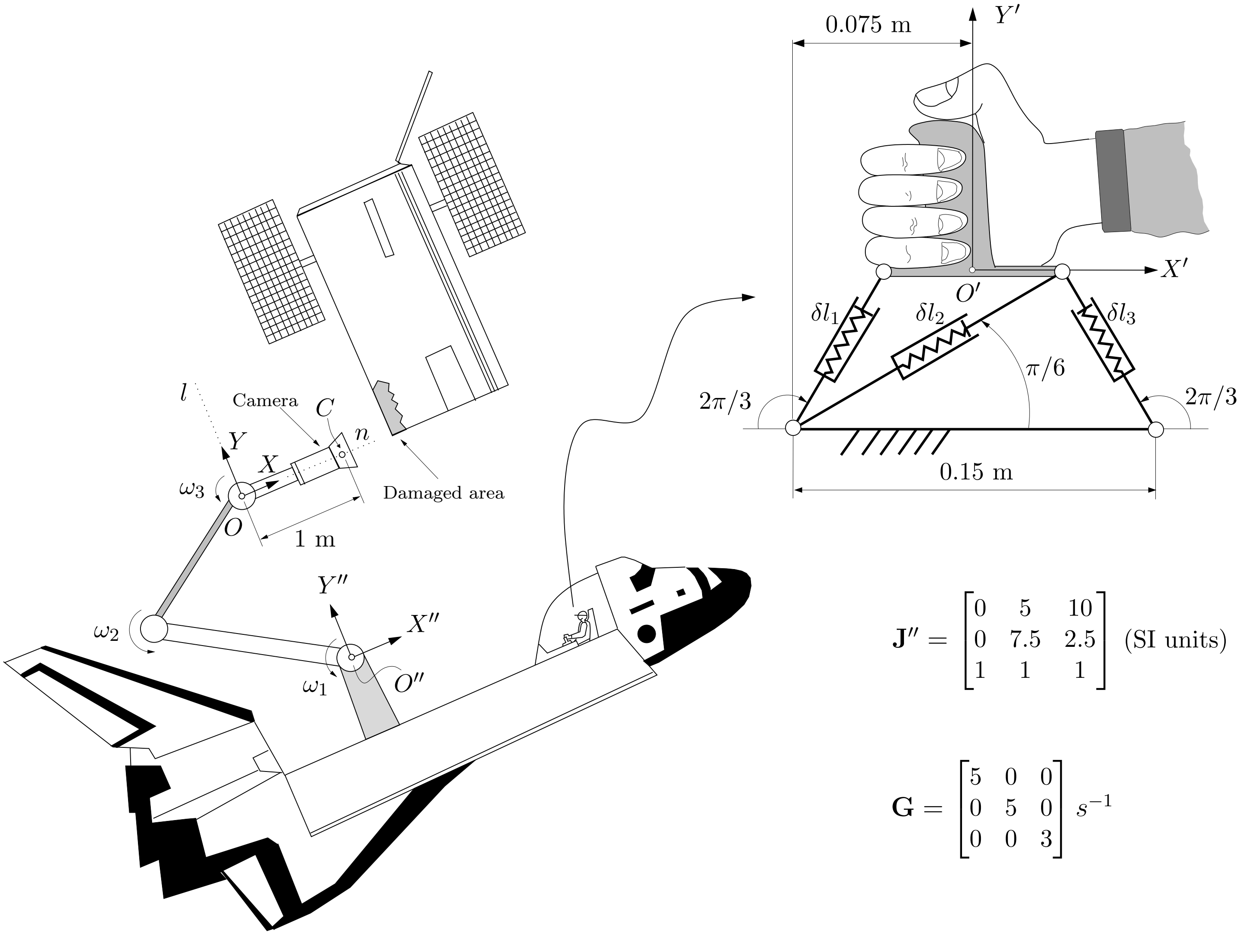
Problem 1 (12/30 points)

The Hubble orbital telescope has received the impact of a small meteorite and the space shuttle Atlantis has been sent to evaluate the damage and to repair it (Fig. 1). The shuttle is equipped with the *Canadarm*, a 3R arm which is used to do inspection and maintenance operations. The actuators of this arm can rotate at maximum angular velocities of 0.5 rad/s in any direction. The kinematic Jacobian $\mathbf{J}''$ of the arm is given in Fig. 1, relative to reference frame $O''X''Y''$.



$$\mathbf{J}'' = \begin{bmatrix} 0 & 5 & 10 \\ 0 & 7.5 & 2.5 \\ 1 & 1 & 1 \end{bmatrix} \text{ (SI units)}$$

$$\mathbf{G} = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 3 \end{bmatrix} s^{-1}$$

Figure 1: The Atlantis shuttle inspecting the Hubble telescope.

The spaceship evolves until it achieves the position of Fig. 1. The end-effector of the Canadarm carries a camera with which the surface of the telescope has to be filmed on the damaged area. From the Atlantis cargo bay, an astronaut commands the motion of the camera with a passive 3-RPR *joystick*, whose P joints have springs with constant $k = 100$ N/m. The control system of the arm guarantees that, at any instant of time, the twist $\hat{T}$ of the camera observed by the astronaut is proportional to the infinitesimal displacement $\delta\hat{D}$ of the joystick from its rest position, according to the equation

$$\hat{T} = \mathbf{G} \cdot \delta\hat{D}, \tag{1}$$

where $\hat{T}$ and $\delta\hat{D}$ are given in the coordinate systems OXY and $O'X'Y'$, respectively, and $\mathbf{G}$ is the gain matrix indicated in Fig. 1. The joystick legs have sensors that measure the small longitudinal displacements $\delta l_1, \delta l_2, \delta l_3$ that correspond to the displacement $\delta\hat{D}$.

In the configuration of Fig. 1, determine:

A **(1.5 points)** The force Jacobian $\mathbf{j}$ of the joystick expressed in coordinate system $O'X'Y'$.

B **(1.5 points)** The kinematic Jacobian $\mathbf{J}$ of the Canadarm expressed in coordinate system OXY.

Starting at the configuration of Fig. 1, the astronaut applies a small displacement $\delta\hat{D}$ to the Joystick, such that its sensors read the values $\delta l_1 = 0.017321$ m, $\delta l_2 = 0.01$ m, and $\delta l_3 = 0.017321$ m.
Determine:

C **(1.5 points)** The twist $\hat{T}$ and the velocity of point C of the camera, as seen by the astronaut.

D **(0.5 points)** The angular velocities ω_1, ω_2, ω_3, that the control system will have to demand to the actuators of the Canadarm to produce the twist $\hat{T}$.

E **(1 point)** The wrench that the astronaut has to apply to the joystick to maintain it in the new position, assuming that all springs work near their unloaded configuration.

During the planning of this mission, the engineers of the Jet Propulsion Lab had to decide a good configuration from which to start filming the damaged area of the telescope. It was necessary to record a video of such area by displacing the distal joint of the Canadarm along a trajectory l parallel to the telescope, with constant velocity of 0.5 m/s, and maintaining the axis n of the camera perpendicular to the trajectory (Fig. 1). The configuration in Fig. 1 was finally chosen, but configurations C_1 and C_2 of Fig. 2 had also been considered. Answer the following questions:

F **(1 point)** Why configuration C_1 was quickly discarded? What happens in this configuration with the velocities?

G **(1.5 points)** Configuration C_2 was potentially adequate, but after a few calculations it was discarded. Reproduce these calculations and justify why this configuration was discarded.

H **(1.5 points)** Give a basis of the space of twists of freedom of the camera in configurations C_1 and C_2 relative to reference frame $O''X''Y''$.

I **(2 points)** If in configuration C_1 the astronaut produces the displacement $\delta\hat{D} = [0.01, 0, 0]^\mathsf{T}$ on the Joystick, can the camera move with the corresponding twist? And what if he produces the displacement $\delta\hat{D} = [0.01, 0.01, 0]^\mathsf{T}$?

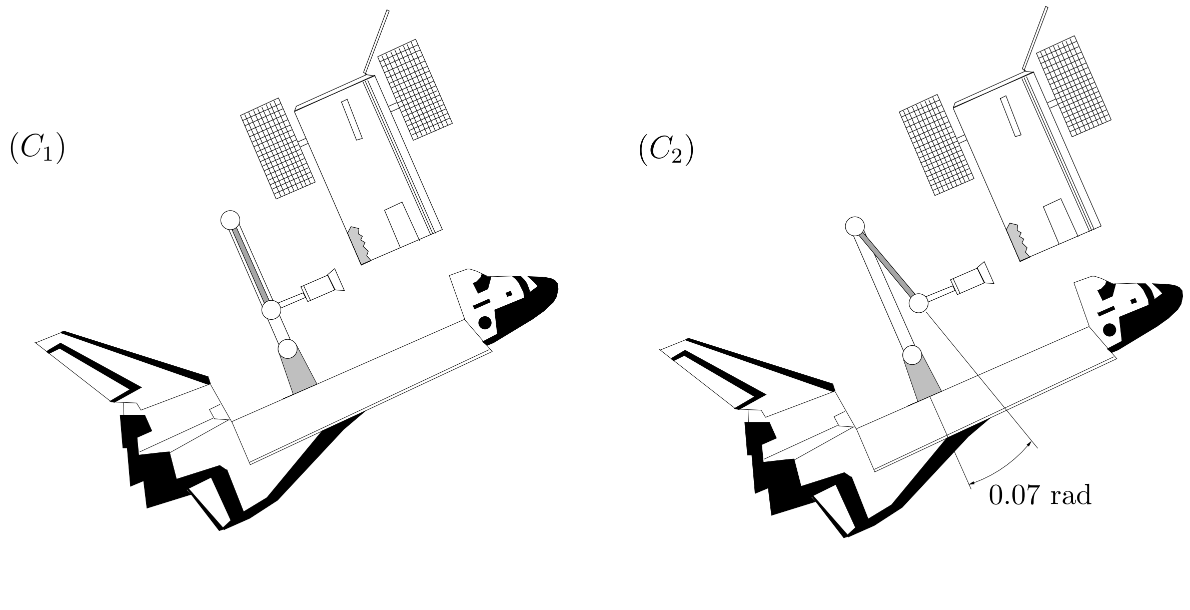
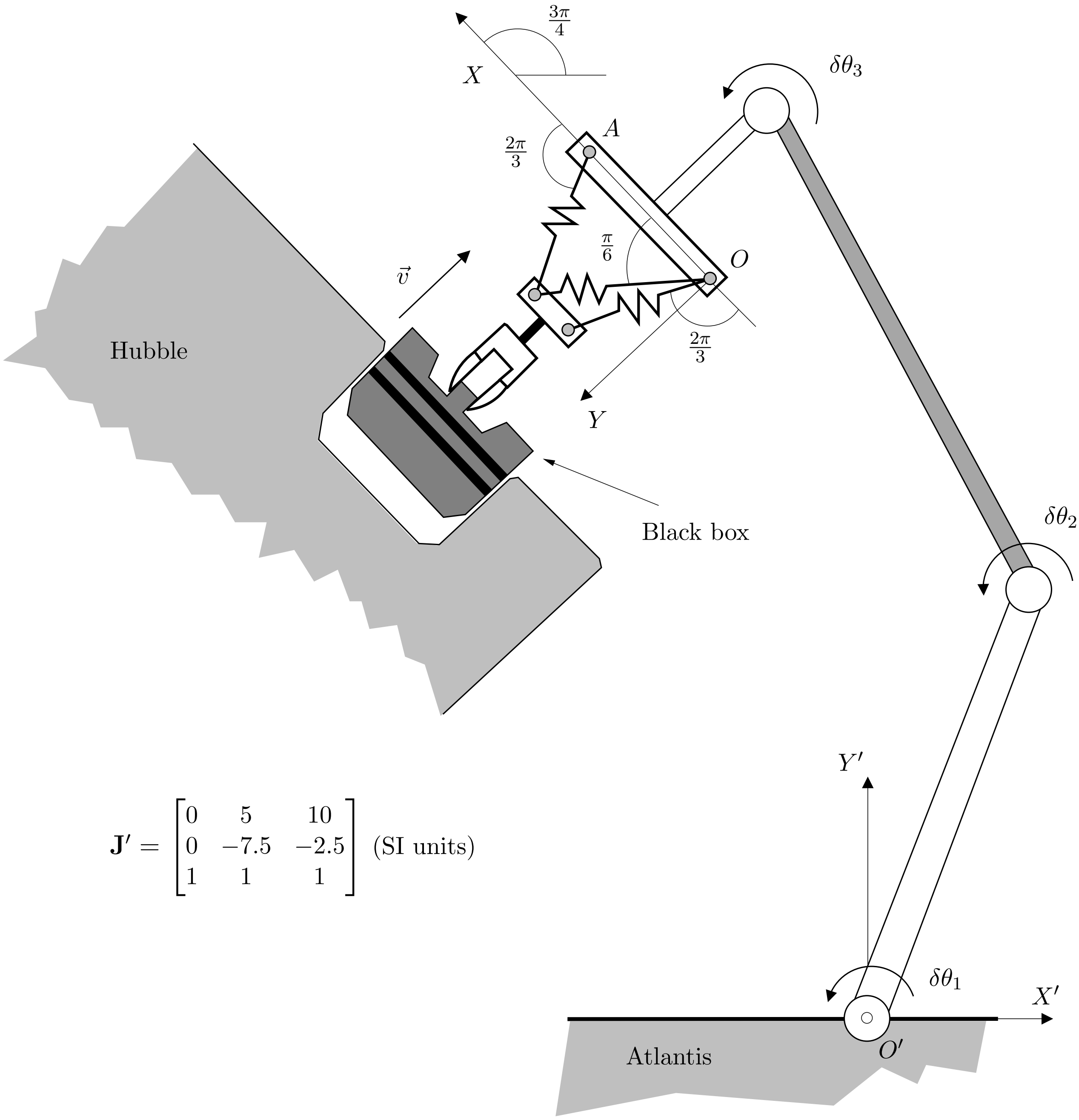


Figure 2: Two alternative initial configurations.

Problem 2 (8/30 points)

The meteorite damaged the telecommunications system of the Hubble and the last observations could not be transmitted to the Earth. To recover the data, the black box of the telescope has to be extracted. The end-effector of the Canadarm is equipped with a gripper and the arm is tele-operated until the handle of the box is hold (Fig. 3). The Hubble frees automatically the box and the Canadarm initiates its extraction, letting it slide along the shown direction $\vec{v}$. The box-Hubble contact is planar without friction.



$$\mathbf{J}' = \begin{bmatrix} 0 & 5 & 10 \\ 0 & -7.5 & -2.5 \\ 1 & 1 & 1 \end{bmatrix} \text{ (SI units)}$$

Figure 3: Extraction of the black box of the Hubble with the Canadarm (the drawing is not at scale).

In order not to affect the orbit, the interaction forces between the box and the telescope have to be minimized using a hybrid-control strategy. The Canadarm carries a flexible 3-RPR wrist that gives the wrench that the Hubble makes on the box at every instant. The Atlantis maintains its position and orientation fixed relative to the Hubble. The springs of the 3-RPR wrist have a length that is very close to the rest length. The spring constants are all equal, of 200 N/m.

In the configuration of the figure, determine:

A **(1.5 points)** The rigidity matrix $\mathbf{K}$ of the 3-RPR wrist in the coordinate system OXY, knowing that the distance OA is of 0.3 m. Note: the wrist is not drawn at scale.

B **(1.5 points)** The Jacobian $\mathbf{J}$ of the Canadarm in the coordinate system OXY, knowing that the Jacobian $\mathbf{J}'$ in $O'X'Y'$ is the one indicated in Fig. 3, and that the coordinates of point O in $O'X'Y'$ are $(0.15, 7.35)$ m.

C **(1 point)** The space of controllable displacements $\delta\hat{D}$ (of the box with respect to the Hubble), in OXY.

D **(1 point)** The space of controllable force variations $\delta\hat{w}$ (of the Hubble on the box), in OXY.

At the time instant of the figure, the Hubble telescope is applying a clockwise pure torque of 0.1 Nm on the box, and we wish to translate the box 0.01 m in the direction $\vec{v}$.

Determine:

E **(1 point)** The wrench increment $\delta\hat{w}$ (of the Hubble telescope on the box) that has to be produced to eliminate the previous torque, and the small displacement $\delta\hat{D}$ associated with the desired translation, both in OXY.

F **(2 points)** The small angular increments $\delta\theta_1$, $\delta\theta_2$, and $\delta\theta_3$ of the Canadarm joints that simultaneously cause $\delta\hat{w}$ and $\delta\hat{D}$.

Figure 4: Image of the real context of problems I and II.