$$

which is the known velocity equation of the 3-R$\underline{\text{P}}$R robot.

Important observation: Note that Eq. (4) can be interpreted in a more general form as follows. Given an arbitrary line through c_i, with normalized coordinates $\hat{s}_i$, then the component of the velocity of point c_i along line $\$_i$ is $v_i = \hat{s}_i^T \cdot \hat{T}$

In particular this allows us to determine the transversal components of the velocity of c_i.

Let $\vec{v}(c_i)$ be the velocity of c_i relative to the ground. Consider the line $\$_{ic}$ through c_i orthogonal to $\$_i$. Then



the component of $\vec{v}(c_i)$ along $\$_{ic}$ can be computed as

$$v_{ti} = \hat{s}_{ic}^T \cdot \hat{T}$$

where $\hat{s}_{ic}$ are the normalized coordinates of $\$_{ic}$

For the three legs we can write, in matrix form

$$\underbrace{\begin{bmatrix} v_{t1} \\ v_{t2} \\ v_{t3} \end{bmatrix}}_{\vec{v}_t} = \underbrace{\begin{bmatrix} \text{---} & \hat{s}_{1c}^T & \text{---} \\ \text{---} & \hat{s}_{2c}^T & \text{---} \\ \text{---} & \hat{s}_{3c}^T & \text{---} \end{bmatrix}}_{C^T} \underbrace{\begin{bmatrix} \hat{T} \end{bmatrix}}_{\hat{T}}$$

i.e.:

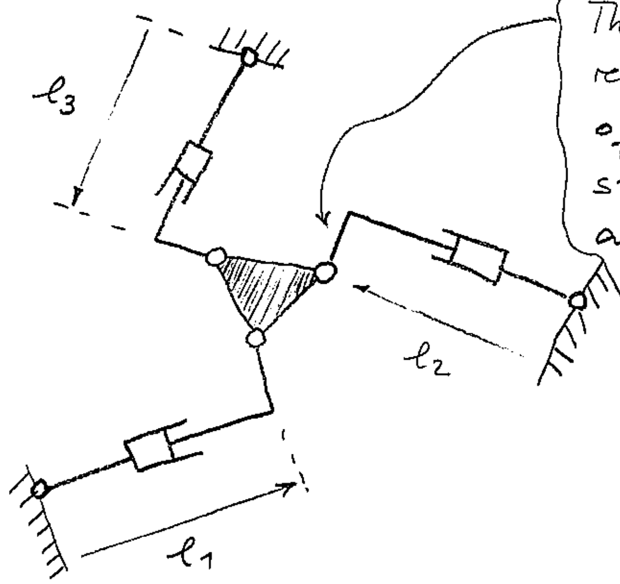
$$\boxed{\vec{v}_t = C^T \cdot \hat{T}} \qquad (7)$$

← Expression needed in module 5.

D. Kinetostatic analysis of fully parallel manipulators

The method in the previous section can be generalised and extended to perform a full kinetostatic analysis of a large family of parallel robots, called <u>fully parallel manipulators</u>. A fully parallel manipulator is a mechanism with an n-degree-of-freedom end effector, connected to the base by n independent kinematic chains, or legs, each having a single activated joint. The following figures and the companion slides of this module show some examples of such robots.

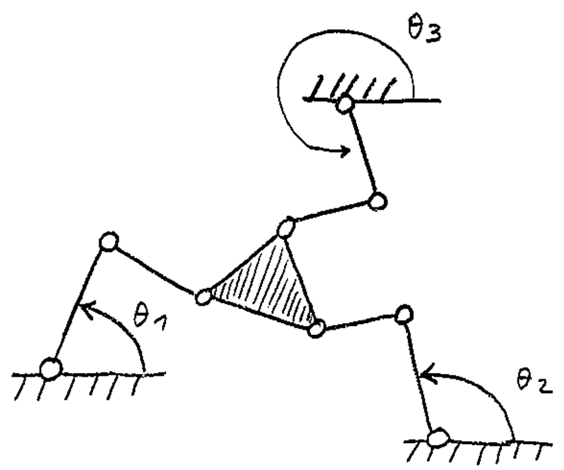
General 3-R<u>P</u>R robot



The platform anchor point has an offset relative to the prismatic joint axis. This offset was zero in the 3-R<u>P</u>R robot studied so far. The equations $\vec{w} = j \cdot \vec{\lambda}$ and $\vec{v} = j^T \cdot \vec{\tau}$ assumed so far are not applicable to this robot, unless the ofsset is zero.

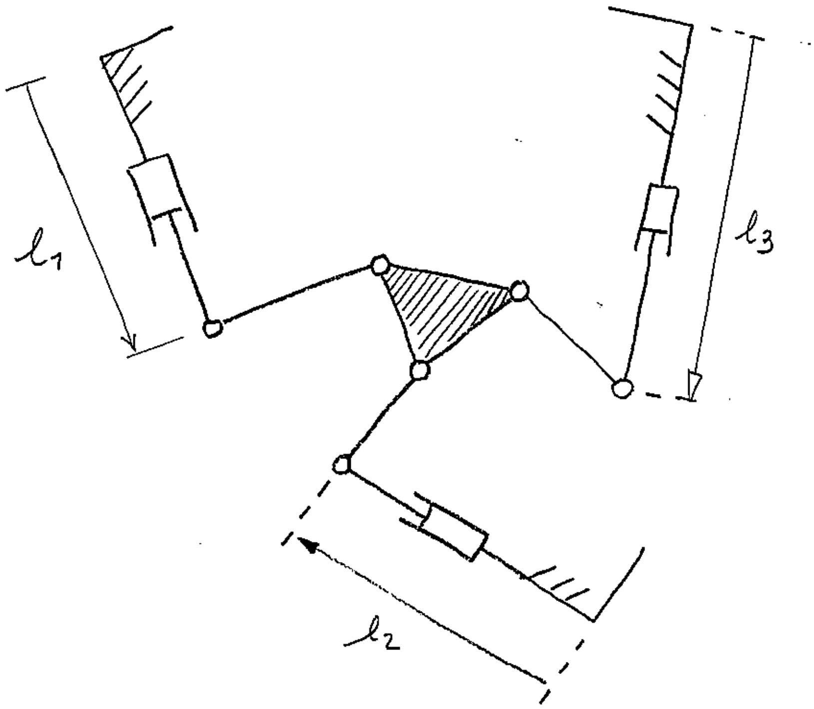
- $n = 3$
- input coords: l_1, l_2, l_3
- output " : 3 pose coords of the moving platform

3-<u>R</u>RR robot



- $n = 3$
- input coords: $\theta_1, \theta_2, \theta_3$
- output " : 3 pose coords. of the moving platform
- example: 3-<u>R</u>RR robot from Leibniz Universität Hannover, Germany.

3 - $\underline{P}$RR robot

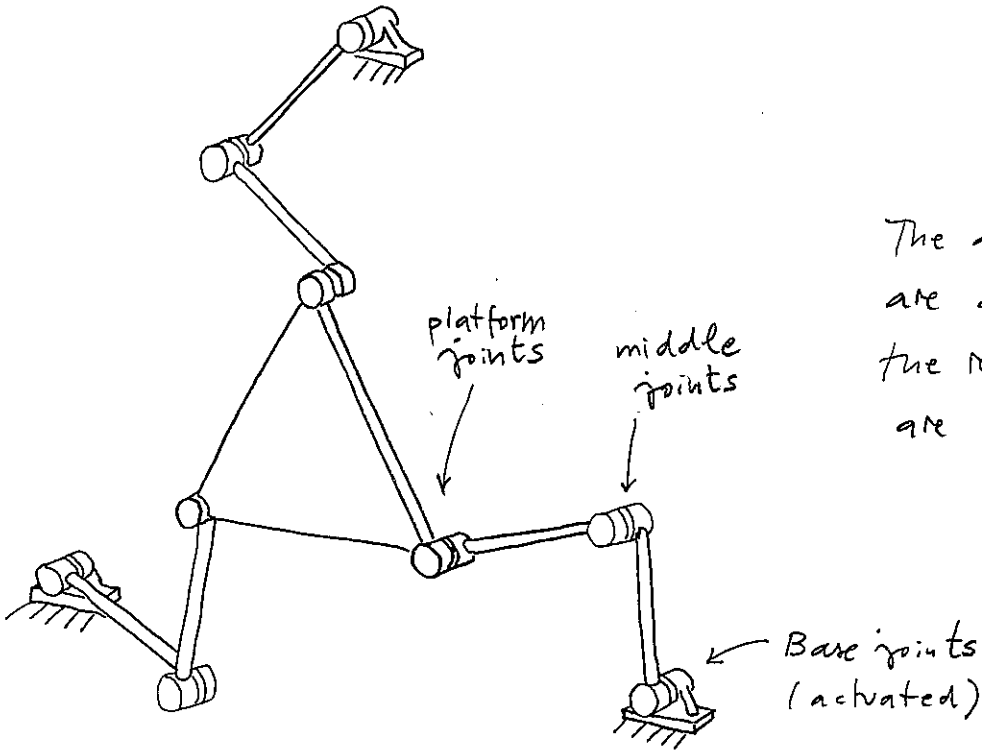


- $n = 3$
- input coords: l_1, l_2, l_3
- output " : the pose coords of the moving platform
- example: 3-$\underline{P}$RR robot from CESTER, University of Cluj Napoka, Romania.

In most cases the actuated joint is the base joint, to reduce the amount of moving mass and avoid robot self-collisions as much as possible. However, the method that follows is applicable to robots where the actuated joint is the middle or platform joint as well.

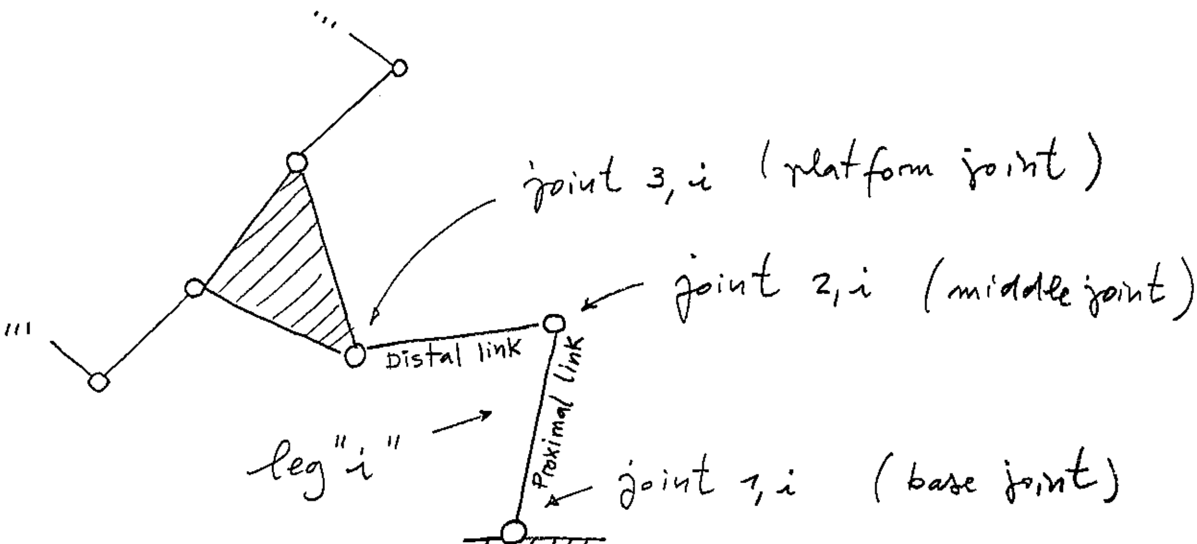
For concreteness, the method will be explained in the particular case of the 3-$\underline{R}$RR robot, but its generalisation to any fully parallel robot will become apparent. See subsection D.3 below, page 18.

Consider the 3-$\underline{R}$RR robot of the following figure



The ground joints are actuated, and the remaining joints are passive.

We label the joints of the i-th leg as follows



and we let $\omega_{j,i}$ be the relative angular velocity of joint j,i $j,i \in \{1,2,3\}$. Our initial goal is to find a velocity equation allowing to solve the forward and inverse instantaneous kinematic problems (FIKP and IIKP respectively):

Given

$$\vec{\gamma} = \begin{bmatrix} \omega_{1,1} \\ \omega_{1,2} \\ \omega_{1,3} \end{bmatrix}$$

input angular speeds

Given

$$\hat{T} = \begin{bmatrix} v_{ox} \\ v_{oy} \\ \omega \end{bmatrix}$$

moving platform twist

FIKP

IIKP

Determine the whole velocity state of the manipulator

(The "whole velocity state" means "all body twists" of the manipulator. To obtain them, it is enough to obtain $\omega_{j,i}$ $j \in \{1,2,3\}$ $i \in \{1,2,3\}$)

Then we will show that, using the principle of virtual power, we can obtain a dual equation allowing to solve the forward and inverse <u>static</u> problems.

D.1. Kinematic analysis of the 3-RRR robot

Note that for each leg of the manipulator we can write

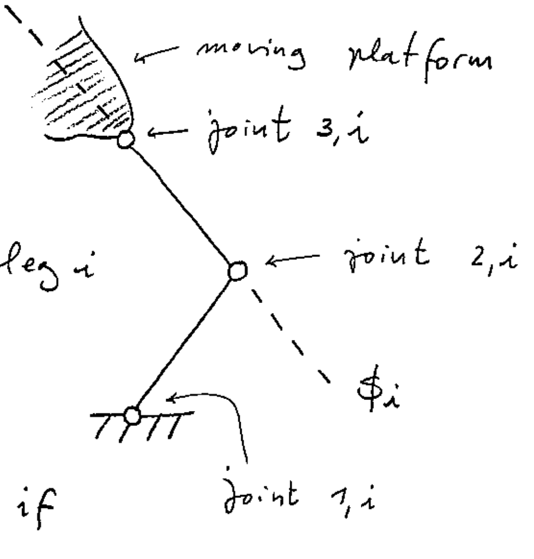
$$\hat{T} = \omega_{1,i} \, \hat{S}_{1,i} + \omega_{2,i} \, \hat{S}_{2,i} + \omega_{3,i} \, \hat{S}_{3,i} \qquad (8)$$

where

$S_{j,i}$ = normalized coordinates of the vertical line through joint j,i.

By writing the previous equation for all legs we obtain a system of linear equations that determines the feasible velocity states of the manipulator. This system has 12 variables (the nine $\omega_{j,i}$, and the three components of $\hat{T}$) and 9 equations (the three scalar equations in Eq. (8) gathered for all legs). Therefore, by fixing $\hat{T}$ or $\vec{\gamma}$ we have a linear system that is in principle compatible and determined, and thus allows to solve the FIKP and IIKP, respectively. This system can also be used to determine the singularities(*) of the manipulator, but this would lead to the analysis of rank conditions on 9×9 matrices which is a bit awkward. Simpler conditions can be obtained using reciprocal products.

Let $\hat{s}_i$ be the coordinate vector of a line $\$_i$ in the XY plane, intersecting joints 2,i and 3,i



IMPORTANT: $\hat{s}_i$ is also called <u>a reciprocal wrench</u> because it can be interpreted as a wrench that, if applied to the platform, it can be resisted by solely activating joint 1,i.

(*) The configurations for which this system is unsolvable or undetermined for some $\vec{\gamma}$ or $\hat{T}$.

Clearly, we can multiply Eq (8) in page 10 by $\hat{s}_i^T$, obtaining

$$\hat{s}_i^T \hat{T} = \omega_{1,i} \, \hat{s}_i^T \hat{S}_{1,i} \qquad i = 1,2,3$$

for $i = 1,2,3$, or, in matrix form

$$\underbrace{\begin{bmatrix} \text{---} & \hat{s}_1^T & \text{---} \\ \text{---} & \hat{s}_2^T & \text{---} \\ \text{---} & \hat{s}_3^T & \text{---} \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} v_{ox} \\ v_{oy} \\ \omega \end{bmatrix}}_{\hat{T}} = \underbrace{\begin{bmatrix} \hat{s}_1^T \hat{S}_{1,1} & 0 & 0 \\ 0 & \hat{s}_2^T \hat{S}_{1,2} & 0 \\ 0 & 0 & \hat{s}_3^T \hat{S}_{1,3} \end{bmatrix}}_{B} \underbrace{\begin{bmatrix} \omega_{1,1} \\ \omega_{1,2} \\ \omega_{1,3} \end{bmatrix}}_{\vec{\gamma}}$$

i.e.

$$A \cdot \hat{T} = B \cdot \vec{\gamma} \qquad (9)$$

where A and B are the 3x3 matrices indicated.

This equation is a necessary and sufficient condition for $\vec{\gamma}$ and $\hat{T}$ to be valid input and output velocities. However, note that it can also be used to solve both the FIKP and IIKP, since fixing $\hat{T}$ allows to compute $\vec{\gamma}$, and vice versa. Strictly speaking, solving the FIKP and the IIKP entails computing not only $\hat{T}$ and $\vec{\gamma}$, but the whole velocity state of the manipulator, but once $\hat{T}$ and $\vec{\gamma}$ are known we can rapidly solve for $\omega_{2,i}$ and $\omega_{3,i}$ for all i using Eq (8).

If both $\det(A) \neq 0$ and $\det(B) \neq 0$ then

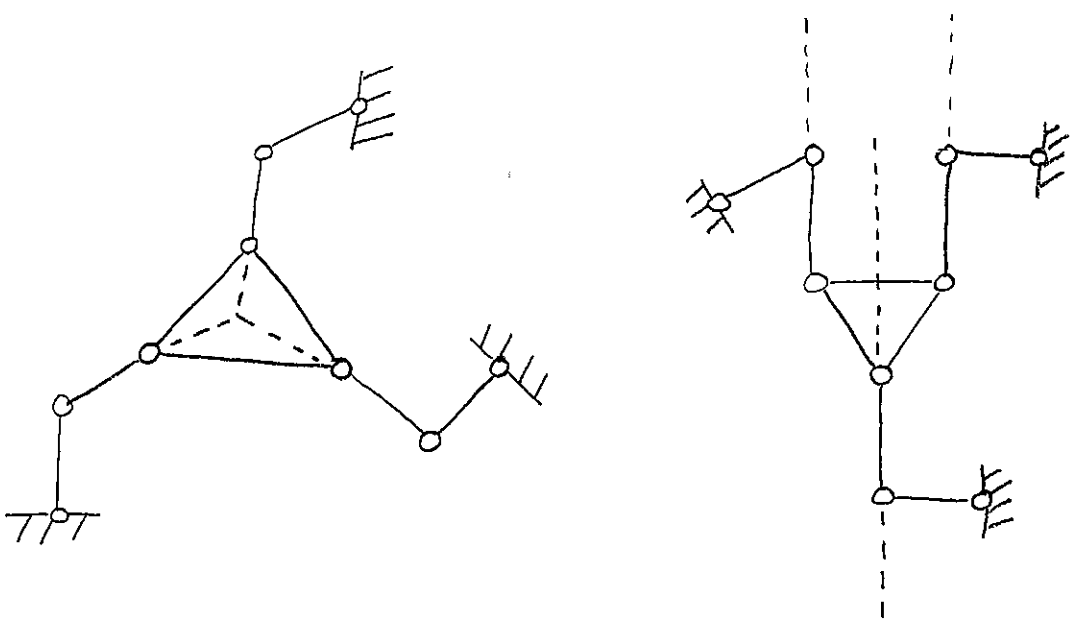
$$\hat{T} = A^{-1} B \, \vec{\gamma}$$

solves the FIKP $\forall \vec{\gamma}$, and

$$\vec{\gamma} = B^{-1} A \, \hat{T}$$

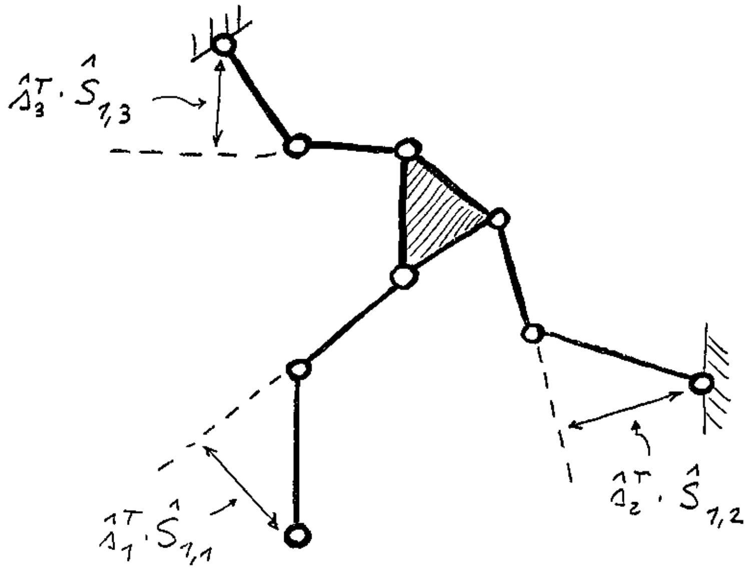
solves the IIKP $\forall \hat{T}$.

If $\det(A) = 0$ then the system $A \cdot \hat{T} = B \vec{\gamma}$ is either unsolvable or undetermined for some values of $\vec{\gamma}$, and the manipulator is said to be in a forward kinematic singularity configuration. Since the rows of A are the line coordinates of the distal links, $\det(A) = 0$ if, and only if, these lines are concurrent or parallel:

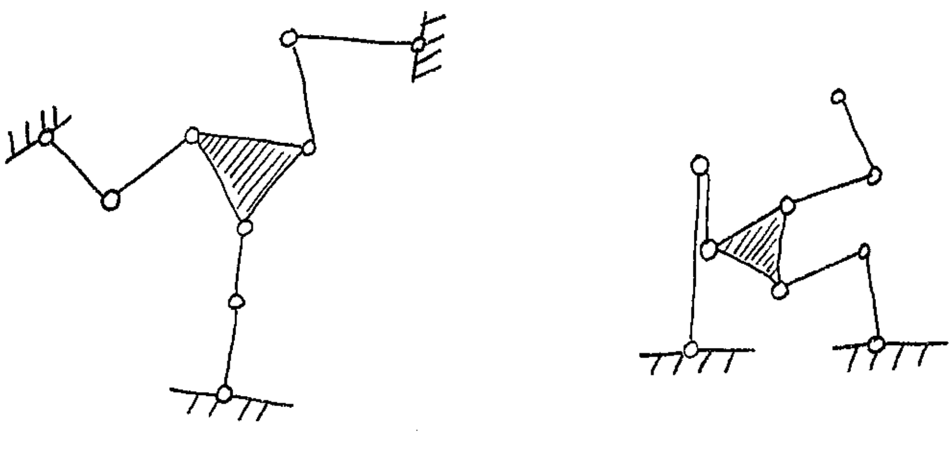


Forward singularities of a 3-RRR manipulator

If det(B) = 0 then the system $A\hat{T} = B\vec{\gamma}$ is either unsolvable or undetermined for some values of $\hat{T}$, and the manipulator is said to be in an <u>inverse kinematic singularity</u> configuration. The elements in the diagonal of B are the distances from the base joints to the distal link lines



and hence inverse singularities arise whenever one of the legs has its three joints aligned, either in full extension, or with the leg "folded back":



Inverse singularities of a 3-$\underline{R}$RR manipulator

Note that if at a forward singularity we set $\vec{\gamma}=0$ (e.g. by locking the actuators), then Eq (9) reduces to

$$A \cdot \hat{T} = 0$$

and since A is singular, its kernel has dimension one or larger. Thus, there will be infinitely-many twists $\hat{T}$ verifying Eq. (9) for $\vec{\gamma}=0$. This phenomenon is observed in practice. The platform "shakes" even though the actuators are locked. We find controllability issues at forward singularities, because the input velocities $\vec{\gamma}$ do not determine the whole velocity state. The set of feasible $\vec{\gamma}$ is a subset of $\mathbb{R}^3$ and there is a dramatic increase in the position error of the platform. All of the critical phenomena that we saw in the case of the 3-R$\underline{P}$R robot arise here, to the point that these singularities are often referred to (a bit misleadingly) as "parallel" singularites in the Literature.

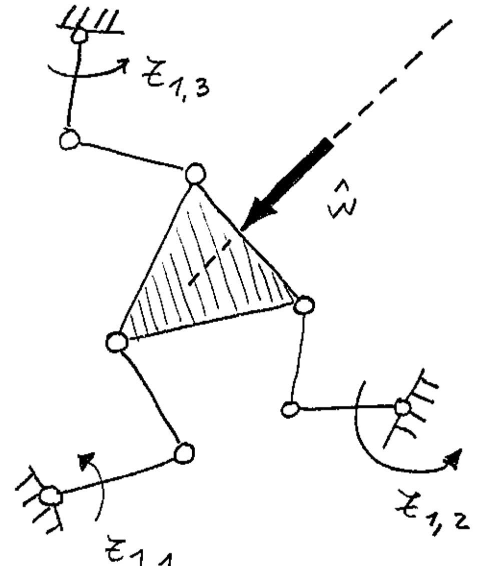
In inverse singularities we have the reverse phenomenon. By setting $\hat{T}=0$ in Eq. (9) we obtain

$$B \cdot \vec{\gamma} = 0$$

and since the kernel of B has dimension ≥ 1 there will be infinitely-many $\vec{\gamma}$ satisfying Eq. (9) for $\hat{T}=0$. In practice, if we lock the platform pose, we see that the legs are "shaky". All of the critical phenomena that we saw in the case of the 3R robot arise here, which justifies why these singularities are often referred to as serial singularities. In them, the platform loses dexterity and there may be overspeeding of the legs when passing near to them.

D. 2. Static analysis of the 3-RRR robot

Suppose that a wrench $\hat{w}$ is acting on the platform of a 3-RRR robot and let $\tau_{1,1}$, $\tau_{1,2}$, $\tau_{1,3}$ be the resultant joint torques. The actuator in joint $1,i$ will exert an equilibrant torque $-\tau_{1,i}$ to keep the manipulator in equilibrium.



We next determine the relationship between $\hat{w}$ and $\vec{\tau} = \begin{bmatrix} \tau_{1,1} \\ \tau_{1,2} \\ \tau_{1,3} \end{bmatrix}$ using the principle of virtual power.

Assume that the platform is moving under a twist $\hat{T}$ induced by the angular velocities $\vec{\gamma}$, where $\hat{T}$ and $\vec{\gamma}$ satisfy Eq. (9), i.e:

$$\underbrace{[A, -B]}_{\substack{\| \\ M}} \cdot \begin{bmatrix} \hat{T} \\ \vec{\gamma} \end{bmatrix} = 0 \qquad (10)$$

We define $M = [A, -B]$

By the principle of virtual power we know that

$$\underbrace{\hat{w}^T \cdot \hat{T}}_{\substack{\text{Power generated} \\ \text{by the wrench} \\ \hat{w} \text{ under twist } \hat{T}}} - \underbrace{\vec{\tau}^T \cdot \vec{\gamma}}_{\substack{\text{Power generated by the equilibrant torques } -\vec{\tau} \\ \text{under the angular velocities } \vec{\gamma}}} = 0 ,$$

or, in matrix form:

$$[\hat{w}^T, -\vec{\tau}^T] \begin{bmatrix} \hat{T} \\ \vec{\gamma} \end{bmatrix} = 0 . \qquad (11)$$

From Eq. (11) we see that the feasible vectors $\begin{bmatrix} \hat{w} \\ -\vec{\tau} \end{bmatrix}$ are those orthogonal to $\begin{bmatrix} \hat{T} \\ \vec{\gamma} \end{bmatrix}$. From Eq. (10) we see that the vectors orthogonal to $\begin{bmatrix} \hat{T} \\ \vec{\gamma} \end{bmatrix}$ are precisely those of the row space of M, i.e., those of the column space of M^T. We therefore conclude that for a vector $\begin{bmatrix} \hat{w} \\ -\vec{\tau} \end{bmatrix}$ to be feasible it must be

$$\begin{bmatrix} \hat{w} \\ -\vec{\tau} \end{bmatrix} = M^T \vec{\beta} \qquad (12)$$

for some β. Expanding Eq. (12) we obtain

$$\begin{bmatrix} \hat{w} \\ -\vec{\tau} \end{bmatrix} = \begin{bmatrix} A^t \\ -B^t \end{bmatrix} \vec{\beta}$$

i.e.:

$$\hat{w} = A^t \vec{\beta} \qquad (13.1)$$

$$\vec{\tau} = B^t \vec{\beta} \qquad (13.2)$$

which are the sought equations describing the static behaviour of the manipulator.

If $\det A \neq 0$, then from Eq. (13.1)

$$\vec{\beta} = (A^t)^{-1}\,\hat{w}$$

and substituting in Eq. (13.2)

$$\vec{\tau} = B^t (A^t)^{-1}\,\hat{w}$$

which solves the inverse statics problem.

If $\det B \neq 0$ then from Eq. (13.2)

$$\vec{\beta} = (B^t)^{-1}\,\vec{\tau} = B^{-1}\,\vec{\tau}$$

B is diagonal

and substituting in Eq. (13.1):

$$\hat{w} = A^t B^{-1}\,\vec{\tau}$$

which solves the forward statics problem.

$\det A = 0$	$\Longleftrightarrow$	forward kinematic singularity	$\Longleftrightarrow$	inverse static singularity
$\det B = 0$	$\Longleftrightarrow$	inverse kinematic singularity	$\Longleftrightarrow$	forward static singularity

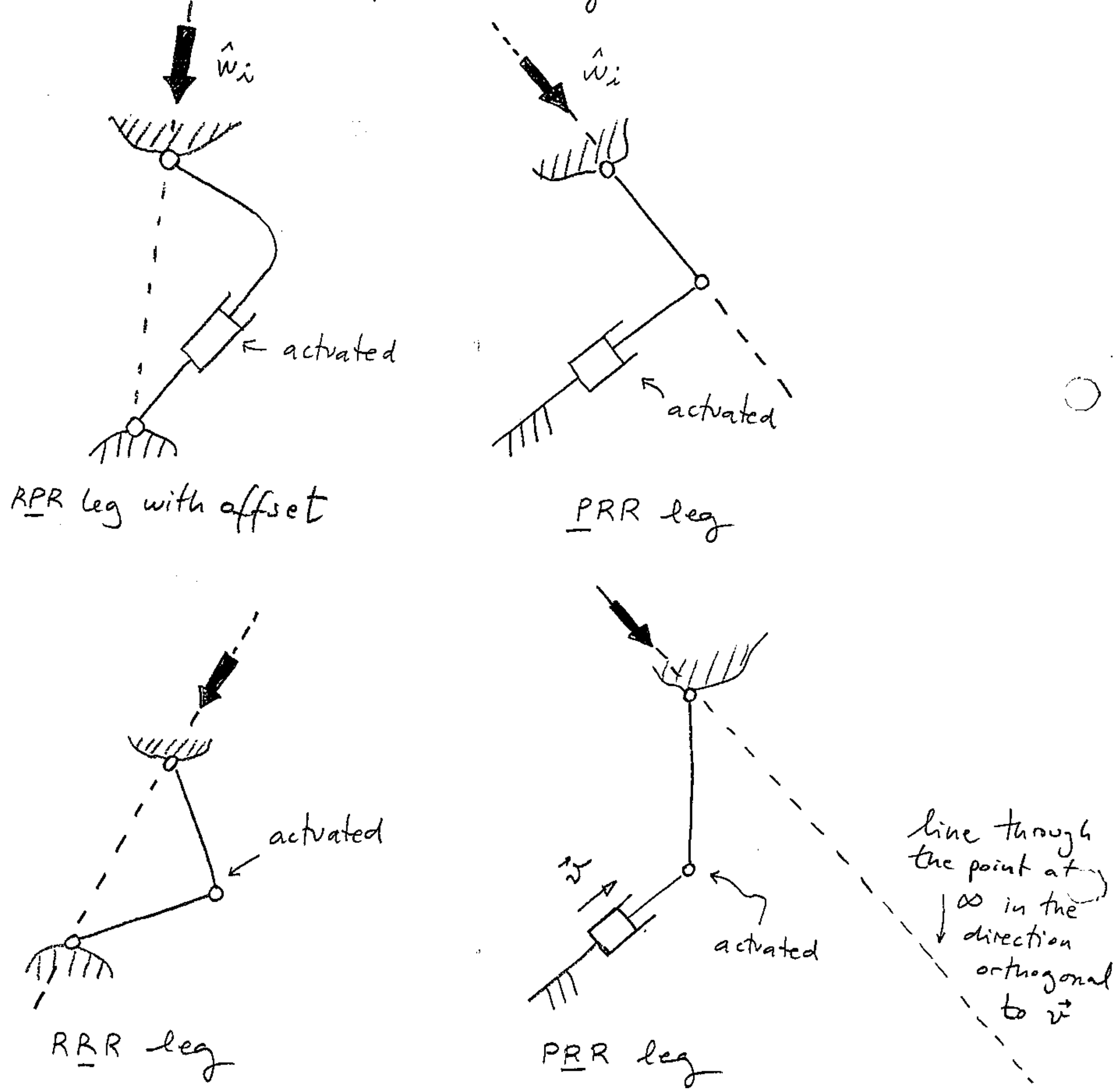
As expected, singularities related to the two problems arise when $\det A = 0$ (inverse static singularity) or $\det B = 0$ (forward static singularity) and again we see that they arise simultaneously with the corresponding kinematic singularities. See, this summary table.

D.3. Other leg architectures

As we said, the method is equally applicable to other leg architectures. We start by writing an equation similar to Eq. (8) for each leg, and then multiply this equation by a reciprocal wrench $\hat{w}_i$ of

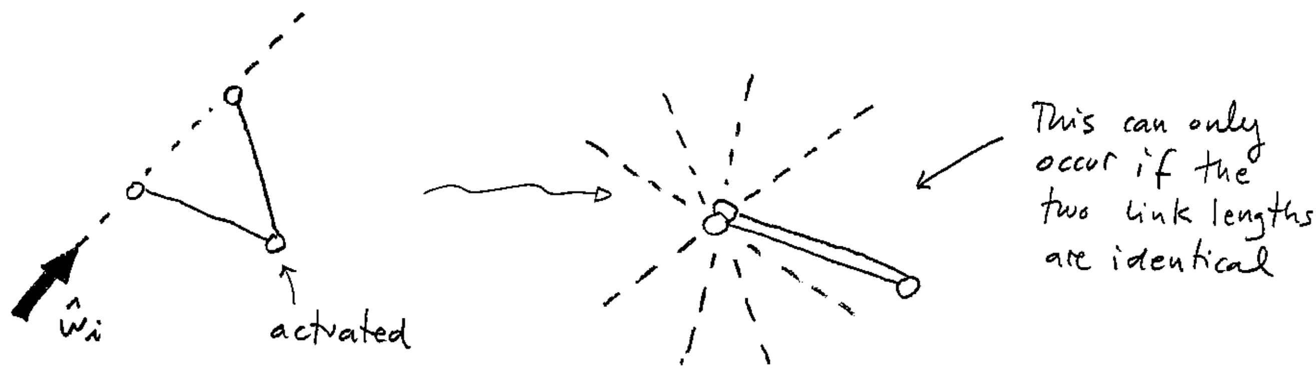
$\hat{w}_i = \hat{s}_i$ in page 11

the leg (see page 12). The following figure shows a reciprocal wrench $\hat{w}_i$ for some leg architectures



R<u>P</u>R leg with offset — <u>P</u>RR leg — R<u>R</u>R leg — P<u>R</u>R leg

Care must be taken in situations for which the space of reciprocal wrenches $\hat{w}_i$ has dimension larger than one. In a R<u>R</u>R leg, for example, this occurs when the two passive joints are coincident:



one-dimensional space of reciprocal wrenches $\hat{w}_i$

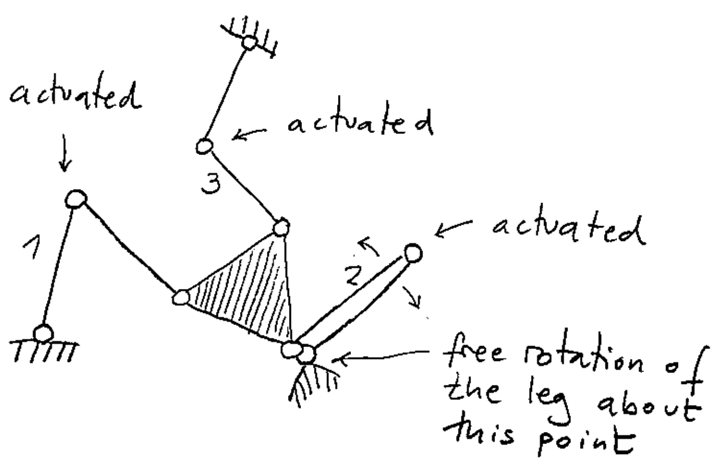
the space of $\hat{w}_i$ is two-dimensional here

When this happens, the manipulator is said to be in a <u>redundant</u> <u>passive motion singularity</u>. These singularities are <u>not captured</u> by the simplified singularity conditions

$$\det A = 0 \qquad \det B = 0$$

In these configurations the overall velocity state is neither determined by the input or output velocities. Certainly, from the following figure we see that, even if we lock the actuators and the platform pose, leg 2 can freely rotate.

Redundant passive motion singularities are forward and inverse singularities at the same time.



A 3-R$\underline{R}$R robot in a redundant passive motion singularity.