Exercises Module 5: Hybrid control

Problem 1

Fig. 1 illustrates the "Renaissance" system from Mazor Robotics, which allows to perform surgery operations on the spinal column. The most common task consists in perforating a vertebra to install a screw, enabling it to be coupled to a neighboring vertebra. The setting to be studied consists of[1]:

- A **3RPR robot** with the base fixed to the ground and to the vertebra to be drilled.
- An **auxiliary arm** coupled to the platform of the 3RPR robot, used for holding a pipe guiding the drill bit during perforation.
- A **3R robot** equipped with a 3RPR compliant wrist, coupled to the perforating drill.

The auxiliary arm, the pipe, and the mobile platform of the 3RPR robot are fixed one to another, and have neglectable mass. Each leg of the 3RPR robot has a load cell mounted on it that provides the magnitude (with sign) of the force f_i that the leg exerts on the platform (if $\boldsymbol{e}_i$ is the unit vector of leg i, pointing from the base towards the platform, this leg makes a force $\boldsymbol{f}_i = f_i \boldsymbol{e}_i$ on the platform). The contact between the drill bit and the pipe is of frictionless planar type.

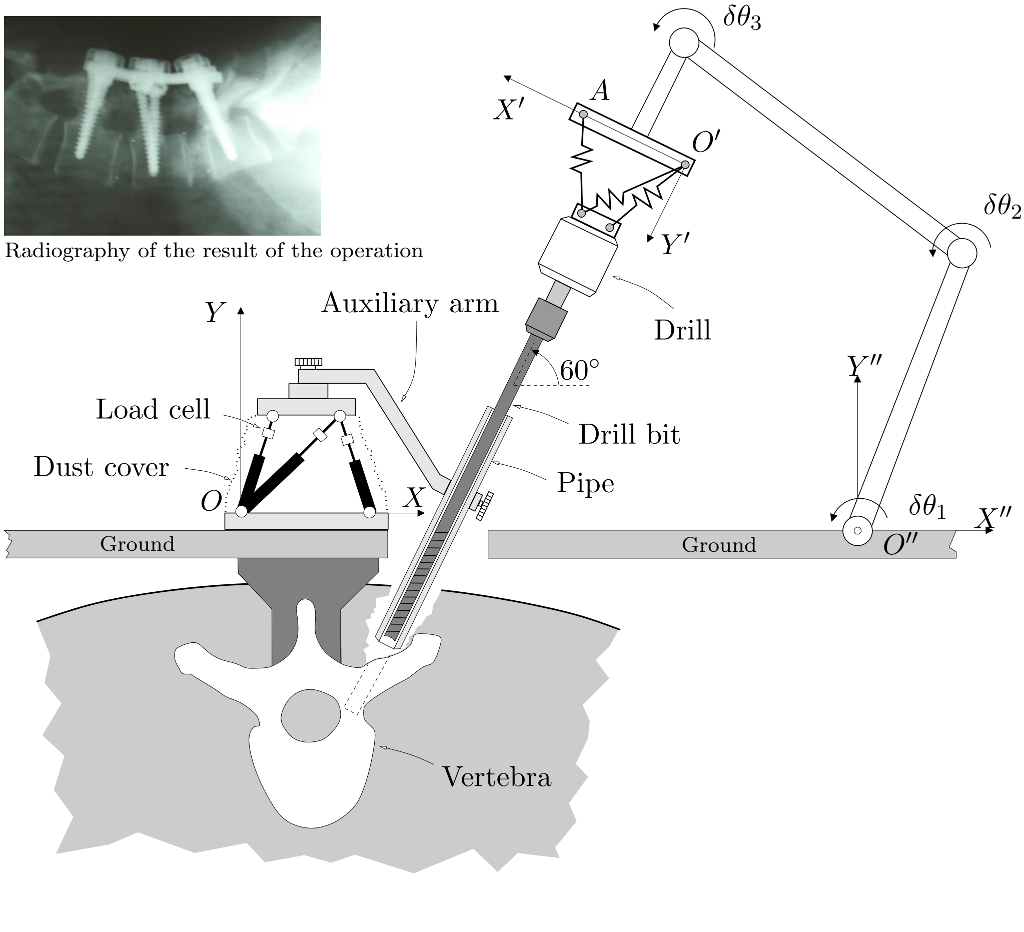


Figure 1: A setting for robotic spinal surgery (not drawn to scale).

At the beginning of the operation, the 3R robot finds itself in a resting position (not shown), and we displace the 3RPR robot to place the pipe in the configuration required by the surgeon. In the situation of Fig. 2 the system is very close to this configuration. To reach it, only a small correcting rotation of the pipe needs to be performed, of 0.01 radians around point P, in counterclockwise sense.

[1]In the real Renaissance system the parallel robot is a Stewart platform, and the drilling operation is currently performed by a human. See Fig. 3 and http://mazorrobotics.com/renaissance/how-it-works for details.

Legs 1, 2, and 3 are appearing from left to right, respectively.

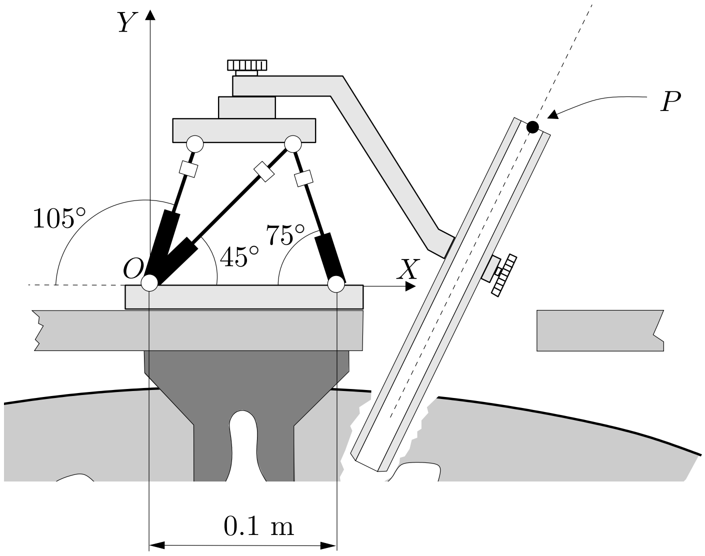


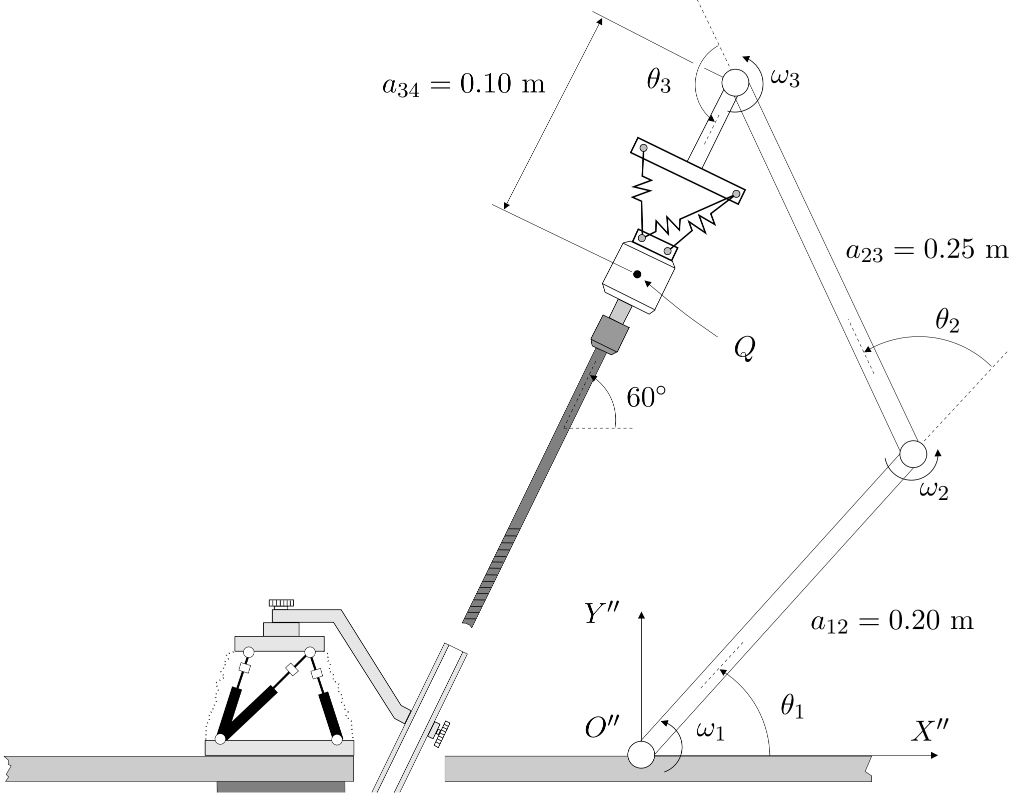
Figure 2: Pose correction of the pipe with the 3RPR robot.

Determine:

A The infinitesimal displacement $\delta\hat{D}$ corresponding to the mentioned correcting rotation, in coordinate system OXY, assuming that the coordinates of P in this coordinate system are $(0.17, 0.05)$ m.

B The small displacements δl_1, δl_2, and δl_3 that the linear actuators of the 3RPR robot will have to perform to achieve the displacement $\delta\hat{D}$.

Next, to insert the drill bit inside the pipe, the 3R arm is moved from the rest configuration to the following configuration, where $Q = (-0.1195, 0.2413)$ m in coordinate system $O''X''Y''$:



For this configuration, and using coordinate system $O''X''Y''$, determine:

C The angles θ_1, θ_2 and θ_3 that allow to place the end effector in the shown position and orientation, assuming that we work with $\theta_2 > 0$.

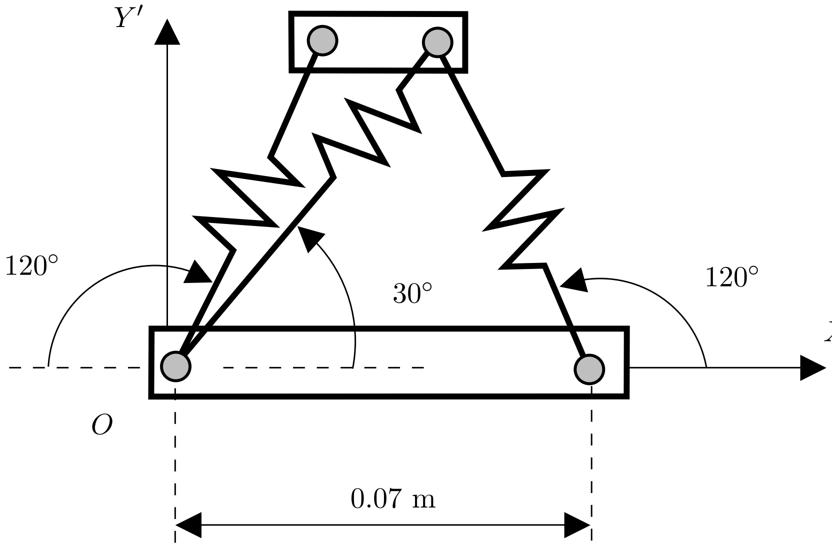
D The twist $\hat{T}$ under which the end effector of the 3R robot will move if the drill bit is introduced at 0.005 m/s along the pipe, and the rotational velocities ω_1, ω_2, and ω_3 needed to produce this twist.

Once the drill bit is inside the pipe, a hybrid control strategy is used to reduce the interaction forces between the drill bit and the pipe, taking advantage of the compliant wrist of the robot. If the system is in the configuration of Fig. 1, where $O' = (0.2675, 0.1489)$ m in the coordinate system OXY, and the load cells of the 3RPR robot are reading the values $f_1 = -0.58699$ N, $f_2 = 1.15539$ N, and $f_3 = -0.77646$ N, determine:

E The wrench $\hat{w}$ that the drill bit applies to the pipe, assuming that the configuration of the 3RPR robot is the same than that in Fig. 2.

F The space $\mathbb{D}$ of displacements $\delta\hat{D}$ (of the drill bit with respect to the pipe) that can be controlled. The space $\mathbb{W}$ of wrenches (applied by the pipe to the drill bit) that can be controlled. Provide a basis of $\mathbb{D}$ and $\mathbb{W}$ in the $O'X'Y'$ coordinate system and prove that, accepting small errors, the wrench $\hat{w}$ obtained in [E] belongs to $\mathbb{W}$.

G The rigidity matrix of the compliant wrist in the coordinate system $O'X'Y'$, assuming that all the springs have a rigidity constant of $k = 5000$ N/m, being their length close to the rest one, and taking into account that the geometry of the wrist is as follows:



H The small angle variations $\delta\theta_1$, $\delta\theta_2$, and $\delta\theta_3$ that the 3R robot joints have to undergo to (1) eliminate the interaction wrench $\hat{w}$ obtained in [E] and simultaneously (2) introduce the drill bit 0.001 m inside the pipe. Assume that the kinematic Jacobian of the 3R robot in coordinate system $O''X''Y''$ is

$$\mathbf{J}'' = \begin{bmatrix} 0 & 0.1679 & 0.2097 \\ 0 & -0.1087 & 0.1378 \\ 1 & 1 & 1 \end{bmatrix} \qquad \text{(SI units)} \tag{1}$$

and that $O' = (-0.1325, 0.1489)$ m in this coordinate system.

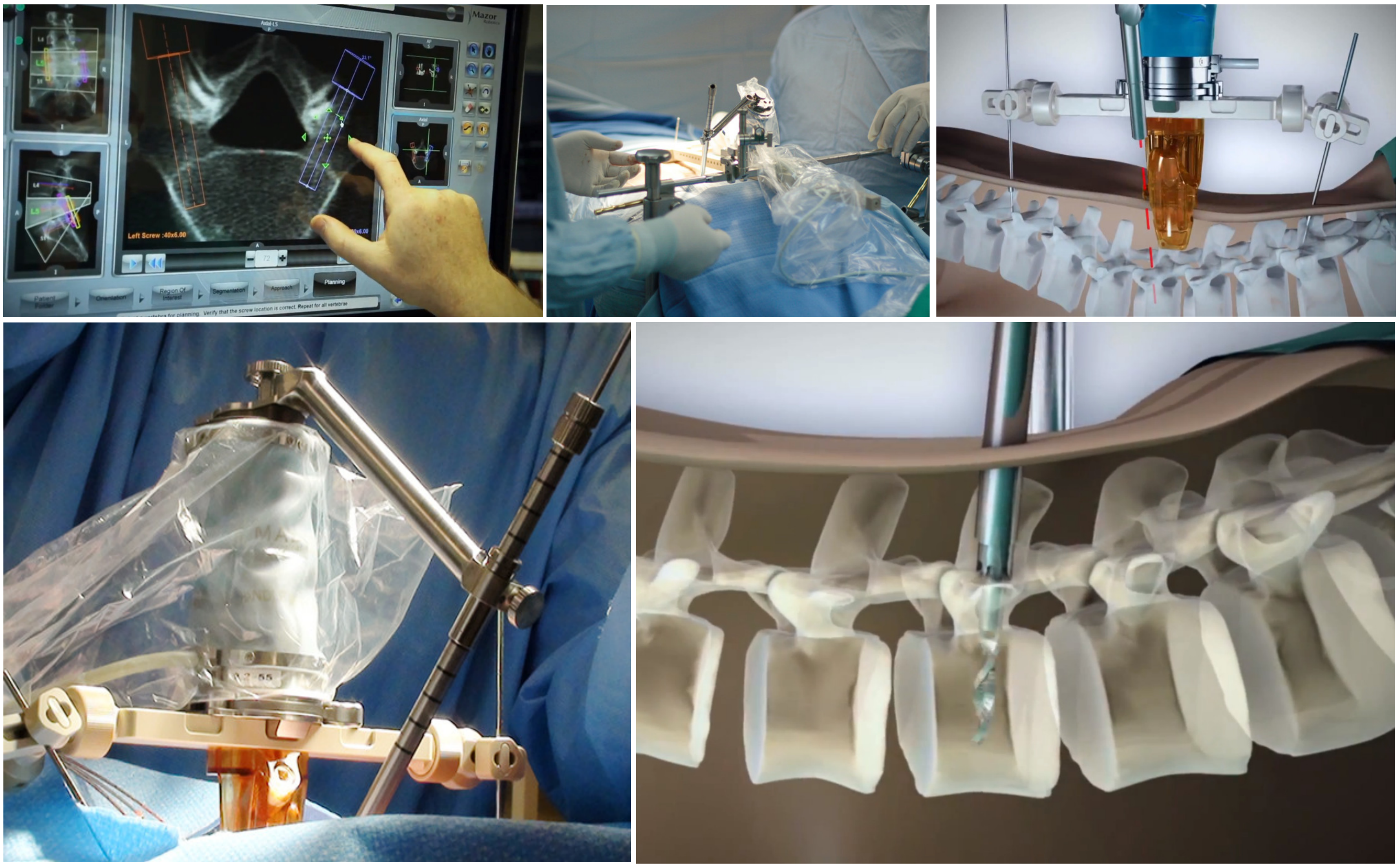
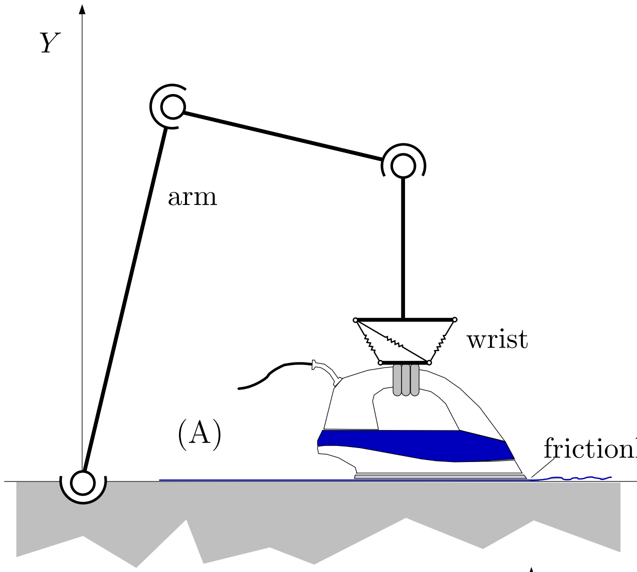


Figure 3: Images of the real “Renaissance” system. The surgeon first plans the operation, then mounts the base and the parallel manipulator, fixing and registering them with the vertebra, and finally triggers the drilling operation. Source: http://www.mazorrobotics.com.

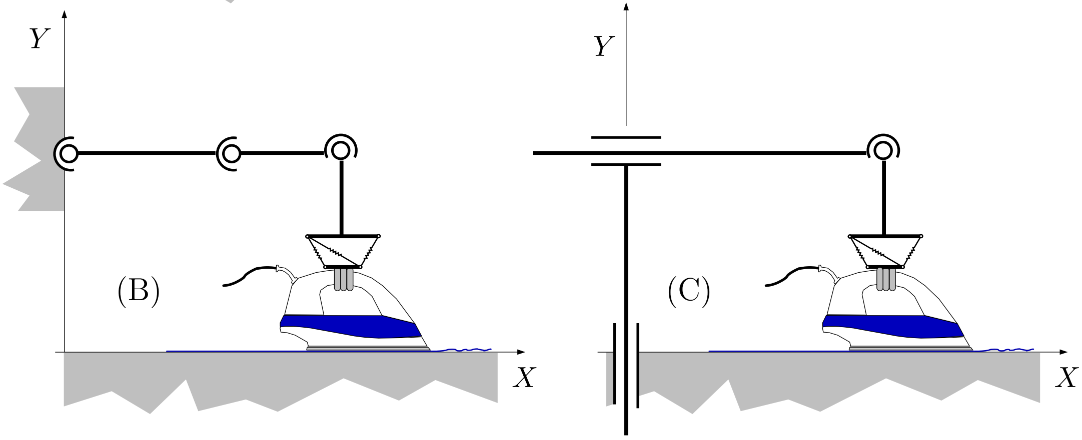
Short questions

Short question 1

Consider an ironing robot working in the plane XY as shown in the figure:



3-dimensional version of the problem



Mark the correct answer, assuming that the first three refer to the setting in (A):

1. The instant center of rotation of the iron with respect to the ground lies at infinity, in the direction of axis X.

2. In the coordinate system OXY, the space of contact wrenches that makes sense to control is

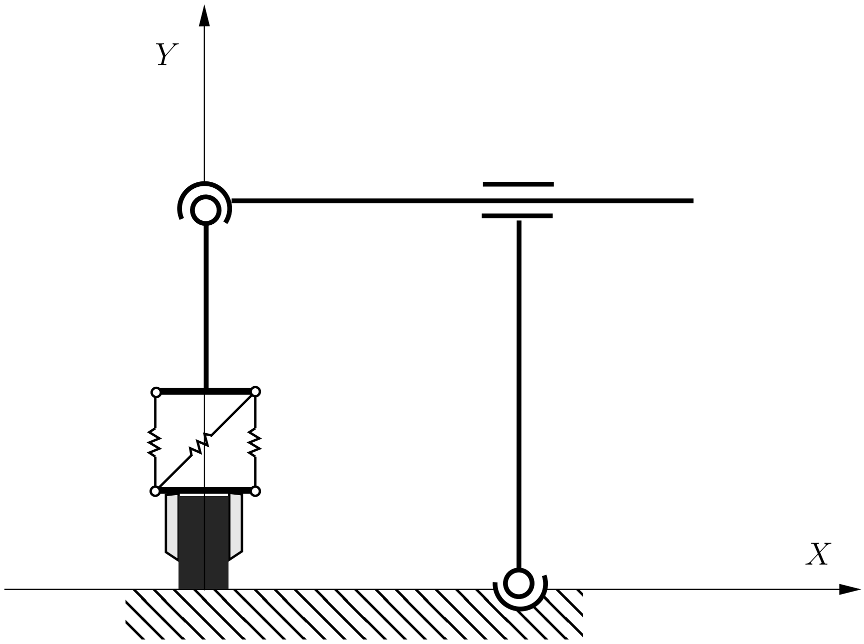
$$\left\langle \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\rangle$$

3. The arm-wrist set can only correct position errors of the iron.

4. At the configuration of the 3R arm shown in (B) not all of the allowed displacement errors of the iron can be corrected.

5. The arm shown in (C) does not allow to perform a hybrid control of force and position on the iron.

Short question 2
The manipulator of the following figure has to maintain the workpiece in planar contact (without friction) with the ground.



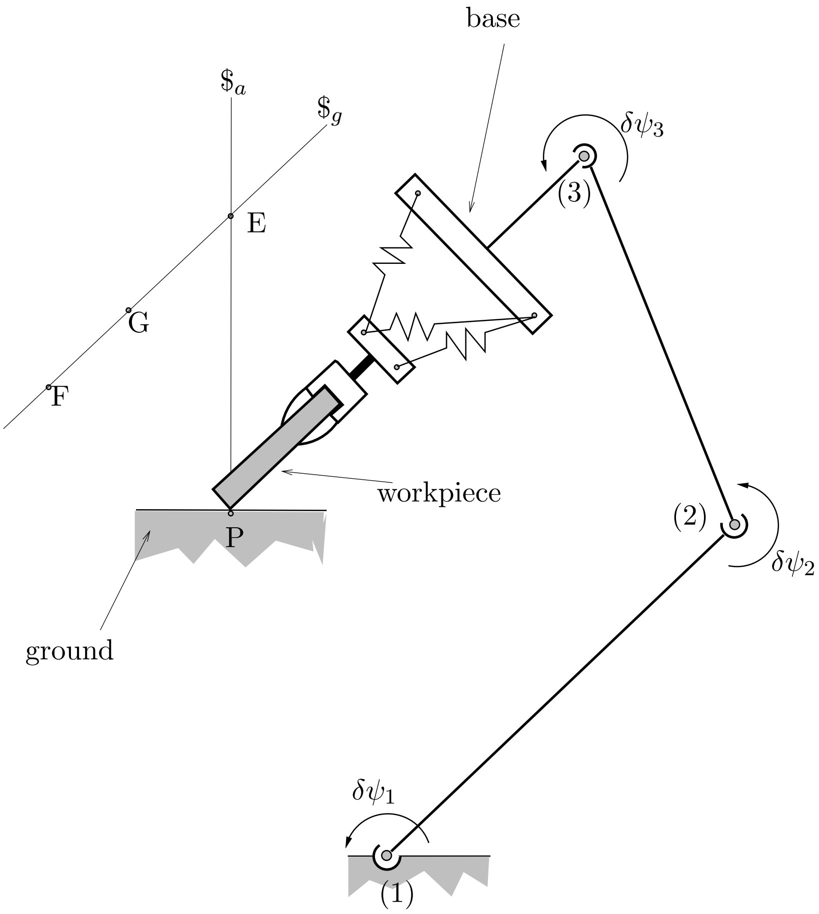
Mark the correct statement:

1. It makes no sense to adopt a hybrid control strategy.
2. The allowed infinitesimal displacements belong to the vector space:

$$\left\langle \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\rangle$$

3. With this serial robot it is not possible to implement any hybrid control strategy.
4. The instant center of rotation of the infinitesimal displacements of the workpiece with respect to the ground is at infinity, in the direction of the Y axis.
5. The compliant mechanism of the wrist does not allow to correct infinitesimal displacements of the force applied on the ground.

Short question 3
The end effector of a 3R manipulator moves a workpiece in contact with the ground, as shown in the following figure:

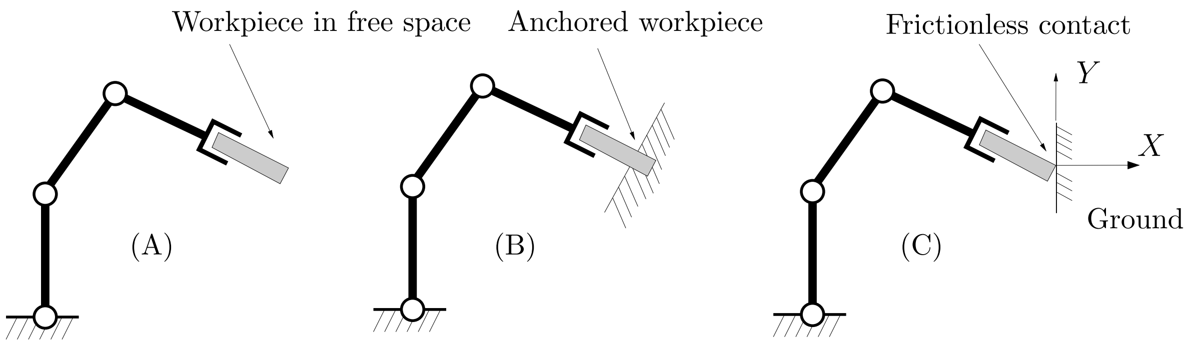


Assume there is no friction at the contact point P. Between the workpiece and the last link of the robot arm, there is a flexible wrist, with the structure of a parallel platform.
Mark the false statement:

1. The instant center of rotation of the workpiece with respect to the ground is a point E on line $\$_a$.
2. If the instant center of rotation of the base with respect to the ground is G, and the instant center of rotation of the workpiece with respect to the ground is E, then the instant center of rotation of the base with respect to the workpiece is a point F, on the line $E - G$.
3. The instant center of rotation of the workpiece with respect to the ground can only be point P.
4. We can control any contact force in P acting on line $\$_a$ and, simultaneously, any twist of the kind $\omega[y_E, -x_E; 1]^T$, where (x_E, y_E) are the coordinates of any point E on the line $\$_a$.
5. Any force that the ground makes on the workpiece is reciprocal to any twist of the workpiece with respect to the ground that maintains the workpiece-ground contact.

Short question 4

Consider the three situations depicted in the figure. In (A) the robot manipulates the workpiece in free space, in (B) the workpiece is fixed to the ground, and in (C) the workpiece maintains a frictionless point contact against the ground.



Mark the incorrect statement (in 3, 4, and 5 assume the coordinate system XY):

1. In (A) it makes sense to perform a velocity control: to make a small displacement $\delta\hat{D}$ at the end effector, the joints have to execute a small rotation $\delta\vec{\theta} = J^{-1}\delta\hat{D}$.

2. In (B) it makes sense to perfom a force control: to change the wrench that the workpiece makes on the ground in an amount equal to $\delta\hat{w}$, the joints have to vary the applied torques in $\delta\vec{\tau} =^{\mathsf{T}} J\delta\hat{w}$.

3. If in (C) we adopt a hybrid control strategy, it makes only sense to command small displacements $\delta\hat{D}$ of the workpiece, and wrench variations of $\delta\hat{w}$ of the workpiece on the ground, such that

$$\delta\hat{D} \in \langle \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \rangle \quad \text{i} \quad \delta\hat{w} \in \langle \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \rangle.$$

4. In (C) it is possible to maintain the workpiece static, in the shown position, and make the workpiece apply the following wrench on the ground:

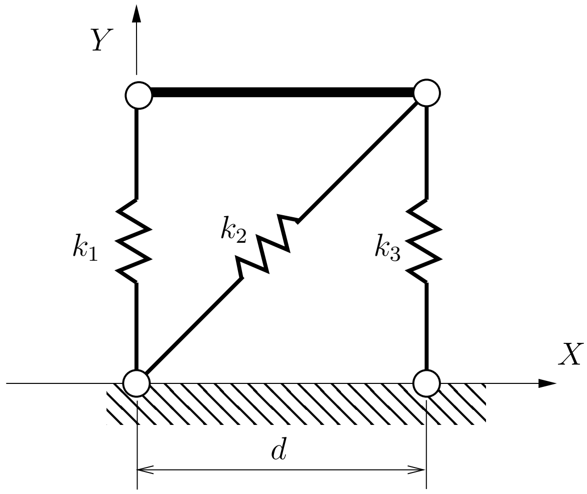
$$\hat{w} = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}.$$

5. In (C) it is possible to maintain a certain force on the ground, and make the workpiece move with respect to the ground with twist

$$\hat{T} = \begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix}.$$

Short question 5

Consider the compliant mechanism shown in the figure, in rest position:



Its rigidity matrix in the coordinate system OXY is:

1.
$$\begin{bmatrix} 0 & \sqrt{2}k_2/2 & 0 \\ k_1 & \sqrt{2}k_2/2 & k_3 \\ 0 & 0 & dk_3 \end{bmatrix}$$

2.
$$\begin{bmatrix} 0 & \sqrt{2}k_1 & 0 \\ k_2/2 & \sqrt{2}k_2/2 & 0 \\ 0 & k_3 & dk_3 \end{bmatrix}$$

3.
$$\begin{bmatrix} k_2/2 & k_2/2 & 0 \\ k_2/2 & k_1 + k_2/2 + k_3 & dk_3 \\ 0 & dk_3 & d^2k_3 \end{bmatrix}$$

4.
$$\begin{bmatrix} k_2 & k_2/2 & k_3 \\ k_2 & k_1 + k_2 + k_3 & dk_3 \\ k_3 & d^2k_3 & d^2k_3 \end{bmatrix}$$

5.
$$\begin{bmatrix} 0 & \sqrt{2}k_2/2 & k_3 \\ \sqrt{2}k_2/2 & k_2 + k_3 & \sqrt{2}k_1 \\ k_3 & \sqrt{2}k_1 & dk_1 \end{bmatrix}$$