Exercises Module 6
Spatial Wrenches and Twists

Problems

Problem 1

Fig. 1 shows the "Hexacrane", a parallel cable-driven robot constructed at Institut de Robòtica i Informàtica Industrial (see http://goo.gl/0TjzLG for videos and details). The robot is formed by a moving *platform* suspended from a rigid octahedral *base* by means of six cables, numbered from 1 to 6 for convenience. Cable i is connected to the base and the platform at points A_i and B_i, respectively. The cable lengths from A_i to B_i can be individually changed by means of servo-motors anchored to the base, which allows to govern the six degrees of freedom of the platform independently. To keep the control of the platform, all cables must be kept in tension during normal operation. The tensions cannot exceed the maximum value supportable by the motors, which is of 6.58 N. The platform mass is of 0.6 Kg, and we operate the robot slowly, so that all inertia forces are negligible.

Consider the reference frames $\mathcal{F}_b = Oxyz$ and $\mathcal{F}_p = O'x'y'z'$ shown in Fig. 1, respectively attached to the base and the platform. The coordinates of A_i and B_i in such frames are given in Table 1. The center of mass of the platform is located in O'. The platform pose is given by the pair $(\boldsymbol{r}, \boldsymbol{R})$, where $\boldsymbol{r} = [x, y, z]^\mathsf{T}$ is the position vector of O' in $\mathcal{F}_b$, and $\mathbf{R}$ is the 3×3 rotation matrix providing the orientation of $\mathcal{F}_p$ relative to $\mathcal{F}_b$. For convenience, we parameterize $\mathbf{R}$ using the tilt-and-torsion convention [1], so that

$$\mathbf{R} = \mathbf{R}_z(\phi)\, \mathbf{R}_z(\theta)\, \mathbf{R}_z(\sigma - \phi). \tag{1}$$

[1] The angles ϕ, θ and σ are called *azimuth*, *tilt*, and *torsion*, respectively. See http://goo.gl/Fx7273 for details.

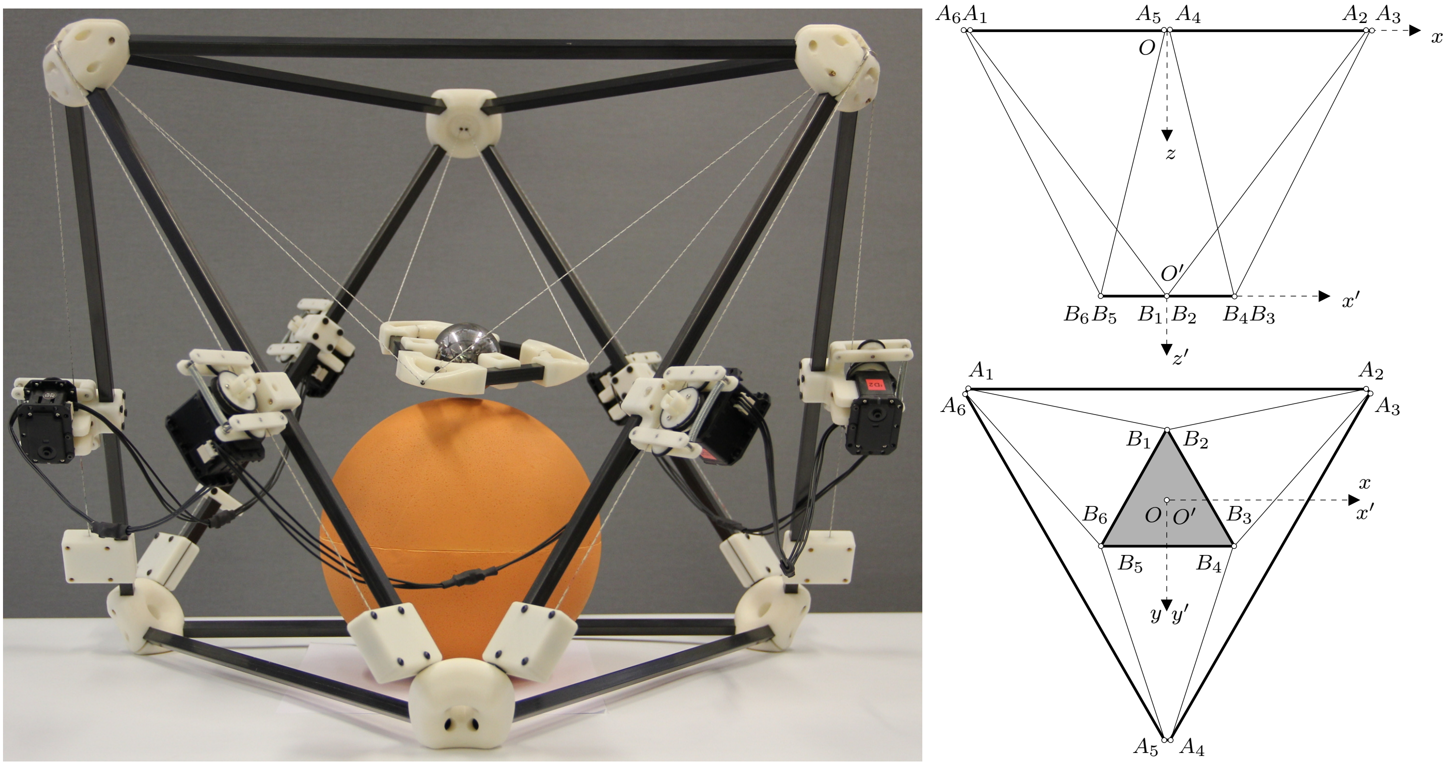


Figure 1: Left: The cable-driven robot "Hexacrane" in a generic view. Right: Its home configuration, with the base and platform reference frames $\mathcal{F}_b = Oxyz$ and $\mathcal{F}_p = O'x'y'z'$ indicated. Only the top triangle of the base octahedral frame is shown.

Base points in $\mathcal{F}_b$	Platform points in $\mathcal{F}_p$
$A_1 = (-231.62, -136.18, 0)$	$B_1 = (0, -89.15, 0)$
$A_2 = (231.62, -136.18, 0)$	$B_2 = (0, -89.15, 0)$
$A_3 = (233.74, -132.50, 0)$	$B_3 = (77.21, 44.57, 0)$
$A_4 = (2.13, 268.67, 0)$	$B_4 = (77.21, 44.57, 0)$
$A_5 = (-2.13, 268.67, 0)$	$B_5 = (-77.21, 44.57, 0)$
$A_6 = (-233.74, -132.50, 0)$	$B_6 = (-77.21, 44.57, 0)$

Table 1: Coordinates of the base and platform points in milimeters.

or, in expanded form,

$$\mathbf{R} = \begin{bmatrix} c_\phi c_\theta c_{\sigma-\phi} - s_\phi s_{\sigma-\phi} & -c_\phi c_\theta s_{\sigma-\phi} - s_\phi c_{\sigma-\phi} & c_\phi s_\theta \\ s_\phi c_\theta c_{\sigma-\phi} + c_\phi s_{\sigma-\phi} & -s_\phi c_\theta s_{\sigma-\phi} + c_\phi c_{\sigma-\phi} & s_\phi s_\theta \\ -s_\theta c_{\sigma-\phi} & s_\theta s_{\sigma-\phi} & c_\theta \end{bmatrix} \tag{2}$$

The platform has to perform inspection operations on the orange sphere shown in Fig. 1 centered in point $C = (x_c, y_c, z_c) = (0, 0, 306)$ mm of $\mathcal{F}_b$. To this end, it is maintained parallel to the sphere at all times, restricting the position of O' to lie on a larger concentric sphere S of radius $r = 130$ mm, centered in C (Fig. 2), and keeping the $O'z'$ axis pointing to C. These constraints can be enforced by moving the platform in the pose space described parametrically by

$$\left.\begin{array}{l} x = x_c + r\cos\alpha_2\cos\alpha_1 \\ y = y_c + r\cos\alpha_2\sin\alpha_1 \\ z = z_c - r\sin\alpha_2 \\ \phi = \alpha_1 \\ \theta = \alpha_2 - \frac{\pi}{2} \\ \sigma = 0 \end{array}\right\}, \tag{3}$$

where α_1 and α_2 are the spherical angular coordinates of O' on S. A platform configuration can be described by the pair $\boldsymbol{q} = (\alpha_1, \alpha_2)$, and it is said to be *wrench-feasible* if the cable tensions equilibrating the weight are all admisible, i.e., within the range $[0, 6.58]$ N.

1. Write a MATLAB routine `ConfigIsFeasible` that determines whether a given configuration $\boldsymbol{q}$ is wrench feasible. Check that the configurations

   $$\boldsymbol{q}_1 = (0.55, \tfrac{\pi}{2})\text{ rad},\quad \boldsymbol{q}_2 = (0.55, 0.75)\text{ rad},\quad \boldsymbol{q}_3 = (2.63, 0.75)\text{ rad},$$

   shown in Fig 2 are wrench feasible.

2. Write a MATLAB routine `PathIsFeasible` that takes two values of $\boldsymbol{q}$ as input, $\boldsymbol{q}_s$ and $\boldsymbol{q}_g$, and determines whether the interpolated path

   $$P(\boldsymbol{q_s}, \boldsymbol{q}_g) = \{\boldsymbol{q} \mid \boldsymbol{q} = \boldsymbol{q}_s + t\,(\boldsymbol{q}_g - \boldsymbol{q}_s)\text{ for } t \in [0,1]\} \tag{4}$$

   is wrench feasible. I.e., whether $P(\boldsymbol{q_s}, \boldsymbol{q}_g)$ only contains wrench feasible configurations. Implement the test approximately by checking a fine discretization of the path with `ConfigIsFeasible`.

3. Using `PathIsFeasible`, verify whether the paths $P(\boldsymbol{q_1}, \boldsymbol{q}_2)$ and $P(\boldsymbol{q_2}, \boldsymbol{q}_3)$ shown in red in Fig. 2 are wrench feasible.

4. Write a MATLAB routine that computes the wrench-feasible workspace of the robot, i.e., the set of values $(\alpha_1, \alpha_2) \in [0, 2\pi] \times [0, 2\pi]$ for which the cable tensions are admisible. Plot the workspace in the (α_1, α_2) plane, or on the sphere S, with the previous two paths shown overlaid.

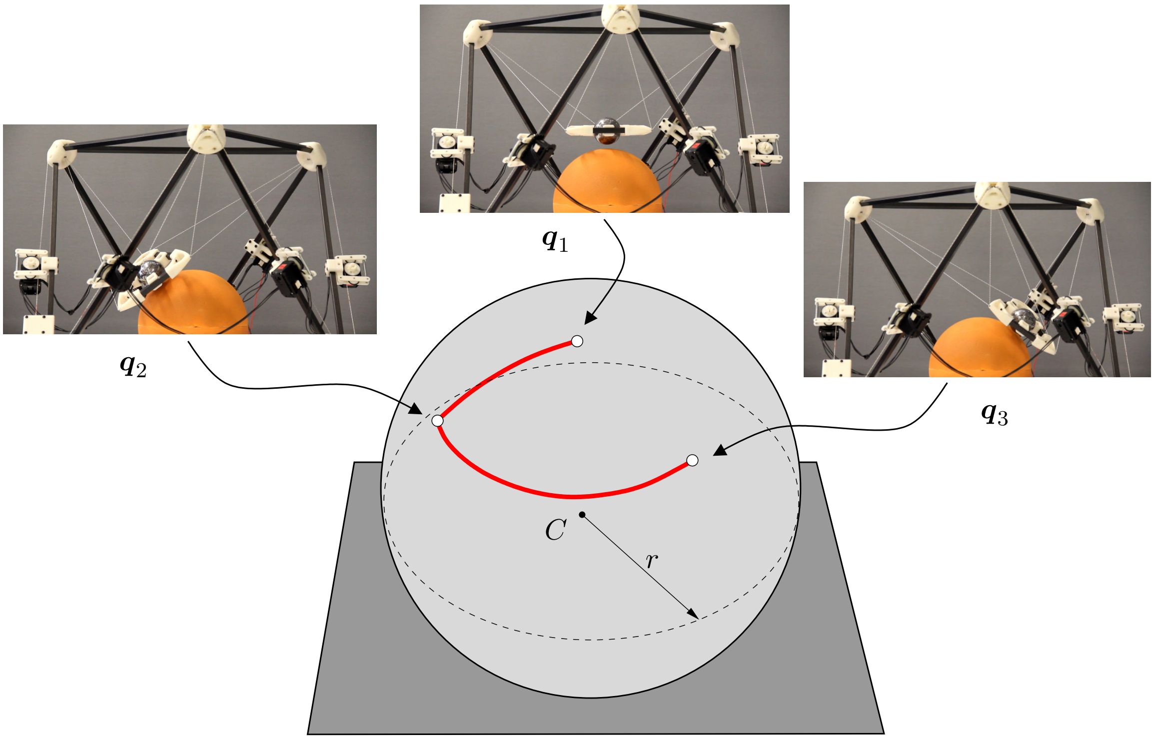


Figure 2: The platform is constrained to move parallel to the orange sphere. Configurations $\boldsymbol{q}_1$, $\boldsymbol{q}_2$ and $\boldsymbol{q}_3$, and their corresponding locations of O' on the concentric sphere S, with the paths $P(\boldsymbol{q_1}, \boldsymbol{q}_2)$ and $P(\boldsymbol{q_2}, \boldsymbol{q}_3)$ shown in red.

Problem 2

Fig. 3 shows an hexaglide robot also constructed at IRI (http://youtu.be/FltBh-g35ZQ). The moving platform is connected to the base by means of PUS legs as indicated, with the P joint actuated. Using the reciprocal product method, find the $\mathbf{A}$ and $\mathbf{B}$ matrices in the kinematic equation $\mathbf{A}\hat{T} = \mathbf{B}\boldsymbol{\gamma}$ of this manipulator. Hint: use the result that if $\hat{w}$ and $\hat{T}$ are a spatial wrench and a spatial twist, written in ray and axis coordinates respectively, then $\hat{w}^{\mathsf{T}} \cdot \hat{T} = 0$ if, and only if, their respective screw axes are either concurrent or parallel.

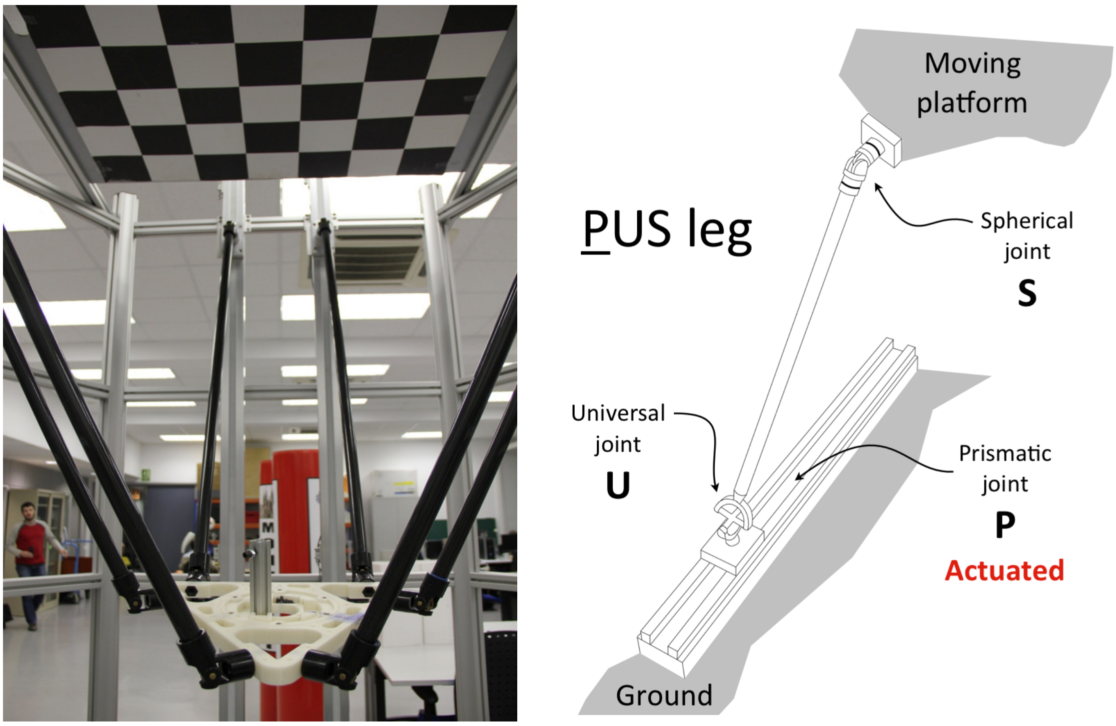


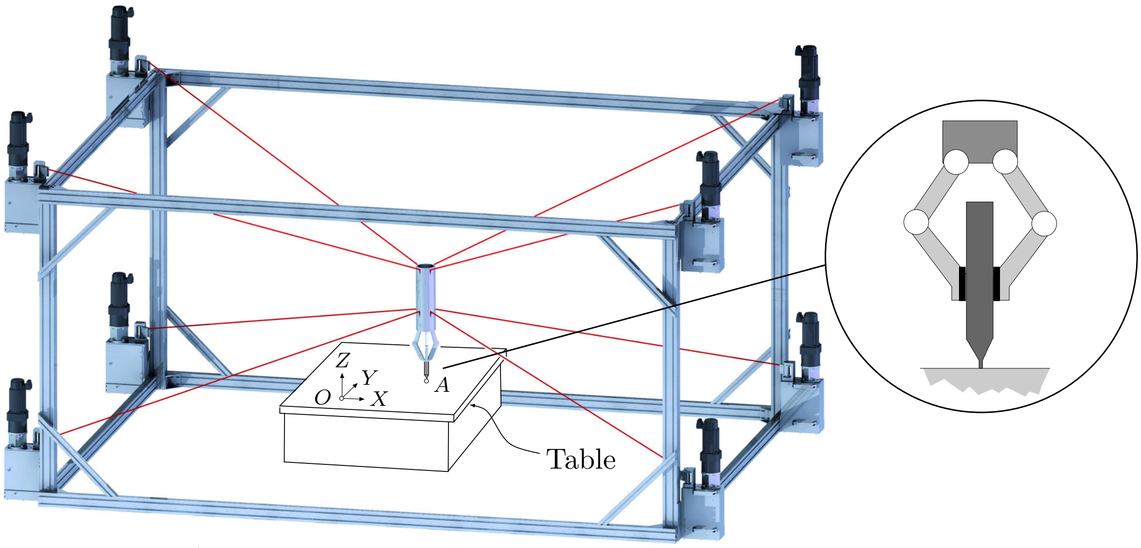
Figure 3: The Hexaglide platform and the structure of one of its PUS legs.

Short questions

Short question 1

The figure shows a robotic gripper hold by eight cables, used in a “demo” to draw on a table with a marker. At the instant depicted in the figure the marker is in equilibrium, and having a point contact with the table at point A. By meassuring the cable tensions, and using the Jacobian of the manipulator, one can find that the wrench that the marker applies to the table is, in the reference frame $OXYZ$,

$$\hat{w} = [0, 0, -2, -6, 6, 0]^{\mathsf{T}} \quad \text{(SI units, ray coordinates).} \tag{5}$$



Robot IPAnema of Fraunhofer Institute for Manufacturing

Assuming that the OXY plane coincides with the plane of the table, mark the incorrect statement:

1. The coordinates of A in $OXYZ$ are $(3, 3, 0)$ m.
2. The pitch of $\hat{w}$ is $h = 0$.
3. The gripper applies a force $[0, 0, -2]$ N to the table in A, plus a couple $[-6, 6, 0]$ Nm.
4. The central axis of the wrench is a line through A aligned with the Z axis.
5. Reducing the wrench $\hat{w}$ to its central axis, we note that $\hat{w}$ encodes a pure force.

Short question 2

The resultant wrench of the system of forces acting on a rigid body is $\hat{w} = [0, 0, 1, 0, -1, 0]^{\mathsf{T}}$ (in ray coordinates and SI units). Mark the incorrect statement:

1. Poinsot’s central axis is a line through point $[2, 0, 0]$, with direction vector $[0, 0, 1]$.
2. The pitch of the wrench is $h = 1$ m.
3. The wrench does not represent a pure force, nor a pure torque.
4. The Plücker coordinates of the central axis are $[0, 0, 2, 0, -1, 0]$.
5. To equilibrate this wrench only a force $[0, 0, -1]^{\mathsf{T}}$ has to be applied at point $[2, 0, 0]$.