Problem (10/20 points)

(SI units are used unless otherwise stated)

Fig. 1 shows the Ictineu 3, a powerful deep-sea submarine designed and constructed by a remarkable Catalan team, with partial Verkami crowdfunding. It is among the top 10 submersibles in the world as for its immersion depth. The submarine has been sent to repair an oil pipe close to the Deepwater Horizon platform wreck, in the Gulf of Mexico. The goal is to cover a hole in the pipe with a shield, using the left arm of the submarine, which has a compliant 3-RPR wrist. The right arm is used to keep the submarine's position fixed relative to the pipe. A human pilot telemanipulates the left arm from inside the submarine's cabin, but the oil emanations make it difficult to govern the shield's position visually, so the use of haptic force feedback is required. Thus, the pilot is set to operate a haptic 3-$\underline{R}$RR joystick (Fig. 2) that can read the small displacements of his hand, and at the same time reflect the contact wrench $\delta\hat{w}_a$ applied by the pipe to the shield. The control loop iterates at a very high frequency over the following steps:

1. The human moves the joystick slightly.
2. The joystick sensors measure the displacement $\delta\hat{D}_j$ of the hand relative to the cockpit.
3. The left arm produces the displacement $\delta\hat{D}_w = \mathbf{G}\,\delta\hat{D}_j$ of the wrist base relative to the pipe, where $\mathbf{G}$ is an appropriate gain matrix.
4. Under contact situations, the compliant wrist deforms slightly producing the reaction wrench $\delta\hat{w}_a$.
5. The haptic device reproduces the wrench $\delta\hat{w}_a$ on the pilot joystick.

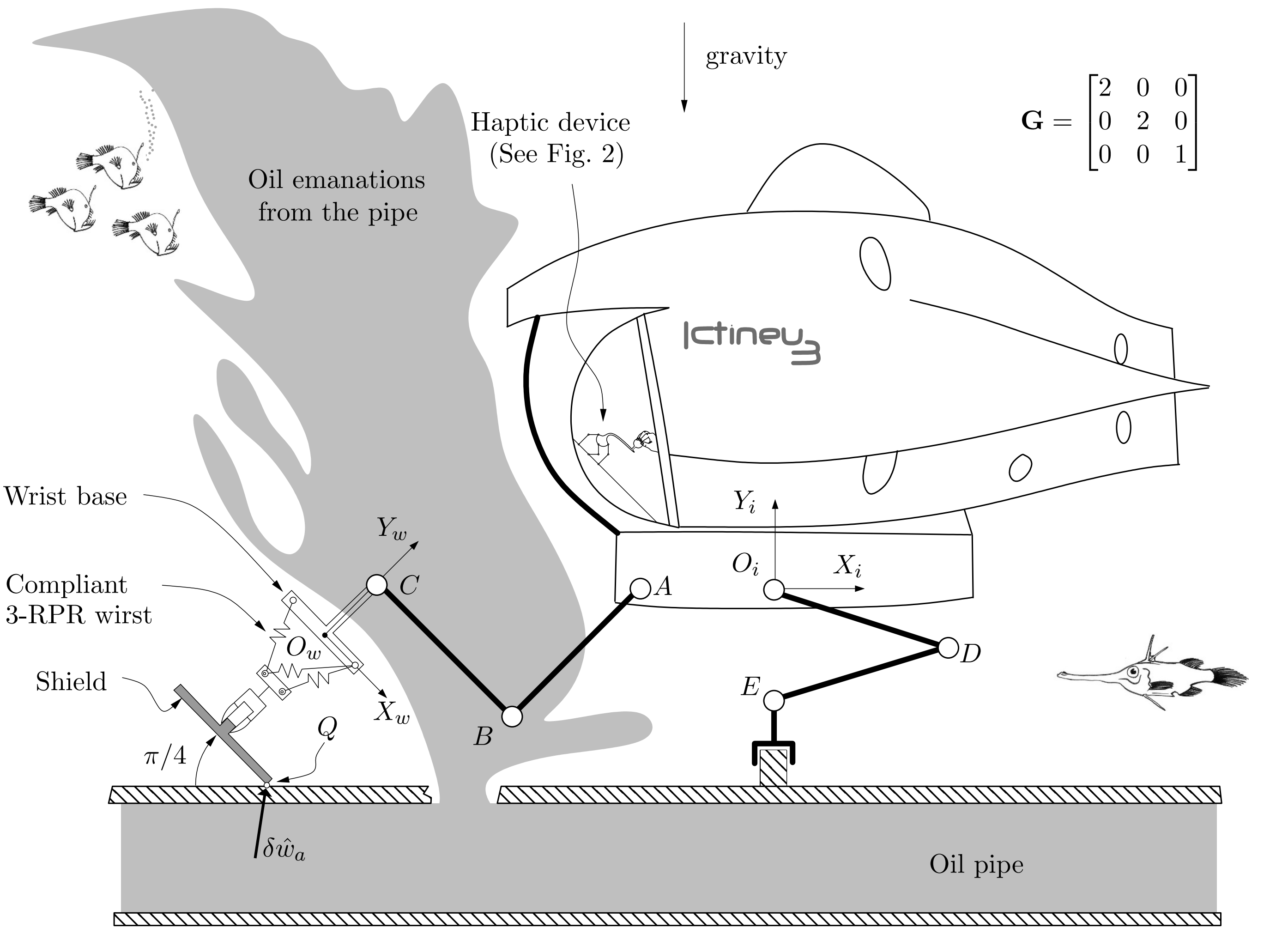

Figure 1: Ictineu 3 during a deap sea mission. (Drawing not to scale.)

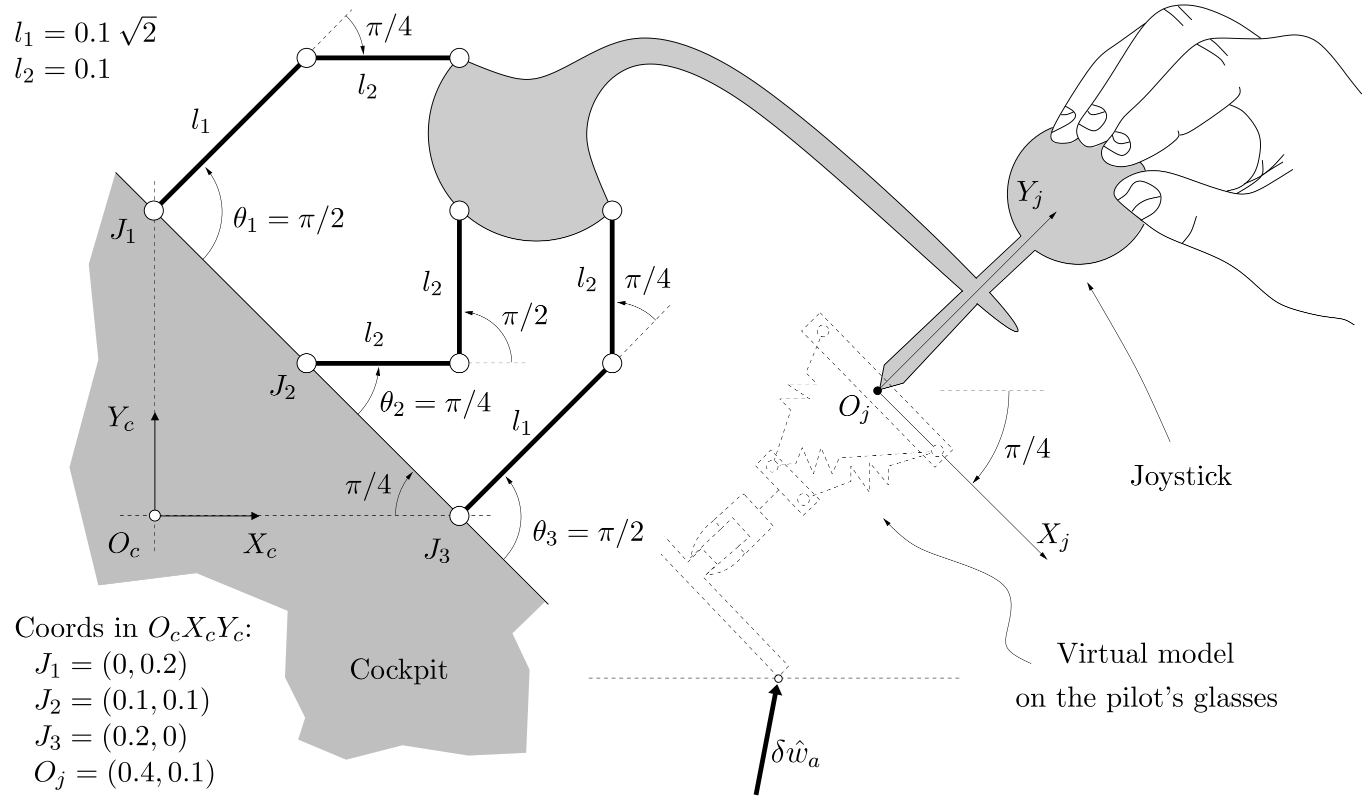

Figure 2: The haptic 3-RRR device at the cockpit. The three joints J_1, J_2, and J_3 have sensors that provide the small angular increments $\delta\theta_1$, $\delta\theta_2$, and $\delta\theta_3$ relative to the previous instant of time. These joints are also actuated to produce the contact wrench $\delta\hat{w}_a$. The compliant end-effector shown dashed is virtual. It is only visualized on augmented-reality glasses weared by the pilot. (Drawing not to scale.)

The following assumptions apply:

1. The weight and buoyancy of the shield have a net effect of a downward vertical wrench of 60 N applied at point $(0, -0.6)$ in frame $O_wX_wY_w$. The weight of the joystick is negligible.
2. The contact of the shield with the pipe at Q is punctual, with friction.
3. The joints of the two arms can supply equilibrant torques in the range $[-100, 100]$ Nm.
4. The springs of the compliant wrist operate close to their rest position.

At the instant shown in Fig. 1, the shield is about to establish contact with the pipe, and the pilot pushes the joystick slightly producing a small displacement $\delta\hat{D}_j$. The left arm replicates the movement on its end-effector (scaled using $\mathbf{G}$), which compresses the 3-RPR compliant wrist, and generates a reaction wrench $\delta\hat{w}_a$ of the pipe on the shield. Determine:

A **(1.5 points)** The $\mathbf{A}$ and $\mathbf{B}$ matrices of the displacement equation $\mathbf{A}\delta\hat{D}_j = \mathbf{B}\delta\boldsymbol{\theta}$ of the haptic device, where $\delta\hat{D}_j$ is the displacement of the joystick relative to the cockpit, expressed in frame $O_jX_jY_j$, and $\delta\boldsymbol{\theta} = [\delta\theta_1, \delta\theta_2, \delta\theta_3]^\mathsf{T}$ is the vector of small angle increments of the actuated joints.

B **(1 point)** The small displacement $\delta\hat{D}_w$ of the wrist base relative to the pipe in frame $O_wX_wY_w$, assuming that $\delta\hat{D}_w = \mathbf{G}\,\delta\hat{D}_j$, where $\mathbf{G}$ is the gain matrix indicated in Fig. 1, and using the fact that the sensors of the haptic device are measuring

$$\delta\theta_1 = 0.00806087, \quad \delta\theta_2 = -0.00252499, \quad \delta\theta_3 = -0.00039261 \quad \text{(radiants)}.$$

C **(1 point)** The angular joint increments of the left arm needed to produce the displacement $\delta\hat{D}_w$, assuming that $A = (1, 1.3)$, $B = (1, 0.3)$, and $C = (0, 0.3)$ in frame $O_wX_wY_w$.

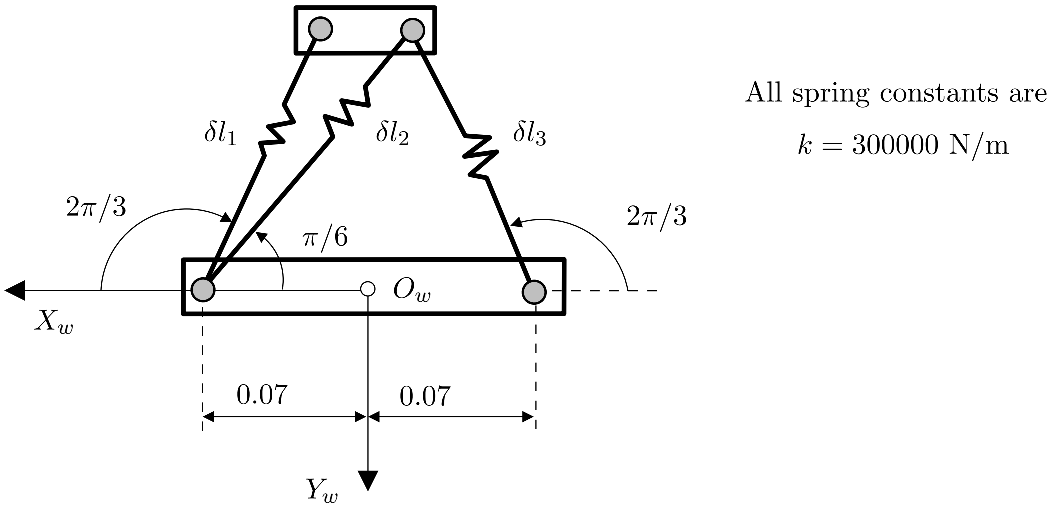

Figure 3: The compliant wrist of the left arm. (Drawing not to scale.)

D **(1 point)** The contact wrench $\delta\hat{w}_a$ that the pipe is applying to the shield, expressed in frame $O_wX_wY_w$, assuming that the compliant wrist has the geometry and parameters indicated in Fig. 3, and that the spring length increments relative to the rest position are

$$\delta l_1 = -0.00114148, \quad \delta l_2 = 0.00093967, \quad \delta l_3 = 0.00076225 \quad \text{(meters)}.$$

E **(1 point)** The spaces $\mathbb{D}$, of infinitesimal displacements of the shield relative to the pipe, and $\mathbb{W}$, of wrenches applied by the pipe to the shield, that could be controlled under a hybrid control strategy. Provide vector bases of such spaces in $O_wX_wY_w$, assuming that $Q = (0.3, -0.7)$ in frame $O_wX_wY_w$. Prove that the wrench $\delta\hat{w}_a$ obtained in [D] belongs to $\mathbb{W}$ up to small errors.

F **(1 point)** The equilibrant joint torques that the haptic device should deliver at J_1, J_2, and J_3, in order to apply the wrench $\delta\hat{w}_a$ obtained in [D] to the joystick, now assumed in frame $O_jX_jY_j$.

The pilot has doubts on whether the sensors measuring δl_1, δl_2, and δl_3 work properly. To verify so:

G **(1.5 points)** Compute $\delta\hat{w}_a$ without using the sensor readings δl_1, δl_2, and δl_3 and compare the results with those obtained in [D]. Use the fact that, knowing the shield-pipe contact model, the displacement $\delta\hat{D}_w$ obtained in [C] can be *uniquely* decomposed into the sum of two relative displacements.

While moving the shield towards the hole, the right arm is kept in the shown configuration, where $D = (0.866, -0.5)$ and $E = (0, -1)$ in frame $O_iX_iY_i$. This arm is used to correct eventual position errors of the submarine relative to the pipe.

H **(1 point)** Determine whether the arm actuators will be able to keep the submarine in equilibrium, assuming that its weight is of 53000 N applied at point $C_g = (0.05, 2)$ (the center of gravity), and its buoyancy is of 52950 N applied at $C_b = (0.05, 2.1)$ (the center of buoyancy), in frame $O_iX_iY_i$.

I **(1 point)** Suppose that we wish to correct position and orientation errors of the submarine in all possible directions. Discuss whether the arm configuration shown in Fig. 1 is better or worse than an alternative configuration with all joints aligned and vertical.

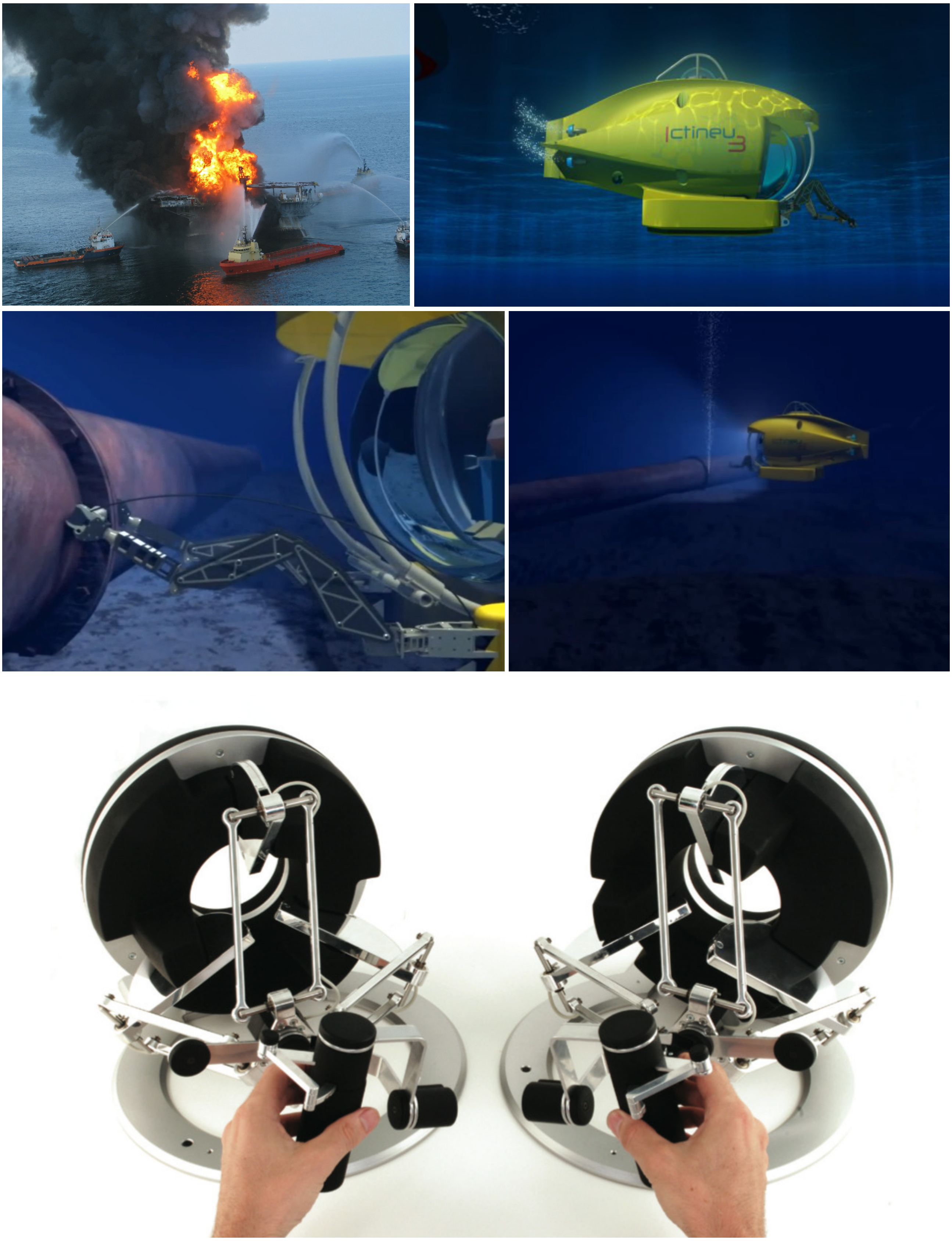

Figure 4: The mission described in this exercise would be plausible in the context of these images: the Deepwater Horizon platform (in flames, after its explosion on 20 April 2010), the Catalan Ictineu 3 submarine in underwater repair missions (images courtesy of Ictineu Submarins, Catalonia), and a pair of omega.7 haptic devices (image courtesy of Force Dimension, Switzerland).